Introduction to the Vainshtein mechanism

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based on arXiv:1107.1569
with C.Deffayet

OUTLINE

- Introduction and motivation
- k-mouflage
- Galileons
- Non-linear massive gravity
- de Rham-Gabadadze-Tolley massive gravity
- Other examples
- Problems

Introduction and motivations:

modifying graivity

- Modifying gravity - explain Dark energy, cosmological constant problem, to cure non-renormalizability problem, theoretical curiosity etc.

- There are many ways to modify gravity: f(R), scalar-tensor theories, Galileons, Horndeski theory, KGB, Fab-four, higher-dimensions, DGP, massive gravity...

We want to recover General Relativity at short distances

When modifying gravity, extra degrees of freedom appear, which alter gravitational interaction between bodies

} A trick to comply with both requirements is needed !

Introduction and motivations:

how to recover GR at small distances

Mechanisms to recover General Relativity:

- ➡ Chameleon (non-linear potential for a canonical extra propagating scalar) - scalar-tensor theories, f(R)
- ➡ Symmetron (coupling to matter depends of the environment)
- ➡ Vainshtein mechanism (nonlinear kinetic term effectively hides extra degree(s) of freedom) - k-essence, DGP, Galileon, Horndeski theory, massive gravity

Introduction and motivations:

Linearized massive graviton

Fierz-Pauli action *(Fierz&Pauli'39)*:

$$S_{PF} = M_P^2 \int d^4x \left[-\frac{1}{2}\hat{h}^{\mu\nu}\mathcal{E}_{\mu\nu}^{\alpha\beta}\hat{h}_{\alpha\beta} - \frac{1}{4}m^2\left(h_{\mu\nu}h^{\mu\nu} - h^2\right) + M_P^{-2}T_{\mu\nu}h^{\mu\nu} \right]$$

Linearized Einstein-Hilbert term

mass term

Minimal coupling to matter

Equations of motion:

$$-\frac{1}{2}\left(\Box - m^2\right)h_{\mu\nu} = \frac{1}{M_P^2}\left(T_{\mu\nu} - \frac{1}{3}T\eta_{\mu\nu}\right) + \frac{1}{3}\frac{\partial_\mu\partial_\nu T}{m^2 M_P^2}$$

To be compared with linearized GR:

$$-\frac{1}{2}\Box h_{\mu\nu} = \frac{1}{M_P^2}\left(T_{\mu\nu} - \frac{1}{2}T\eta_{\mu\nu}\right)$$

Introduction and motivations:

vDVZ discontinuity

Solution for a point-like source in GR:

$$h_{tt} = \frac{M}{M_P}\frac{1}{4\pi r}$$

$$h_{ij} = \frac{M}{M_P}\frac{1}{4\pi r}\delta_{ij}$$

Tested with high precision !

Solution for a point-like source in massive gravity:

$$h_{tt} = \frac{4}{3}\frac{M}{M_P}\frac{1}{4\pi r}$$

$$h_{ij} = \frac{2}{3}\frac{M}{M_P}\frac{1}{4\pi r}\delta_{ij}$$

van Dam-Veltman-Zakharov discontinuity ! *(vanDam&Veltman'70&Zakharov'70)*

How to fit observation in MG ?

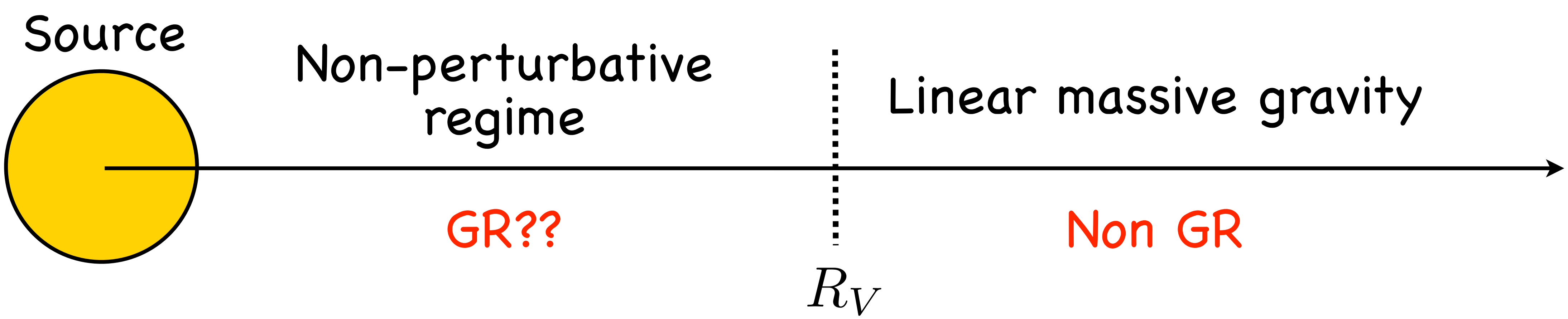
Introduction and motivations:

Vainshtein's idea

Vainshtein'72: In non-linear massive gravity GR can be restored!

The linear approximation breaks down at the Vainshtein radius $r_V = \left(\frac{r_S}{m^4}\right)^{1/5}$

Inside the Vainshtein radius GR is restored, outside -- linear MG



k-mouflage

simple way to understand the Vainshtein mechanism

<u>k-mouflage</u> *(EB,Deffayet,Ziour'09)*

For the Vainshtein mechanism it is (generically) sufficient to have a non-linear (non-canonical) kinetic term.

$$S_{k-mouflage} = M_P^2 \int d^4x \sqrt{-g} \left[R + \varphi R + m^2 K_{NL}(\varphi, \partial\varphi, \partial^2\varphi \ldots) \right] + S_m[g]$$

EH term

modification of gravity ("Brans-Dicke term")

Non-linear kinetic term

Do not expand

Expansion: $"h\, \partial^2 h"$ $"\varphi \partial^2 h"$ $"m^2 K_{NL}"$

- There are two scales in the action: m and the Planck mass
- Composite scales
- Nonlinear regimes happen for different scales

k-mouflage

simple way to understand the Vainshtein mechanism

Expanded action $S_{k-mouflage} \sim M_P^2 \int h\,\partial^2 h + \varphi\partial^2 h + m^2 K_{NL} + hT$

$$\mathcal{E}_\varphi = \frac{\delta K_{NL}}{\delta\varphi}$$

Variation wrt $h_{\mu\nu}$: $\partial^2 h + \partial^2\varphi = \frac{T}{M_P^2}$

Variation wrt φ : $\partial^2 h + m^2\mathcal{E}_\varphi = 0$

$\Rightarrow$ $\partial^2\varphi + \mathcal{E}_\varphi = \frac{T}{M_{\rm P}^2}$

Two regimes:

$\partial^2\varphi = M_{\rm P}^{-2}T$

$h \neq h_{\rm GR}$

$\partial^2\varphi \ll \mathcal{E}_\varphi = M_{\rm P}^{-2}T$

$h \approx h_{\rm GR}$

k-mouflage

simple explanation 2

Demix kinetic terms for φ and $h : h \to \hat{h} + \varphi$

$$S_{k-mouflage} \sim M_P^2 \int h\, \partial^2 h + \varphi \partial^2 \varphi + m^2 K_{NL} + hT + \varphi T$$

Variation wrt $h_{\mu\nu}$: $\partial^2 \hat{h} = T/M_P^2$

Variation wrt φ : $\partial^2 \varphi + m^2 \mathcal{E}_\varphi = T/M_P^2$

$$\partial^2 \varphi = M_\mathrm{P}^{-2} T$$

$$h \neq h_\mathrm{GR}$$

$$\partial^2 \varphi \ll \mathcal{E}_\varphi = M_\mathrm{P}^{-2} T$$

$$h \approx h_\mathrm{GR}$$

k-mouflage

action

$$S_{k-mouflage} = M_P^2 \int d^4x \sqrt{-g} \left[R + \frac{\varphi}{2M_P} R + m^2 K_{NL}(\partial\varphi, \partial^2\varphi, ...) \right] + S_m[g]$$

Expand in perturbations $h_{\mu\nu}$ around Minkowski:

- keep the Einstein-Hilbert term up to h^2,

- keep only the first order in h in the mixing term $\sim \varphi R$ (because φ/M_P will be of order of h or of higher order as we will verify later),

- do not expand K_{NL}, because the Vainshtein mechanism relies precisely on the non-linearity of this term,

- normalize the spin-2 perturbations as $h_{\mu\nu} \to \hat{h}_{\mu\nu}/M_P$.

