Cosmological perturbations in the most general scalar-tensor theories

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What are the origins of early time (inflation) and late time (dark energy) cosmic accelerations?

1. For inflation, the exact scale-invariant spectrum is ruled out at more than 5sigma C.L. from the Planck data.

⇨ This means that inflation should have a dynamics to end the epoch of cosmic acceleration.

2. The dark energy equation of state is constrained to be

$$w = -1.13^{+0.13}_{-0.10} \qquad \text{(Planck +BAO)}$$

⇨ We do not have the strong evidence for dynamical dark energy, but the cosmological constant has a mild tension with the recent data.

Inflationary models

Many inflationary models have been proposed so far.

- **Curvature inflation** (first model of inflation)

The higher-order curvature term leads to inflation.

Lagrangian: $f(R) = R + R^2/(6M^2)$ Starobinsky (1980)

- **"Old" inflation**

Inflation occurs due to the first-order phase transition of a vacuum.

Kazanas (1980), Sato (1981), Guth (1981)

- **Slow-roll inflation** Inflation is driven by the potential energy of a scalar field.

New, chaotic, power-law, hybrid, natural, extra-natural, eternal, D-term, F-term, brane, oscillating, tachyon, hill-top, KKLMMT, … (too many)

- **K-inflation** Inflation is driven by the kinetic energy of a scalar field.

Ghost condensate, DBI, Galileon,…

- **Inflation in extended theories of gravity.**

Brans-Dicke, non-minimal couplings, derivative couplings,….

Dark energy models

1. Modified matter models

- Quintessence: Acceleration driven by the potential energy $V(\phi)$ of a field ϕ

  Slow-roll inflation $\quad \mathcal{L} = X - V(\phi) \quad X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$

- K-essence: Acceleration driven by the kinetic energy X of a field ϕ

  K-inflation $\quad \mathcal{L} = K(\phi, X)$

  e.g. Dilatonic ghost condensate: $K = -X + ce^{\lambda\phi}X^2$

2. Modified gravity models

- $f(R)$ gravity: The Lagrangian is the function of a Ricci scalar R.

  Curvature inflation

- Scalar-tensor gravity: $\mathcal{L} = F(\phi)R + K(\phi, X)$

  Higgs inflation

- DGP model: Acceleration by the gravitational leakage to extra dimensions.
- Galileon gravity: The Lagrangian is constructed to satisfy the Galilean symmetry $\partial_\mu\phi \to \partial_\mu\phi + b_\mu$ in the flat spacetime.

  Galileon inflation

  Such as $X\Box\phi$

- Massive gravity

Most general single-field scalar-tensor theorr with second-order equations of motion: Horndeski's theory

$$S=\int d^4x\sqrt{-g}\,[G_2(\phi,X)-G_3(\phi,X)\Box\phi+\mathcal{L}_4+\mathcal{L}_5]$$

Horndeski (1974)
Deffayet et al (2011)
Charmousis et al (2011)
Kobayashi et al (2011)

$$\mathcal{L}_4=G_4(\phi,X)\,R+G_{4,X}\,[(\Box\phi)^2-(\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)]$$

$$\mathcal{L}_5=G_5(\phi,X)\,G_{\mu\nu}\,(\nabla^\mu\nabla^\nu\phi)-\frac{1}{6}G_{5,X}[(\Box\phi)^3-3(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)+2(\nabla^\mu\nabla_\alpha\phi)\,(\nabla^\alpha\nabla_\beta\phi)\,(\nabla^\beta\nabla_\mu\phi)]$$

This action covers most of the single-field inflation and dark energy models proposed in literature.

- LCDM: $G_2=-\Lambda,\quad G_3=0,\quad G_4=M_{\rm pl}^2/2,\quad G_5=0$
- Quintessence and K-essence: $G_2=G_2(\phi,X),\quad G_3=0,\quad G_4=M_{\rm pl}^2/2,\quad G_5=0$
- f(R) gravity and scalar-tensor gravity: $G_4=F(\phi),\quad G_3=0,\quad G_5=0$
- Galileon: $G_2=-c_2X,\quad G_3=c_3X/M^3,\quad G_4=M_{\rm pl}^2/2-c_4X^2/M^6,\quad G_5=3c_5X^2/M^9$
- Gauss-Bonnet coupling $\xi(\phi)\mathcal{G}$:

$$G_2=8\xi^{(4)}(\phi)X^2(3-\ln X),\quad G_3=4\xi^{(3)}(\phi)X(7-3\ln X)$$
$$G_4=4\xi^{(2)}(\phi)X(2-\ln X),\quad G_5=-4\xi^{(1)}(\phi)\ln X$$

Horndeski's paper in 1973

At the age of 25 when he was the PhD student of Lovelock, he wrote this valuable paper.

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Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space

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Received: 10 *July* 1973

Abstract

Lagrange scalar densities which are concomitants of a pseudo-Riemannian metric-tensor, a scalar field and their derivatives of arbitrary order are considered. The most general second-order Euler–Lagrange tensors derivable from such a Lagrangian in a four-dimensional space are constructed, and it is shown that these Euler–Lagrange tensors may be obtained from a Lagrangian which is at most of second order in the derivatives of the field functions.

Horndeski became an artist in 1981 when he saw Gogh's arts!

Horndeski was born in 1948 and he got the PhD degree in 1973.

Horndeski: The First Thirty Years

February 11, 2012 through March 3, 2012

Artist's Reception
6:00 – 8:00pm, Saturday, February 11, 2012

Before he became an artist, Mr. Horndeski was a tenured professor of applied mathematics at the University of Waterloo in Ontario, Canada.
While on sabbatical in the Netherlands in 1981,
he saw a van Gogh exhibition and was deeply moved.

"I was never that interested in art," he states.
"Then I stumbled onto van Gogh. I never knew art could be like that. I had always thought of it as very representational and not very interesting. But then I thought, 'This is something I eventually want to do.' When I saw van Gogh I was sure I could paint."

