Waves in the Galileon theory

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Why modify gravity?

Why modify gravity?
- cosmological constant problems,
- non-renormalizability problem,
- benchmarks for testing General Relativity
- theoretical curiosity.

Many ways to modify gravity:
- f(R), scalar-tensor theories,
- Galileons, Horndeski (and beyond) theory, KGB, Fab-four,
- higher-dimensions,
- DGP,
- Horava, Khronometric
- massive gravity

- Most general scalar-tensor theory leading to equations of motions with no more than 2 derivatives;
- Cancellation of Lambda (Fab-Four), Self-tuning, Self-acceleration;
- Vainshtein mechanism

Linear theory

Canonical kinetic term + quadratic mass:

$$S = \int d^4x \left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 \right)$$

$$\rightarrow \quad g^{\mu\nu}\nabla_\mu\nabla_\nu\phi + m^2\phi = 0$$

Linear partial differential equation of second order (one degree of freedom).
Need to specify two conditions, ϕ and $\dot{\phi}$

1+1 case, no mass:

$$-\ddot{\phi} + \phi'' = 0$$

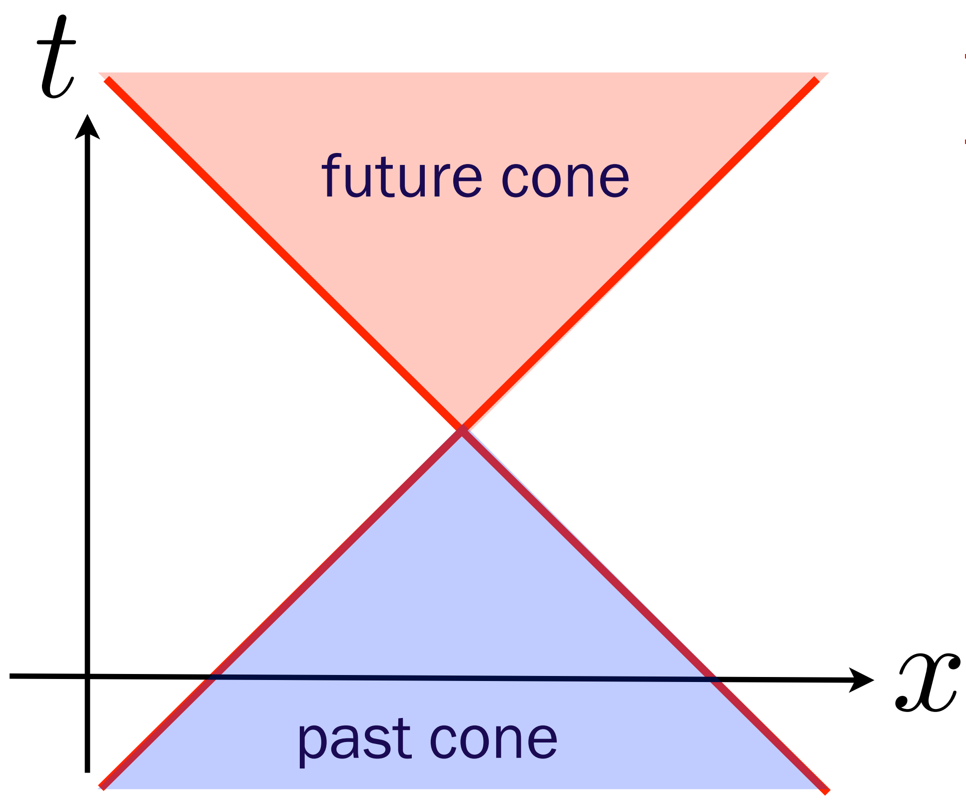
general solution:

$$\phi = f_1(t-x) + f_2(t+x)$$

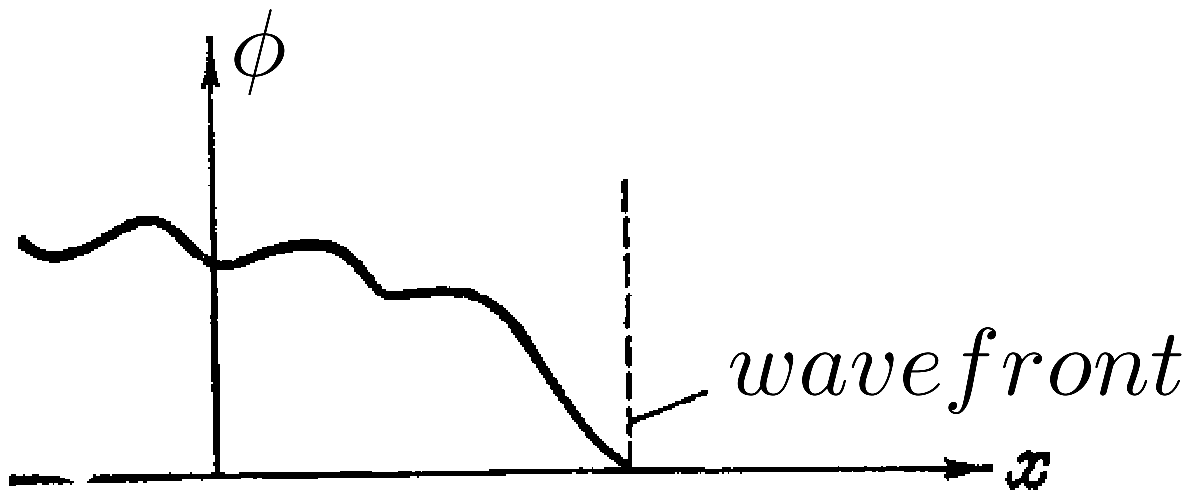
Linear theory

causal structure

$$-\ddot{\phi} + \phi'' = -m^2\phi$$ 1+1 case



- All characteristics are straight lines (45 degrees)
- Characteristics are determined by the kinetic part



