Classical and Quantum Aspects of Galileon Theories

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European Union
European Social Fund
OPERATIONAL PROGRAMME
EDUCATION AND LIFELONG LEARNING
investing in knowledge society
MINISTRY OF EDUCATION & RELIGIOUS AFFAIRS
MANAGING AUTHORITY
Co-financed by Greece and the European Union
NSRF 2007-2013
EUROPEAN SOCIAL FUND

- The cubic Galileon theory describes the dynamics of the scalar mode that survives in the decoupling limit of the DGP model (Dvali, Gabadadze, Porrati).
- The action contains a higher-derivative term, cubic in the field $\pi(x)$, with a dimensionful coupling that sets the scale Λ at which the theory becomes strongly coupled.
- The action is invariant under the Galilean transformation $\pi(x) \to \pi(x) + b_\mu x^\mu + c$, up to surface terms.
- In the Galileon theory additional terms can also be present, but the theory is ghost-free: EOM is second order (Nicolis, Rattazzi, Trincherini).
- The most general ghost-free theory, without the Galilean symmetry, is the Horndeski or generalized Galileon theory.
- The effective theory of four-dimensional surfaces (branes), embedded in five-dimensional flat space, belongs to the Horndeski class.
- The brane theory reduces to the Galileon theory in the nonrelativistic limit (de Rham, Tolley).

Outline

- Classical solutions of higher-derivative theories that describe surfaces embedded in Minkowski space.
- Branes with throats or shock fronts. Brane annihilation. Classicalization?
- Connection with solutions of the Galileon theory. Vainshtein mechanism.
- Renormalization of the cubic Galileon theory.
- Renormalization of theories that describe surfaces and connection with the Galileon theory.
- Renormalization-group evolution and asymptotic safety.

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- N.T.
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- A. Codello, N.T., O. Zanusso
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Brane effective action

- Consider a (3+1)-dimensional surface (brane) embedded in (4+1)-dimensional Minkowski space.
- Induced metric in the static gauge: $g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu \pi \, \partial_\nu \pi$
- Extrinsic curvature: $K_{\mu\nu} = -\partial_\mu \partial_\nu \pi / \sqrt{1 + (\partial \pi)^2}$.
- Leading terms in the effective action (de Rham, Tolley)

$$S_\lambda = -\lambda \int d^4 x \sqrt{-g} = -\lambda \int d^4 x \sqrt{1 + (\partial \pi)^2}$$

$$S_K = -M_5^3 \int d^4 x \sqrt{-g}\, K = M_5^3 \int d^4 x \; \left([\Pi] - \gamma^2 [\phi] \right)$$

$$S_R = (M_4^2/2) \int d^4 x \sqrt{-g}\, R$$

$$= (M_4^2/2) \int d^4 x \, \gamma \left([\Pi]^2 - [\Pi^2] + 2\gamma^2 ([\phi^2] - [\Pi][\phi]) \right)$$

- Notation: $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$, $\gamma = 1/\sqrt{-g} = 1/\sqrt{1 + (\partial \pi)^2}$, $\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi$, square brackets represent the trace, $[\phi^n] \equiv \partial \pi \cdot \Pi^n \cdot \partial \pi$.

Galileon theory

- The Galileon theory can be obtained in the nonrelativistic limit $(\partial\pi)^2 \ll 1$.
- The action becomes

$$S^{NR} = \int d^4x \left\{ -\frac{\lambda}{2}(\partial\pi)^2 + \frac{M_5^3}{2}(\partial\pi)^2\Box\pi + \frac{M_4^2}{4}(\partial\pi)^2 \left((\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2\right) \right\}$$

- Invariant under the Galilean symmetry $\delta\pi = c + v_\mu x^\mu$.
- The term of highest order in the Galileon theory, omitted here, can be obtained by including in the brane action the Gibbons-Hawking-York term associated with the Gauss-Bonnet term of $(4+1)$-dimensional gravity.
- Generalized Galileon or Horndeski (1974) theory: Up to second derivatives in the EOM, but no Galilean symmetry.