Horndeski: The First Three Years as a physicist
Paros (Greece), September 23-28, 2013

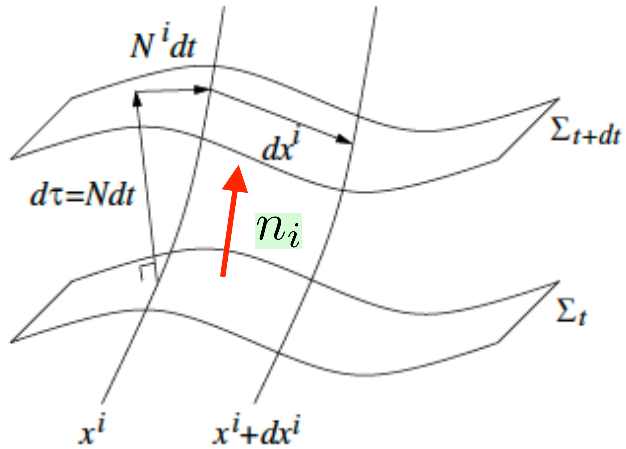
Relation between Horndeski's theory and effective field theory of inflation and dark energy

The Horndeski's theory belongs to a sub-class of effective field theory described by the action

$$S = \int d^4x\sqrt{-g}\, L(N, K, \mathcal{S}, \mathcal{R}, \mathcal{Z}; t)$$

where the 3+1 splitting in unitary gauge is performed with the ADM metric

$$ds^2 = -N^2dt^2 + h_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$



The Lagrangian depends on the Lapse N as well as the 4 scalar quantities

$$K \equiv K^{\mu}_{\ \mu}\,, \qquad \mathcal{S} \equiv K_{\mu\nu}K^{\mu\nu},$$

$$\mathcal{R} \equiv {}^{(3)}R \equiv {}^{(3)}R^{\mu}_{\mu}, \qquad \mathcal{Z} \equiv {}^{(3)}R^{\mu\nu}\,{}^{(3)}R_{\mu\nu}$$

$K_{\mu\nu}$ is the extrinsic curvature and
${}^{(3)}R$ is the 3-dimensional Ricci scalar.

Extrinsic curvature: $K_{ij} = n_{i;j}$

Expansion of the Lagrangian up to second order in the perturbations on the flat cosmological background

Gleyzes et al (2013)

$$L = \bar{L} - \dot{\mathcal{F}} - 3H\mathcal{F} + (\dot{\mathcal{F}} + L_N)\delta N + L_{\mathcal{R}}\delta\mathcal{R}$$

$$+ \left(\frac{1}{2}L_{NN} - \dot{\mathcal{F}}\right)\delta N^2 + L_{N\mathcal{R}}\delta N\delta\mathcal{R} + \cdots .$$

→ Up to first order

→ Second order terms

where $\mathcal{F} \equiv 2HL_{\mathcal{S}} + L_K$ $\quad$ $L_K = \partial L/\partial K$ etc.

The Lagrangian up to first order gives rise to the background equations

$$\bar{L} - \dot{\mathcal{F}} - 3H\mathcal{F} = 0, \qquad \dot{\mathcal{F}} + L_N = 0$$

We describe the spatial metric in terms of curvature perturbations, as

$$h_{ij} = a^2(t)e^{2\zeta}\delta_{ij}$$

ζ: curvature perturbations

The second order Lagrangian does not contain the terms (such as $(\nabla^2\zeta)^2$) which give rise to the equations of motion higher than the second order, provided that

$$4H^2L_{\mathcal{SS}} + 4HL_{\mathcal{S}K} + L_{KK} + 2L_{\mathcal{S}} = 0$$

$$2HL_{\mathcal{S}\mathcal{R}} + L_{K\mathcal{R}} = 0$$

$$4L_{\mathcal{RR}} + 3L_{\mathcal{Z}} = 0$$

Horndeski's theory should satisfy these conditions.

Second order action for cosmological perturbations

$$\mathcal{L}_2 = a^3 Q \left[\dot{\zeta}^2 - c_s^2 \frac{(\partial \zeta_i)^2}{a^2} \right]$$

where

$$Q = (L_N + L_{NN}/2 - 3HB - 6H^2 L_{\mathcal{S}})D^2 + 6L_{\mathcal{S}}$$

$$Qc_s^2 = 2\left[\frac{1}{a}\frac{d}{dt}(aM) - L_{\mathcal{R}}\right]$$

and

$$B = 2HL_{\mathcal{S}N} + L_{KN}, \quad D = 4L_{\mathcal{S}}/(B + 4HL_{\mathcal{S}}), \quad M = D(L_{\mathcal{R}} + L_{N\mathcal{R}})$$

The conditions for the avoidance of scalar ghosts and Laplacian instabilities are

$$Q > 0, \quad c_s^2 > 0$$

The curvature perturbation obeys the second order equation of motion

$$\frac{d}{dt}(a^3 Q \dot{\zeta}) - aQc_s^2 \partial^2 \zeta = 0$$

Effective field theory (EFF) language

Weinberg (2008)

Cheung et al (2008)

$$S=\int d^4x\sqrt{-g}\left[\frac{M_*^2}{2}fR-\Lambda-cg^{00}+\frac{M_2^4}{2}(\delta g^{00})^2-\frac{\bar{m}_1^3}{2}\delta K\delta g^{00}\right.$$
$$\left.-\frac{\bar{M}_2^2}{2}\delta K^2-\frac{\bar{M}_3^2}{2}\delta K^{\mu}{}_{\nu}\,\delta K_{\mu}{}^{\nu}+\frac{\mu_1^2}{2}{}^{(3)}\!R\delta g^{00}+\frac{\bar{m}_5}{2}{}^{(3)}\!R\delta K+\frac{\lambda_1}{2}{}^{(3)}\!R^2+\frac{\lambda_2}{2}{}^{(3)}\!R^{\mu}{}_{\nu}\,{}^{(3)}\!R_{\mu}{}^{\nu}\right]$$

where $g^{00}=-1/N^2$

The conditions for the absence of higher spatial derivatives translate to

$$\bar{M}_3^2=-\bar{M}_2^2,\quad \bar{m}_5=0,\quad 3\lambda_2=-8\lambda_1$$

Under these conditions the EFF Lagrangian with second order equations of motion at linear order reads

$$L=\frac{M_*^2}{2}f(t)R-\Lambda(t)-c(t)g^{00}+\frac{M_2^4(t)}{2}(\delta g^{00})^2-\frac{m_3^3(t)}{2}\delta K\delta g^{00}$$
$$-m_4^2(t)\left(\delta K^2-\delta K^{\mu}{}_{\nu}\,\delta K^{\nu}{}_{\mu}\right)+\frac{\tilde{m}_4^2(t)}{2}{}^{(3)}\!R\,\delta g^{00}\,,$$

This should cover the Horndeski's Lagrangian.

where

$$m_3^3\equiv\bar{m}_1^3\,,\qquad m_4^2\equiv\frac{1}{4}(\bar{M}_2^2-\bar{M}_3^2)\,,\qquad \tilde{m}_4^2\equiv\mu_1^2$$

In fact, the Horndeski's theory can be accommodated in the EFF Lagrangian

$$L = \frac{M_*^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} + \frac{M_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2} \delta K \delta g^{00} - m_4^2(t) \left(\delta K^2 - \delta K^{\mu}_{\ \nu} \delta K^{\nu}_{\ \mu}\right) + \frac{\tilde{m}_4^2(t)}{2} {}^{(3)}R \, \delta g^{00} ,$$

with the additional condition

$$m_4^2 = \tilde{m}_4^2$$

The Horndeski's theory automatically satisfies the other conditions for the avoidance of higher spatial derivatives:

$$\bar{M}_3^2 = -\bar{M}_2^2, \quad \bar{m}_5 = 0, \quad 3\lambda_2 = -8\lambda_1$$

When $m_4^2 \neq \tilde{m}_4^2$, higher-order derivatives are expected to appear beyond linear order.

