Introduction to Chern-Simons forms in Physics - II

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Lecture I:

1. Topological invariants
2. Minimal coupling
3. Chern-Simons
4. Brane couplings
5. Quantization
6. Summary

Lecture II:

1. Spacetime geometry
2. Metric and affine structures: e^a , $\omega^{a}b$
3. Building blocks
4. Gravity actions: L-L, T, CS
5. CS gravities, BHs, SUGRAs
6. Summary

The Euler characteristic belongs to a family of famous invariants:

- Sum of exterior angles of a polygon
- Residue theorem in complex analysis
- Winding number of a map
- Poincaré-Hopff theorem ("*one cannot comb a sphere*")
- Atiyah-Singer index theorem
- Witten index
- Dirac's monopole quantization
- Aharonov-Bohm effect
- Gauss' law
- Bohr-Sommerfeld quantization
- Soliton/Instanton topologically conserved charges
- etc…

All of these examples involve topological invariants called the Chern characteristic classes and Chern-Simons forms.

Chern-Simons forms

$$C_{2n+1} = \left\langle A \wedge (dA)^n + \alpha_1 A^3 \wedge (dA)^{n-1} + \cdots + \alpha_n A^{2n+1} \right\rangle \equiv \left\langle \tilde{C}_{2n+1} \right\rangle$$

- *No dimensionful constants:* α_k = *fixed rational numbers*
- *No adjustable coefficients:* α_k *cannot get renormalized*
- *No metric needed; scale invariant*
- *Entirely determined by the Lie algebra and the dimension*
- *Only defined in odd dimensions*
- *Unique gauge quasi-invariants:* $\delta C_{2p+1} = d\Omega_{2p}$
- *Related to the Chern characteristic classes*

Is “CS gravity” an oxymoron?

What gauge symmetry is this?

How are the α’s found?

Chern-Simons recipe

- Lie algebra G, generators J_a
- Invariant tensor $\tau_{a_1 \cdots a_n} = \langle J_{a_1} \cdots J_{a_n} \rangle$
- G-valued connection 1-form A, $F = dA + A^2$
- Invariant polynomial (characteristic class)

$$P_{2n}[A] = \frac{1}{n}\left\langle (F\wedge)^n \right\rangle = \frac{1}{n}\langle F \wedge \cdots \wedge F \rangle$$

- Odd-dimensional manifold M^{2n-1}

CS action: $I[A] = \kappa \int_{\mathcal{M}^{2n-1}} C_{2n-1}(A)$

where $dC_{2n-1}(A) = P_{2n}[A]$

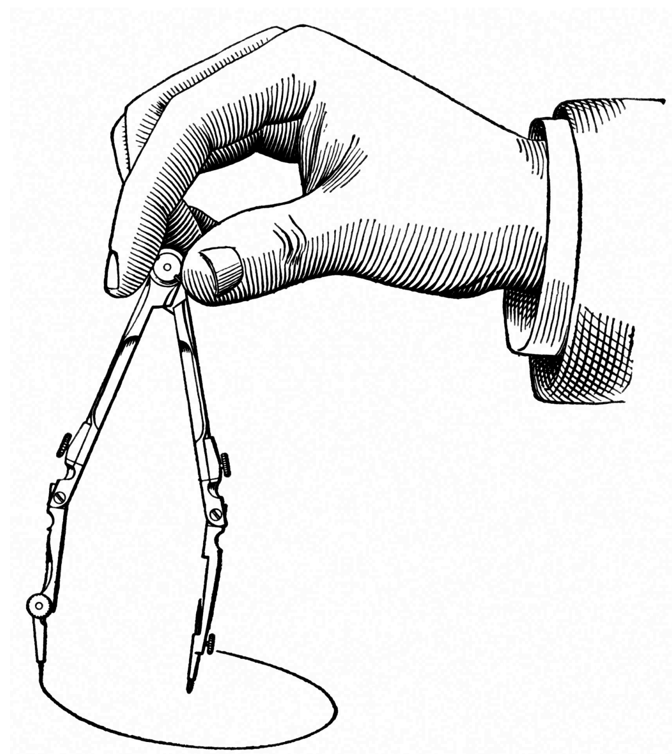
$$L(e,\omega) = (n+1)\int_0^1 dt \left\langle A \wedge F_t^{\,n} \right\rangle,$$

