# Black Hole Solutions in Massive Gravity

Mikhail S. Volkov

LMPT, University of Tours, FRANCE

7th Aegean Summer School, Paros, 25th September 2013

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# Part I: Black holes in massive gravity

# Black holes and no-hair conjecture

# No-hair conjecture

- All stationary black holes are completely characterized by their mass, angular momenum, and electric charge measurable from infinity.
- They cannot support hair= external fields distributed close to the horizon but not seen from infinity /Wheeler 1969/

Logic: only the exact local symmetries (Lorentz, U(1)) can survive in the gravitational collapse. Associated to them global charges (mass, etc.) remain attached to the black hole. All other symmetries will be broken ⇒ no associated charges ⇒ no extra black hole parameters. Everything that can be radiated or absorbed will be radiated and absorbed. Exact charges can be neither swallowed nor radiated away.

Black holes have no hair

# Evidence for ho-hair

- Uniqueness theorems: all electrovacuum black holes are Kerr-Newman /Israel '68/, /Carter '73/, /Mazur '82/ ... .
- No-hair theorems for black holes coupled to other fields

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}(\Psi), \qquad \Box\Psi = V(\Psi)$$

$\Psi$ as a scalar, spinor, massive vector etc. /Bekenstein '72/.
If $\Psi \neq 0$ at the horizon, then it diverges $\Rightarrow$ singular horizon $\Rightarrow$ no black holes other than Kerr-Newman.

No black holes with massive hair.

# First evidence against ho-hair

Local SU(2) is also an exact symmetry – can one have black holes with Yang-Mills fields ?

- U(1)∈SU(2) ⇒ all electrovacuum Kerr-Newman black holes are contained in the Einstein-Yang-Mills theory /Yasskin '75/.
- Theory also admits other black holes with Yang-Mills field $\sim 1/r^3$ at large $r$ ⇒ no charges. They are labeled by the horizon radius $r_H$ and by the number $n = 1, 2, \ldots$ of nodes of the Yang-Mills field for $r > r_H$ /M.S.V., Gal'tsov '89/

⇒ first example of hairy black hoes

# Hairy black holes

- In systems with Yang-Mills coupled to other fields: black holes inside magnetic monopoles, sphalerons, Skyrmions, etc. /M.S.V., Gal'tsov Phys.Rept. **319** (1999) 1/
- In systems with stringy-inspired (Gauss-Bonnet, axion, dilaton, fluxes etc. fields). In $D > 4$. In non-asymptotically flat systems (AdS) /many papers after the year 2000/

Nowadays hairy black holes are common in physics.
They usually support massless hair.

Typically they are either unstable or microscopically small $\Rightarrow$ the no-hair conjecture holds for astrophysical black holes.

What about black holes in massive gravity ?

# Theories with massive gravitons

# The ghost-free bigravity

$$S = \frac{1}{2\kappa_g^2}\int R\sqrt{-g}\,d^4x + \frac{1}{2\kappa_f^2}\int \mathcal{R}\,\sqrt{-f}\,d^4x - \frac{m^2}{\kappa^2}\int \mathcal{U}\sqrt{-g}\,d^4x$$

$$\kappa_g = \kappa\cos\eta, \quad \kappa_f = \kappa\sin\eta, \quad \gamma^\mu_{\ \nu} = \sqrt{g^{\mu\alpha} f_{\alpha\nu}}$$

$$\mathcal{U} = \sum_k \beta b_k \mathcal{U}_k = \beta_0 + \beta_1 \sum_A \lambda_A + \beta_2 \sum_{A<B} \lambda_A\lambda_B + \beta_3 \sum_{A<B<C} \lambda_A\lambda_B\lambda_C + \beta_4\,\lambda_0\lambda_1\lambda_2\lambda_3$$

Flat space is the solution if only $\beta_0 = 4c_3 + c_4 - 6$, $\beta_1 = 3 - 3c_3 - c_4$, $\beta_2 = 2c_3 + c_4 - 1$, $\beta_3 = -(c_3 + c_4)$, $\beta_4 = c_4$. Two gravitons, propagates 7 degrees of freedom