↓

k-mouflage

expanded action

$$S_{k-m} = \int d^4x \Big\{ -\frac{1}{2}\hat{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\hat{h}_{\alpha\beta} + \hat{h}^{\mu\nu}\varphi_{,\mu\nu} - \hat{h}\Box\varphi + M_P^2 m^2 K_{NL} + T_{\mu\nu}\hat{h}^{\mu\nu} \Big\}$$

$$\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} = -\frac{1}{2}\partial_\mu\partial_\nu h - \frac{1}{2}\Box h_{\mu\nu} + \frac{1}{2}\partial_\rho\partial_\mu h^\rho_\nu + \frac{1}{2}\partial_\rho\partial_\nu h^\rho_\mu - \frac{1}{2}\eta_{\mu\nu}(\partial^\rho\partial^\sigma h_{\rho\sigma} - \Box h)$$

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redefine the spin-2 mode as $\hat{h}_{\mu\nu} = \tilde{h}_{\mu\nu} - \eta_{\mu\nu}\phi$

$$S_{k-m} = \int d^4x \Big\{ -\frac{1}{2}\tilde{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\tilde{h}_{\alpha\beta} + \frac{3}{2}\varphi\Box\varphi + M_P^2 m^2 K_{NL} + \frac{1}{M_P}\left(T_{\mu\nu}\tilde{h}^{\mu\nu} - T\varphi\right) \Big\}$$

k-mouflage

expanded action

$$S_{k-m} = \int d^4x \Big\{ -\frac{1}{2}\hat{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\hat{h}_{\alpha\beta} + \hat{h}^{\mu\nu}\varphi_{,\mu\nu} - \hat{h}\Box\varphi + M_P^2 m^2 K_{NL} + T_{\mu\nu}\hat{h}^{\mu\nu} \Big\}$$

$$\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} = -\frac{1}{2}\partial_\mu\partial_\nu h - \frac{1}{2}\Box h_{\mu\nu} + \frac{1}{2}\partial_\rho\partial_\mu h^\rho_\nu + \frac{1}{2}\partial_\rho\partial_\nu h^\rho_\mu - \frac{1}{2}\eta_{\mu\nu}(\partial^\rho\partial^\sigma h_{\rho\sigma} - \Box h)$$

redefine the spin-2 mode as $\hat{h}_{\mu\nu} = \tilde{h}_{\mu\nu} - \eta_{\mu\nu}\phi$

$$S_{k-m} = \int d^4x \Big\{ -\frac{1}{2}\tilde{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\tilde{h}_{\alpha\beta} + \frac{3}{2}\varphi\Box\varphi + M_P^2 m^2 K_{NL} + \frac{1}{M_P}\Big(T_{\mu\nu}\tilde{h}^{\mu\nu} - T\varphi\Big) \Big\}$$

- the spin-0 and spin-2 modes decouple
- a non-minimal scalar-matter coupling appears

k-mouflage

two regimes

EOMs:

$$\mathcal{E}^{\alpha\beta}_{\mu\nu}\tilde{h}_{\alpha\beta} = \frac{T_{\mu\nu}}{M_P}$$

$$3\Box\varphi + \mathcal{E}_\varphi = \frac{T}{M_P}$$

where $\mathcal{E}_\varphi \equiv M_P^2 m^2 \frac{\delta K_{NL}}{\delta\varphi}$

Linear regime

$$\left.\begin{array}{c} \mathcal{E}^{\alpha\beta}_{\mu\nu}\tilde{h}_{\alpha\beta} = T_{\mu\nu}/M_P \\ 3\Box\varphi = T/M_P \end{array}\right\} \varphi \sim \tilde{h}$$

the (normalized) physical metric
$\hat{h} \sim \tilde{h} + \tilde{\phi}$ receives corrections $\mathcal{O}(1)$

non-GR

Non-linear regime

$$\left\{\begin{array}{c} \mathcal{E}^{\alpha\beta}_{\mu\nu}\tilde{h}_{\alpha\beta} = T_{\mu\nu}/M_P \\ \mathcal{E}_\varphi = T/M_P \end{array}\right.$$

$$\partial^2\varphi \ll \mathcal{E}_\varphi = M_{\mathrm{P}}^{-1}T$$

$$h \approx h_{\mathrm{GR}}$$

GR is restored

k-mouflage

scalings

$$\mathcal{E}_\varphi \sim \partial^{n-k+3}\varphi^k/\Lambda_n^n$$

- Schematic form of kinetic self-interacting

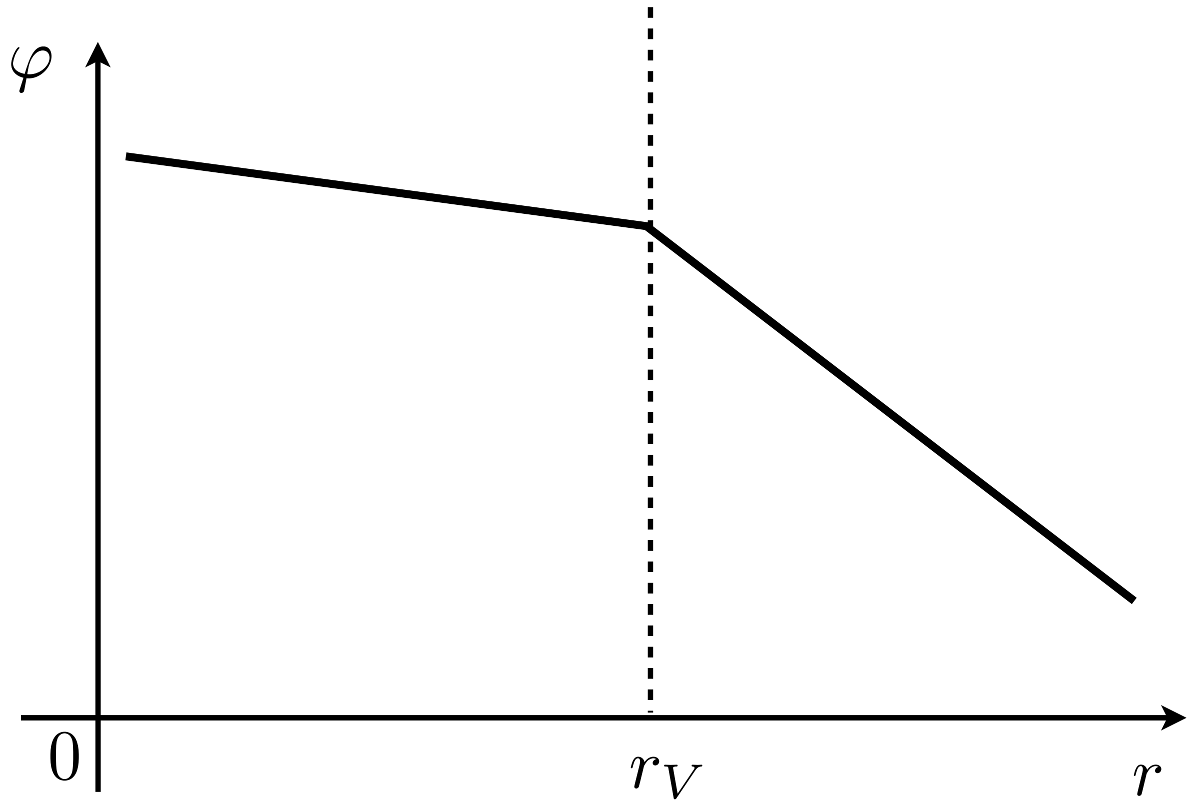
Spherical symmetry: $\partial \to 1/r$ $\quad \mathcal{E}_\varphi \sim (\Lambda_n)^{-n} r^{k-n-3}\varphi^k$ n>k