Exact analytical solutions

- DBI action: surface area

$$\mathcal{L} = -\frac{1}{\lambda}\sqrt{1+\lambda\left(\partial_\mu \pi\right)^2}.$$

- Exact solutions ($c > 0$):

$$d\pi/dr = \pm\frac{c}{\sqrt{r^4-\lambda c^2}}.$$

- $\lambda < 0$: Field configuration induced by a δ-function source resulting from the concentration of energy around $r = 0$ (Dvali, Giudice, Gomez, Kehagias).
  Static classicalons: Similar to BIons (Gibbons).
- $\lambda > 0$: The solutions have a square-root singluarity at $r_s = \lambda^{1/4}c^{1/2}$. They can be joined smoothly in a continuous double-valued function of r for $r \geq r_s$: throat connecting two (3)-branes embedded in $(4+1)$-dimensional Minkowski space. The field π corresponds to the Goldstone mode of the broken translational invariance (Gibbons).

- Exact dynamical solutions $\pi = \pi(z)$, with $z = r^2 - t^2$, satisfying

$$d\pi/dz = \pm \frac{1}{\sqrt{cz^4 - 4\lambda z}}.$$

- For both signs of λ, the solutions display square-root singularities at $z = 0$ and at the value z_s that satisfies $z_s^3 = 4\lambda/c \quad (c > 0)$.
- For $\lambda > 0$, the singularity is located at $r_s^2 = t^2 + (4\lambda/c)^{1/3}$.
- Shock fronts associated with meson production (Heisenberg).
- They display strong scattering at a length scale $\sim (4\lambda/c)^{1/6}$. Classicalization?
- The general solution of the equation of motion does not display scattering at large length scales.

Brane picture

- By joining solutions with opposite signs, one can create evolving networks of throats or wormholes, connecting two branes.
- When the throat expands, the worldvolume of the part of the branes that is eliminated reappears as energy distributed over the remaining part of the branes.
- Interpretation: Annihilating branes, bouncing Universe.
- The solutions can be generalized in the context of higher-derivative effective actions that describe surfaces embedded in Minkowski space.

Equations of motion

- Brane theory

$$\lambda\gamma\Big\{([\Pi]-\gamma^2[\phi]\Big\}-M_5^3\gamma^2\Big\{[\Pi]^2-[\Pi^2]+2\gamma^2\left([\phi^2]-[\Pi][\phi]\right)\Big\}$$
$$-\frac{M_4^2}{2}\gamma^3\Big\{[\Pi]^3+2[\Pi^3]-3[\Pi][\Pi^2]$$
$$+3\gamma^2\left(2\left([\Pi][\phi^2]-[\phi^3]\right)-\left([\Pi]^2-[\Pi^2]\right)[\phi]\right)\Big\}=0.$$

- Galileon theory

$$\lambda[\Pi]-M_5^3\left([\Pi]^2-[\Pi^2]\right)-\frac{M_4^2}{2}\left([\Pi]^3+2[\Pi^3]-3[\Pi][\Pi^2]\right)=0,$$

- They have solutions of the form
  1. $\pi=\pi(r^2)$
  2. $\pi=\pi(r^2-t^2)$.

Solutions $\pi(w)$, with $w = r^2$

- Brane theory
  a) For $M_5 = M_4 = 0$ and $c > 0$

$$\pi_w = \pm \frac{c}{\sqrt{w^3 - 4c^2 w}},$$

  b) For $M_4 = 0$ and $\kappa = 12M_5^3/\lambda$

$$\pi_w = \frac{\pm\sqrt{6}c}{\sqrt{3w^3 + \sqrt{9w^6 \mp 12\kappa c\, w^{9/2}} - 24c^2 w \mp 2\kappa c\, w^{3/2}}}.$$

- Galileon theory
  For $M_4 = 0$ and $\kappa = 12M_5^3/\lambda$

$$\pi_w = \frac{3}{2\kappa}\left(1 - \sqrt{1 \mp \frac{4}{3}\frac{\kappa c}{w^{3/2}}}\right).$$