# Field equations

$$G^{\rho}_{\lambda} = m^2 \cos^2\eta\, T^{\rho}_{\lambda}$$

$$\mathcal{G}^{\rho}_{\lambda} = m^2 \sin^2\eta\, \mathcal{T}^{\rho}_{\lambda}$$

$$T^{\rho}_{\lambda} = \tau^{\rho}_{\lambda} - \delta^{\rho}_{\lambda}\mathcal{U}\,, \qquad \mathcal{T}^{\rho}_{\lambda} = -\frac{\sqrt{-g}}{\sqrt{-f}}\,\tau^{\rho}_{\lambda}\,,$$

$$\begin{aligned}
\tau^{\rho}_{\lambda} &= \{b_1\mathcal{U}_0 + b_2\mathcal{U}_1 + b_3\mathcal{U}_2 + b_4\mathcal{U}_3\}\gamma^{\mu}{}_{\nu} \\
&- \{b_2\mathcal{U}_0 + b_3\mathcal{U}_1 + b_4\mathcal{U}_2\}(\gamma^2)^{\mu}{}_{\nu} \\
&+ \{b_3\mathcal{U}_0 + b_4\mathcal{U}_1\}(\gamma^3)^{\mu}{}_{\nu} \\
&- b_4\mathcal{U}_0\,(\gamma^4)^{\mu}{}_{\nu}
\end{aligned}$$

For $\eta \to 0$ one can set $f_{\mu\nu} = \eta_{\mu\nu}$ and the theory reduces to the dRGT massive gravity theory with one massive graviton.

# Black hole solutions

# Black holes with non-bidiagonal metrics

$$ds_g^2 = -D(r)dt^2 + \frac{dr^2}{D(r)} + r^2 d\Omega^2 \qquad \text{Schwarzschild-dS}$$

$$ds_f^2 = -\Delta(U)\,dT^2 + \frac{dU^2}{\Delta(U)} + U^2 d\Omega^2 \qquad \text{AdS}$$

where

$$D(r) = 1 - \frac{2M}{r} - \frac{\Lambda_g}{3}\,U^2, \qquad \Delta(U) = 1 - \frac{\Lambda_f}{3}\,U^2$$

$$T = ut - u\int \frac{D-\Delta}{D\Delta}\,dr, \qquad U = ur, \quad u = \frac{1}{b_3}\left(-b_2 \pm \sqrt{b_2^2 - b_1 b_3}\right)$$

For $\eta \to 0$ one has $\Lambda_f \to 0$, $f_{\mu\nu} \to \eta_{\mu\nu}$, which gives black holes in dRGT massive gravity. There are no asymptotically flat black holes in the theory. A possible explanation: black holes in massive gravity are non-stationary. For $M = 0 \Rightarrow$ self-accelerated cosmologies (a large family of solutions /M.V. QCG **30** 2013/).

# Hairy black holes with bidiagonal metrics

$$ds_g^2 = Q^2 dt^2 - \frac{dr^2}{N^2} - r^2 d\Omega^2, \quad ds_f^2 = A^2 dt^2 - \frac{U'^2}{Y^2} dr^2 - U^2 d\Omega^2$$

$Q, N, Y, U, A$ are 5 functions of $r$, they fulfill 5 equations

$$\begin{aligned}
G_0^0 &= m^2 \cos^2\eta \, T_0^0, \\
G_r^r &= m^2 \cos^2\eta \, T_r^r, \\
\mathcal{G}_0^0 &= m^2 \sin^2\eta \, \mathcal{T}_0^0, \\
\mathcal{G}_r^r &= m^2 \sin^2\eta \, \mathcal{T}_r^r, \\
T_r^{r\prime} &+ \frac{Q'}{Q}(T_r^r - T_0^0) + \frac{2}{r}(T_\vartheta^\vartheta - T_r^r) = 0.
\end{aligned}$$

/M.S.V.Phys.Rev. D85 (2012) 124043/

# Simplest solutions

$$\boxed{f_{\mu\nu} = C^2 g_{\mu\nu}}$$

$$A_4 C^4 + A_3 C^3 + A_2 C^2 + A_1 C + A_0 = 0 \qquad (*)$$

$$G^{\mu}_{\nu} + \Lambda_g(C) = 0$$

There are 4 roots of $(*)$, $C = \{C_k\}$.
$C = 1$ is a root, $\Lambda(C) = 0 \Rightarrow$ Schwarzschild black holes
For other roots, $\Lambda(C) \neq 0 \Rightarrow$ Schwarzschild-dS,AdS

The idea is to deform these solutions by changing their horizon boundary conditions.