Dictionary between EFF and Horndeski's theories

Gleyzes et al (2013)

In unitary gauge where the uniform field hypersurfaces are constant time hypersurfaces, the unit vector orthogonal to hypersurfaces is

$$n_\mu = -\phi_{;\mu}/\sqrt{2X} \qquad \text{where} \qquad X = -\phi^{;\mu}\phi_{;\mu}/2 = \dot{\phi}^2/(2N^2)$$

Using the property such as $K_{\mu\nu} = h^\sigma_\mu n_{\nu;\sigma}$, we can express the field derivatives in terms of three dimensional quantities, say

$$\Box\phi = -\sqrt{2X}K + \frac{\phi^{;\mu}}{2}\frac{X_{;\mu}}{X}$$

For example, one of the Horndeski's Lagrangian

$$L_4 = G_4(\phi, X)R + G_{4,X}[(\Box\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}]$$

can be expressed as

$$L_4 = G_4\mathcal{R} + (2XG_{4,X} - G_4)(K^2 - \mathcal{S}) - 2\sqrt{2X}G_{4,\phi}K$$

⇒ From this we can evaluate the EFF parameters like

$$m_4^2 = \frac{1}{2}\left(L_{\mathcal{S}} - 2L_{\mathcal{R}} - 2H^2L_{\mathcal{S}\mathcal{S}} - 2HL_{\mathcal{S}K} - \frac{1}{2}L_{KK}\right)$$

$$\tilde{m}_4^2 = L_{N\mathcal{R}}\ ,$$

$$m_4^2 = \tilde{m}_4^2 = -2XG_{4,X}$$

In summary, the Horndeski's theory corresponds to the EFF Lagrangian

$$L = \frac{M_*^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} + \frac{M_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2} \delta K \delta g^{00} - m_4^2(t) \left(\delta K^2 - \delta K^\mu{}_\nu \, \delta K^\nu{}_\mu\right) + \frac{\tilde{m}_4^2(t)}{2} \, {}^{(3)}R \, \delta g^{00} ,$$

with the functions

$$M_*^2 f = 2G_4 - 2XG_{5,\phi} - 2XG_{5,X}\ddot{\phi}$$

$$\Lambda = XG_{2,X} - G_2 + (\ddot{\phi} + 3H\dot{\phi})XG_{3,X} + \cdots$$

$$c = XG_{2,X} - (\ddot{\phi} - 3H\dot{\phi})XG_{3,X} - 2XG_{3,\phi} + \cdots$$

$$M_2^4 = X^2 G_{2,XX} + \frac{1}{2}(\ddot{\phi} + 3H\dot{\phi})XG_{3,X} + 3HX^2 G_{3,XX}\dot{\phi} - X^2 G_{3,\phi X} + \cdots$$

$$m_3^3 = 2\dot{\phi}XG_{3,X} + 2\dot{\phi}\ddot{\phi}G_{4,X} + \cdots$$

$$m_4^2 = \tilde{m}_4^2 = -2XG_{4,X} + 2XG_{5,\phi} - HXG_{5,X}\dot{\phi} + XG_{5,X}\ddot{\phi}$$

We only need the three variables f, Λ, c to describe the background evolution and the three additional variables M_2^4, m_3^3, m_4^2 to describe the linear perturbations.

Background equations of motion

In the presence of matter with the density ρ_m and the pressure P_m, we have

$$\Lambda + c = 3M_*^2(fH^2 + \dot{f}H) - \rho_m$$

$$\Lambda - c = M_*^2(2f\dot{H} + 3fH^2 + 2\dot{f}H + \ddot{f}) - P_m$$

These equations can be written in the forms

$$3M_{\rm pl}^2 H^2 = \rho_{\rm DE} + \rho_m$$

$$M_{\rm pl}^2(2\dot{H} + 3H^2) = -P_{\rm DE} - P_m$$

where

$$\left.\begin{aligned}\rho_{\rm DE} &= 3H^2(M_{\rm pl}^2 - M_*^2 f) - 3M_*^2\dot{f}H + c + \Lambda \\ P_{\rm DE} &= -(2\dot{H} + 3H^2)(M_{\rm pl}^2 - M_*^2 f) + M_*^2(2H\dot{f} + \ddot{f}) + c - \Lambda\end{aligned}\right\}$$

Satisfying the continuity equation

$$\dot{\rho}_{\rm DE} + 3H(\rho_{\rm DE} + P_{\rm DE}) = 0$$

The dark energy equation of state is given by

$$w_{\rm DE} = \frac{P_{\rm DE}}{\rho_{\rm DE}} = -1 - \frac{2\dot{H}(M_{\rm pl}^2 - M_*^2 f) - M_*^2(\ddot{f} - H\dot{f}) - 2c}{3H^2(M_{\rm pl}^2 - M_*^2 f) - 3M_*^2\dot{f}H + c + \Lambda}$$

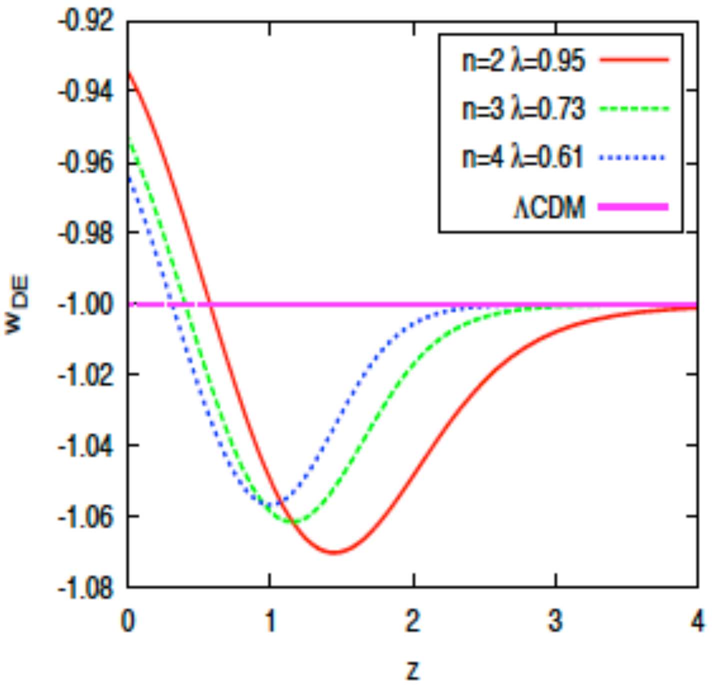
In modified gravity one can realize the phantom equation of state without having ghosts and instabilities.

