Instability of black holes in massive gravity

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with A.Fabbri

Introduction

- de Rham-Gabadadze-Tolley massive gravity: 5 propagating degrees of freedom (no Boulware-Deser ghost)
- Many applications: cosmology, spherically symmetric solutions, black holes.

(talk by Mikhail Volkov on black holes)

Action

Action for BD-ghost-free massive gravity *(de Rham-Gabadadabze-Tolley'10,Hassan-Rosen'11)*

$$S = M_P^2 \int d^4x \sqrt{-g}\left(\frac{R_g}{2} - m^2\mathcal{U}\right) + \frac{\kappa M_P^2}{2}\int d^4x\sqrt{-f}\mathcal{R}_f$$

$$\mathcal{U} = \mathcal{U}_2 + \alpha_3\mathcal{U}_3 + \alpha_4\mathcal{U}_4 \qquad \mathcal{K}^\mu_\nu \equiv \delta^\mu_\nu - \sqrt{g^{\mu\alpha}f_{\alpha\nu}}$$

$$\mathcal{U}_2 = -\frac{1}{2!}\left(\left(\mathcal{K}^\mu_\mu\right)^2 - \mathcal{K}^\mu_\nu\mathcal{K}^\nu_\mu\right)$$

$$\mathcal{U}_3 = \frac{1}{3!}\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\sigma}\mathcal{K}^\mu_\alpha\mathcal{K}^\nu_\beta\mathcal{K}^\rho_\gamma$$

$$\mathcal{U}_4 = \frac{1}{4!}\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}\mathcal{K}^\mu_\alpha\mathcal{K}^\nu_\beta\mathcal{K}^\rho_\gamma\mathcal{K}^\sigma_\delta$$

Equations of motion

$$G_{\mu\nu} = m^2 T^{\mathcal{U}}_{\mu\nu}$$

$G_{\mu\nu}$ is the Einstein tensor for metric $g_{\mu\nu}$

$$\mathcal{G}_{\mu\nu} = \frac{\sqrt{-g}}{\sqrt{-f}} \frac{m^2}{\kappa} \mathcal{T}^{\mathcal{U}}_{\mu\nu}$$

$\mathcal{G}_{\mu\nu}$ is the Einstein tensor for metric $f_{\mu\nu}$

No matter action, the energy-momentum tensor from interaction term:

$$T_{\mu\nu} = \mathcal{U} g_{\mu\nu} - 2\frac{\delta \mathcal{U}}{\delta g^{\mu\nu}} =$$

$$- g_{\mu\sigma}\gamma^{\sigma}_{\alpha} \left(\mathcal{K}^{\alpha}_{\nu} - [\mathcal{K}]\delta^{\alpha}_{\nu}\right) + \alpha_3 g_{\mu\sigma}\gamma^{\sigma}_{\alpha} \left(\mathcal{U}_2 \delta^{\alpha}_{\nu} - [\mathcal{K}]\mathcal{K}^{\alpha}_{\nu} + (\mathcal{K}^2)^{\alpha}_{\nu}\right)$$

$$+ \alpha_4 g_{\mu\sigma}\gamma^{\sigma}_{\alpha} \left(\mathcal{U}_3 \delta^{\alpha}_{\nu} - \mathcal{U}_2 \mathcal{K}^{\alpha}_{\nu} + [\mathcal{K}](\mathcal{K}^2)^{\alpha}_{\nu} - (\mathcal{K}^3)^{\alpha}_{\nu}\right) + \mathcal{U}$$

$$\mathcal{T}_{\mu\nu} = -2\frac{\delta \mathcal{U}}{\delta f^{\mu\nu}} =$$

$$f_{\mu\sigma}\gamma^{\sigma}_{\alpha} \left(\mathcal{K}^{\alpha}_{\nu} - [\mathcal{K}]\delta^{\alpha}_{\nu}\right) - \alpha_3 f_{\mu\sigma}\gamma^{\sigma}_{\alpha} \left(\mathcal{U}_2 \delta^{\alpha}_{\nu} - [\mathcal{K}]\mathcal{K}^{\alpha}_{\nu} + (\mathcal{K}^2)^{\alpha}_{\nu}\right)$$

$$- \alpha_4 f_{\mu\sigma}\gamma^{\sigma}_{\alpha} \left(\mathcal{U}_3 \delta^{\alpha}_{\nu} - \mathcal{U}_2 \mathcal{K}^{\alpha}_{\nu} + [\mathcal{K}](\mathcal{K}^2)^{\alpha}_{\nu} - (\mathcal{K}^3)^{\alpha}_{\nu}\right)$$