$\Lambda_n^n = M_P m^{n-1}$ - strong coupling scale

$$\varphi \sim \frac{M_P r_S}{r},\ r > r_V$$

$$\varphi \sim \frac{1}{r}\left(M_P r_S\right)^{1/k}\left(\Lambda_n r\right)^{n/k},\ r < r_V$$

$$r_V = \frac{1}{\Lambda_n}\left(M_P r_S\right)^{(k-1)/n}$$



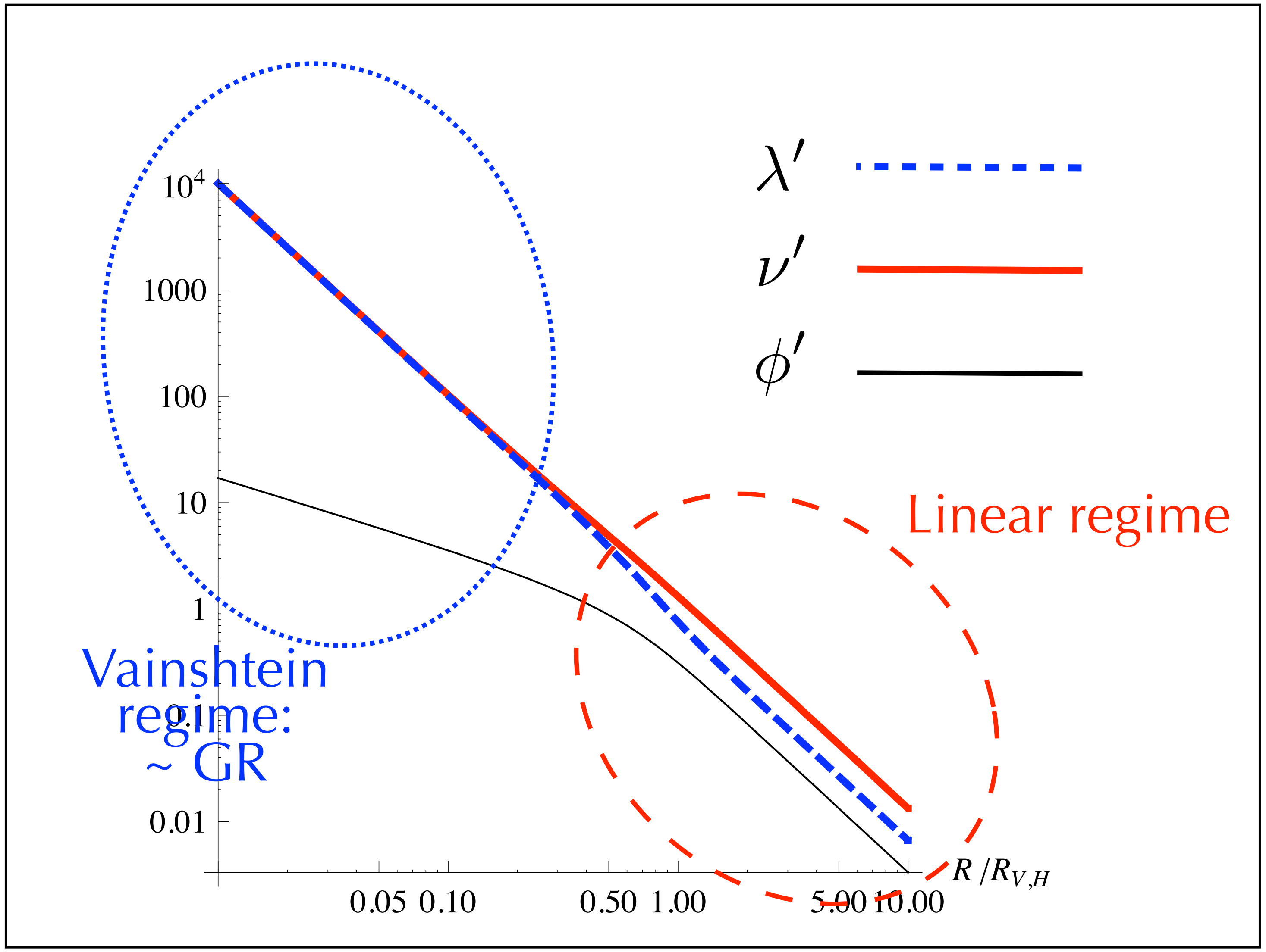
k-mouflage

k-essence

$$\mathcal{L} = \frac{1}{m^3 M_P} (\partial\varphi)^4 \quad \rightarrow \quad k = 3, n = 4, \Lambda_4 = \left(M_P m^3\right)^{1/4}$$

$$r_V = \frac{1}{\Lambda_4} \left(M_P r_S\right)^{1/2}$$

$$g_{\mu\nu} dx^\mu dx^\nu = -e^{\nu(R)} dt^2 + e^{\lambda(R)} dR^2 + R^2 d\Omega^2$$



Galileons

cubic galileon

$$S_{k-m} = \int d^4x \Big\{ -\frac{1}{2}\tilde{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\tilde{h}_{\alpha\beta} + \frac{1}{2}\varphi\Box\varphi + M_P^2 m^2 K_{NL} + \frac{1}{M_P}\left(T_{\mu\nu}\tilde{h}^{\mu\nu} - T\varphi\right) \Big\}$$

$$M_P^2 m^2 K_{NL} = \frac{1}{2m^2 M_P}(\partial\varphi)^2\Box\varphi \quad \rightarrow \quad \Lambda_3 = \left(m^2 M_P\right)^{1/3}, \; r_V = \left(\frac{r_S}{m^2}\right)^{1/3}$$

current: $$J^\mu = \partial^\mu\varphi - \frac{1}{M^2}\Box\varphi\partial^\mu\varphi + \frac{1}{2M^2}\nabla^\mu\left((\partial_\lambda\varphi)^2\right)$$

EOM: $$\nabla_\mu J^\mu = \frac{T}{M_P} \quad \rightarrow \quad \frac{1}{r^2}\partial_r\left(r^2 J^r\right) = \frac{T}{M_P}$$

$$\varphi' = \frac{m^2 M_P r}{4}\left(1 \pm \sqrt{1 + \frac{8 r_S}{m^2 r^3}}\right) = \begin{cases} \varphi' = -\frac{M_P r_S}{r^2}, & r > r_V \\ \varphi' = -\frac{M_P m}{\sqrt{2}}\left(\frac{r_S}{r}\right)^{1/2}, & r < r_V \end{cases}$$

$$|\varphi| \sim |h|, \text{ for } r > r_V$$
$$|\varphi| \ll |h|, \text{ for } r < r_V$$

Galileons

other galileons

$$S_g = \int d^4x \Big\{ \frac{1}{2}\varphi\Box\varphi + M_P^2 m^2 K_{NL} - \frac{1}{M_P} T\varphi \Big\}$$

Galileon Lagrangians *(Horndeski'74,Fairlie et al'92, Nicolis'09, Deffayet et al'09+many others)*

$$\mathcal{L}_2 = K\left(X\right), \qquad X = \frac{1}{2}(\partial\varphi)^2$$

$$\mathcal{L}_3 = G^{(3)}\left(X\right)\Box\varphi$$

$$\mathcal{L}_4 = G^{(4)}_{,X}(X)\left[(\Box\varphi)^2 - (\nabla\nabla\varphi)^2\right] + R\,G^{(4)}(X),$$

$$\mathcal{L}_5 = G^{(5)}_{,X}\left(X\right)\left[(\Box\varphi)^3 - 3\Box\varphi\left(\nabla\nabla\varphi\right)^2 + 2\left(\nabla\nabla\varphi\right)^3\right] - 6G_{\mu\nu}\nabla^\mu\nabla^\nu\varphi\,G^{(5)}\left(X\right)$$

The Vainshtein mechanism works for a generic galileon:

➡ non-covariant galileons *(Nicolis'09)*

➡ ...