# Event horizon at $r = r_h$

Local solutions near the horizon ($N^2 = g^{rr}$, $Y^2 = f^{rr}$)

$$N^2 = \sum_{n\geq 1} a_n(r-r_h)^n, \quad Y^2 = \sum_{n\geq 1} b_n(r-r_h)^n, \quad U = u r_h + \sum_{n\geq 1} c_n(r-r_h)^n,$$

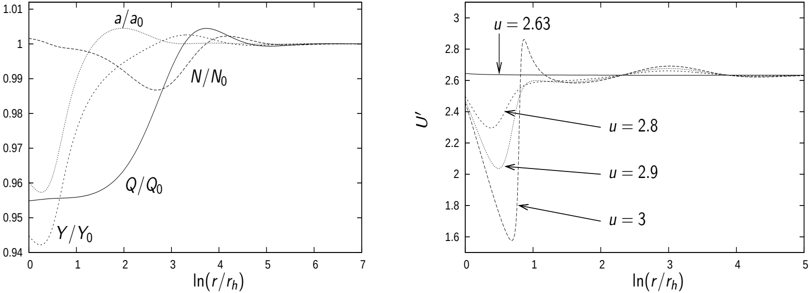
contain one free parameter $u = U(r_h)/r_h$, the ratio of the horizon radius measured by $f_{\mu\nu}$ to that measured by $g_{\mu\nu}$.

- Horizon is common for both metrics
- Horizon temperatures and surface gravities are the same with respect to both metrics /Deffayet, Jackobson '12/

For $u = C_k$ (root of the algebraic equation) one obtains the Schwarzschild-(A)dS black holes.
For $u \neq C_k$ one obtains more general solutions.

# Deforming Schwarzschild-AdS

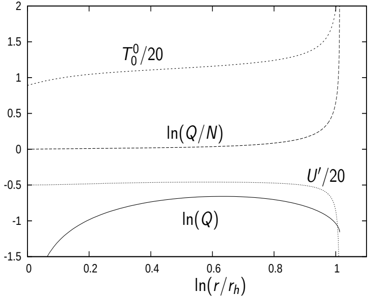
Deformations stay close to the horizon and tend to zero for $r \to \infty$. Solutions approach AdS for large $r$.

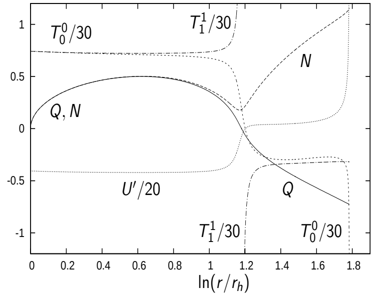



$N_0, Q_0, Y_0, a_0$ correspond to the background AdS.

Hair is localized close to horizon $\Rightarrow$ hairy black holes in AdS

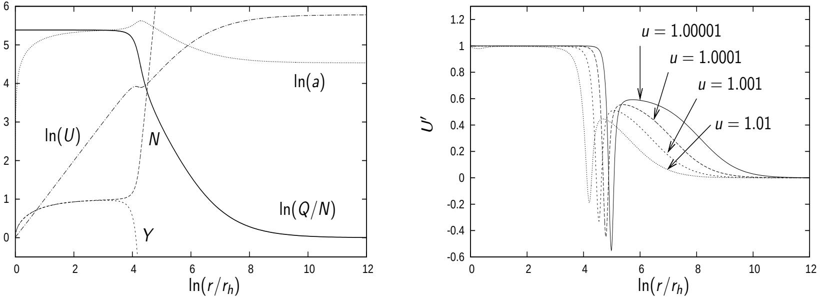
# Deforming Schwarzschild-dS




Deformations become singular at a finite distance from the horizon. Solutions are compact and singular.