Vainshtein mechanism

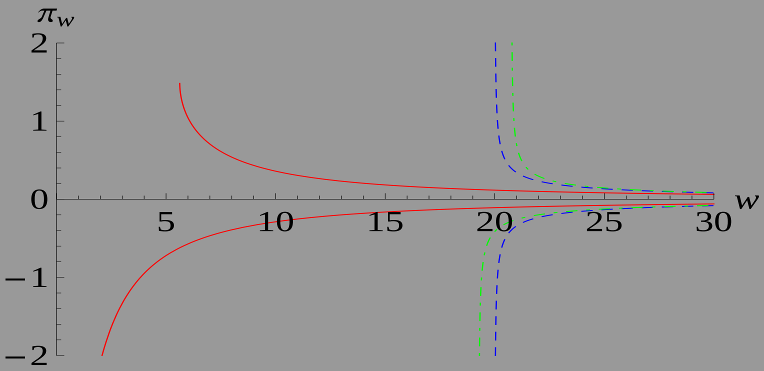


Figure : The solution $\pi_w = d\pi/dw$ for:
a) The brane theory with $M_4 = M_5 = 0$, $c = 10$ (blue).
b) The brane theory with $M_4 = 0$, $12M_5^3/\lambda = \kappa = 1$, $c = 10$ (green).
c) The Galileon theory with $M_4 = 0$, $12M_5^3/\lambda = \kappa = 1$, $c = 10$ (red).

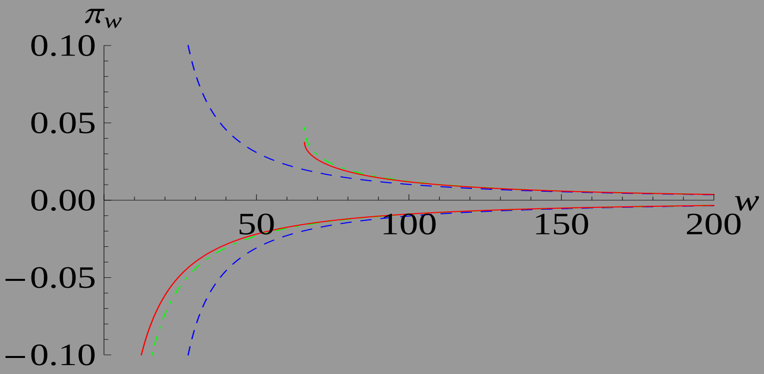


Figure : The solution $\pi_w = d\pi/dw$ for:
a) The brane theory with $M_4 = M_5 = 0$, $c = 10$ (blue).
b) The brane theory with $M_4 = 0$, $12M_5^3/\lambda = \kappa = 40$, $c = 10$ (green).
c) The Galileon theory with $M_4 = 0$, $12M_5^3/\lambda = \kappa = 40$, $c = 10$ (red).

Solutions $\pi(z)$, with $z = r^2 - t^2$

- Brane theory
  a) For $M_5 = M_4 = 0$ and $c > 0$

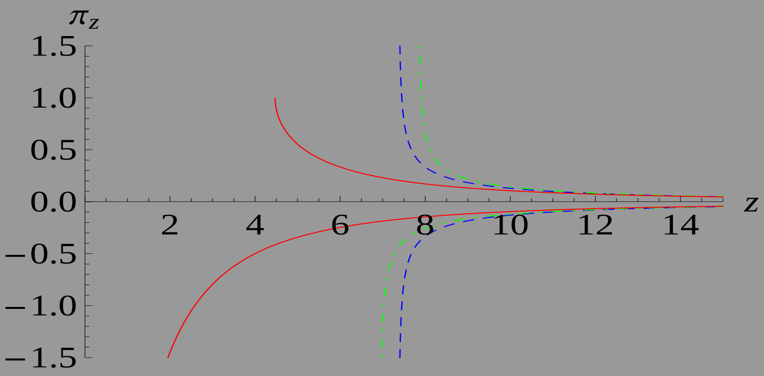
$$\pi_z = \pm \frac{c}{\sqrt{z^4 - 4c^2 z}}.$$

  b) For $M_4 = 0$ and $\kappa = 12 M_5^3/\lambda$

$$\pi_z = \frac{\pm\sqrt{2}c}{\sqrt{z^4 + z^3\sqrt{z^2 \mp 2\kappa c} - 8c^2 z \mp \kappa c\, z^2}}.$$

- Galileon theory
  For $M_4 = 0$ and $\kappa = 12 M_5^3/\lambda$

$$\pi_z = \frac{1}{\kappa}\left(1 - \sqrt{1 \pm \frac{2\kappa c}{z^2}}\right).$$



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b) The brane theory with $M_4 = 0$, $12M_5^3/\lambda = \kappa = 1$, $c = 10$ (green).
c) The Galileon theory with $M_4 = 0$, $12M_5^3/\lambda = \kappa = 1$, $c = 10$ (red).