Dark energy equation of state: modified gravity models

(1) f(R) gravity

Hu and Sawicki (2007)

$$f(R) = R - \lambda R_0 \frac{(R/R_0)^{2n}}{(R/R_0)^{2n}+1} \quad (n>0)$$

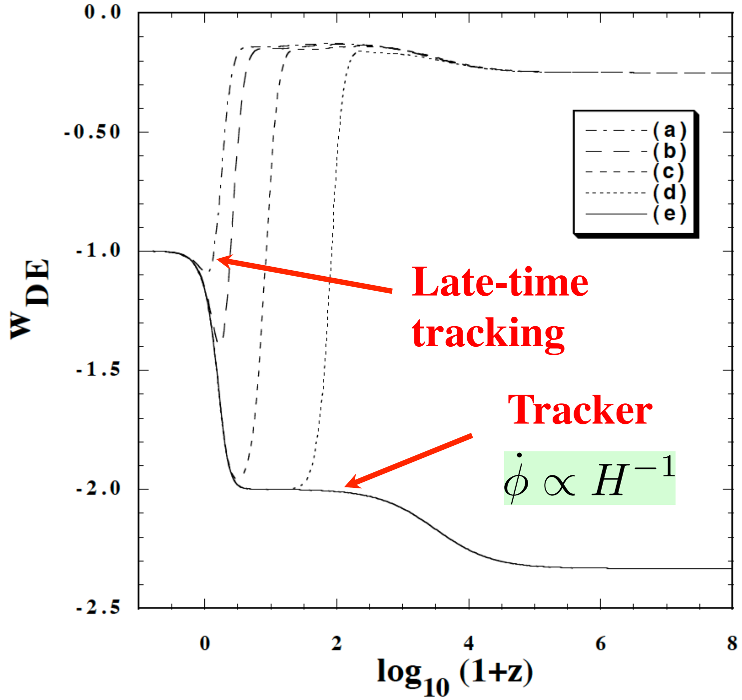


(2) Covariant Galileons

Deffayet et al (2009)

$G_2 = -c_2 X, \quad G_3 = c_3 X/M^3$

$G_4 = M_{\rm pl}^2 - c_4 X^2/M^6, \quad G_5 = 3c_5 X^2/M^9$

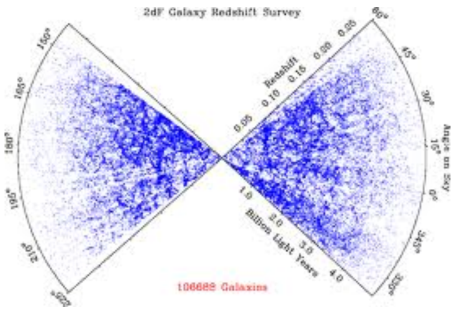


The equation of state $w_{\rm DE} < -1$ is a good signature to discriminate modified gravity models from the ΛCDM.

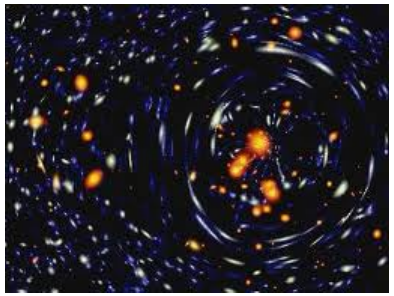
Discrimination between dark energy models from cosmological perturbations

The dark energy models can be further distinguished from the observations of large-scale structure, weak lensing, CMB (ISW effect) etc.

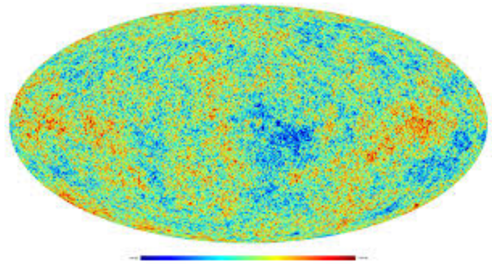
LSS



Weak lensing



CMB



Perturbed metric: $ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)\delta_{ij}dx^i dx^j$ (Newtonian gauge)

Non-relativistic matter: $\rho_m = \rho_m(t) + \delta\rho_m(t, \mathbf{x})$

with the four velocity $u^\mu = (1-\Psi, \nabla^i v)$

v is the rotational-free velocity potential.

Matter perturbations in the Horndeski's theory

$\delta \equiv \delta\rho_m/\rho_m$ and $\theta \equiv \nabla^2 v$ obey

$$\dot{\delta} = -\theta/a - 3\dot{\Phi}$$

$$\dot{\theta} = -H\theta + (k^2/a)\Psi$$

The growth rate of matter perturbations is related with the peculiar velocity.

We introduce the gauge-invariant density contrast: $\delta_m \equiv \delta + \frac{3aH}{k^2}\theta$

$$\ddot{\delta}_m + 2H\dot{\delta}_m + \frac{k^2}{a^2}\Psi = 3\left(\ddot{I} + 2H\dot{I}\right)$$

where $I \equiv (aH/k^2)\theta - \Phi$

The two gravitational potentials Φ and $-\Psi$ are generally different:

$$B_6\Phi + B_8\Psi = -B_7\delta\phi$$

There are other perturbation equations. See De Felice, Kobayashi, S.T. (2011).

where

$$B_6 = 4[G_4 - X(\ddot{\phi}G_{5,X} + G_{5,\phi})]$$

$$B_8 = 4[G_4 - 2XG_{4,X} - X(H\dot{\phi}G_{5,X} - G_{5,\phi})]$$

$$B_7 = -4G_{4,X}H\dot{\phi} - 4(G_{4,X} + 2XG_{4,XX})\ddot{\phi} + 4G_{4,\phi} - 8XG_{4,\phi X} + 4(G_{5,\phi} + XG_{5,\phi X})\ddot{\phi}$$
$$-4H[(G_{5,X} + XG_{5,XX})\ddot{\phi} - G_{5,\phi} + XG_{5,\phi X}]\dot{\phi} + 4X[G_{5,\phi\phi} - (H^2 + \dot{H})G_{5,X}]$$

In GR ($G_4 = M_{\rm pl}^2/2$) one has $B_6 = B_8 = 2M_{\rm pl}^2$ and $B_7 = 0$. $\Rightarrow$ $\Phi = -\Psi$

Quasi-static approximation on sub-horizon scales

De Felice, Kobayashi, S.T. (2011).