# Deforming Schwarzschild



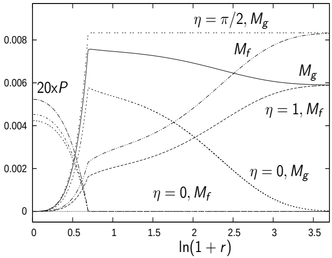

Deformations are small close to horizon but then grow and change completely the asymptotic behavior at $r \to \infty$.

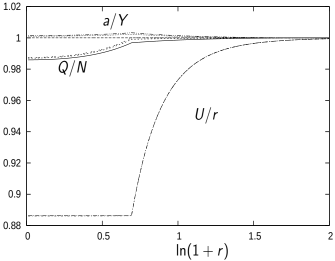
The only asymptotically flat solution one finds is pure Schwarzschild.

# Regular stars and Vainstein mechanism

$$G^{\rho}_{\lambda} = m^2 \cos^2\eta\, T^{\rho}_{\lambda} + T^{[\mathrm{m}]\rho}_{\lambda} \qquad \mathcal{G}^{\rho}_{\lambda} = m^2 \sin^2\eta\, \mathcal{T}^{\rho}_{\lambda}$$

$$T^{[\mathrm{m}]\rho}_{\lambda} = \delta^{\rho}_{0}\delta^{0}_{\lambda}\,\rho_\star\Theta(r - r_\star), \qquad \frac{r_h}{r_\star} = \frac{8\pi G}{3} {r_\star}^2 \rho_\star < 0.3$$




$$g^{rr} = N^2 = 1 - 2M_g(r)/r, \qquad f^{rr} = Y^2/U'^2 = 1 - 2M_f(r)/r$$

$m \ll 1 \Rightarrow M_g, M_f \approx const$ for $r_\star < r < r_{\mathrm{V}} = \left(\frac{\rho_\star r_\star^3}{m^2}\right)^{1/3}$

$\Rightarrow$ GR recovery in a finite region, any $\eta$

## Are there other asymptotically flat black holes ?

There are 3 first coupled order equations for $N, U, Y$. At the horizon the solutions depend on one parameter $u$. At infinity, if they are asymptotically flat, they are Yukawa + Newton,

$$
\begin{aligned}
N &= 1 - \frac{C_1 \sin^2 \eta}{r} + C_2 \cos^2 \eta \, \frac{mr + 1}{r} \, e^{-mr}, \\
U &= r + C_2 \, \frac{m^2 r^2 + mr + 1}{m^2 r^2} \, e^{-mr}, \\
Y &= 1 - \frac{C_1 \sin^2 \eta}{r} - C_2 \sin^2 \eta \, \frac{1 + mr}{r} \, e^{-mr}
\end{aligned}
$$

$\Rightarrow$ two other parameters $C_1, C_2$. One can try to adjust $\{u, C_1, C_2\}$ such that $\{N, U, Y\}$ fulfill correct boundary conditions at the horizon and infinity $\Rightarrow$ 3 conditions for 3 free parameters $\Rightarrow$ solutions can only comprise a discrete set. One cannot get them deforming Schwarzschild $\Rightarrow$ a good initial guess for $\{u, C_1, C_2\}$ is needed.

# Black hole stability

# Perturbing the Schwarzschild black hole

$$g_{\mu\nu} = g^{\rm BH}_{\mu\nu} + \delta g_{\mu\nu}, \qquad f_{\mu\nu} = g^{\rm BH}_{\mu\nu} + \delta f_{\mu\nu}$$

Linear combinations of $\delta g_{\mu\nu}$ and $f_{\mu\nu}$ describe the massive and massless gravitons. The massive graviton fulfills

$$\Box h_{\mu\nu} + 2R_{\mu\alpha\nu\beta}h^{\alpha\beta} = m^2 h_{\mu\nu} \qquad (*)$$
$$\nabla^\mu h_{\mu\nu} = h^\mu_\mu = 0$$

Eq.($*$) is exactly the same as the one describing the Gregory-Laflamme instability. With $h_{\mu\nu} = e^{i\omega t}H_{\mu\nu}(r,\vartheta,\varphi)$ there are bound state solutions with $\omega^2 < 0$ if

$$m\, r_H = \frac{\text{black hole radius}}{\text{graviton's Compton length}} < 0.86$$

$\Rightarrow$ small black holes are unstable. /Babichev,Fabbri '13/ /Brito,Cardoso,Pani '13/.
$m\, r_H = 0.86 \Rightarrow$ zero mode = bound state with $\omega = 0$.