- The throat solutions of the DBI theory can be generalized to solutions of the (quantum corrected) brane theory.
- The Galileon theory reproduces correctly the shape of the throats at large distances, but fails to do so at short distances.
- The solutions of the brane and Galileon theories coincide in the formal limits $\kappa c \to \infty$ with c fixed, or $c \to 0$, with κc fixed.
- Similar solutions exist in the context of the generalized Galileon theory, and in particular in theories with kinetic gravity braiding.
- Possible cosmological applications: brane annihilation, bouncing Universe.

Renormalizaiton of the Galileon theory

- If a momentum cutoff is used, of the order of the fundamental scale Λ of the theory, and the couplings are taken of order Λ, the one-loop effective action of the Galileon theory is, schematically, (Luty, Porrati, Nicolis, Rattazzi)

$$\Gamma_1 \sim \int d^4x \sum_m \left[\Lambda^4 + \Lambda^2 \partial^2 + \partial^4 \log\left(\frac{\partial^2}{\Lambda^2}\right)\right] \left(\frac{\partial^2 \pi}{\Lambda^3}\right)^m .$$

- Non-renormalization of the Galileon couplings (de Rham, Gabadadze, Heisenberg, Pirtskhalava, Hinterbichler, Trodden, Wesley).
- Explicit one-loop calculation using dimensional regularization (Paula Netto, Shapiro).

One-loop corrections to the cubic Galileon

- Tree-level action in Euclidean d-dimensional space

$$S_0 = \int d^d x \left\{ \frac{\mu_0}{2} (\partial \pi)^2 - \frac{\nu_0}{2} (\partial \pi)^2 \Box \pi \right\}.$$

- Field fluctuation $\delta\pi$ around the background π. The quadratic part is

$$S_0^{quad} = \int d^d x \left\{ -\frac{1}{2} \delta\pi \Box \delta\pi + \frac{\nu_0}{2} \delta\pi \left[2(\Box\pi)\Box\delta\pi - 2(\partial^\mu \partial^\nu \pi)\partial_\mu \partial_\nu \delta\pi \right] \right\}.$$

- Define

$$K = -\Box \qquad \Sigma_1 = 2\nu_0 (\Box\pi)\,\Box \qquad \Sigma_2 = -2\nu_0 (\partial_\mu \partial_\nu \pi)\,\partial^\mu \partial^\nu$$

- One-loop contribution to the effective action

$$S_1 = \frac{1}{2} \mathrm{tr} \log\left(K + \Sigma_1 + \Sigma_2\right) = \frac{1}{2} \mathrm{tr} \log\left(1 + \Sigma_1 K^{-1} + \Sigma_2 K^{-1}\right) + \mathcal{N}.$$

- Expanding the logarithm

$$\mathrm{tr}\left(\Sigma_1 K^{-1}\Sigma_1 K^{-1}\right) = 4\frac{\nu_0^2}{\mu_0^2}(2\pi)^d \int d^d k\, k^4 \tilde{\pi}(k)\tilde{\pi}(-k)\int \frac{d^d p}{(2\pi)^d}$$

$$\mathrm{tr}\left(\Sigma_1 K^{-1}\Sigma_2 K^{-1}\right) = -4\frac{\nu_0^2}{\mu_0^2}(2\pi)^d \int d^d k\, k^4 \tilde{\pi}(k)\tilde{\pi}(-k)\frac{1}{d}\int \frac{d^d p}{(2\pi)^d}$$

$$\mathrm{tr}\left(\Sigma_2 K^{-1}\Sigma_2 K^{-1}\right) = 4\frac{\nu_0^2}{\mu_0^2}(2\pi)^d \int d^d k\, \tilde{\pi}(k)\tilde{\pi}(-k)\left\{\frac{3}{d(d+2)}k^4\int \frac{d^d p}{(2\pi)^d} + \frac{(d-8)(d-1)}{d(d+2)(d+4)}k^6\int \frac{d^d p}{(2\pi)^d}\frac{1}{p^2} - \frac{(d-24)(d-2)(d-1)}{d(d+2)(d+4)(d+6)}k^8\int \frac{d^d p}{(2\pi)^d}\frac{1}{p^4}\right\}.$$