Equations of motion

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No matter action, the energy-momentum tensor from interaction term:

$$T^{\mathcal{U}}_{\mu\nu} = -g_{\mu\beta}\left(\delta^{\beta}_{\alpha} - \mathcal{K}^{\beta}_{\alpha}\right)\left(\mathcal{K}^{\alpha}_{\nu} - \mathcal{K}^{\lambda}_{\lambda}\delta^{\alpha}_{\nu}\right) + \mathcal{O}(\mathcal{K}^3)$$
$$\mathcal{T}^{\mathcal{U}}_{\mu\nu} = -T^{\mathcal{U}}_{\mu\nu} + \mathcal{O}(\mathcal{K}^3)$$

In original dRGT model no EH term for f -> the second EOM is absent

Background solution

Take identical metrics, $g_{\mu\nu} = f_{\mu\nu} \Rightarrow \mathcal{K} = 0$.
Hence $\mathcal{T}^{\mathcal{U}}_{\mu\nu} = T^{\mathcal{U}}_{\mu\nu} = 0$.
Any vacuum GR solution $g_{\mu\nu}$ is also a solution
for the bi-gravity theory provided that $f_{\mu\nu} = g_{\mu\nu}$

Simplest black hole solution in bigravity, bi-Schwarzschild solution

$$\begin{aligned} g^{(0)}_{\mu\nu} dx^\mu dx^\nu &= \\ f^{(0)}_{\mu\nu} dx^\mu dx^\nu &= \end{aligned} \quad ds^2 = -\left(1 - \frac{r_S}{r}\right) dt^2 + \left(1 - \frac{r_S}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Perturbations

Perturbations of both metrics

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}, \quad f_{\mu\nu} = f^{(0)}_{\mu\nu} + \tilde{h}_{\mu\nu}$$

$$g^{\mu\alpha} f_{\alpha\nu} = \delta^{\mu}_{\nu} + \left(\tilde{h}^{\mu}_{\nu} - h^{\mu}_{\nu}\right)$$

$$\mathcal{K}^{\mu}_{\nu} = \frac{1}{2} h^{(-)\mu}{}_{\nu} + \mathcal{O}(h^2)$$

$$h^{(-)\mu}{}_{\nu} = h^{\mu}_{\nu} - \tilde{h}^{\mu}_{\nu}$$

$$h^{(+)\mu}{}_{\nu} = h^{\mu}_{\nu} + \kappa \tilde{h}^{\mu}_{\nu}$$

Perturbations

linearized equations

Einstein eq for g

Einstein eq for f

$$\mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + \frac{m^2}{2}\left(h^{(-)}_{\mu\nu} - g_{\mu\nu} h^{(-)}\right) = 0,$$

$$\mathcal{E}^{\alpha\beta}_{\mu\nu} \tilde{h}_{\alpha\beta} - \frac{m^2}{2\kappa}\left(h^{(-)}_{\mu\nu} - g_{\mu\nu} h^{(-)}\right) = 0$$

$$R_{\mu\nu} = R = 0$$

$$\mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} = -\frac{1}{2}\big(\nabla_\mu \nabla_\nu h - \nabla_\nu \nabla_\sigma h^\sigma_\mu - \nabla_\mu \nabla_\sigma h^\sigma_\nu + \Box h_{\mu\nu} - g_{\mu\nu}\Box h + g_{\mu\nu}\nabla_\alpha \nabla_\beta h^{\alpha\beta} + 2R^{\sigma}{}_{\mu}{}^{\lambda}{}_{\nu} h_{\lambda\sigma}\big)$$

Perturbations

linearized equations

Einstein eq for g

Einstein eq for f

$$\mathcal{E}^{\alpha\beta}_{\mu\nu} h^{(-)}_{\alpha\beta} + \frac{m'^2}{2}\left(h^{(-)}_{\mu\nu} - g_{\mu\nu} h^{(-)}\right) = 0,$$