- Perturbations propagate along characteristics
- Signals (wave fronts) propagate along characteristics
- Cones of influence are defined by characteristics

Non-linearity (mild)

Non-linear potential term

Canonical kinetic term + arbitrary potential:

$$S = \int d^4x \left(-\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi) \right)$$

$$\rightarrow \quad \nabla_\mu \nabla^\mu \phi + \frac{dV(\phi)}{d\phi} = 0$$

Non-linear partial differential equation of second order (inflationary models, quintessence).
However, because the kinetic term is canonical, the characteristic structure is the same.

Non-linearity

Non-linear kinetic term

Armendariz-Picon, Damour, Mukhanov'99

Non-linear kinetic term?

- General Relativity
- QCD
- Hydrodynamics
- ...

$$X \equiv \frac{1}{2}\left(\partial_\mu \phi \partial^\mu \phi\right)$$

k-essence: $$S = \int d^4x \, \mathcal{K}(X)$$

example:
canonical scalar:

$$S = \int d^4x \; X$$

k-essence

equations of motion & causal structure

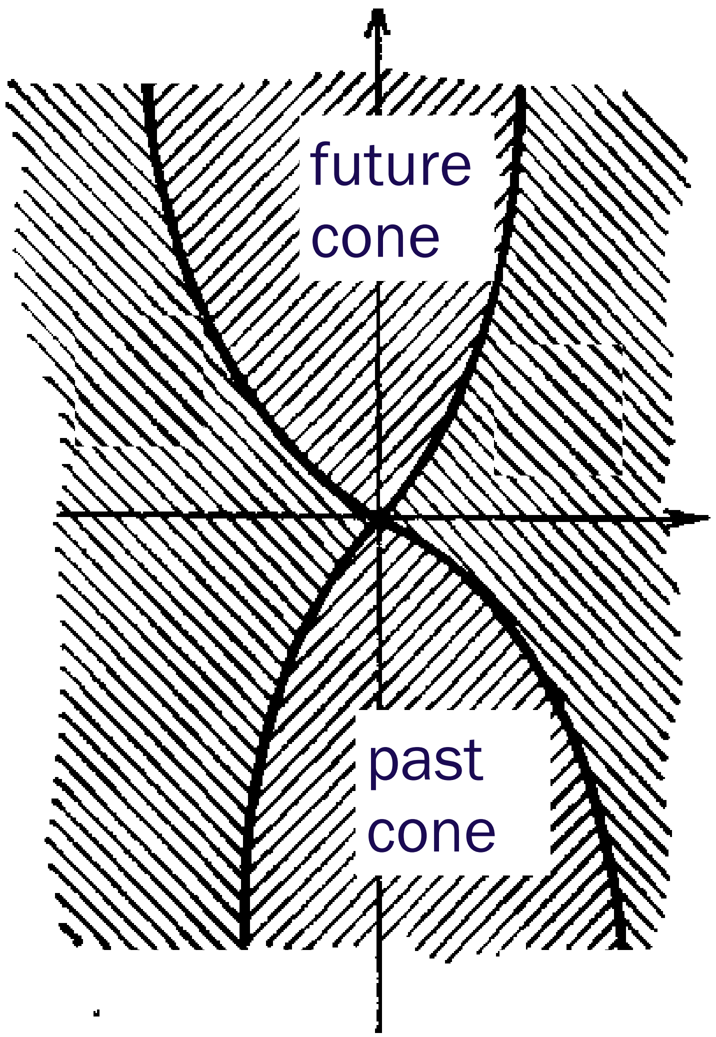
$$S_\phi = \int d^4x\sqrt{-g}\mathcal{L}\left(X,\phi\right) \qquad X = \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$$

Variation with respect to the scalar field gives:

- quasi-linear equations
- second order in derivatives

$$-\frac{1}{\sqrt{-g}}\frac{\delta S_\phi}{\delta\phi} = \tilde{G}^{\mu\nu}\nabla_\mu\nabla_\nu\phi + 2X\mathcal{L}_{,X\phi} - \mathcal{L}_{,\phi} = 0,$$

$$\tilde{G}^{\mu\nu}\left(\phi,\nabla\phi\right) \equiv \mathcal{L}_{,X}g^{\mu\nu} + \mathcal{L}_{,XX}\nabla^\mu\phi\nabla^\nu\phi.$$