- Putting everything together, we obtain in position space, the one-loop correction to the effective action

$$S_1 = \frac{\nu_0^2}{\mu_0^2} \int d^d x\, \pi(x) \Bigg\{ -\frac{d^2-1}{d(d+2)} \left(\int \frac{d^d p}{(2\pi)^d} \right) \Box^2 + \frac{(d-8)(d-1)}{d(d+2)(d+4)} \left(\int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2} \right) \Box^3 + \frac{(d-24)(d-2)(d-1)}{d(d+2)(d+4)(d+6)} \left(\int \frac{d^d p}{(2\pi)^d} \frac{1}{p^4} \right) \Box^4 \Bigg\} \pi(x).$$

- The momentum integrals are defined with UV and IR cutoffs.
- If dimensional regularization near $d = 4$ is used, the first two terms are absent. The third one corresponds to a counterterm $\sim 1/\epsilon$ (Paula Netto, Shapiro).
- No corrections to the Galileon couplings.
- Terms outside the Galileon theory are generated.

Brane effective action

- Leading terms in the effective action (Euclidean space)

$$S_\lambda = \mu \int d^4x \sqrt{g} = \mu \int d^4x \sqrt{1+(\partial\pi)^2}$$

$$S_\nu = \nu \int d^4x \sqrt{g}\, K = -\nu \int d^4x \; \left([\Pi] - \gamma^2[\phi]\right)$$

$$S_\kappa = (\kappa/2) \int d^4x \sqrt{g}\, K^2 = (\kappa/2) \int d^4x \sqrt{g} \left([\Pi] - \gamma^2[\phi]\right)^2$$

$$S_{\bar\kappa} = (\bar\kappa/2) \int d^4x \sqrt{g}\, R$$

$$= (\bar\kappa/2) \int d^4x\, \gamma \left([\Pi]^2 - [\Pi^2] + 2\gamma^2([\phi^2] - [\Pi][\phi])\right)$$

- The action $S_\lambda + S_\nu + S_{\bar\kappa}$ belongs the generalized Galileon (Horndeski) class. It reduces to the Galileon theory in the nonrelativistic limit.
- The first Gauss-Codazzi equation gives $R = K^2 - K^{\mu\nu}K_{\mu\nu}$.
- The term S_κ becomes $\sim \pi\Box^2\pi$ in the nonrelativistic limit $(\partial\pi)^2 \ll 1$. This term is not included in the Galileon theory.

One-loop corrections to the brane action

- Brane theory with $\mu = \mu_0$, $\nu = \nu_0$, $\kappa = \bar{\kappa} = 0$
- The one-loop correction is

$$S_{b1} = \frac{1}{2} \mathrm{tr} \log \left(S_{b0}^{(2)} \right) ,$$

with

$$S_{b0}^{(2)} = \mu_0 \Delta + \nu_0 V^{\mu\nu} \nabla_\mu \nabla_\nu + \mu_0 U + \mathcal{O}(K^4, \nabla K) ,$$

- Covariant derivatives are evaluated with the induced metric $g^{\mu\nu}$. $\Delta = -g^{\mu\nu}\nabla_\mu\nabla_\nu$, $V^{\mu\nu} = 2(K^{\mu\nu} - Kg^{\mu\nu})$, $U = K^2 - K^{\mu\nu}K_{\mu\nu} = R$.
- Expanding the logarithm

$$\begin{aligned} S_{b1} \quad = \quad & \frac{1}{2}\mathrm{tr}\log(\mu_0\Delta) + \frac{1}{2}\frac{\nu_0}{\mu_0}\mathrm{tr}\left(\frac{1}{\Delta}V^{\mu\nu}\nabla_\mu\nabla_\nu\right) + \frac{1}{2}\mathrm{tr}\left(\frac{1}{\Delta}U\right) \\ & -\frac{1}{4}\frac{\nu_0^2}{\mu_0^2}\mathrm{tr}\left(\frac{1}{\Delta}V^{\mu\nu}\nabla_\mu\nabla_\nu\frac{1}{\Delta}V^{\alpha\beta}\nabla_\alpha\nabla_\beta\right) + \mathcal{O}(K^4, \nabla K) . \end{aligned}$$