$$F_t = dA_t + A_t^{\,2}, \;\; A_t \equiv tA$$

1. Spacetime geometry

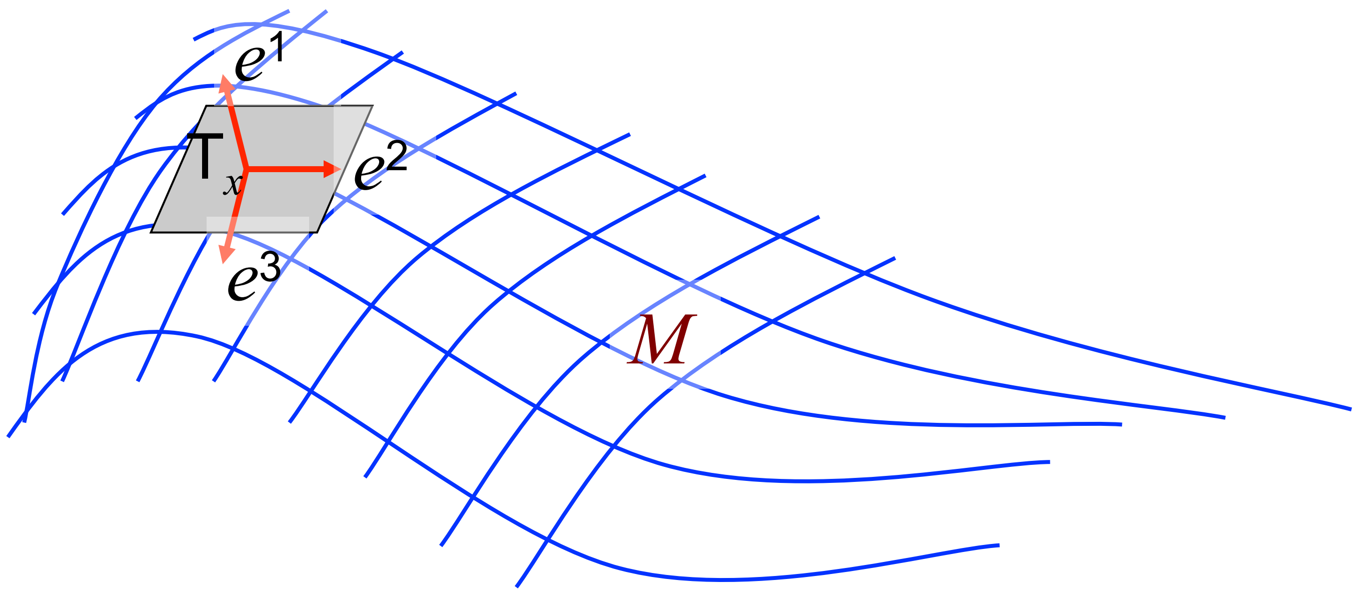
Geometry has two ingredients:

- Metric structure (length/area/volume, scale) $\longrightarrow e^a$
- Affine structure (parallel transport, congruence) $\longrightarrow \omega^a{}_b$



- *Spacetime M is a D-dimensional differentiable manifold (up to isolated singularities in sets of zero-measure).*

- *M admits a D-dimensional tangent space* T_x *at each point.*



- *An open set around any point x is diffeomorphic to an open set in the tangent.*

2. Metric & affine structures

Metric Structure

- *Each tangent space* T_x *is a copy of Minkowski space.*
- *Since Minkowski space is endowed with the Lorentzian metric, the diffeomorphism induces a metric structure on M:*

$$dz^a = e^a_\mu(x)dx^\mu \equiv e^a$$

(e^a: *vielbein, soldering form, local orthonormal frame*).

$$\begin{aligned} ds^2 &= \eta_{ab}dz^a dz^b \\ &= \eta_{ab}e^a_\mu e^b_\nu dx^\mu dx^\nu \equiv g_{\mu\nu}dx^\mu dx^\nu \end{aligned}$$