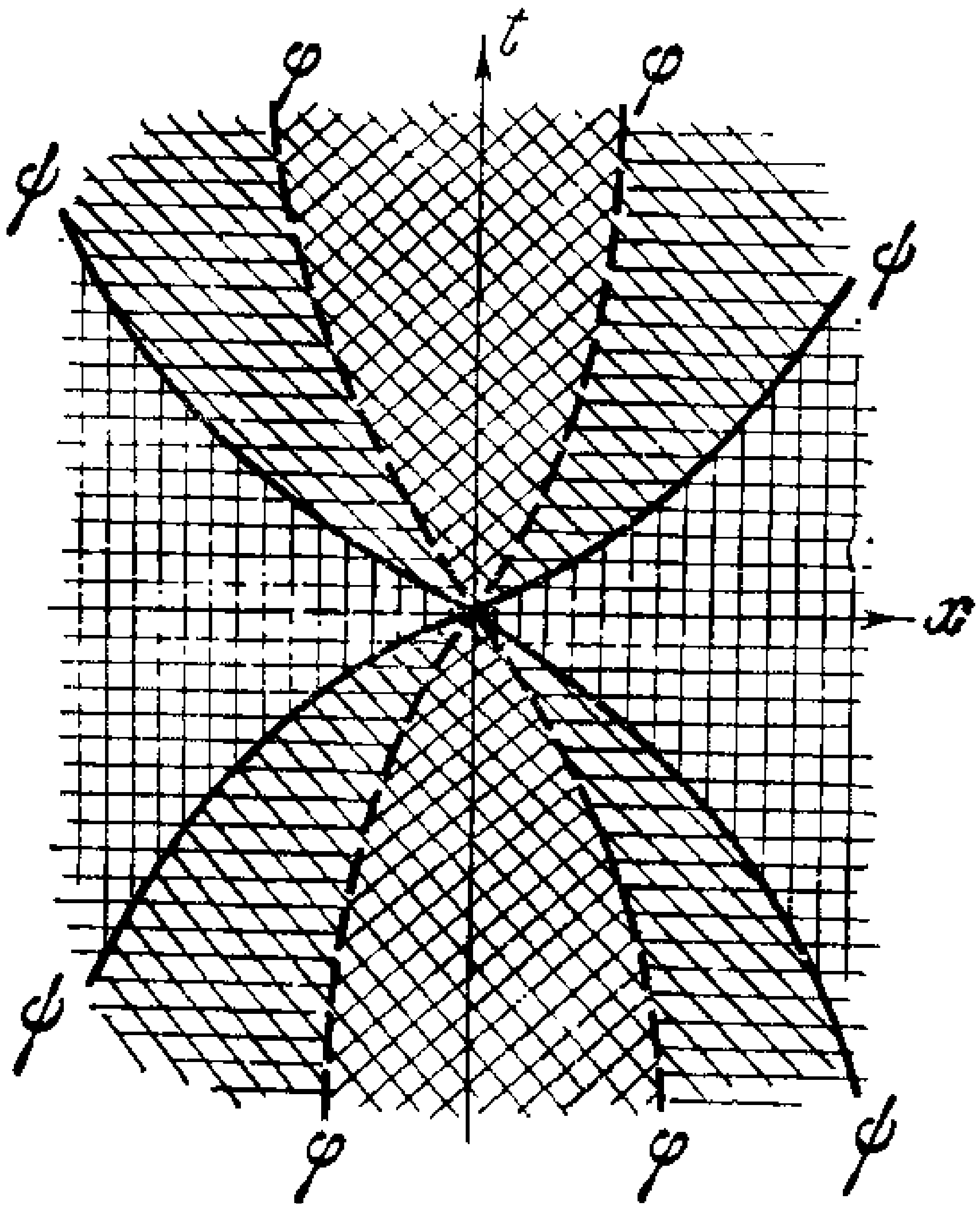


Cones of influence for the scalar field do not coincide with those of the photons and gravitons.

k-essence

causal structure

Two scalar fields:



Pure k-essence

k-essence as perfect fluid

$$S_\phi = \int d^4x \sqrt{-g}\, p(X)$$

$$X = \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$$

Definitions:

$$u_\mu = \frac{\nabla_\mu \phi}{\sqrt{2X}}$$

(gradient of scalar field is timelike)

$$\varepsilon(X) = 2X p_{,X}(X) - p(X)$$

Stress tensor:

$$T_{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta S_\phi}{\delta g^{\mu\nu}} = (\epsilon + p)\, u_\mu u_\nu - g_{\mu\nu} p$$

pure kinetic k-essence is perfect fluid !

Even more non-linear?

Monge-Ampere equaiton

Monge'1784, Ampère'1820

Monge-Ampère equation

$$A(u_{xx}u_{yy} - u_{xy}^2) + Bu_{xx} + Cu_{xy} + Du_{yy} + E = 0$$

- to find a surface with a prescribed Gaussian curvature
- optimizing transportation costs
-...

$$u_{xx}u_{yy} - u_{xy}^2$$

first galileon in history

Even more non-linear?

galileons, Horndeski

Horndeski'1974

The most generic scalar-tensor theory in 4D, whose equations of motion contain no more than second derivatives

$$S = \int d^4x \, F\left[g, \partial g, \partial^2 g, \varphi, \partial\varphi, \partial^2\varphi\right]$$

$$\Big\downarrow ?$$

$$E[g, \partial g, \partial^2 g, \varphi, \partial\varphi, \partial^2\varphi] = 0$$

Horndeski theory

Why no more than 2 derivatives in EOMs?

Ostrogradski ghost

Ostrogradski'1850

$$S = \int L(q, \dot{q}) dt \qquad \rightarrow \qquad \frac{dL}{dq} - \frac{d}{dt}\frac{dL}{d\dot{q}} = 0$$

$$S = \int L(q, \dot{q}, \ddot{q}) dt \qquad \rightarrow \qquad \frac{dL}{dq} - \frac{d}{dt}\frac{dL}{d\dot{q}} + \frac{d^2}{dt^2}\frac{dL}{d\ddot{q}} = 0$$

- Generically Hamiltonian is unbounded from below.

- New propagating degree of freedom appear. It is a ghost.

- Avoiding the theorem ?

Even more non-linear?

