3D non-relativistic supergravity

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Motivation

- *Fermions* are ubiquitous in description of the 'real' world; conceptually they are nicely incorporated via *supersymmetry*
- Most physics (for all practical purposes) is best described *non-relativistically*, e.g. *Newtonian gravity*

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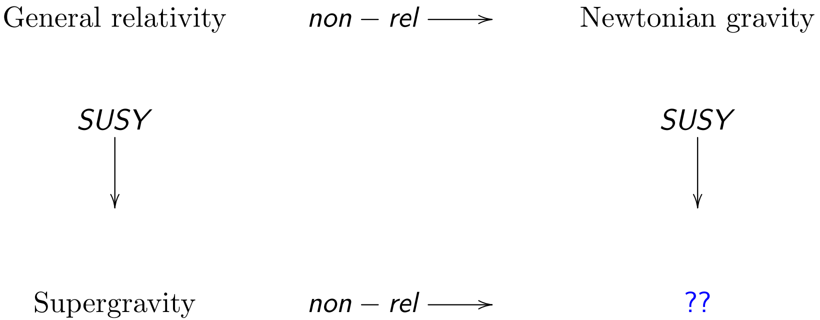
A simplification:

- to make our task easier we start with lower-dimensional 3D toy models

Diagrammatic motivation



Outline

- Motivation
- Our setup:
  - Gauge the super-Bargmann algebra
  - Impose contraints to obtain

    Newton–Cartan super-gravity [Andringa,Bergshoeff,Rosseel,Sezgin '13]
  - Fix Stückelberg symmetries to obtain supergravity for the Galilean observer
- Results obtained so far
- Future directions

The setup

- Gauge the $\mathcal{N} = 2$ super-Bargmann algebra.
- Impose contraints, e.g. to convert time- and space-translations into general coordinate transformations
- At this point we have *Newton–Cartan supergravity*
- The constraints turn some fields into pure Stückelberg fields that can be gauged away
- Doing so will lead to supergravity for the Galilean observer, thus to a supersymmetric version of Newtonian gravity

Gauging the $\mathcal{N} = 2$ super-Bargmann algebra

- Bargmann algebra in 3D:

$$\Big(H, P_a, G_a, J_{ab}, Z, Q_\alpha^+, Q_\alpha^-\Big)$$

$$\text{Why } \mathcal{N} = 2: \qquad \{Q_\alpha^+, Q_\beta^+\} = 2\delta_{\alpha\beta} H\,,$$
$$\{Q_\alpha^+, Q_\beta^-\} = -[\gamma^{a0}]_{\alpha\beta} P_a\,.$$

- Gauged algebra: for each generator (e.g. Z) we have associated
  – gauge fields (m_μ),
  – gauge parameters ($\delta m_\mu = \partial_\mu \sigma + \lambda^a e_{\mu a}$+fermionic) and
  – curvatures ($\hat{R}_{\mu\nu}(Z) = 2\partial_{[\mu} m_{\nu]} - 2\omega_{[\mu}{}^a e_{\nu]a}$+fermionic).

From Bargmann to Newton–Cartan to Galileo

- Gauged algebra: gauge fields, gauge parameters and curvatures.
- Impose constraints: $\hat{R}_{\mu\nu}{}^{a}(P) = 0$, $\hat{R}_{\mu\nu}{}^{ab}(J) = 0$, $\hat{R}_{\mu\nu}(H) = 0$, $\hat{R}_{\mu\nu}(Z) = 0$ and $\hat{\psi}_{\mu\nu+} = 0$.
- This allows us to convert H- and P^a-transformations into general coordinate transformations.
- Setting some of the gauge fields to zero, while fixing constant values for others we can remove all residual gauge freedom.
- The only fields left are the Newton force $\Phi_i = \partial_i \Phi = \partial_i m_\emptyset$ and its supersymmetric partner $\Psi = \psi_{\emptyset -}$.
- It is not possible to write down a closed algebra in terms of Φ and Ψ!

Results obtained so far

- We know the supersymmetric partner of the Newton force Φ_i.
- We found a representation of the $\mathcal{N} = 2$ super-Bargmann algebra realized on the "Newton potentials" Φ_i and Ψ:

$$\delta\Phi_i = \ldots - \lambda^m{}_n x^n \partial_m \Phi_i + \bar{\epsilon}_-(t)\gamma^0\partial_i\Psi + \frac{1}{2}\bar{\epsilon}_+\gamma_i\Psi$$

$$\delta\Psi = \ldots - \lambda^i{}_j x^j \partial_i \Psi + \frac{1}{4}\lambda^{ab}\gamma_{ab}\Psi + \dot{\epsilon}(t) - \frac{1}{2}\Phi^k\gamma_{k0}\epsilon_+$$

- We can write the algebra in terms of Φ thereby introducing the dual potentials χ and Ξ: $\Psi = (1/2)\gamma^i\partial_i\chi$ and $\partial_i\Phi = \varepsilon_{ij}\partial^j\Xi$.
- Ξ is a bosonic dual to Φ that is needed to close the algebra.

Future directions

- Repeat the analysis in 4D, i.e. find the supersymmetric partner(s) of Φ at the level of the algebra.
- Find other 'multiplets' in 3D.
- Find an action for a (3D) superparticle:

$$I = \frac{m}{2} \int \mathrm{d}t \, \{\dot{x}^i \dot{x}^i + 2\Phi + \text{SUSY extension}\}$$

Summary

$$\begin{array}{ccc} \text{GR} & \mathit{non-rel} \longrightarrow & \text{Newtonian gravity}, \Phi \\ \mathit{SUSY} \downarrow & & \mathit{SUSY} \downarrow \\ \text{SUGRA} & \mathit{non-rel} \longrightarrow & \Phi, \chi, \Xi \quad (\text{algebra}) \ \text{particle?} \end{array}$$