$$\mathcal{E}^{\alpha\beta}_{\mu\nu} h^{(+)}_{\alpha\beta} = 0,$$

$R_{\mu\nu} = R = 0$

$$\mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} = -\frac{1}{2}\left(\nabla_\mu \nabla_\nu h - \nabla_\nu \nabla_\sigma h^\sigma_\mu - \nabla_\mu \nabla_\sigma h^\sigma_\nu \right.$$
$$\left. + \Box h_{\mu\nu} - g_{\mu\nu} \Box h + g_{\mu\nu} \nabla_\alpha \nabla_\beta h^{\alpha\beta} + 2 R^\sigma{}_\mu{}^\lambda{}_\nu h_{\lambda_\sigma} \right)$$

$$m' \equiv m\sqrt{1 + 1/\kappa}$$

$h^{(-)}_{\mu\nu}$ is massive

$h^{(+)}_{\mu\nu}$ is massless

Perturbations

massive modes

$$\mathcal{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta}^{(-)} + \frac{m'^2}{2}\left(h_{\mu\nu}^{(-)} - g_{\mu\nu} h^{(-)}\right) = 0$$

Acting with ∇^ν gives $\nabla^\nu h_{\mu\nu}^{(-)} = \nabla_\mu h^{(-)}$ and hence $\nabla^\nu \nabla^\mu h_{\mu\nu}^{(-)} = \Box h^{(-)}$
Using this last relation in the trace $\rightarrow$

$$\nabla^\nu h_{\mu\nu}^{(-)} = h^{(-)} = 0$$

$$\Box h_{\mu\nu}^{(-)} + 2R^{\sigma}{}_{\mu}{}^{\lambda}{}_{\nu} h_{\lambda\sigma}^{(-)} = m'^2 h_{\mu\nu}^{(-)}$$

EFI-93-02

January 1993

BLACK STRINGS AND p-BRANES ARE UNSTABLE

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ABSTRACT

We investigate the evolution of small perturbations around black strings and branes which are low energy solutions of string theory. For simplicity we focus attention on the zero charge case and show that there are unstable modes for a range of time frequency and wavelength in the extra $10-D$ dimensions.

Perturbations

Gregory-Laflamme instability

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$\delta g_{ab} = h_{ab}$, the Lichnerowicz equation, is essentially a wave equation

$$\Delta_L h_{ab} = \left(\delta_a^c \delta_b^d \Box + 2R_a{}^c{}_b{}^d\right) h_{cd} = 0. \tag{1.1}$$

Because of the symmetries of the background $\mathrm{Sch}_4 \times \mathbb{R}$ metric, this reduces to a four-dimensional Lichnerowicz operator plus a ∂_z^2 piece. Performing a Fourier decomposition of h_{ab} in the fifth dimension yields

$$\Delta_L h_{ab} = \left(\Delta_4 - m^2\right) h_{ab} = 0. \tag{1.2}$$

We investigate the evolution of small perturbations around black strings and branes which are low energy solutions of string theory. For simplicity we focus attention on the zero charge case and show that there are unstable modes for a range of time frequency and wavelength in the extra $10 - D$ dimensions.

Gregory-Laflamme instability

The solutions to this equations has been studied already.
In the context of Gregory-Laflamme instability.

Five-dimensional black string:

$$ds^2 = -\left(1 - \frac{r_S}{r}\right) dt^2 + \left(1 - \frac{r_S}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 + dz^2$$

Fourier decomposition around the infinite 5th flat dimension
$h^{(4)}_{\mu\nu}$ satisfy the same massive spin two equations with $m'^2 = k^2$

$$h^{(4)}_{\mu\nu} = e^{\Omega t} \begin{pmatrix} H_{tt}(r) & H_{tr}(r) & 0 & 0 \\ H_{tr}(r) & H_{rr}(r) & 0 & 0 \\ 0 & 0 & K(r) & 0 \\ 0 & 0 & 0 & K(r)\sin^2\theta \end{pmatrix}$$