Universal equations

"Universal field equations"

Fairlie et al'1991

$$\mathcal{L}_n = F_n(\partial\varphi)W_{n-1}, \quad W_0 = 1 \qquad \mathcal{L}_1 = (\partial\varphi)^2 \to W_1 = \Box\varphi \to$$

$$W_n = \mathcal{E}\mathcal{L}_n \qquad \mathcal{L}_2 = (\partial\varphi)^2\Box\varphi \to \mathcal{E}\mathcal{L}_2 = (\Box\varphi)^2 - (\nabla\nabla\varphi)^2$$

Galileons: flat case

first non-standard term

DGP: brane model of gravity

Dvali et al'00

Particular limit of the theory (decoupling limit) gives scalar field Lagrangian,

Luty et al'03

$$\mathcal{L}_{DGP} = -\frac{M_P^2}{4} h^{\mu\nu} \left(\mathcal{E}h\right)_{\mu\nu} - 3(\partial\pi)^2 - \frac{r_c^2}{M_P}(\partial\pi)^2\Box\pi + \frac{1}{2}h^{\mu\nu}T_{\mu\nu} + \frac{1}{M_P}\pi T$$

direct coupling to matter

$$(\Box\varphi)^2 - (\nabla\nabla\varphi)^2$$

Monge-Ampère type

Galileons: flat case

generalisation

Generalization of DGP scalar:
- direct coupling to matter
- Galilean symmetry
- up to second order derivatives in EOM

Nicolis et al'09

$$\mathcal{L}_\pi = \sum_{i=1}^{i=5} c_i \mathcal{L}_i ,$$

$$\mathcal{L}_i \sim \pi^i$$

$$\mathcal{L}_1 = \pi ,$$

$$\mathcal{L}_2 = -\frac{1}{2}\partial_\mu \pi \partial^\mu \pi ,$$

$$\mathcal{L}_3 = -\frac{1}{2}\left(\partial \pi\right)^2 \Box \pi ,$$

$$\mathcal{L}_4 = -\frac{1}{4}(\Box\pi)^2 \partial_\mu \pi \partial^\mu \pi + \frac{1}{2}\Box\pi \partial_\mu \pi \partial_\nu \pi \partial^\mu \partial^\nu \pi + \ldots$$

$$\mathcal{L}_5 = -\frac{1}{5}(\Box\pi)^3 \partial_\mu \pi \partial^\mu \pi + \frac{3}{5}(\Box\pi)^2 \partial_\mu \pi \partial_\nu \pi \partial^\mu \partial^\nu \pi + \ldots$$

Galileons: flat case

equations of motion

Equations of motion (in flat space-time)

$$
\begin{aligned}
\mathcal{E}_1 &= 1 \\
\mathcal{E}_2 &= \Box\pi \\
\mathcal{E}_3 &= (\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2 \\
\mathcal{E}_4 &= (\Box\pi)^3 - 3\,\Box\pi(\partial_\mu\partial_\nu\pi)^2 + 2(\partial_\mu\partial_\nu\pi)^3 \\
\mathcal{E}_5 &= (\Box\pi)^4 - 6(\Box\pi)^2(\partial_\mu\partial_\nu\pi)^2 + 8\,\Box\pi(\partial_\mu\partial_\nu\pi)^3 + 3\left[(\partial_\mu\partial_\nu\pi)^2\right]^2 - 6(\partial_\mu\partial_\nu\pi)^4
\end{aligned}
$$

Nonlinear second-order equations of motion !

No additional degree of freedom => no Ostrogradski ghost

Galileons: flat case

generalisation

Naive covariantization leads to higher order derivatives in EOMs

Galileons: covariant case

Covariant Galileon: adding non-minimal scalar-matter coupling to flat Galileon.

Deffayet et al'09
+ many other works

Most general galileon Shift-symmetric action:

$$\mathcal{L}_2 = K\left(X\right)$$

$$\mathcal{L}_3 = G^{(3)}\left(X\right)\Box\varphi$$

$$\mathcal{L}_4 = G^{(4)}_{,X}(X)\left[(\Box\varphi)^2 - (\nabla\nabla\varphi)^2\right] + R\,G^{(4)}(X),$$