➡ Horndeski model *(Koyama et al'13, Kase&Tsujikawa'13)*

Galileons

induced coupling

The Vainshtein mechanism with time dependent boundary conditions *(EB&Esposito-Fasere'12)*

$$S = 2M_P^2 \int d^4x \sqrt{-g} \left\{ \frac{R}{4} - \frac{k_2}{2}(\partial_\mu \varphi)^2 - \frac{k_3}{2M^2} \Box\varphi (\partial_\mu \varphi)^2 \right\} + S_{\rm m}\left[\psi_{\rm m}; \tilde{g}_{\mu\nu}\right]$$

$$\varphi = \phi(r) + \dot{\varphi}_c t + \varphi_0$$

- because of cosmological evolution (e.g. in KGB, *deffayet et al'10*)

Solution for a spherically symmetric source

$$\varphi' = -\frac{k_2 M^2 r}{4k_3}\left(1 \pm \sqrt{1 + \frac{4k_3 r_S}{k_2^2 M^2 r^3}\,\alpha_{\rm eff}}\right)$$

$$\alpha_{\rm eff} \equiv \alpha + \frac{k_3 \dot{\varphi}_c^2}{M^2}$$

Naturally $\varphi \sim H \sim M \Rightarrow$ the induced coupling is of the order of 1 !

Non-linear massive gravity

potential for metric

Need to construct a mass term -> introduce an extra metric

$g_{\mu\nu}$:physical metric, matter couples to it

$f_{\mu\nu}$:an extra metric (may be dynamical or fixed)

Construct a potential, following the rules:
- general covariance under diffeomorphisms (common to the two metrics)
- has flat spacetime as solution for physical metric
- when expanding around flat metric the potential takes a specific form, the Pauli-Fierz form

building block: $\mathbf{g^{-1}f}$

$$S_{int}^{(2)} \equiv -\frac{1}{8} m^2 M_P^2 \int d^4x \, \sqrt{-f} \; H_{\mu\nu} H_{\sigma\tau} \left(f^{\mu\sigma} f^{\nu\tau} - f^{\mu\nu} f^{\sigma\tau} \right)$$

(Boulware & Deser'72)

$$S_{int}^{(3)} \equiv -\frac{1}{8} m^2 M_P^2 \int d^4x \, \sqrt{-g} \; H_{\mu\nu} H_{\sigma\tau} \left(g^{\mu\sigma} g^{\nu\tau} - g^{\mu\nu} g^{\sigma\tau} \right)$$

(Arkani-Hamed et al'03)

where $H_{\mu\nu} = g_{\mu\nu} - f_{\mu\nu}$

Non-linear massive gravity

Boulware-Deser ghost

There are two propagating scalars: one is a ghost ! *(Boulware & Deser'72)*

The presence of ghosts is not connected to the Vainshtein mechanism.

Non-linear massive gravity

equations of motion

EOMs $$M_P^2 G_{\mu\nu} = \left(T_{\mu\nu} + T^g_{\mu\nu}\right)$$

$$T^g_{\mu\nu}(x) = -\frac{2}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}(x)} S_{int}(f,g)$$

ansatz

$$g_{\mu\nu}dx^\mu dx^\nu = -e^{\nu(R)}dt^2 + e^{\lambda(R)}dR^2 + R^2 d\Omega^2 \ ,$$

$$f_{\mu\nu}dx^\mu dx^\nu = -dt^2 + \left(1 - \frac{R\mu'(R)}{2}\right)^2 e^{-\mu(R)}dR^2 + e^{-\mu(R)}R^2 d\Omega^2$$

⇓

$$T^g_{tt} = m^2 M_P^2\, f_t, \quad T^g_{RR} = m^2 M_P^2\, f_R, \quad \nabla^\mu T^g_{\mu R} = -m^2 M_P^2 f_g,$$

Non-linear massive gravity

equations of motion

EOMs $$M_P^2 G_{\mu\nu} = \left(T_{\mu\nu} + T^g_{\mu\nu}\right)$$

$$T^g_{\mu\nu}(x) = -\frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}(x)} S_{int}(f,g)$$

$$f_t = \frac{e^{-\lambda-2\mu}}{4}$$
$$\times \left[\left(3e^{\mu+\nu} + e^{\mu} - 2e^{\nu}\right)\left(1 - \frac{R\mu'}{2}\right)^2 + e^{\lambda}\left(2e^{\mu} - e^{\nu}\right) - 3e^{\lambda+\mu}\left(2e^{\mu+\nu} + e^{\mu} - 2e^{\nu}\right)\right],$$

$$f_R = \frac{e^{-\nu-2\mu}}{4}$$
$$\times \left[\left(3e^{\mu+\nu} - e^{\mu} - 2e \ldots \left(-2e^{\mu+\nu} + e^{\mu} + 2e^{\nu}\right)\right],$$

NUMERICS ?
SIMPLIFY ?

$$f_g = -\left(1 - \frac{R\mu'}{2}\right)\frac{e^{-\lambda-2\mu-\nu}}{8R}$$
$$\times \left[8\left(e^{\lambda} - 1\right)\left(3e^{\mu+\nu} - e^{\mu} - e^{\nu}\right) + 2R\left(\left(3e^{\mu+\nu} - 2e^{\nu}\right)\left(\lambda' + 4\mu' - \nu'\right) - e^{\mu}\left(\lambda' + 4\mu' + \nu'\right)\right)\right.$$
$$\left. -R^2\left(\left(3e^{\mu+\nu} - 2e^{\nu}\right)\left(\lambda'\mu' - 2\mu'' - \mu'\nu' + (\mu')^2\right) - e^{\mu}\left(\lambda'\mu' - 2\mu'' + \mu'\nu' + (\mu')^2\right) - 2e^{\nu}(\mu')^2\right)\right]$$

Non-linear massive gravity

Stuckelberg approach

Stuckelberg approach *(Arkani-Hamed et al'03)*

$$X^A \to x^\mu$$

$$f_{AB}(X) \to f_{\mu\nu}(x) = \partial_\mu X^A(x) \partial_\nu X^B(x) f_{AB}(X(x))$$

consider X^A as 4 dynamical fields

Unitary gauge: $f_{AB} = \eta_{AB} = diag(-1,1,1,1)$

in non-unitary gauge $g_{\mu\nu}$ and X^A

expansion around unitary gauge: $X_0^A(x) \equiv \delta_\mu^A x^\mu$

"pion" fields, $X^A(x) = X_0^A(x) + \pi^A(x)$

$$\pi^A(x) = \delta_\mu^A \left(A^\mu(x) + \eta^{\mu\nu} \partial_\nu \phi \right).$$