Modes regular at the future horizon, not growing at infinity

Gregory-Laflamme instability

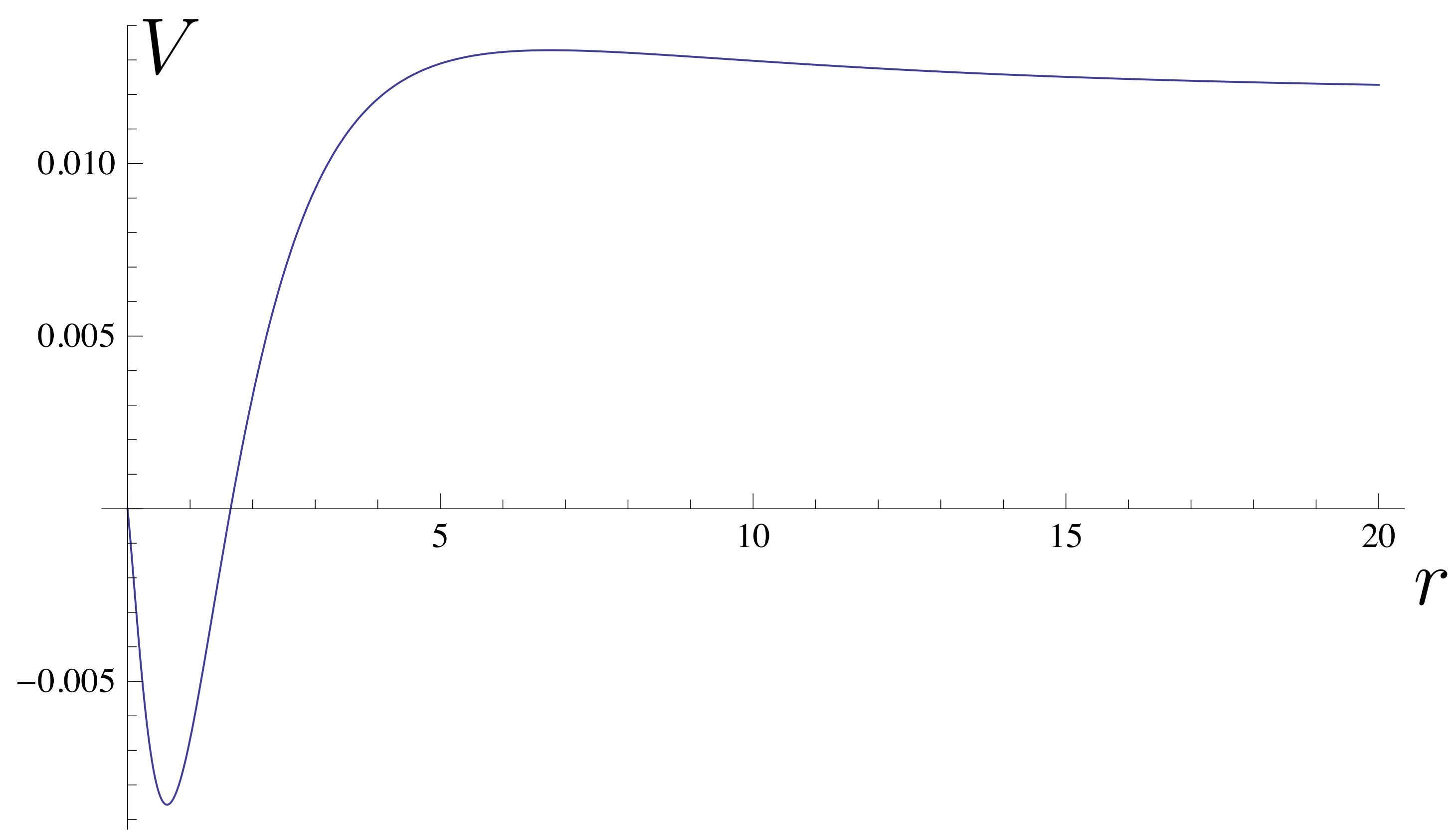
a singe second-order equation

A system of equations of second order plus 2 constraints on $H_{tt},\ H_{tr},\ H_{rr},\ K$

Playing with equations we can obtain a single equation on $\hat{H}$ (a combination of H_{tt}, H_{rr} and H_{tr})

$$\frac{d^2\hat{H}}{dr_*^2} = \left(V(r) + \Omega^2\right)\hat{H}$$

Unstable ($\Omega > 0$) mode, satisfying boundary conditions?