Metric on M:

$$g_{\mu\nu}(x) = \eta_{ab}e^a_\mu(x)e^b_\nu(x)$$

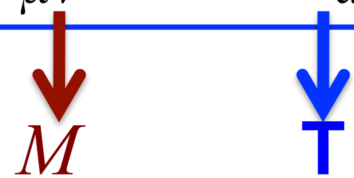
- *The vielbein can be viewed as the Jacobian matrix that relates differential forms in* T_x *and M.*

$$\boxed{dz^a = e^a_\mu(x)dx^\mu}$$

- *This is a mapping between differentials in two distinct spaces.*

- *It can also be viewed as the operator that relate tensors in each space, for example, the metric:*

$$\boxed{g_{\mu\nu}(x) = \eta_{ab}e^a_\mu(x)e^b_\nu(x)}$$



- *The metric is a composite*
- e^a_μ *is the square root of the metric,* $e^a_\mu \sim \sqrt{g_{\mu\nu}}$.

Equivalence Principle

A sufficiently small vicinity of any point of M can be accurately approximated by T_x .

- *In a sufficiently small region of spacetime a reference frame can always be found in which the laws of physics are those of special relativity* (*free fall*).

- *The laws of physics are invariant under local Lorentz transformations.*

- *General Reativity is a nonabelian gauge theory for the group* $SO(3,1)$ *in* 4*D*. (40 *years before Yang & Mills!*)

Affine Structure

- *The differential operations appropriate for the theory should be Lorentz covariant, e.g.,*

$$D_\mu u^a = \partial_\mu u^a + \omega^a{}_{b\mu} u^b$$

where u^a *is a Lorentz vector and* $\omega^a{}_{b\mu}$ *is the connection for the Lorentz group.*

- *This means that under a Lorentz transformations,* u^a *and* $D_\mu u^a$ *must transform in the same manner:*

$$u^a \to u'^a = \Lambda^a{}_b u^b,\ \Lambda^a{}_b \in SO(D-1,1)$$

$$Du^a \to (Du^a)' = \Lambda^a{}_b (Du^b)$$

- *Geometric interpretation*

$$u^a(x)$$: *vector field at x*

$$u^a(x+dx)$$: *vector field at x+dx*

$$u^a{}_{\parallel}(x+dx) = [\delta^a{}_b + \omega^a{}_{b\mu} dx^\mu] u^b(x+dx)$$
$$= u^a(x+dx) + \omega^a{}_{b\mu} dx^\mu u^b(x)$$: *u^a parallel-transported from x+dx to x*

Hence, $$u^a{}_{\parallel}(x+dx) - u^a(x)$$

$$= u^a(x+dx) + \omega^a{}_{b\mu} dx^\mu u^a - u^a$$

$$= dx^\mu [\partial_\mu u^a + \omega^a{}_{b\mu} u^a] = dx^\mu D_\mu u^a$$

- *The covariant derivative and the curvature 2-form are defined uniquely by the Lorentz connection 1-form*

$$\omega^{a} = \omega^{a}{}_{b\mu} dx^{\mu}$$

- *Covariant derivative:*

$$Du^{a} = du^{a} + \omega^{a}{}_{b} u^{a}$$

- *Curvature:*

$$DDu^{a} = R^{a}{}_{b} u^{b}, \qquad R^{a}{}_{b} = d\,\omega^{a}{}_{b} + \omega^{a}{}_{c} \omega^{c}{}_{b}$$

- *Bianchi identity:*

$$DR^{a}{}_{b} u^{b} = R^{a}{}_{b} Du^{b}$$

where $DR^{a}{}_{b} = dR^{a}{}_{b} + \omega^{a}{}_{c} R^{c}{}_{b} - \omega^{a}{}_{c} R^{c}{}_{b} \equiv 0$

3. Building Blocks

- *So far, we have defined the fundamental elements to construct a dynamical theory of the spacetime geometry:*

e^a	*vector, 1-form*
$\omega^a{}_b$	*connection 1-form*
$R^a{}_b = d\omega^a{}_b + \omega^a{}_c\omega^c{}_b$	*curvature 2-form*
$T^a = de^a + \omega^a{}_c e^c$	*torsion 2-form*
$\varepsilon_{a_1 a_2 \cdots a_D}, \ \eta_{ab}$	*invariant tensors 0-forms*