Non-linear massive gravity

Stuckelberg approach

$$H_{\mu\nu} = h_{\mu\nu} - \partial_\mu A_\nu - \partial_\nu A_\mu - 2\partial_\mu\partial_\nu\phi - \partial_\mu A_\sigma \partial_\nu A^\sigma$$
$$- \partial_\mu\partial_\sigma\phi\, \partial_\nu\partial^\sigma\phi - \partial_\nu A^\sigma \partial_\mu\partial_\sigma\phi - \partial_\mu A^\sigma \partial_\nu\partial_\sigma\phi$$

$$\hat{h}_{\mu\nu} = M_P h_{\mu\nu},\ \tilde{A}^\mu = M_P m A^\mu,\ \tilde{\phi} = M_P m^2 \phi$$

$$\hat{h}_{\mu\nu} = \tilde{h}_{\mu\nu} - \eta_{\mu\nu}\tilde{\phi}$$

$$S = \frac{1}{8}\int d^4x \Big\{ 2\tilde{h}^{\mu\nu}\partial_\mu\partial_\nu\tilde{h} - 2\tilde{h}^{\mu\nu}\partial_\nu\partial_\sigma\tilde{h}^\sigma_\mu + \tilde{h}^{\mu\nu}\Box\tilde{h}_{\mu\nu} - \tilde{h}\Box\tilde{h}$$
$$+ m^2\left(\tilde{h}^2 - \tilde{h}_{\mu\nu}\tilde{h}^{\mu\nu}\right) - \tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - 4m\left(\tilde{h}\partial\tilde{A} - \tilde{h}_{\mu\nu}\partial^\mu\tilde{A}^\nu\right)$$
$$+ 6\tilde{\phi}(\Box + 2m^2)\tilde{\phi} - m^2\tilde{h}\phi + 2m\tilde{\phi}\partial\tilde{A}\Big\} + \frac{1}{2}\frac{T_{\mu\nu}}{M_P}\tilde{h}^{\mu\nu} - \frac{1}{2}\frac{T}{M_P}\tilde{\phi}$$

Expand the action in $\tilde{\phi}$, $\tilde{A}$ and $\tilde{h}_{\mu\nu}$ to next orders, cubic self-interactions suppressed by Λ_5, the strongest interactions:

$$\Lambda_5 = \left(m^4 M_P\right)^{1/5}$$

Non-linear massive gravity

action and EOM in decoupling limit

Vainshtein mechanism in decoupling limit of massive gravity (*EB,Deffayet,Ziour'09*)

Decoupling limit

$$M_P \to \infty,\ m \to 0,\ \Lambda_5 \sim \text{constant},\ T_{\mu\nu}/M_P \sim \text{constant}$$

$$S = \frac{1}{2}\int d^4x\, \left\{\frac{3}{2}\tilde{\phi}\Box\tilde{\phi} + \frac{1}{\Lambda_5^5}\left[\alpha\, (\Box\tilde{\phi})^3 + \beta\, (\Box\tilde{\phi}\ \tilde{\phi}_{,\mu\nu}\ \tilde{\phi}^{,\mu\nu})\right] - \frac{1}{M_P}T\tilde{\phi}\right\}$$

k-mouflage: $$S_{km} = \int d^4x\left\{\frac{1}{2}\varphi\Box\varphi + M_P^2 m^2 K_{NL} - \frac{1}{M_P}T\varphi\right\}$$

$$3\Box\tilde{\phi} + \frac{1}{\Lambda^5}\left[3\alpha\,\Box\left(\Box\tilde{\phi}\right)^2 + \beta\,\Box\left(\tilde{\phi}_{,\mu\nu}\ \tilde{\phi}^{,\mu\nu}\right) + 2\beta\ \partial_\mu\partial_\nu\left(\Box\tilde{\phi}\ \tilde{\phi}^{,\mu\nu}\right)\right] = \frac{1}{M_P}T$$

Non-linear massive gravity

EOM - spherical symmetry

$$\frac{2}{\Lambda_5^5} Q(\tilde{\mu}) + \frac{3}{2}\,\tilde{\mu} = \frac{M_P r_S}{r^3}, \qquad \tilde{\mu} = -\frac{2}{r}\tilde{\phi}'$$

Q is a non-linear function

$$Q(\mu) = -\frac{1}{2R}\left\{3\alpha\left(6\mu\mu' + 2R\mu'^2 + \frac{3}{2}R\mu\mu'' + \frac{1}{2}R^2\mu'\mu''\right)\right.$$
$$\left. +\beta\left(10\mu\mu' + 5R\mu'^2 + \frac{5}{2}R\mu\mu'' + \frac{3}{2}R^2\mu'\mu''\right)\right\}$$

$$S^{(2)}_{int} \equiv -\frac{1}{8}m^2 M_P^2 \int d^4x\,\sqrt{-f}\;H_{\mu\nu}H_{\sigma\tau}\left(f^{\mu\sigma}f^{\nu\tau} - f^{\mu\nu}f^{\sigma\tau}\right)$$

$$S^{(3)}_{int} \equiv -\frac{1}{8}m^2 M_P^2 \int d^4x\,\sqrt{-g}\;H_{\mu\nu}H_{\sigma\tau}\left(g^{\mu\sigma}g^{\nu\tau} - g^{\mu\nu}g^{\sigma\tau}\right)$$

$$Q(\tilde{\mu}) = \frac{\tilde{\mu}'^2}{4} + \frac{\tilde{\mu}\tilde{\mu}''}{2} + \frac{2\tilde{\mu}\tilde{\mu}'}{r}$$

$$Q(\tilde{\mu}) = -\frac{\tilde{\mu}'^2}{4} - \frac{\tilde{\mu}\tilde{\mu}''}{2} - \frac{2\tilde{\mu}\tilde{\mu}'}{r}$$

Non-linear massive gravity

solutions in decoupling limit

$$\frac{2}{\Lambda_5^5} Q(\tilde{\mu}) + \frac{3}{2}\,\tilde{\mu} = \frac{M_P r_S}{r^3}, \quad Q(\tilde{\mu}) = -\frac{\tilde{\mu}'^2}{4} - \frac{\tilde{\mu}\tilde{\mu}''}{2} - \frac{2\tilde{\mu}\tilde{\mu}'}{r}$$

Newtonian gauge:

$$ds^2 = -(1 + \Psi/M_P)dt^2 + (1 - \Phi/M_P)(dr^2 + r^2 d\Omega^2)$$

$$\Psi = -\frac{4M_P}{3}\frac{r_S}{r}\left[1 + \mathcal{O}\left(\frac{r_V}{r}\right)^5\right],$$

$$\Phi = -\frac{2M_P}{3}\frac{r_S}{r}\left[1 + \mathcal{O}\left(\frac{r_V}{r}\right)^5\right],$$

$$\tilde{\phi} = \frac{M_P}{3}\frac{r_S}{r}\left[1 + \mathcal{O}\left(\frac{r_V}{r}\right)^5\right],$$

$$\Psi = -\frac{M_P r_S}{r} + \frac{M_P r_S}{r} \times \mathcal{O}\left(\frac{r}{r_V}\right)^{5/2}$$