Instability of black holes

$$0 < m' < \frac{\mathcal{O}(1)}{r_S}$$

Instability

Confirmed independently by Brito, Cardoso, Pani arXiv:1304.6725

Instability of black holes

extended result

Proportional metrics $f_{\mu\nu} = \omega^2 g_{\mu\nu}$

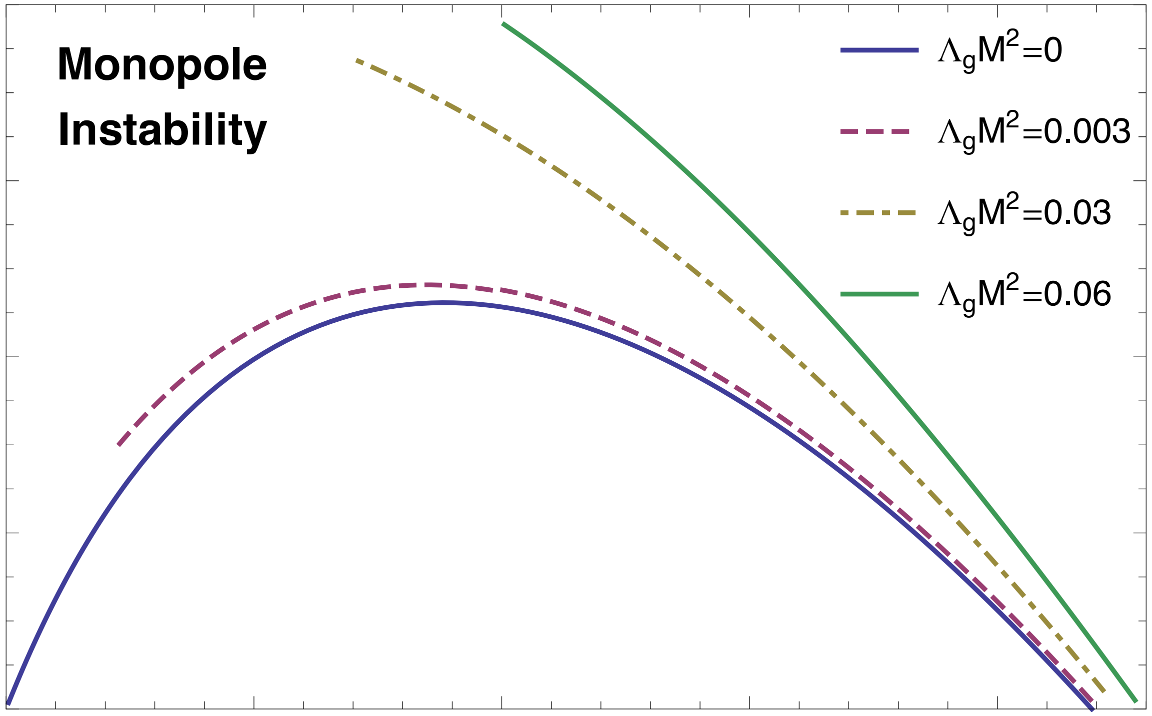
By appropriate choice of the parameters of the mass term the extended bi-Schwarzschild solution exists.

The mass of perturbations is modified by a factor depending on $\alpha_3,\ \alpha_4,\ \omega$

The result is the same: there is instability in the range $0 < \hat{m} < \frac{\mathcal{O}(1)}{r_S}$