- Evaluation of the traces with heat kernel techniques

$$\mathrm{tr}\log(\mu_0\Delta) = \left(\int \frac{d^d p}{(2\pi)^d} \ln\left(\mu_0 p^2\right)\right)\int d^d x\sqrt{g} + \frac{d-2}{12}\left(\int \frac{d^d p}{(2\pi)^d}\frac{\ln\left(\mu_0 p^2\right)}{p^2}\right)\int d^d x\sqrt{g}R$$

$$\mathrm{tr}\left(\frac{1}{\Delta}V^{\mu\nu}\nabla_\mu\nabla_\nu\right) = \frac{d-1}{d}\left(\int \frac{d^d p}{(2\pi)^d}\right)\int d^d x\sqrt{g}K$$

$$\mathrm{tr}\left(\frac{1}{\Delta}U\right) = \left(\int \frac{d^d p}{(2\pi)^d}\frac{1}{p^2}\right)\int d^d x\sqrt{g}R$$

$$\mathrm{tr}\left(\frac{1}{\Delta}V^{\mu\nu}\nabla_\mu\nabla_\nu\frac{1}{\Delta}V^{\alpha\beta}\nabla_\alpha\nabla_\beta\right) = \frac{4(d^2-1)}{d(d+2)}\left(\int \frac{d^d p}{(2\pi)^d}\right)\int d^d x\sqrt{g}K^2 - \frac{8}{d(d+2)}\left(\int \frac{d^d p}{(2\pi)^d}\right)\int d^d x\sqrt{g}R\,.$$

- The couplings at one-loop level are

$$\mu = \mu_0 + \frac{1}{2}\int \frac{d^d p}{(2\pi)^d} \ln\left(\mu_0 p^2\right)$$

$$\nu = \nu_0 + \frac{d-1}{2d}\frac{\nu_0}{\mu_0}\int \frac{d^d p}{(2\pi)^d}$$

$$\kappa = -\frac{2(d^2-1)}{d(d+2)}\frac{\nu_0^2}{\mu_0^2}\int \frac{d^d p}{(2\pi)^d}$$

$$\bar{\kappa} = \frac{4}{d(d+2)}\frac{\nu_0^2}{\mu_0^2}\int \frac{d^d p}{(2\pi)^d} + \int \frac{d^d p}{(2\pi)^d}\frac{1}{p^2} + \frac{d-2}{12}\int \frac{d^d p}{(2\pi)^d}\frac{\ln\left(\mu_0 p^2\right)}{p^2}.$$

- Terms outside the Galileon theory are generated.
- The couplings of the brane (generalized Galileon) theory are renormalized.

Renormalization-group evolution

- We use the Wilsonian (exact) renormalization group.
- Heat-kernel techniques.
- The evolution equations for the couplings take the form

$$\partial_t \mu_k = \frac{k^d}{(4\pi)^{d/2}\Gamma\left(\frac{d}{2}+1\right)} \frac{2\kappa_k k^2+\mu_k}{\kappa_k k^2+\mu_k}$$

$$\partial_t \nu_k = -\frac{k^d}{(4\pi)^{d/2}\Gamma\left(\frac{d}{2}+2\right)}(d-1)\frac{(2\kappa_k k^2+\mu_k)\nu_k}{(\kappa_k k^2+\mu_k)^2}$$

$$\partial_t \kappa_k = \frac{2k^d}{(4\pi)^{d/2}\Gamma\left(\frac{d}{2}+2\right)}\left\{\frac{d+4}{4}\frac{(2\kappa_k k^2+\mu_k)\kappa_k}{(\kappa_k k^2+\mu_k)^2} + \frac{4(d^2-1)}{d+4}\frac{(2\kappa_k k^2+\mu_k)\nu_k^2}{(\kappa_k k^2+\mu_k)^3}\right\}$$