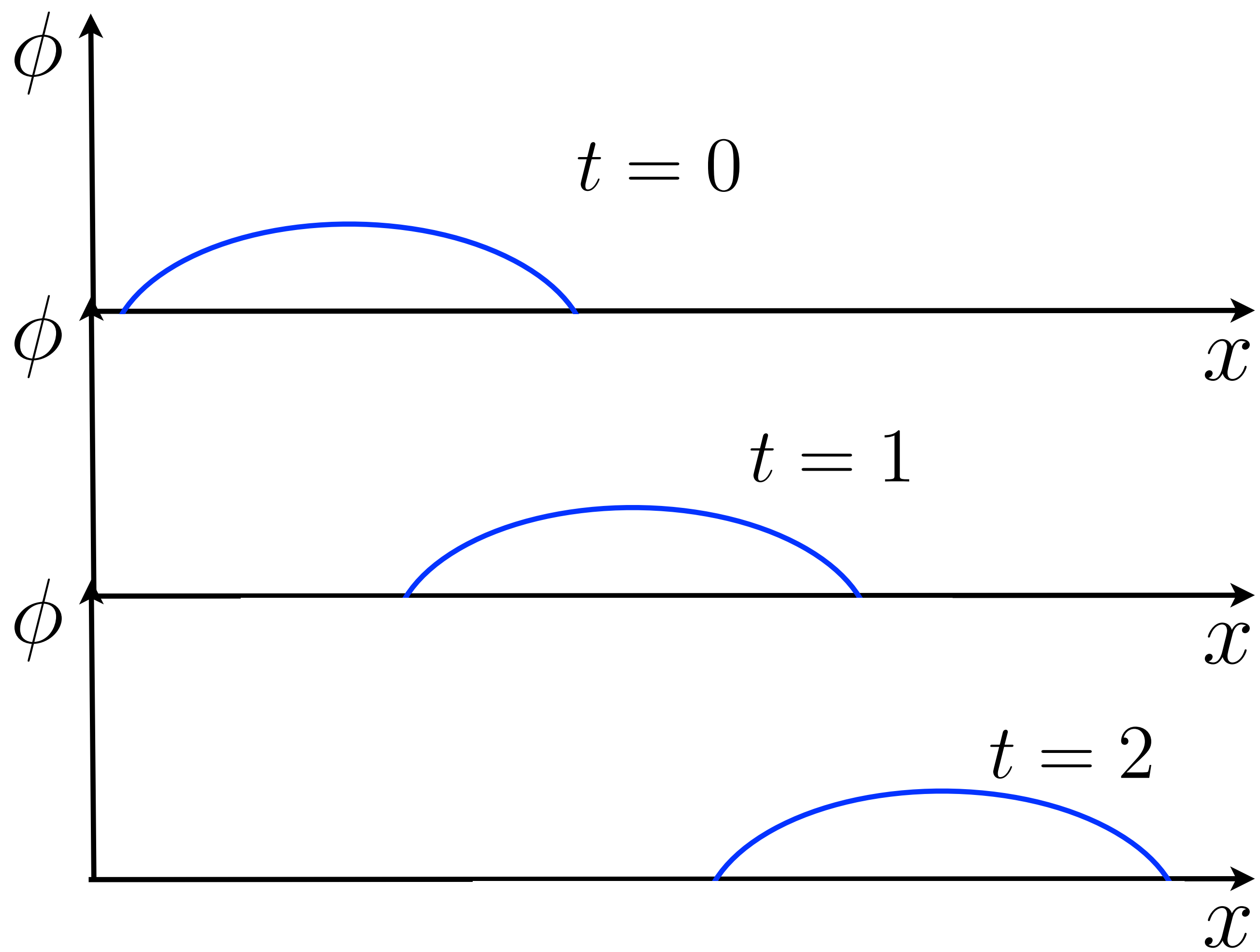
$$\mathcal{L}_5 = G^{(5)}_{,X}\left(X\right)\left[(\Box\varphi)^3 - 3\Box\varphi\left(\nabla\nabla\varphi\right)^2 + 2\left(\nabla\nabla\varphi\right)^3\right] - 6G_{\mu\nu}\nabla^{\mu}\nabla^{\nu}\varphi\, G^{(5)}\left(X\right)$$

Waves: canonical scalar

1+1 case:

$$-\ddot{\phi} + \phi'' = 0$$

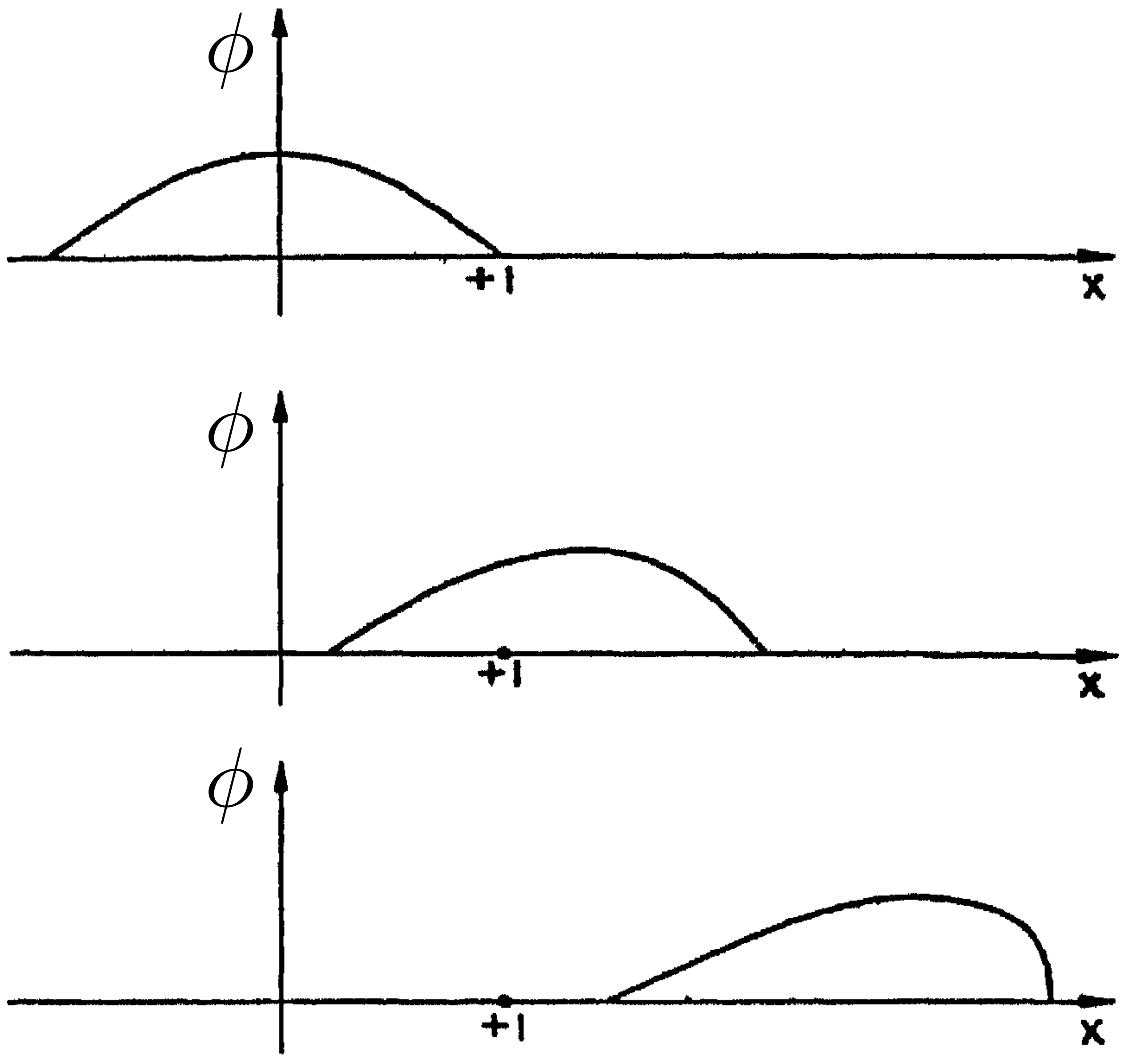
$$\phi = f_1(t-x) + f_2(t+x)$$



$$\phi = f_1(t-x)$$

Waves: non-linear example

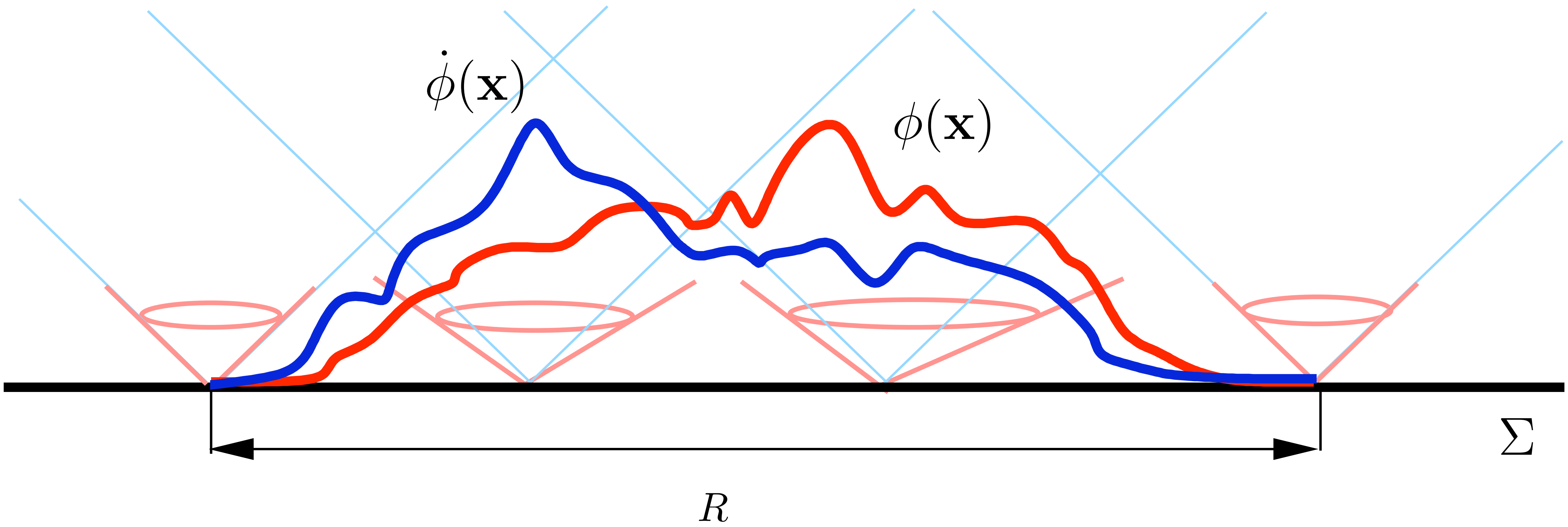
hydrodynamics



Waves in k-essence

flat metric

EB, Mukhanov, Vikman'07



Waves in k-essence

flat metric

EB, Mukhanov, Vikman et al'07

Fixed metric!

"pure" k-essence $\mathcal{L} = \mathcal{K}(X)\,, \quad X \equiv \frac{1}{2}\left(\partial\varphi_\mu \partial\varphi^\mu\right)$

assuming that solution depends on $\theta \equiv x + vt$

EOM : $\mathcal{L}_{,X}\varphi_{,\theta\theta}\left(v^2-1\right) + \mathcal{L}_{,XX}\varphi_{,\theta\theta}\varphi_{,\theta}^2\left(v^2-1\right)^2 = 0$

any $\varphi\left(t \pm x\right)$ is a solution

Waves in simplest galileon

flat metric

Evslin'11
Masoumi&Xiao'12

2D case: $\mathcal{E}_2 = -(\pi_{tt} - \pi_{xx})$

$$\mathcal{E}_3 = 2\left(\pi_{tx}^2 - \pi_{tt}\pi_{xx}\right)$$

easy to see that any $\pi(t \pm x)$ is a solution of $\mathcal{E}_2 + \mathcal{E}_3 = 0$

moreover $\mathcal{E}_4 = \mathcal{E}_5 = 0$ in 2D case

Waves in generalized galileon

Dynamical metric

EB'12

$$\mathcal{L}^{(2)} = K\left(X\right)$$
$$\mathcal{L}^{(3)} = G^{(3)}\left(X\right)\Box\varphi$$
$$\mathcal{L}^{(4)} = G^{(4)}_{,X}\left(X\right)\left[\left(\Box\varphi\right)^2 - \left(\nabla\nabla\varphi\right)^2\right] + R\,G^{(4)}\left(X\right)$$
$$\mathcal{L}^{(5)} = G^{(5)}_{,X}\left(X\right)\left[\left(\Box\varphi\right)^3 - 3\Box\varphi\left(\nabla\nabla\varphi\right)^2 + 2\left(\nabla\nabla\varphi\right)^3\right] - 6G_{\mu\nu}\nabla^{\mu}\nabla^{\nu}\varphi\, G^{(5)}\left(X\right)$$