For the modes deep inside the Hubble radius ($k \gg aH$) we can employ the quasi-static approximation under which the dominant terms are those including k^2/a^2, δ_m, and $M^2 \equiv -K_{,\phi\phi}$.

See e.g., Starobinsky (1998), Boisseau et al (2000)

$$\ddot{\delta}_m + 2H\dot{\delta}_m + \frac{k^2}{a^2}\Psi \simeq 0 \quad \text{and} \quad \frac{k^2}{a^2}\Psi \simeq -4\pi G_{\rm eff}\rho_m\delta_m$$

where the effective gravitational coupling $G_{\rm eff}$ is

$$G_{\rm eff} = \frac{2M_{\rm pl}^2[(B_6D_9 - B_7^2)(k/a)^2 - B_6M^2]}{(A_6^2B_6 + B_8^2D_9 - 2A_6B_7B_8)(k/a)^2 - B_8^2M^2}G$$

Schematically

$$G_{\rm eff} = \frac{a_0(k/a)^2 + a_1}{b_0(k/a)^2 + b_1}$$

where

$$A_6 = -2XG_{3,X} - 4H(G_{4,X} + 2XG_{4,XX})\dot{\phi} + 2G_{4,\phi} + 4XG_{4,\phi X} + 4H(G_{5,\phi} + XG_{5,\phi X})\dot{\phi} - 2H^2X(3G_{5,X} + 2XG_{5,XX})$$

$$D_9 = -K_{,X} + \text{derivative terms of } G_3, G_4, G_5$$

In GR, $G_4 = M_{\rm pl}^2/2$, $B_6 = B_8 = 2M_{\rm pl}^2$, $A_6 = B_7 = 0$, $D_9 = -K_{,X}$ $\Rightarrow$ $G_{\rm eff} = G$

In the massive limit ($M^2 \to \infty$) with $B_6 \simeq B_8 \simeq 2M_{\rm pl}^2$ we also have $G_{\rm eff} \simeq G$

In the massless limit $M^2 \to 0$ we have

$$G_{\rm eff} = \frac{2M_{\rm pl}^2(B_6D_9 - B_7^2)}{A_6^2B_6 + B_8^2D_9 - 2A_6B_7B_8}G$$

$\Rightarrow$ The effect of modified gravity manifests itself.

e.g., $G_{\rm eff} = 4G/3$ in $f(R)$ gravity

More general theories based on the EFF of dark energy

$$L = \frac{M_*^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} + \frac{M_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2} \delta K \delta g^{00} - m_4^2(t) \left(\delta K^2 - \delta K^\mu{}_\nu \delta K^\nu{}_\mu\right) + \frac{\tilde{m}_4^2(t)}{2} {}^{(3)}R \, \delta g^{00} ,$$

This gives the second-order equations at linear order.

plus

$$L_{\rm h.s.d.} = - \bar{m}_4^2(t) \, \delta K^2 + \frac{\bar{m}_5(t)}{2} {}^{(3)}R \, \delta K + \frac{\bar{\lambda}(t)}{2} {}^{(3)}R^2$$

→ This gives the spatial derivatives higher than second order.

In the presence of non-relativistic matter, the effective gravitational coupling can be schematically expressed as

$$G_{\rm eff} = \frac{a_{-2}(k/a)^{-2} + a_0 + a_2 (k/a)^2 + a_4 (k/a)^4}{b_{-2}(k/a)^{-2} + b_0 + b_2 (k/a)^2}$$

⇒ $a_2 = a_4 = b_2 = 0$ in Horndeski's theory

Gleyzes et al (2013)

For the theories with $a_4 \neq 0$, $G_{\rm eff} \to a_4 (k/a)^2 / b_2$ in the small-scale limit, so such theories should be tightly constrained.

Constraints from large-scale structure

The galaxy perturbation δ_g is related with δ_m via the bias factor b, i.e., $\delta_g = b\delta_m$.

$\theta = \nabla^2 v$ is related with $f_m \equiv \dot{\delta_m}/(H\delta_m)$ via

$$\theta/(aH) \simeq -f_m \delta_m$$

The galaxy power spectrum in the redshift space can be modelled as

$$\mathcal{P}_g^s(\mathbf{k}) = \mathcal{P}_{gg}(\mathbf{k}) + 2\mu^2 \mathcal{P}_{g\theta}(\mathbf{k}) + \mu^4 \mathcal{P}_{\theta\theta}(\mathbf{k})$$

μ is the cosine of the angle of the $\boldsymbol{k}$ vector and along the line of sight.

The real space galaxy power spectrum: $(b\sigma_8)^2$

The cross power spectrum: $(b\sigma_8)(f_m\sigma_8)$

The real space velocity power spectrum: $(f_m\sigma_8)^2$

σ_8 is the rms mass fluctuations in spheres within the radius $8\,h^{-1}$ Mpc.

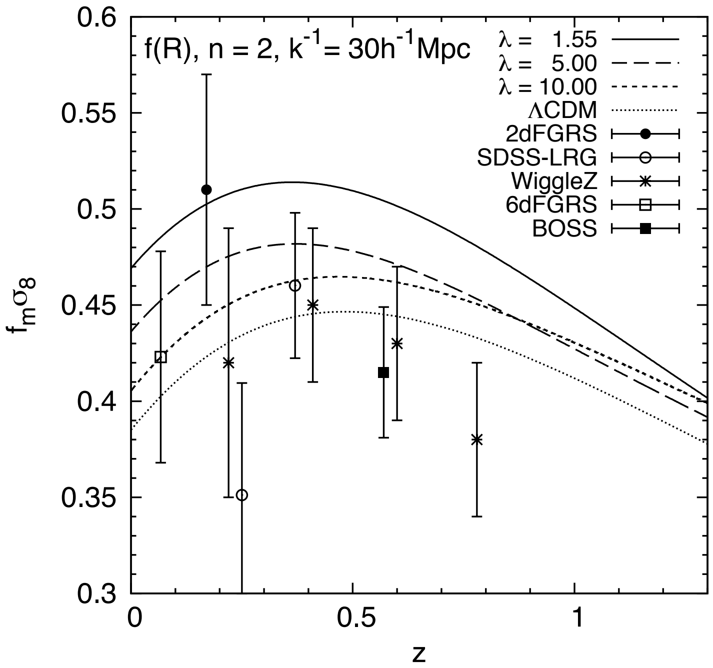
The redshift space distorsions are known as an additive component by observing $b\sigma_8$ and $f_m\sigma_8$.