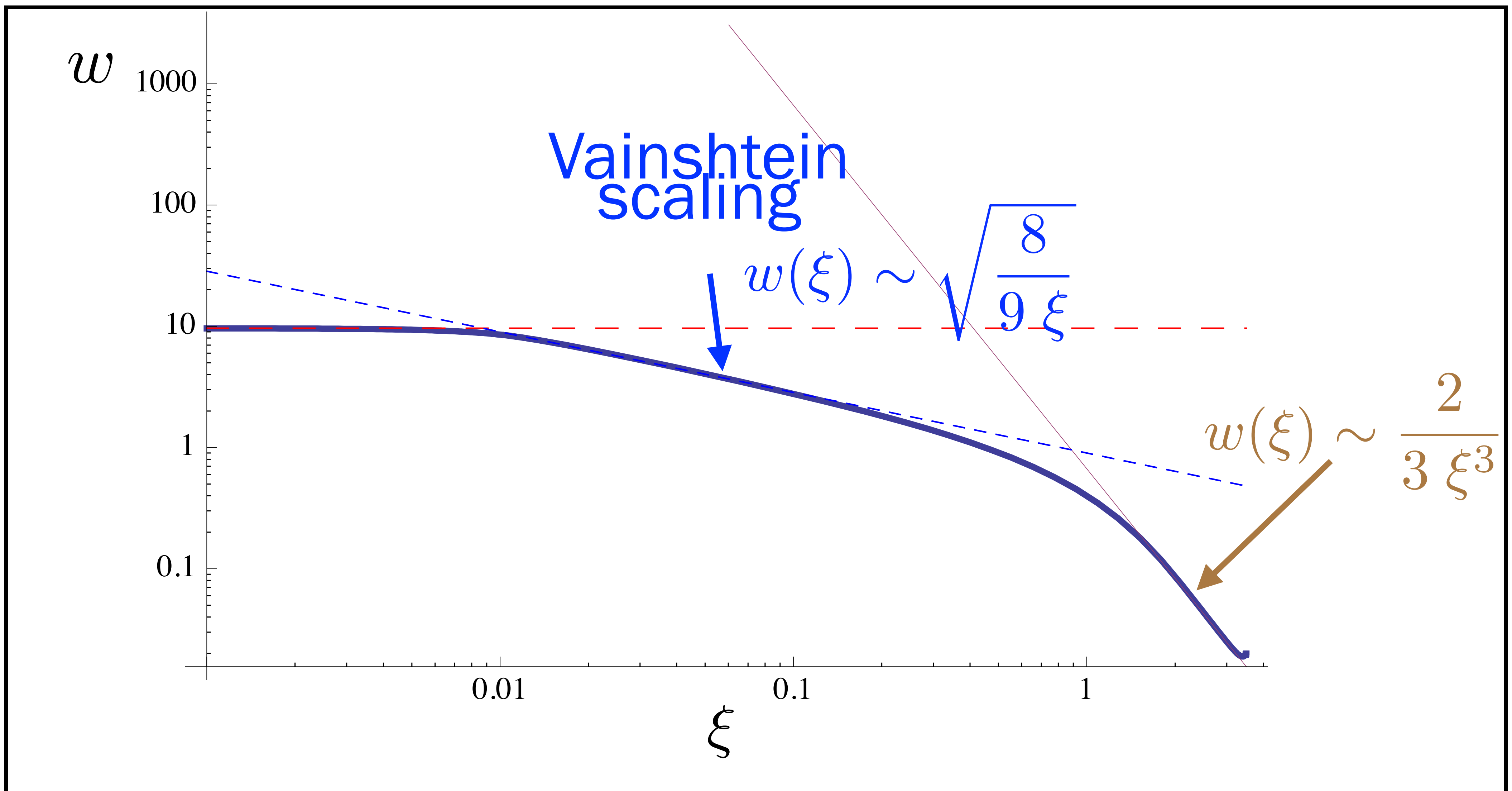
$$\Phi = \Psi$$

$$\tilde{\phi} = -\frac{2\sqrt{2}M_P}{9}\frac{r_S}{r}\left(\frac{r}{r_V}\right)^{5/2}$$

Non-linear massive gravity

solutions in decoupling limit

$$\frac{2}{\Lambda_5^5} Q(\tilde{\mu}) + \frac{3}{2}\,\tilde{\mu} = \frac{M_P r_S}{r^3}, \quad Q(\tilde{\mu}) = -\frac{\tilde{\mu}'^2}{4} - \frac{\tilde{\mu}\tilde{\mu}''}{2} - \frac{2\tilde{\mu}\tilde{\mu}'}{r}$$



dRGT

the model

Massive gravity without Boulware-Deser ghost *(de Rham-Gabadabze-Tolley'10)*

$$\mathcal{K} = \mathbb{I} - \sqrt{\mathbf{g^{-1}f}}.$$

$$S = M_P^2 \int d^4x \sqrt{-g} \left[R + 2m^2 \left(e_2(\mathcal{K}) + \alpha_3 e_3(\mathcal{K}) + \alpha_4 e_4(\mathcal{K}) \right) \right]$$

$$e_2(\mathcal{K}) = \frac{1}{2}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)$$

$$e_3(\mathcal{K}) = \frac{1}{6}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right)$$

$$e_4(\mathcal{K}) = \frac{1}{24}\left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]\right)$$

dRGT

decoupling limit

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{\delta^\mu_\nu - g^{\mu\alpha} H_{\alpha\nu}}$$

Expand action in powers of $\hat{H} \equiv H^\mu_\nu = (g^{-1}H)$

$$H_{\mu\nu} = h_{\mu\nu} - \partial_\mu A_\nu - \partial_\nu A_\mu - 2\partial_\mu\partial_\nu\phi - \partial_\mu A_\sigma \partial_\nu A^\sigma - \partial_\mu\partial_\sigma\phi\,\partial_\nu\partial^\sigma\phi - \partial_\nu A^\sigma \partial_\mu\partial_\sigma\phi - \partial_\mu A^\sigma \partial_\nu\partial_\sigma\phi$$

$$\hat{h}_{\mu\nu} = M_P h_{\mu\nu},\ \tilde{A}^\mu = M_P m A^\mu,\ \tilde{\phi} = M_P m^2 \phi$$

$$\hat{h}_{\mu\nu} = \tilde{h}_{\mu\nu} - \eta_{\mu\nu}\tilde{\phi}$$

The same procedure as in case of NLMG

$\sim (\partial\tilde{\phi})^3/(M_P m^4)$ cancel out

$$\Lambda_3 = \left(m^2 M_P\right)^{1/3}$$

Decoupling limit :

$$M_P \to \infty,\ m \to 0,\ \Lambda_3 \sim \text{const},\ T_{\mu\nu}/M_P \sim \text{const}$$

dRGT

decoupling limit

$$S = \int d^4x \Big\{ -\frac{1}{2}\hat{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\hat{h}_{\alpha\beta} + \hat{h}^{\mu\nu}X^{(1)}_{\mu\nu} + \frac{\tilde{\alpha}}{\Lambda_3^3}\hat{h}^{\mu\nu}X^{(2)}_{\mu\nu} + \frac{\tilde{\beta}}{\Lambda_3^6}\hat{h}^{\mu\nu}X^{(3)}_{\mu\nu} + T_{\mu\nu}\hat{h}^{\mu\nu} \Big\}$$

$$X^{(1)}_{\mu\nu} = \frac{1}{2}\epsilon_\mu{}^{\alpha\rho\sigma}\epsilon_\nu{}^\beta{}_{\rho\sigma}\Phi_{\alpha\beta},$$

$$X^{(2)}_{\mu\nu} = -\frac{1}{2}\epsilon_\mu{}^{\alpha\rho\gamma}\epsilon_\nu{}^{\beta\sigma}{}_\gamma\Phi_{\alpha\beta}\Phi_{\rho\sigma},$$

$$X^{(3)}_{\mu\nu} = \frac{1}{6}\epsilon_\mu{}^{\alpha\rho\gamma}\epsilon_\nu{}^{\beta\sigma\delta}\Phi_{\alpha\beta}\Phi_{\rho\sigma}\Phi_{\gamma\delta}$$

$$\Phi_{\alpha\beta} \equiv \varphi_{,\alpha\beta}$$

$$S_{k-m} = \int d^4x \Big\{ -\frac{1}{2}\hat{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\hat{h}_{\alpha\beta} + \hat{h}^{\mu\nu}\varphi_{,\mu\nu} - \hat{h}\Box\varphi + M_P^2 m^2 K_{NL} + T_{\mu\nu}\hat{h}^{\mu\nu} \Big\}$$

dRGT

decoupling limit

Nonlinear field redefinition:

$$\hat{h}_{\mu\nu} = \tilde{h}_{\mu\nu} - \eta_{\mu\nu}\tilde{\phi} - \tilde{\alpha}\frac{\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi}}{\Lambda_3^3},$$

$$S = \int d^4x \Big\{ -\frac{1}{2}\tilde{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\tilde{h}_{\alpha\beta} + \frac{3}{2}\tilde{\phi}\Box\tilde{\phi} - \frac{\tilde{\alpha}}{\Lambda_3^3}\tilde{\phi}^{,\mu}\tilde{\phi}^{,\nu}X^{(1)}_{\mu\nu} - \frac{1}{\Lambda_3^6}\left(\frac{\tilde{\alpha}^2}{2} + \frac{\tilde{\beta}}{3}\right)\tilde{\phi}^{,\mu}\tilde{\phi}^{,\nu}X^{(2)}_{\mu\nu} + \frac{\tilde{\beta}}{\Lambda_3^6}\left(h^{\mu\nu} - \frac{\tilde{\alpha}}{\Lambda_3^3}\tilde{\phi}^{,\mu}\tilde{\phi}^{,\nu}\right)X^{(3)}_{\mu\nu} + \frac{1}{M_P}\left(T_{\mu\nu}\tilde{h}^{\mu\nu} - T\tilde{\phi} - \frac{\tilde{\alpha}}{\Lambda_3^3}T_{\mu\nu}\partial^\mu\tilde{\phi}\partial^\nu\tilde{\phi}\right)\Big\}.$$

Additional constraint because of stability *(Berezhiani et al'13)*

decoupling limit - special case

Vainshtein mechanism for $\tilde{\beta} = 0$ *(Koyama et al'11)*

$$\frac{3}{2}\tilde{\mu} + \frac{3\tilde{\alpha}}{2\Lambda_3^3}\tilde{\mu}^2 + \frac{\tilde{\alpha}^2}{4\Lambda_3^6}\tilde{\mu}^3 = \frac{M_P r_S}{r^3}, \quad \tilde{\mu} = -\frac{2}{r}\tilde{\phi}'$$

$$r_V = \left(r_S/m^2\right)^{1/3}$$

Solution inside the Vainshtein radius

$$\Psi = -\frac{M_P r_S}{r} + \frac{M_P r_S}{r} \times \mathcal{O}\left(\frac{r}{r_V}\right)^2, \Phi = -\frac{M_P r_S}{r} + \frac{M_P r_S}{r} \times \mathcal{O}\left(\frac{r}{r_V}\right),$$

$$\tilde{\phi} = -\frac{M_P r_S}{(2\tilde{\alpha}^2)^{1/3} r}\left(\frac{r}{r_V}\right)^2$$

dRGT

decoupling limit - general case

Vainshtein mechanism for $\tilde{\beta} \neq 0$ *(Chkareuli et al'11)*

EOMs:

$$\mathcal{E}^{\alpha\beta}_{\mu\nu}\tilde{h}_{\alpha\beta} - \frac{\tilde{\beta}}{\Lambda_3^6}X^{(3)}_{\mu\nu} = \frac{T_{\mu\nu}}{M_P},$$

$$3\Box\tilde{\phi} + \frac{3\tilde{\alpha}}{\Lambda_3^3}\Phi^{\mu\nu}X^{(1)}_{\mu\nu} + \frac{4}{\Lambda_3^6}\left(\frac{\tilde{\alpha}^2}{2} + \frac{\tilde{\beta}}{3}\right)\Phi^{\mu\nu}X^{(2)}_{\mu\nu} + \frac{5\alpha\beta}{\Lambda_3^3}\Phi^{\mu\nu}X^{(3)}_{\mu\nu}$$

$$+ \frac{\tilde{\beta}}{2\Lambda_3^6}\partial_\alpha\partial_\beta\tilde{h}^{\mu\nu}\epsilon_\mu{}^{\alpha\rho\gamma}\epsilon_\nu{}^{\beta\sigma\delta}\Phi_{\rho\sigma}\Phi_{\gamma\delta} = \frac{T}{M_P} - \frac{2\tilde{\alpha}}{M_P\Lambda_3^3}\Phi^{\mu\nu}T_{\mu\nu}$$

dRGT

decoupling limit - general case

Vainshtein mechanism for $\tilde{\beta} \neq 0$ *(Chkareuli et al'11)*

EOMs:

$$\mathcal{E}^{\alpha\beta}_{\mu\nu}\tilde{h}_{\alpha\beta} - \frac{\tilde{\beta}}{\Lambda_3^6}X^{(3)}_{\mu\nu} = \frac{T_{\mu\nu}}{M_P},$$

$$3\Box\tilde{\phi} + \frac{3\tilde{\alpha}}{\Lambda_3^3}\Phi^{\mu\nu}X^{(1)}_{\mu\nu} + \frac{4}{\Lambda_3^6}\left(\frac{\tilde{\alpha}^2}{2} + \frac{\tilde{\beta}}{3}\right)\Phi^{\mu\nu}X^{(2)}_{\mu\nu} + \frac{5\alpha\beta}{\Lambda_3^3}\Phi^{\mu\nu}X^{(3)}_{\mu\nu}$$

$$+ \frac{\tilde{\beta}}{2\Lambda_3^6}\partial_\alpha\partial_\beta\tilde{h}^{\mu\nu}\epsilon_\mu{}^{\alpha\rho\gamma}\epsilon_\nu{}^{\beta\sigma\delta}\Phi_{\rho\sigma}\Phi_{\gamma\delta} = \frac{T}{M_P} - \frac{2\tilde{\alpha}}{M_P\Lambda_3^3}\Phi^{\mu\nu}T_{\mu\nu}$$

Solution inside the Vainshtein radius

$$\Psi = \Phi = -\frac{M_P r_S}{r} + \frac{M_P r_S}{r}\times\mathcal{O}\left(\frac{r}{r_V}\right)^3, \quad \tilde{\phi} = -\frac{M_P m^2}{\sqrt{\tilde{\beta}}}r^2$$

$$\frac{3}{2}\tilde{\mu} + \frac{3\tilde{\alpha}}{2\Lambda_3^3}\tilde{\mu}^2 + \left(\frac{\tilde{\alpha}^2}{2} + \frac{\tilde{\beta}}{3}\right)\frac{\tilde{\mu}^3}{2\Lambda_3^3} - \frac{\tilde{\beta}^2\tilde{\mu}^5}{96\Lambda_3^3} = \frac{M_P r_S}{r^3}\left(1 - \frac{\tilde{\beta}\mu^2}{4\Lambda_3^3}\right)$$

Vainshtein mechanism in bi-gravity

the model

Action for bi-gravity *(Hassan&Rosen'11)*

$$S = M_P^2 \int d^4x \sqrt{-g} \left(\frac{R[g]}{2} + m^2 \mathcal{U}[g,f] \right) + S_m[g] + \frac{\kappa M_P^2}{2} \int d^4x \sqrt{-f} \mathcal{R}[f].$$

Decoupling limit ?