- *By taking successive derivatives no new building blocks are produced:*

$$DR^a{}_b \equiv 0, \ DT^a = R^a{}_b e^b, \ dd = 0.$$

- *The most general D-dimensional Lorentz invariant Lagrangian built with these ingredients is a polynomial in e^a, $\omega^a{}_b$, and their exterior derivatives.*

- *We assume nothing about the invertibility of the vielbein, or the metric, and do not include the *Hodge.*

- *The use of exterior derivatives guarantees that all field equations will be first order. If the torsion is set to zero, the equations for the metric will be second-order at most.*

- *This recipe gives all the Lovelock series and a lot more.*

If no further ingredients or assumptions are added, there is a finite family of Lagrangians that can be constructed with these elements in each dimension.

4. Gravity actions

D - dimensional gravity

Local frames and parallelism are the essential features of geometry.

- They are logically independent.
- They should be dynamically independent.

Dynamical fields

Vielbein: $e^a = e^a_\mu dx^\mu$

Lorentz (spin) connection: $\omega^a{}_b(x) = \omega^a{}_{b\mu}(x) dx^\mu$.

Local symmetry:

Local Lorentz transformations,

$$e^a(x) \to e'^a(x) = \Lambda^a{}_b(x) e^b(x)$$

$$\omega^a{}_b(x) \to \omega'^a{}_b(x) = (\Lambda^{-1})^a{}_c \left[\omega^c{}_d(x) \Lambda^d{}_b + d\Lambda^c{}_b \right]$$

$$\Lambda^a{}_b(x) \in SO(D-1,1)$$

Manifestly Lorentz invariant theory

Coordinate independence is achieved naturally using coordinate-free one-form fields e^a, $\omega^a{}_b$.

The Lagrangian for *D*-dimensional gravity is assumed to be:

- A *D*-form constructed out of e^a, $\omega^a{}_b$, their exterior derivatives, and the Lorentz invariant tensors η_{ab} and $\varepsilon_{a_1 a_2 \ldots a_D}$,

$$L = L(e, de, \omega^a{}_b, d\omega^a{}_b)$$

- (Quasi*-) invariant under local Lorentz transformations, in order to ensure Lorentz covariance of the field equations.

(* Up to total derivatives)

Using only exterior derivatives limits the number of options:
Taking exterior derivatives of e and ω, two tensors are found

$$De^a = de^a + \omega^a{}_b \wedge e^b = T^a \quad \text{Torsion}$$

$$d\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b = R^a{}_b \quad \text{Curvature}$$

Torsion and Curvature: 2-forms

Bianchi identities

$$DR^a{}_b \equiv 0$$

$$DT^a \equiv R^a{}_b \wedge e^b$$

This limits the number of ingredients in *L* to:

$$\underbrace{\eta_{ab},\ \varepsilon_{a_1 a_2 \ldots a_D},}_{\text{Zero-forms}} \quad \underbrace{e^a,\ \omega^a{}_b,}_{\text{one-forms}} \quad \underbrace{T^a, R^a{}_b}_{\text{two-forms}}$$

Three series are found:

A. Lovelock — B. Torsion — C. Lorentz C-S

(B and C: Exotic gravities)

A ↔ **Euler classes**

B, C ↔ **Pontryagin classes**

A: Generalization of E-H for all *D*, parity even.

B: Some *D* only, vanish for $T^a=0$ or $\Lambda=0$, parity odd.

C: For $D=4k-1$ only, parity odd.

$$L(e,\omega) = L_{Lovelock} + L^{T} + L^{CS}_{Lorentz}$$

This series includes:

$$L_0 = \varepsilon_{a_1 \ldots a_D} e^{a_1} \cdots e^{a_D}$$

Cosmological constant term (volume)

$$L_1 = \varepsilon_{a_1 \ldots a_D} R^{a_1 a_2} e^{a_3} \cdots e^{a_D}$$

Einstein-Hilbert

$$L_2 = \varepsilon_{a_1 \ldots a_D} R^{a_1 a_2} R^{a_3 a_4} e^{a_5} \cdots e^{a_D}$$

Gauss-Bonnet

.