# Hairy black holes

Zero mode for $m\, r_H = 0.86$ provides a perturbative description of a new solution branch. Starting from it one can iteratively decrease $r_H$, which leads to fully non-linear stationary black hole solutions different from Schwarzschild = asymptotically flat hairy black holes /Brito,Cardoso,Pani arXiv:1309.0818/.

For given parameters $\beta_k$ and for $m\, r_H < 0.86$ one finds 2 different asymptotically flat solutions: the Schwarzschild black hole and the hairy black hole.

It seems that hairy black holes can exist also for small values of $r_H$ if only $c_3 = -c_4 = 2$.

# Summary of Part I

- In dRGT massive gravity there are only Schwarzschild-dS black holes, no asymptotically flat solutions, perhaps because black holes should be time-dependent.
- In bigravity there is a continuous family of asymptotically AdS hairy black holes.
- In bigravity there are also asymptotically flat black holes: Schwarzschild and its hairy counterpart with very large size. Small hairy black holes exist only for special values of the theory parameters.
- In bigravity there are also Schwarzschild-dS black holes, whose their hairy analogs are generically singular. It is not known if some special solutions of this type could be regular.

# Part II: Energy in the ghost-free massive gravity

# ADM formulation of GR

$$L = \sqrt{-g}R$$

$3+1$ decomposition

$$ds^2 = -N^2 dt^2 + h_{ik}(dx^i + N^i dt)(dx^k + N^k dt)$$

Momenta

$$\pi^{ik} = \frac{\partial L}{\partial \dot{h}_{ik}} = \sqrt{h}(K^{ik} - Kh^{ik}), \qquad \frac{\partial L}{\partial \dot{N}_\mu} = 0$$

here $N^\mu = (N, N^k)$. Hamiltonian

$$H = \pi^{ik}\dot{h}_{ik} - L = N^\mu \mathcal{H}_\mu$$

with

$$\mathcal{H}_0 = -\sqrt{h}R^{(3)} + \frac{1}{\sqrt{h}}(\pi^{ik}\pi_{ik} - \frac{1}{2}\pi^2), \quad \mathcal{H}_k = -2\nabla_i^{(3)} h_k^i$$

$N^\mu$ are non-dynamical, phase space is spanned by 12 $(\pi^{ik}, h_{ik})$.

# Constraints

$$\frac{\partial H}{\partial N^\mu} = \mathcal{H}_\mu(\pi^{ik}, h_{ik}) = 0 \qquad \text{4 constraints}$$

Since

$$\{\mathcal{H}_\mu, \mathcal{H}_\nu\} \sim \mathcal{H}_\alpha$$

they are first class and generate gauge symmetries $\Rightarrow$ one can impose 4 gauge condition. There remain

$$12 - 4 - 4 = 4 = 2 \times (2 \text{ DoF})$$

independent phase space variables describing 2 graviton polarizations.

Energy is zero on the constraint surface,

$$H = N^\mu \mathcal{H}_\mu = 0$$

# Generic massive gravity

Hamiltonian

$$H = N^{\mu}\mathcal{H}_{\mu} + m^2 V(N^{\alpha}, \pi^{ik}, h_{ik}) \qquad (*)$$

Varying with respect to $N^{\mu}$ gives

$$\frac{\partial H}{\partial N^{\mu}} = \mathcal{H}_{\mu}(\pi^{ik}, h_{ik}) + m^2 \frac{\partial V(N^{\alpha}, \pi^{ik}, h_{ik})}{\partial N^{\mu}} = 0$$

These are not constraints but equations for $N^{\mu}$ whose solution is $N^{\mu}(\pi^{ik}, h_{ik})$. No constraints $\Rightarrow$ all 12 phase space variables are independent $\Rightarrow 6 = 5 + 1$ degrees of freedom.

Inserting $N^{\mu}(\pi^{ik}, h_{ik})$ to $(*)$ gives a non-positive-definite quadratic form in $\pi^{ik}$ $\Rightarrow$ energy is non-zero and is unbounded from below. Among the 6 degrees of freedom 5 correspond to graviton polarizations and 1 is a BD ghost which should be excluded.