$$\partial_t \bar{\kappa}_k = \frac{k^d}{(4\pi)^{d/2}\Gamma\left(\frac{d}{2}+2\right)}\left\{\frac{d(d+2)}{12}\frac{2\kappa_k k^2+\mu_k}{(\kappa_k k^2+\mu_k)k^2} - \frac{16}{d+4}\frac{(2\kappa_k k^2+\mu_k)\nu_k^2}{(\kappa_k k^2+\mu_k)^3}\right.$$

$$\left. -\left[(d+2)\frac{\mu_k}{k^2} + 2d\kappa_k + \frac{3(d-2)}{2}\bar{\kappa}_k\right]\frac{2\kappa_k k^2+\mu_k}{(\kappa_k k^2+\mu_k)^2}\right\}.$$

- We can obtain the β-functions of κ_k, $\bar{\kappa}_k$ for two-dimensional fluid membranes for which the volume (now area) term is considered subleading.
- We set $d = 2$, $\mu_k = \nu_k = 0$ and obtain

$$\partial_t \kappa_k = \frac{3}{4\pi}, \qquad \partial_t \bar{\kappa}_k = -\frac{5}{6\pi}. \tag{1}$$

- These expressions reproduce known results (Polyakov, Kleinert, Forster) for the renormalization of the bending and Gaussian rigidities of fluctuating membranes in a three-dimensional bulk space.

Asymptotic safety

- Consider the theory with $d = 4$, $\nu = \kappa = 0$. It includes a cosmological constant and an Einstein term.
- Define the dimensionless cosmological and Newton's constants through

$$\frac{\mu_k}{k^4} = \frac{\Lambda_k}{8\pi G_k}, \qquad \frac{\bar{\kappa}_k}{k^2} = -\frac{1}{8\pi G_k}. \tag{2}$$

- Their scale dependence is given by

$$\partial_t \Lambda_k = -2\Lambda_k + \frac{1}{6\pi} G_k (3 - 2\Lambda_k) \tag{3}$$

$$\partial_t G_k = 2G_k + \frac{1}{12\pi} \frac{G_k^2}{\Lambda_k} (3 - 4\Lambda_k). \tag{4}$$

- This system of equations has two fixed points at which the β-functions vanish:
  a) the Gaussian one, at $\Lambda_k = G_k = 0$, and
  b) a nontrivial one, at $\Lambda_k = 9/8$, $G_k = 18\pi$.
- The flow diagram is similar to the scenario of asymptotic safety.

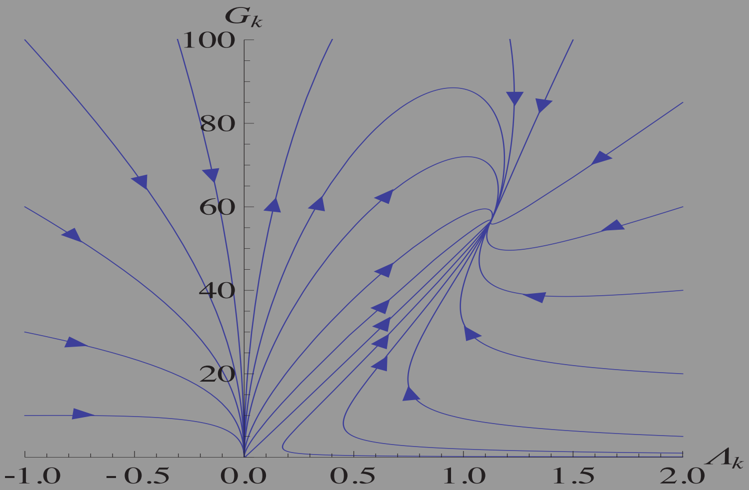

Figure : The flow diagram.

Conclusions

- The DBI action and its generalizations have exact classical solutions that can be interpreted as shock fronts that scatter at length scales much larger than the fundamental scale (classicalization). This is possible only for very specific initial conditions.
- The same solution can be interpreted as wormholes or throats connecting a pair of branes.
- They can also be viewed as bouncing Universe solutions. Cosmological applications?
- The couplings of the Galileon theory do not get renormalized. However, the Galileon theory is not stable under quantum corrections. Additional terms are generated.
- The nonrenormalization of couplings is not a feature of the generalized Galileon theories.
- The brane theory displays RG evolution very similar to that in the asymptotic safety scenario.