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.

$$L_{D-1} = \varepsilon_{a_1 \ldots a_D} R^{a_1 a_2} \cdots R^{a_{D-2} a_{D-1}} e^{a_D}$$

Poincaré invariant term (odd D) [see below]

$$L_D = \varepsilon_{a_1 \ldots a_D} R^{a_1 a_2} \cdots R^{a_{D-2} a_D} = \mathrm{E}_D$$

Euler density (even D) [see above]

B. Torsion series

If $T^a \neq 0$, more invariant terms are allowed:

$$t_p = T_{a_1} R^{a_1}{}_{a_2} R^{a_2}{}_{a_3} \cdots R^{a_{2p-1}}{}_{a_{2p}} T^{a_{2p}}, \ [p = even]$$

$$e_p = e_{a_1} R^{a_1}{}_{a_2} R^{a_2}{}_{a_3} \cdots R^{a_{2p-1}}{}_{a_{2p}} e^{a_{2p}}, \ [p = odd]$$

$$k_p = T_{a_1} R^{a_1}{}_{a_2} R^{a_2}{}_{a_3} \cdots R^{a_{2p-1}}{}_{a_{2p}} e^{a_{2p}}$$

..and products thereof.

These traces are related to the Pontryagin form (Chern class),

$$P_{4k} = R^{a_1}{}_{a_2} R^{a_2}{}_{a_3} \cdots R^{a_{2k}}{}_{a_1} = \mathrm{Tr}(R^{2k}),$$

whose integral is a topological invariant in *4k* dimensions.

C. Lorentz Chern-Simon series

These are quasi-invariant terms involving ω^{ab} explicitly:

$$C_3 = d\omega^a{}_b \omega^b{}_a + \frac{2}{3}\omega^a{}_b \omega^b{}_c \omega^c{}_a ,$$

Fixed numbers

and in general,

$$C_{4n-1} = \left(d\omega^a{}_b\right)^{2n-1} \omega^b{}_a + \cdots + \gamma_{4n-1}\left(\omega^b{}_a\right)^{4n-1}$$

- They are also related to the Pontryagin forms:

$$dC_{4n-1} = \boldsymbol{P}_{4n}$$

- They do not vanish if $T^a \equiv 0$ or if $\Lambda = 0$
- Products of these and torsional terms are also acceptable Lagrangians, provided they are *D*-forms .

The Action

The most general lagrangin for gravity in *D* dimensions is

$$L_G[e,w] = {}_{M_D} L^D_{Lovelock} + L^D_{Exotic} \, ,$$

Arbitrary coefficients!

$$\text{where} \quad L^D_{Lovelock} = \sum_{p=0}^{[D/2]} a_p L_p$$

$$\text{with} \quad L_p = e_{a_1 \ldots a_D} R^{a_1 a_2} \ldots R^{a_{2p-1} a_{2p}} e^{a_{2p+1}} \ldots e^{a_D} \, ,$$

The exotic part contains torsional and Lorentz C-S terms, with more arbitrary constant coefficients,

$$\beta_1, \beta_2, \ldots \beta_s \, .$$

Features

General action for a D - dimensional geometry.

- 1^{st} order field equations for e, ω.

- Torsion-free sector :

$$T^a = de^a + \omega^a{}_b e^b = 0 \Rightarrow \omega^{ab} = \omega^{ab}(e, \partial e)$$

  - Second order theory for $g_{\mu\nu}$
  - Same degrees of freedom as ordinary gravity

- Diffeomorphic invariant by construction

- Invariant under local Lorentz transformations

- For D=4 Einstein-Hilbert is the only option

Puzzles / problems

- The action has a number of cosmological constants

$$\lambda_i = \lambda_i(\alpha_p, \beta_q), \quad i = 1, 2, \ldots, [(D-1)/2]$$

whose values *are not fixed* (the cosmological constant problem raised to a large number)

- Violently different behavior for different choices of $\alpha_{p,}$ β_q.

- Hopeless quantum scenario: $\alpha_{p,}$ β_q dimensionful and unprotected from renormalization.

- How could this be so if gravity descends from a fundamental renormalizable or finite theory?

- Dynamics: Non-uniform phase space (??)