# dRGT massive gravity

$V$ is chosen such that the equations

$$\frac{\partial H}{\partial N^\mu} = \mathcal{H}_\mu(\pi^{ik}, h_{ik}) + m^2 \frac{\partial V(N^\alpha, \pi^{ik}, h_{ik})}{\partial N^\mu} = 0$$

determine only $N^k(\pi^{ik}, h_{ik})$ while $N$ remains free. $H$ becomes

$$H = \mathcal{E}(\pi^{ik}, h_{ik}) + NC(\pi^{ik}, h_{ik})$$

Varying with respect to $N$ gives $C = 0$ while $dC/dt = 0$ gives a secondary constraint $C_2 = 0$. The two constraints eliminate one DoF. The remaining 5 DoF are healthy in the decoupling limit and in the flat space limit. The energy is

$$H = \mathcal{E}(\pi^{ik}, h_{ik})$$

where $(\pi^{ik}, h_{ik})$ should fulfill the conditions

$$C(\pi^{ik}, h_{ik}) = 0, \quad C_2(\pi^{ik}, h_{ik}) = 0.$$

Is H positive ?

# Spherical symmetry

$$ds_g^2 = -N^2 dt^2 + \frac{1}{\Delta^2}(dr + N^r dt)^2 + R^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)$$
$$ds_f^2 = -dt^2 + dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)$$

where $N, N^r, \Delta, R$ depend on $t, r$. $H = H(\pi_\Delta, \pi_R, \Delta, R, N, N^r, r)$:

$$H = N\mathcal{H}_0 + N^r\mathcal{H}_r + m^2 V$$
$$\mathcal{H}_0 = \frac{\Delta^3}{4R^2}\pi_\Delta^2 + \frac{\Delta^2}{2R}\pi_\Delta\pi_R + \Delta(R'^2 + 2R'') + 2R\Delta' R' - \frac{1}{\Delta}$$
$$\mathcal{H}_r = \Delta\pi'_\Delta + 2\Delta'\pi_\Delta + R'\pi_R$$
$$V = \frac{N}{\Delta}P_0 + \frac{R^2}{\Delta}P_1\sqrt{(N\Delta + 1)^2 - (N^r)^2} + R^2 P_2$$

$P_m = \beta_m + 2\beta_{m+1}\frac{r}{R} + \beta_{m+2}\frac{r^2}{R^2}$ $(m = 0, 1, 2)$. Phase space is 4D.
If $m = 0 \Rightarrow 2$ constraints $\mathcal{H}_0 = \mathcal{H}_r = 0 \Rightarrow 4 - 2 - 2 = 0$ DoF.
If $m \neq 0$ and generic $V \Rightarrow 2$ DoF=scalar graviton+ghost

# Excluding $N^r$

Varying with respect to $N^r$ gives $(\forall \beta_k)$

$$N^r = (N\Delta + 1)/\sqrt{1 + (m^2 \frac{R^2 P_1}{\Delta \mathcal{H}_r})^2}$$

Inserting back to $H$ gives $H = \mathcal{E}(p,q) + N\, C(p,q)$ with

$$C = \mathcal{H}_0 + \sqrt{(\Delta \mathcal{H}_r)^2 + (m^2 R^2 P_1)^2} + m^2 P_0 \frac{R^2}{\Delta}$$

$$\mathcal{E} = -\frac{1}{\Delta}\mathcal{H}_1 + m^2 R^2 (P_2 - \frac{P_0}{\Delta^2})$$

The kinetic part of $\mathcal{E}$ is a non-positive-definite quadratic form in $\pi_\Delta, \pi_R$. However, $\pi_\Delta, \pi_R$ are not arbitrary, they should fulfill $C = 0$ and also the secondary constraint

$$C_2 = \{C, \int H\, dr\}; \quad \{C(r_1), C(r_2)\} = 0 \;\Rightarrow\; C_2 = \{C, \int \mathcal{E}\, dr\}$$