Perturbation growth in two modified gravity models

Okada, Totani, S.T. (2012)

(1) f(R) gravity

$$f(R) = R - \lambda R_0 \frac{(R/R_0)^{2n}}{(R/R_0)^{2n}+1} \quad (n>0)$$

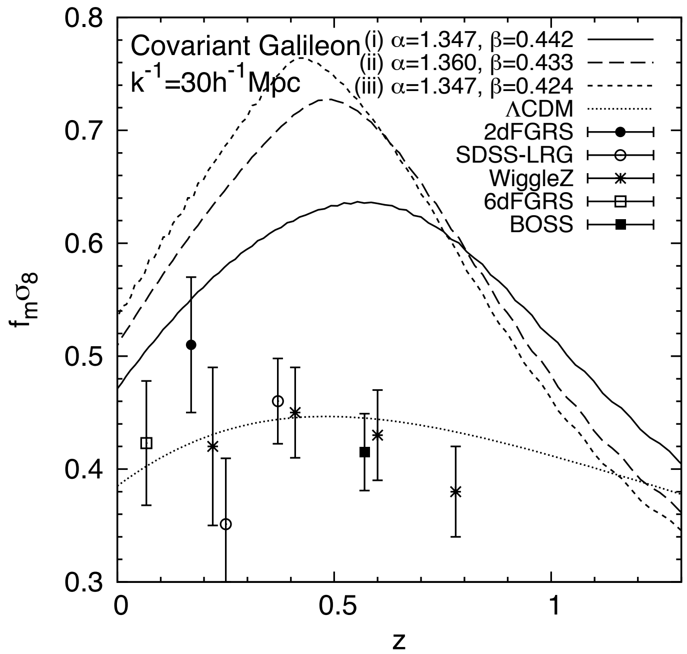


There are some allowed parameter spaces for larger values of n and λ.

(2) Covariant Galileons

$$G_2 = -c_2 X, \quad G_3 = c_3 X/M^3$$

$$G_4 = M_{\rm pl}^2 - c_4 X^2/M^6, \quad G_5 = 3c_5 X^2/M^9$$



The growth rate of matter perturbations is quite large, but there is still a viable parameter space if we do not assume the existence of late-time de Sitter attractor.

Neveu et al (2013)

Cosmological perturbations in general single-field inflation

In either EFF of inflation without higher derivatives or in Horndeski's theory, the second-order action for perturbations is

$$\mathcal{L}_2 = a^3 Q \left[\dot{\zeta}^2 - c_s^2 \frac{(\partial \zeta_i)^2}{a^2} \right] \quad \Longrightarrow \quad \frac{d}{dt}(a^3 Q \dot{\zeta}) - a Q c_s^2 \partial^2 \zeta = 0$$

The curvature perturbation is conserved after the Hubble radius crossing, by which we obtain the scalar power spectrum

$$\mathcal{P}_\zeta = \frac{H^2}{8\pi^2 Q c_s^3} \quad \Longrightarrow$$

The scalar spectral index: $n_s - 1 = \left. \frac{d \ln \mathcal{P}_\zeta}{d \ln k} \right|_{c_s k = aH}$

The second-order tensor Lagrangian is

$$\mathcal{L}_t = \sum_{\lambda = +, \times} a^3 Q_t \left[\dot{h}_\lambda^2 - \frac{c_t^2}{a^2} (\partial h_\lambda)^2 \right]$$

$$Q_t = \frac{1}{4}(M_*^2 f + 2m_4^2), \quad c_t^2 = \frac{M_*^2 f}{M_*^2 f + 2m_4^2}$$

in the language of EFF

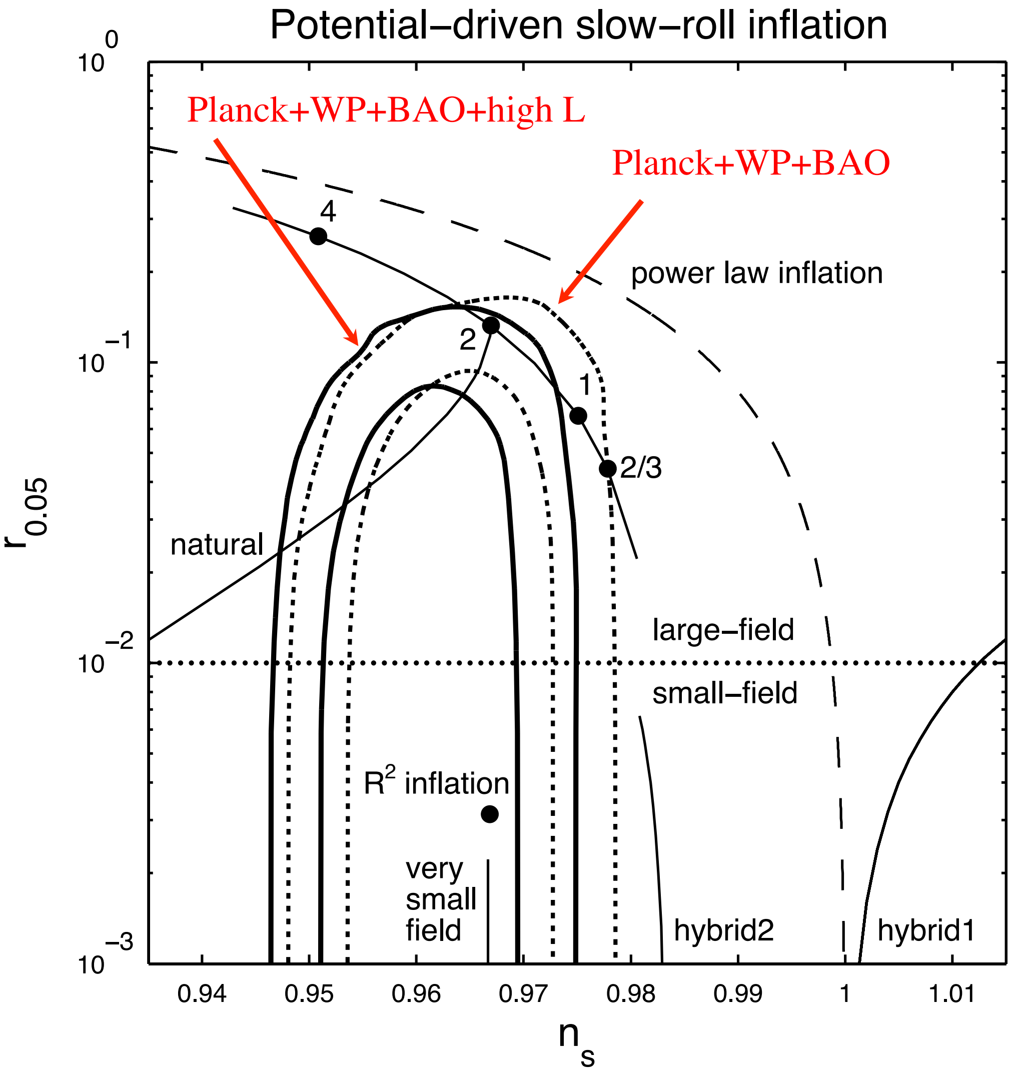
$$\mathcal{P}_t = \frac{H^2}{2\pi^2 Q_t c_t^3} \quad \Longrightarrow$$