Weak-field approximation *(EB,Deffayet,Ziour'10)*

Vainshtein mechanism in bi-gravity *(EB,Crisostomi'13)*

$$ds^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2 d\Omega^2,$$

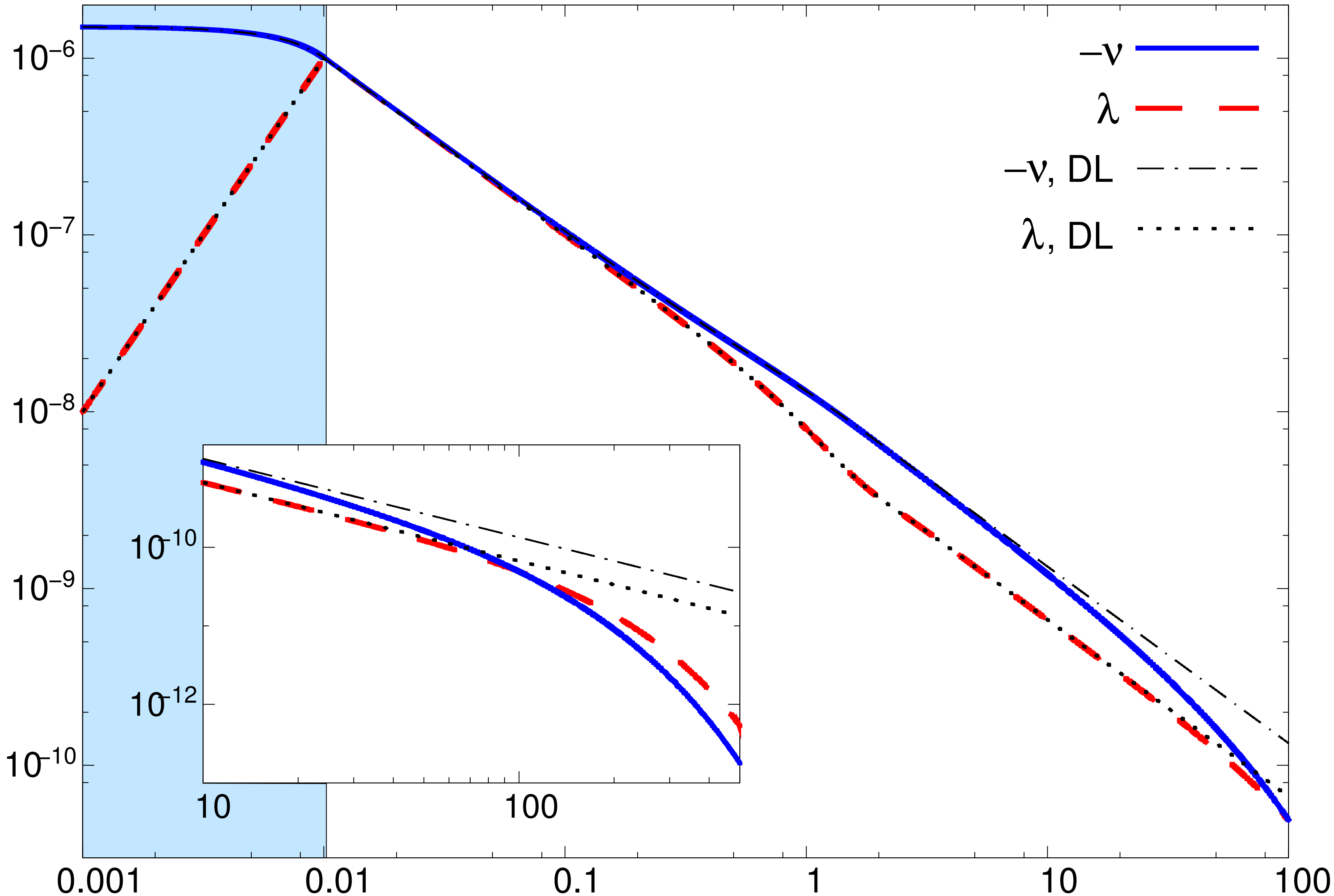
$$df^2 = -e^n dt^2 + e^l \left(r + r\mu\right)'^2 dr^2 + (r + r\mu)^2 d\Omega^2$$

$$\{\lambda,\, \nu,\, l,\, n\} \ll 1, \quad \{r\,\lambda',\, r\,\nu',\, r\,l',\, r\,n'\} \ll 1$$

see talk by Marco Crisostomi

Vainshtein mechanism in full NLMG

Numerical solution for the full system of equations *(EB,Deffayet,Ziour'09)*



Vainshtein mechanism in full dRGT

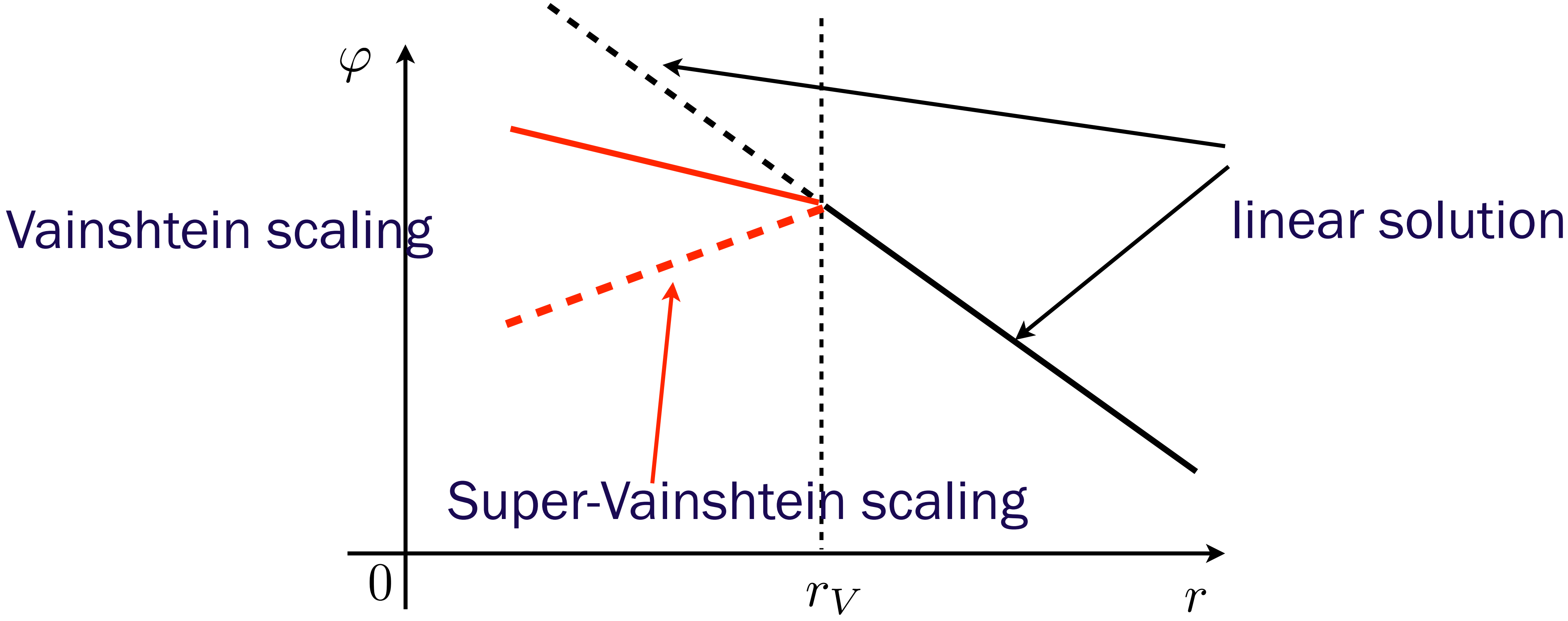
Numerical solution for the full system of equations *(Volkov'12,Gruzinov& Mirbabayi'12)*

Also in bi-gravity *(Volkov'12)*

The Vainshtein mechanism does work !

Super-Vainshtein mechanism and MOND

Improving MOND *(EB,Deffayet,Esposito-Farese'11)*



CONCLUSIONS

- The Vainshtein mechanism as k-mouflage
  - k-essence
  - galileons
- The Vainshtein mechanism in the decoupling limit of the non-linear massive gravity
- The Vainshtein mechanism in the decoupling limit of the dRGT theory
- Other examples: bi-gravity, full NLMG and full dRGT
- Stability of solutions (ghosts, Laplace instabilities?)
- Strong coupling and problems on quantum level ?
- Superluminality
- Very compact stars (no solutions for far) ?