$K, G^{(3)}, G^{(4)}, G^{(5)}$ are arbitrary functions of the canonical kinetic term X

only shift-symmetric Lagrangians!
The full theory is more general

Second-order derivative EOMs:
"generalized" Galileons
or
Horndeski theory

Ansatz for metric

EB'12

$$ds^2 = -F(\tau, y)d\tau^2 - 2d\tau dx + dy^2 + dz^2$$

$$\begin{aligned} R_{0202} = R_{2020} = -R_{2002} = -R_{0220} &= \frac{1}{2}F_{yy} \\ R &= 0 \\ R_{\tau\tau} &= \frac{1}{2}F_{yy} \\ G_{\tau\tau} &= \frac{1}{2}F_{yy}. \end{aligned}$$

Ansatz for scalar field

EB'12

plane-wave solution : $\quad \varphi = \varphi(\tau).$

$$J_\mu^{(2)} = \{K_X \varphi', 0, 0, 0\}, \; J_\mu^{(3)} = 0, \;\; J_\mu^{(4)} = 0, \;\; J_\mu^{(5)} = 0.$$

$$T_{\tau\tau}^{(2)} = K_X(0)\varphi'^2, \quad \text{other components} = 0$$

$$T_{\mu\nu}^{(3)} = 0$$

$$T_{\mu\nu}^{(4)} = -2G^{(4)}(0)\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) = -2G^{(4)}(0)G_{\mu\nu}$$

$$T_{\mu\nu}^{(5)} = 0$$

Solution

EB'12

scalar field equation: eom is satisfied automatically, $\nabla_\mu J^\mu_{(n)} = 0$

non-trivial component of the Einstein equations : $G^{(4)}(0) F_{yy} = K_X(0) \left(\varphi'_\tau\right)^2$

$$F(\tau, y) = \eta \left(\varphi'_\tau\right)^2 y^2 + F_1(\tau) y + F_0(\tau), \quad \eta \equiv K_X(0)/G^{(4)}(0)$$

appropriate coordinate transformation => eliminate F_0 and F_1

$$ds^2 = -\eta \left(\frac{d\varphi}{d\tau}\right)^2 y^2 d\tau^2 - 2 d\tau dx + dy^2 + dz^2,$$
$$\varphi = \varphi(\tau).$$

Causality?

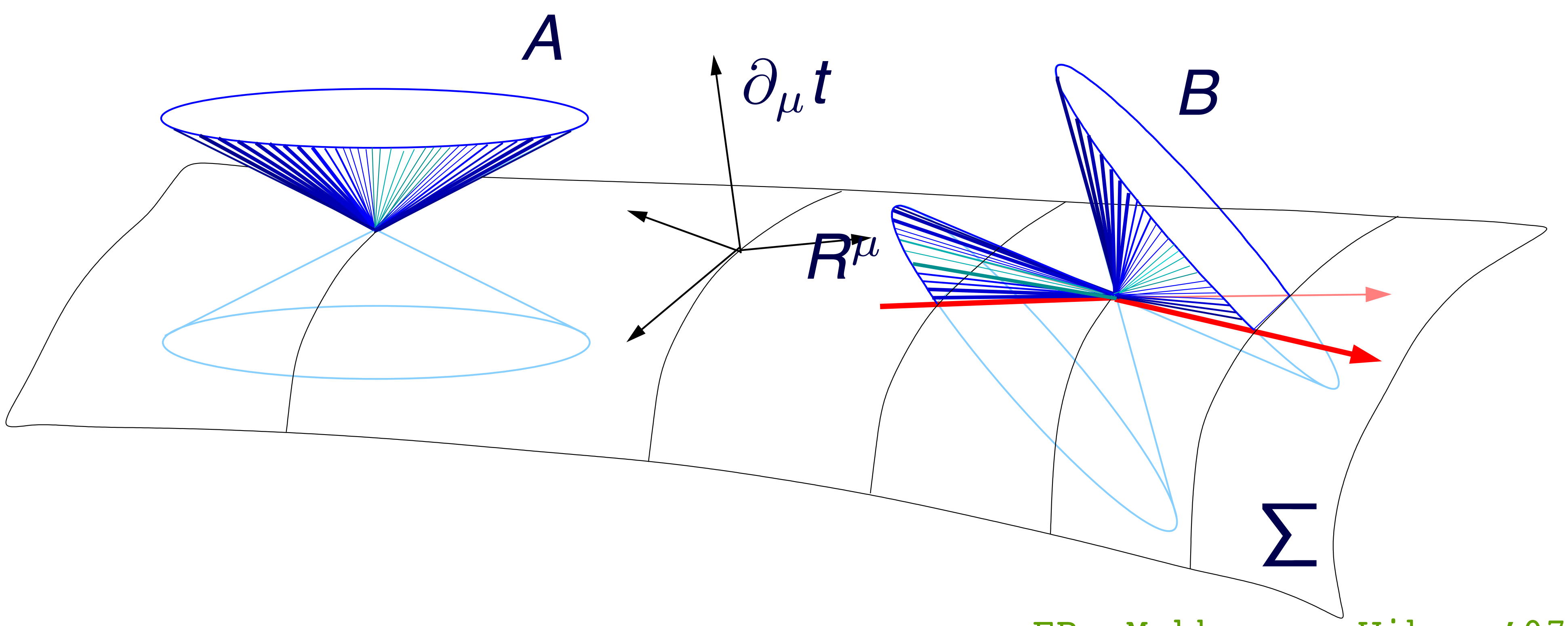
problems with superluminal propagation?