At the point in parameter space where all the cosmological constants coincide, a <u>miracle occurs</u>

- All the dimensionful coupling constants can be absorbed by a redefinition of the vielbein

$$e^a \to \frac{e^a}{l}$$

- The vielbein and the spin connection can be combined as components of a single connection 1-form,

$$A = l^{-1} e^a J_a + \tfrac{1}{2} \omega^{ab} J_{ab}$$

where $J_{ab} =$ Lorentz generator

$J_a =$ generators of (A)dS boosts/translations

} (A)dS/Poincaré algebra

- In odd dimensions, the action describes a gauge theory for the corresponding group. *Symmetry enhancement and no dimensionful coupling constants*!

???

The new curvature is

$$F^{AB} = dW^{AB} + W^{A}{}_{C}W^{CB} = \begin{bmatrix} R^{ab} \pm l^{-2}e^{a}e^{b} & l^{-1}T^{a} \\ -l^{-1}T^{b} & 0 \end{bmatrix}$$

+ AdS

– dS

The Euler (E_{2n}) and Pontryagin (P_{4k}) invariants can be constructed with F^{AB}. These invariants are closed:

$$d\mathbf{E}_{2n} = 0 \Rightarrow \mathbf{E}_{2n} = d\mathbf{C}^{E}{}_{2n-1}$$

$$d\mathbf{P}_{4k} = 0 \Rightarrow \mathbf{P}_{4k} = d\mathbf{C}^{P}{}_{4k-1}$$

- Hence, they can be (locally) written as the exterior derivative of "something else".

$C^{E}{}_{2n-1}$, $C^{P}{}_{4k-1}$ are functions of e, ω^{ab} and their derivatives.
E_{2n}, P_{4k} invariant under (A)dS, $\Rightarrow$ so are the C's.

These are the combinations we're looking for!!

4.a Special choices (*Lovelock series*)

For D =2n+1, choosing the coefficients in the Lovelock series as

$$\alpha_p = \alpha_p^{AdS} = \frac{l^{2p-D}}{D-2p}\begin{pmatrix} \frac{D-1}{2} \\ p \end{pmatrix},$$

produces the particular form of the Lovelock lagrangian

$$L_{\mathrm{AdS}} = l^{-1} R^n e + \frac{l^{-3}}{3}\, {}_n R^{n-1} e^3 + \cdots + \frac{l^{-(1+2n)}}{2n+1} e^{2n+1},$$

with an enhanced symmetry, Lorentz ⟶ AdS(*).

The secret of the enhancement:

SO(D-1,2) invariant

$$dL_{\mathrm{AdS}} = \mathsf{E}_{2n+2}$$

(*) Alternate signs for dS.

In the limit $l \longrightarrow \infty$, only one constant survives

$$\alpha_p = \begin{cases} 1 & \text{if } p = \frac{D-1}{2} \\ 0 & \text{otherwise} \end{cases}$$

The result is a Poincaré-invariant theory,

$$L_{Poincaré} = \varepsilon_{a_1 \ldots a_D} R^{a_1 a_2} \cdots R^{a_{D-2} a_{D-1}} e^{a_D}$$

with the enhancement *SO(D-1,1)* → *ISO(D-1,1)*

N.B. The Einstein-Hilbert action,

$$L_{E-H} = \varepsilon_{a_1 \ldots a_D} R^{a_1 a_2} e^{a_3} \cdots e^{a_D},$$

is not Poincaré - invariant.

4b. Special choices (*torsion series*)

In *D=4k-1*, there is a particular combination of the torsion series invariant under (A)dS, related to the Pontryagin class.

$$dL^*_{AdS} = F^{A_1}{}_{A_2} F^{A_2}{}_{A_3} \cdots F^{A_{2k}}{}_{A_1}$$

$= \boldsymbol{P_{4k}}$ Pontryagin invariant of 4*k* dimensions

(The explicit form of L^*_{AdS} is not very interesting and it involves torsion explicitly.)