The tensor-to-scalar ratio: $r = \frac{\mathcal{P}_t}{\mathcal{P}_\zeta} \simeq 16 c_s \epsilon_s$

where $\epsilon_s = \frac{Q c_s^2}{M_*^2 f + 2m_4^2}$

Planck constraints on potential-driven slow-roll inflation

S.T., J. Ohashi, S. Kuroyanagi, A. De Felice (2013)



$$n_s - 1 = -6\epsilon_V + 2\eta_V$$

$$r = 16\epsilon_V$$

where

$$\epsilon_V = \frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \qquad \eta_V = \frac{M_{\rm pl}^2 V_{,\phi\phi}}{V}$$

1. Chaotic inflation with $n = 2, 1, 2/3$ is under an observational pressure.
2. In natural inflation with the potential $V(\phi) = V_0[1+\cos(\phi/f)]$, the scale f is constrained to be $5.1M_{\rm pl} < f < 7.9M_{\rm pl}$ (68 % CL).
3. The hybrid models are outside the 95 % CL border.
4. Very small-field models such as $V(\phi) = V_0(1-e^{-\alpha\phi})^2$ are most favoured.

Higgs inflation

The Higgs potential $V(\phi) = \lambda(\phi^2 - v^2)^2/4$ ($v \sim 100$ GeV) can be accommodated for inflation?
The self coupling λ is about 0.1, but the WMAP normalization gives the constraint $\lambda \sim 10^{-13}$.

There are several ways to accommodate Higgs for inflation.

1. Nonminimal coupling: $\mathcal{L} = X - V(\phi) - \xi\phi^2 R/2$

Bezrukov and Shaposhnikov (2008)

$\lambda \approx 10^{-10}\xi^2$ for $|\xi| \gg 1$

In this limit $n_s = 1 - 2/N$ and $r = 12/N^2$ ($n_s \simeq 0.96$, $r \simeq 10^{-3}$).

[same as those in the model $f(R) = R + R^2/(6M^2)$]

2. Running kinetic coupling: $\mathcal{L} = \omega(\phi)X - V(\phi)$

Nakayama and Takahashi (2010),
De Felice, S.T., Elliston, Tavakol (2011)

The evolution of the field slows down with the coupling $\omega(\phi) = \phi^n$ or $\omega(\phi) = e^{\mu\phi}$.
This leads to the suppressed tensor-to-scalar ratio.

3. Derivative coupling to the Einstein tensor: $\mathcal{L} = X - V(\phi) + G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/(2M^2)$

Germani and Keghagias (2010)

$\lambda = 6 \times 10^{-32}(M_{\rm pl}/M)^4$ $\quad$ $n_s = 0.972$, $\quad r = 0.088$ $\quad$ ($N = 60$)

r is smaller than the value 0.262 in standard inflation

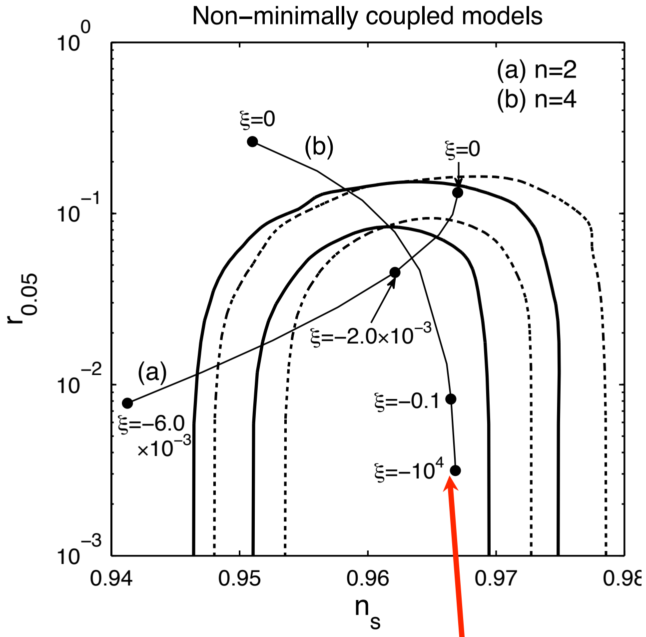
4. Galileon-like field self-interaction: $\mathcal{L} = X - V(\phi) - \epsilon\phi^n X\Box\phi/M^{3+n}$

Kamada, Kobayashi,
Yamaguchi, Yokoyama (2010)

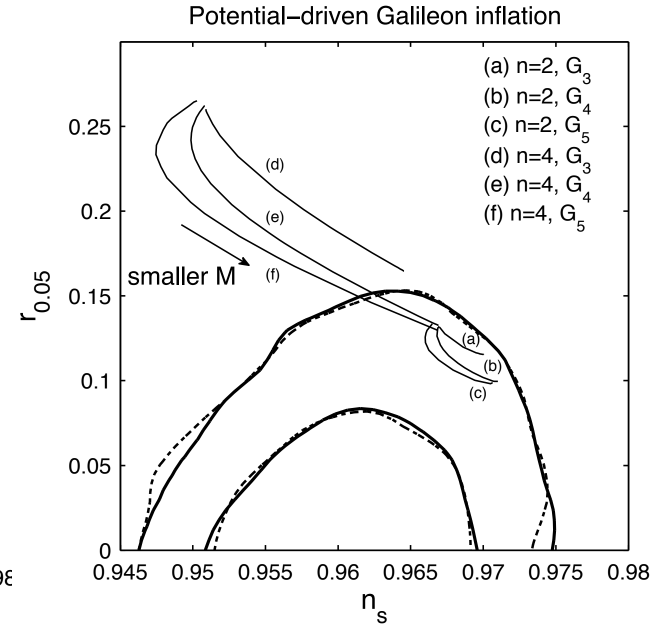
For $n = 0$ $\quad$ $\lambda = 2.4 \times 10^{-26}(M_{\rm pl}/M)^4$ $\quad$ $n_s = 0.965$, $\quad r = 0.164$ $\quad$ ($N = 60$)