- The theorem on stable causality: *A spacetime $(M, g_{\mu\nu})$ is stably causal if and only if there exists a differentiable function f on M such that $\nabla^\mu f$ is a future directed timelike vector field.*
- The scalar field ϕ itself serves as such a global time function.

Causality?

Cauchy problem

A) Good initial surface
B) Bad initial surface

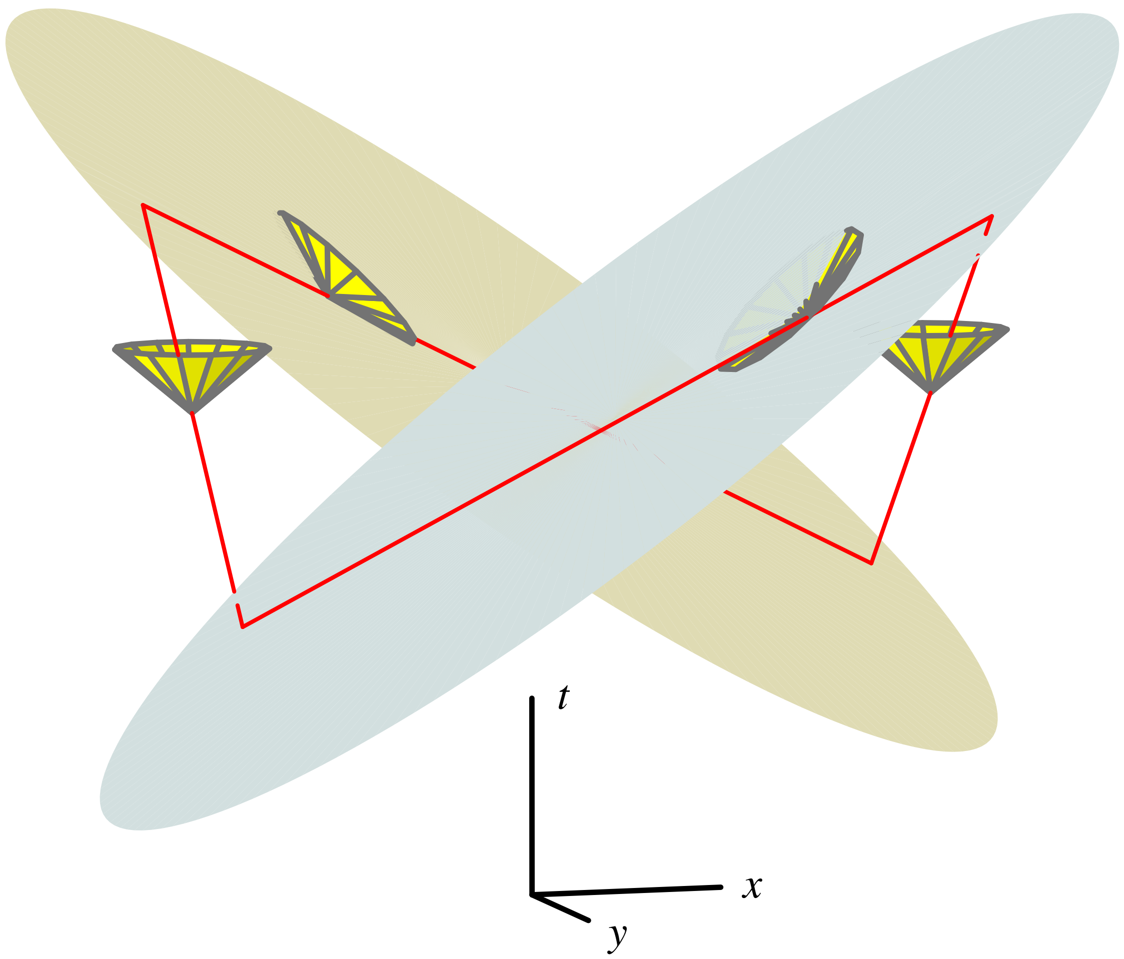


EB, Mukhanov, Vikman'07

Causality?

Time machine (closed time-like curves for non-homogeneous backgrounds) ?

Adams et al'06



Causality?

Chronology protection?

Hawking'92

- wormholes
- Goedel cosmological solution
- Stockum's rotating dust cylinders
- Gott's solution for two infinitely long cosmic strings
- Ori's time machine
- ...

Chronology protection conjecture:
Laws of physics must prohibit appearance of closed causal curves

Causality?

Chronology protection for k-essence?

EB, Mukhanov, Vikman'07

Chronology protection for galileons?

Burrage et al'11

Further study?

- More general solutions of k-essence and galileons.
- Formation of caustics in k-essence and galileons.
- Chronology protection.