- In the limit $l \to \infty$, the only surviving term in L^*_{AdS} is the Lorentz Chern-Simons term

$$L^L{}_{4n-1} = \left(d\omega^a{}_b\right)^{2n-1} \omega^b{}_a + \cdots + c_{4n-1}\left(\omega^b{}_a\right)^{4n-1}$$

5. CS actions

AdS Theory as Chern-Simons

The AdS invariant theory can be expressed in more standard form in terms of its Lie algebra valued connection and curvature:

$$A = l^{-1} e^a J_a + \tfrac{1}{2} \omega^{ab} J_{ab}$$

$$\begin{aligned} F &= dA + A^2 \\ &= l^{-1} T^a J_a + \tfrac{1}{2} (R^{ab} + l^{-2} e^a e^b) J_{ab} \end{aligned}$$

Both the Euler and Pontryagin Chern-Simons can be written as

$$dL^{(*)}_{AdS} = \langle \mathbf{F}^n \rangle$$

Appropriate multilinear invariant product (trace)

Dynamics (*generic C-S case*)

For $D \leq 3$ all CS theories are topological: No propagating degrees of freedom.

For $D = 2n+1 \geq 5$, a CS theory for a gauge algebra with N generators can as many as $\Delta = Nn - N - n$ local d. of f. (Bañados, Garay, Henneaux)

Phase space is <u>not homogeneous</u>:

$$\Delta = \begin{cases} Nn-N-n & \text{generic configurations (most states, no symmetry)} \\ 0 & \text{maximally degenerate (measure zero, max. symmetric)} \end{cases}$$

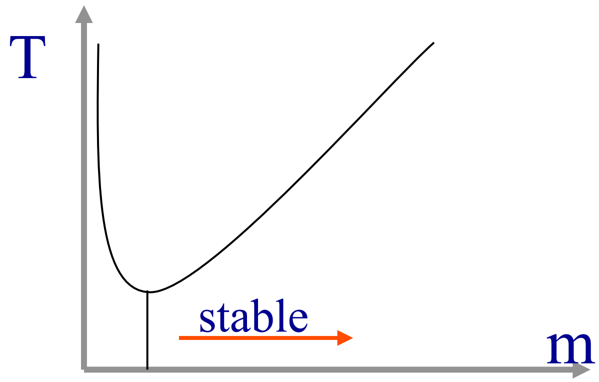
Degeneracy: The system may evolve from an initial state with Δ_1 d.o.f., reching a state with $\Delta_2 < \Delta_1$ d.o.f. in a finite time. This is an irreversible process in which there is no record of the initial conditions in the final state.

Dynamics of C-S Gravity

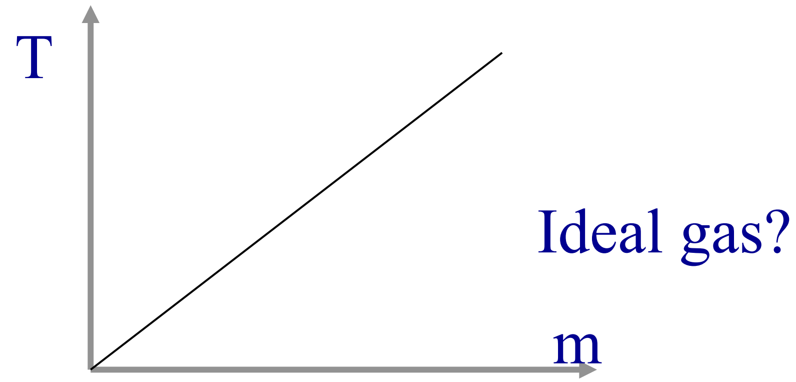
Around a flat background, $T^a = 0, R^{ab} = 0$, Chern-Simons theories behave as the standard Einstein-Hilbert theory. (same d.o.f., propagators, etc.

- But, *D*-dimensional Minkowski space is not a classical solution!!
- The natural vacuum is, *D*-dimensional AdS space (max. symmetric).
- But around *D*-dimensional AdS space the theory has no d.o.f.!!
- Black holes are particularly sensitive to the theory:

Generic Lovelock theory



Chern-Simons theory



Supersymmetry

Supersymmetry is the only nontrivial extension of the spacetime symmetry [Lorentz, Poincaré, (A)dS].

New ingredient: Gravitino $\psi = \psi_\mu dx^\mu$

This is also a 1-form, which suggests a new connection,

$$\mathbf{A} = l^{-1} e^a \mathbf{J}_a + \frac{1}{2}\omega^{ab}\mathbf{J}_{ab} + \frac{1}{2}(\overline{\psi}\mathbf{Q} + \overline{\mathbf{Q}}\psi) + b \cdot \mathbf{X}$$

Necessary to close the algebra.
Depends on the dimension.