Planck constraints on Higgs inflation

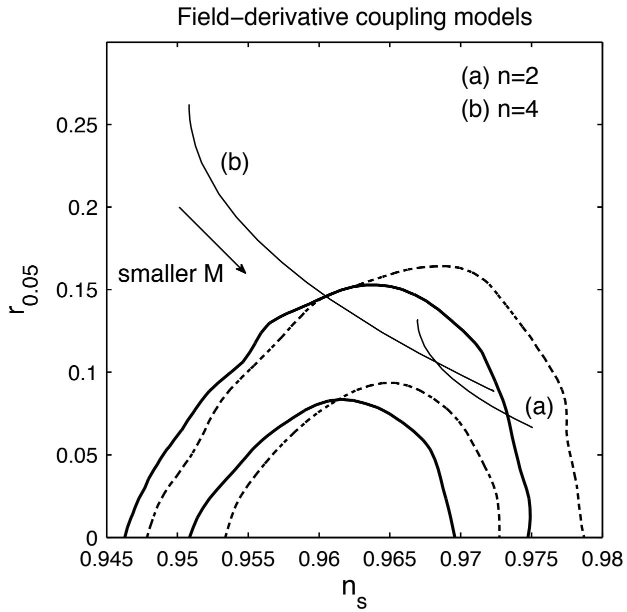
$\mathcal{L} = X - \lambda\phi^n/n - \xi\phi^2 R/2$



$\mathcal{L} = X - \lambda\phi^n/n + \text{Galileon terms}$



$\mathcal{L} = X - \lambda\phi^n/n + G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/(2M^2)$



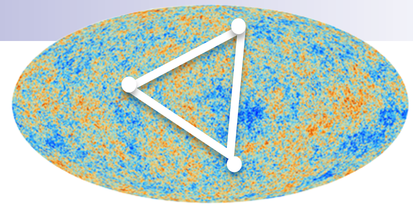
$n_s = 1 - 2/N$

$r = 12/N^2$

⇒ Same as the Starobinsky's model $f(R) = R + R^2/(6M^2)$

The potential which fits the data well has a form $V(\phi) = V_0(1 - e^{-\alpha\phi})^2$ in the Einstein frame.

Scalar non-Gaussianities



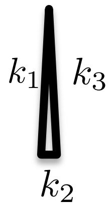
The three-point correlation functions of scalar no-Gaussainities in the Horndeski's theory or the EFF of inflation have been also derived.

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7 \delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)(\mathcal{P}_\zeta)^2 \frac{\mathcal{A}_\zeta(k_1,k_2,k_3)}{\prod_{i=1}^3 k_i^3}$$

$$f_{\rm NL} = \frac{10}{3}\frac{\mathcal{A}_\zeta}{\sum_{i=1}^3 k_i^3}$$

Planck constraints on the nonlinear estimator

1. Local shape → $f_{\rm NL}^{\rm local} = 2.7 \pm 5.8$



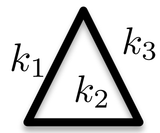
In the Horndeski's theory the local nonlinear parameter is

$$f_{\rm NL}^{\rm local} = \frac{5}{12}(1-n_s) \ll 1$$

→ Consistent with the Planck data as long as the slow-roll conditions are satisfied.

De Felice and S.T. (2013)

2. Equilateral shape → $f_{\rm NL}^{\rm equil} = -42 \pm 75$



In the Horndeski's theory or the EFF of inflation, the equilateral nonlinear parameter can be large for $c_s^2 \ll 1$.

Planck constraints on power-law k-inflation

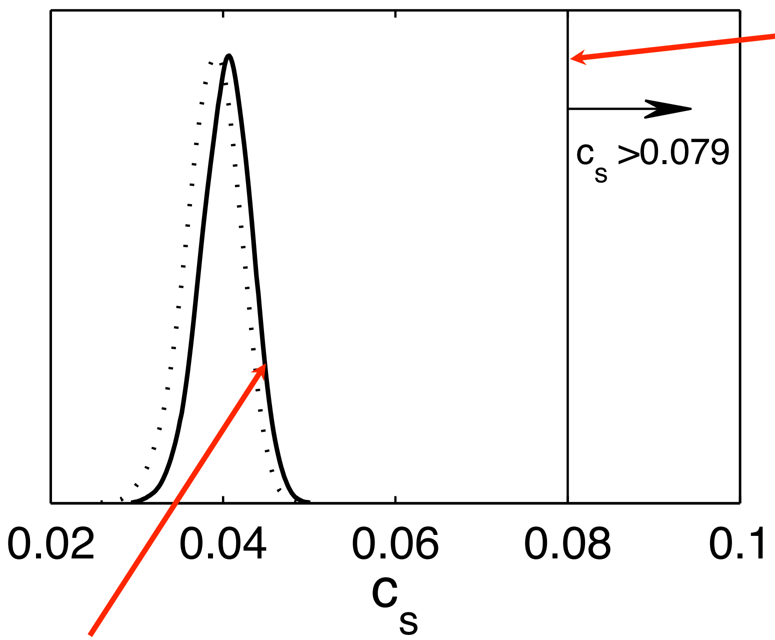
The power-law inflation can be realized by the Lagrangian

$$P = -X + ce^{\lambda\phi}X^2$$

(dilatonic ghost condensate)

In this case the observables are

$$n_s - 1 = n_t = -\frac{24c_s^2}{1+3c_s^2}\,, \qquad r = \frac{192c_s^3}{1+3c_s^2}\,, \qquad f_{\rm NL}^{\rm equil} = -\frac{85}{324}\frac{1}{c_s^2} + \frac{5}{81}c_s^2 + \frac{65}{324}$$



Constraints from NG

Constraints from n_s and r

The constraint on c_s from n_s and r is not compatible with the bound $c_s > 0.079$ coming from NG.

Generally the k-inflation models can be tightly constrained from the equilateral type NG.

Summary

1. We have shown the correspondence between the Horndenski's theory and effective field theory of inflation and dark energy.

2. The full linear perturbation equations including higher spatial derivatives were recently derived.

3. Dark energy models such as f(R) gravity and Galileons can be tightly constrained from the redshift-space distortions in the galaxy surveys.

4. Using the recent Planck data, our general formulas have been used to place observational constraints on many single-field inflationary models.

The Vainshtein screening in the Horndeski's theory was studied by Kase and me (also by Kimura et al).

➡ Please listen to the Kase's talk.