de Sitter: instability is still there

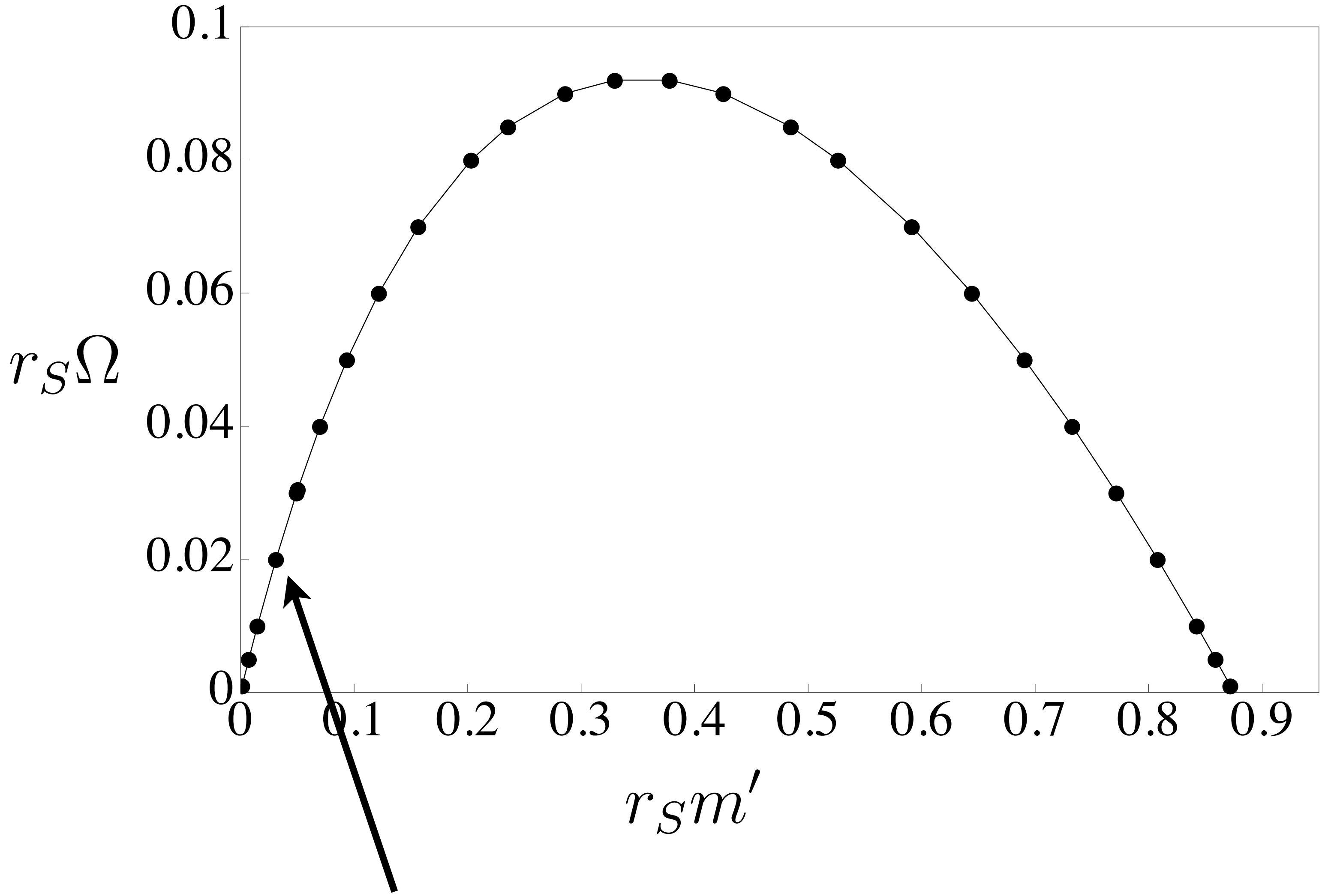
(Brito, Cardoso, Pani arXiv:1304.6725)



Instability of black holes

rate of instability

Rate of instability



Approximately linear dependance

$$r_S \ll 1/m'$$

$$\Omega = m'$$

for $m' \sim H \rightarrow \boxed{\tau \sim H^{-1}}$

Very slow instability !

Black holes in dRGT model

Schwarzschild black hole in dRGT model (one dynamical metric, the fiducial metric is Minkowski): physical singularity at the horizon for bi-diagonal solutions *(Deffayet, Jacobson'11)*

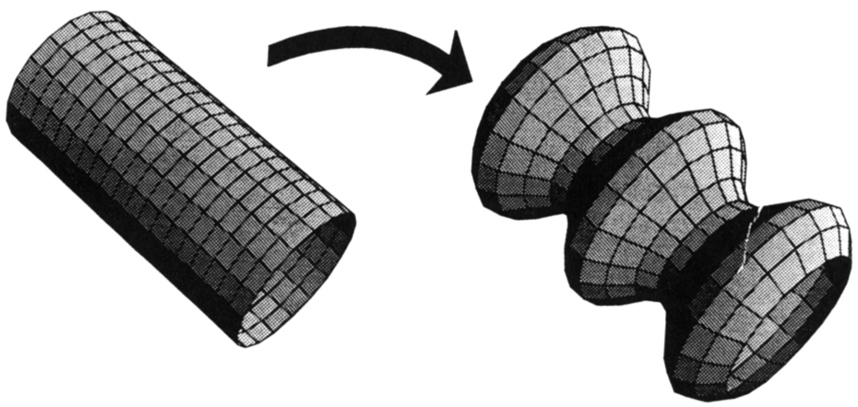
Time-dependent black hole *(Mirbabayi,Gruzinov'13)*

The black hole is "discharging" by accretion of the massive hair.
Analogous to the black hole discharge in massive electrodynamics.
The final point is Minkowski space-time.

Instability of black holes

rate of instability

What is the fate of such black holes?



5D Gregory-Laflamme instability

Black holes with massive hair *(Brito,Cardoso,Pani'13) ?*
only very massive hairy black holes

Do such black holes form during the
gravitational collapse ?

CONCLUSIONS

- The simplest black holes in bi-gravity are unstable
- The rate of instability is extremely small
- The fate of black holes? The endpoint of gravitational collapse?