# Second constraint

$$
\begin{aligned}
C_2 &= \Delta\mathcal{H}_r' + \frac{\mathcal{H}_0}{\Delta^2}\partial_{\pi_\Delta}(\mathcal{H}_0) - \frac{\Delta'}{Y}\mathcal{H}_0\mathcal{H}_r + \frac{\mathcal{H}_r^2}{Y}\partial_{\pi_\Delta}(\mathcal{H}_0) + \frac{\Delta\mathcal{H}_r}{Y}\mathcal{H}_0' \\
&+ m^2\left(\frac{2P_0R^2}{\Delta^3} + \partial_R(R^2P_2 - \frac{R^2P_0}{\Delta^2})\,\partial_{\pi_R}\right)\mathcal{H}_0 \\
&- m^2\frac{\mathcal{H}_r}{Y}\left(\frac{2\Delta'}{\Delta}R^2P_0 - (R^2P_0)' - R^2\partial_r P_0\right) \\
&+ \frac{m^4}{2\Delta Y}\mathcal{H}_1'\,\partial_R(R^4P_1^2)\partial_{\pi_R}(\mathcal{H}_0),
\end{aligned}
$$

with $Y = \sqrt{\Delta^2\mathcal{H}_r^2 + (m^2R^2P_1)^2}$. Requiring further that $\frac{dC_2}{dt} = 0$ gives an equation for $N$.

For flat space values, $\Delta = 1$, $R = r$, $\pi_\Delta = \pi_R = 0$ one has

$$
C = C_2 = \mathcal{E} = 0 \quad \text{if} \quad \beta_k = \beta_k(c_3, c_4)
$$

# Exciting the flat space

$$\text{Let} \quad \Delta = 1, \quad R = r, \quad \pi_\Delta = \pi_\Delta(r), \quad \pi_R = \pi_R(r)$$

With $z = \pi_\Delta^2/r$ both constraints are fulfilled if $z$ fulfills

$$\begin{aligned} 4r^3 m^2 (z - 4r) z'' \quad + \quad & 4m^6 r^6 z - 4r^3 m^4 (z^2 - r^2 z' + 3rz) \\ - \quad & m^2 r^2 (16r + 11z) z' - z'^2 z - 4zz'^2 \\ - \quad & (4m^4 r^4 - 8z' - 7m^2 rz) Q = 0, \qquad (*) \end{aligned}$$

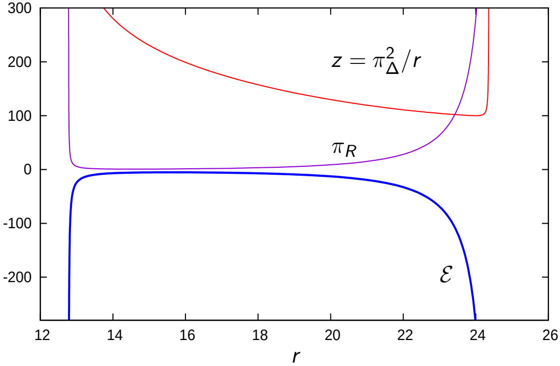
with $Q = \pm\sqrt{4m^4 r^4 z^2 + 8m^2 r^3 z' z + z'^2 z}$ and if

$$\pi_R = \frac{4m^2 r^3 z + 4rz' + 4rz - z^2 + 2rQ}{2\sqrt{rz}(z - 4r)} \,.$$

The energy is

$$\mathcal{E} = -\frac{2m^2 r^2 z + 2rz' - Q}{z - 4r}$$

Eq.$(*)$ has singular points at $r = 0, \infty$ and $r = z/4$ (movable).
Near a regular $r = r_0$ the solution is determined by $z(r_0)$, $z'(r_0)$.




Solution on a compact interval, $\mathcal{E}$ is everywhere negative, $E = \int \mathcal{E} dr = -\infty$. Perhaps one can have $E > 0$ for globally regular solutions on $r \in [0, \infty)$, but it is unclear if they exist.

# Summary of Part II

- The dRGT massive gravity contains two constraints which remove one degree of freedom $\Rightarrow$ only 5 degrees propagate.
- However, the energy can be negative and infinite $\Rightarrow$ it is unclear if it is the ghost and not something else which is removed. Perhaps there are several ghosts, not seen in the DL.
- This raises concerns about stability of the theory.
- There are also positive energy solutions of the constraint equations.
- It possible that the positive and negative energy solutions belong to disjoint sectors, but more analysis is needed to claim this.