The generators $\{\mathbf{J}_a, \mathbf{J}_{ab}, \mathbf{Q}, \overline{\mathbf{Q}}, \mathbf{X}\}$ should form a (super) algebra.

Supersymmetry transformations

$$\delta e^{a} = \bar{\varepsilon}\Gamma^{a}\psi$$

$$\delta\omega^{ab} = \bar{\varepsilon}\Gamma^{ab}\psi$$

$$\delta\psi = \nabla\varepsilon = (d + e^{a}\mathbf{J}_{a} + \frac{1}{2}\omega^{ab}\mathbf{J}_{ab})\varepsilon$$

⇓

$$\delta L(e,\omega) = \delta L_{F}(e,\omega,\psi,\bar{\psi})$$

Tentative fermionic Lagrangian

⇓

$$\delta L_{F}(e,\omega,\psi,\bar{\psi}) = \delta L_{B}^{New}(e,\omega,b^{[a_1 a_2 \ldots]})\cdots$$ etc.

… this eventually yields a supersymmetric Lagrangian.

The construction is straightforward and the resulting theories are Chern-Simons systems . The field content and the superalgebras for the first cases are

D	Algebra	Field content	Standard Supergravity
5	*su(2,2|1)*	$e^a_\mu, \omega^{ab}_\mu, A_\mu, \psi^\alpha_\mu, \overline{\psi}_{\alpha\mu}$	$g_{\mu\nu}, A_\mu, \psi^\alpha_\mu, \overline{\psi}_{\alpha\mu}$
7	*osp(2|8)*	$e^a_\mu, \omega^{ab}_\mu, A^i_{j\mu}, \psi^{\alpha i}_\mu \ (i,j = 1,2)$	$g_{\mu\nu}, A_{[3]}, A^i_{j\mu}, \lambda^\alpha, \phi, \psi^{\alpha i}_\mu$
11	*osp(1|32)*	$e^a_\mu, \omega^{ab}_\mu, b^{abcde}_\mu, \psi^\alpha_\mu$	$g_{\mu\nu}, A_{[3]}, \psi^\alpha_\mu$

Local SUSY (off-shell algebra):

$$\delta e^a = \bar{\varepsilon}\,\Gamma^a\psi, \quad \delta\omega^{ab} = 0, \quad \delta\psi = D\varepsilon,$$

$$\delta b^{ab} = \bar{\varepsilon}\,\Gamma^{ab}\psi, \quad \delta b^{abcde} = \bar{\varepsilon}\,\Gamma^{abcde}\psi, \quad \text{etc.}$$

This is an exact local symmetry: no auxiliary fields, no on-shell conditions, no assumptions on the background ($T^a \equiv 0$, etc.)

Summary

- The equivalence principle leads to a geometric action where the vielbein and the spin connection are the dynamical fields.
- These theories are invariant under local Lorentz transformations and invariant (by construction) under diffeomorphisms.
- They give rise to 2nd order field eqs. for $g_{\mu\nu}$, and spin $S \leq 2$.
- Around a flat background ($R^{ab}=0=T^{a}$) these theories behave like ordinary GR (Einstein-Hilbert). However, this is just one of the many sectors of the theory.
- These theories possess a number of indeterminate dimensionful parameters $\{\alpha_p, \beta_q\}$.
- In odd dimensions and for a particular choice of these parameters, the theory has no dimensionful constants and its symmetry is enhanced to (A)dS. The resulting action is a Chern-Simons form for the (A)dS or Poincaré groups.

- The classical evolution can go from one sector to another with fewer degrees of freedom in an irreversible manner.
- The supersymmetric extensions only exist for special combinations of AdS or Poincaré invariant gravitational actions.
- The maximally (super)symmetric configuration is a vacuum state that has no propagating degrees of freedom around it. At that point the theory is topological.
- This shows that the standard supergravity of Cremmer, Julia and Sherk is not the only consistent supersymmetric field theory containing gravity in *11D*. There exist at least 3 more:
  - Super-AdS [Osp(32|1)]
  - Super-Poincaré
  - M-Algebra

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Ευχαριστω!