Higher spin black holes in 3D

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Beyond Einstein’s Theory of Gravity

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Motivation

- **Inconsistency of fundamental interacting fields with $s>2$**
  - Well-known claims in supergravity: $N \leq 8 \leftrightarrow$ (KK) $\leftrightarrow D \leq 11$
  - Rely on supposed inconsistency of $s>2$
  - No-go theorems (Aragone and Deser): massless higher spin fields, minimally coupled: gauge symmetries are incompatible with interactions (in particular with gravity)
- **Circumventing the obstacles (Vasiliev)**
  - Nonminimal couplings
  - $\Lambda \neq 0$
  - Whole tower of higher spin fields $s=0,2,3,4,\ldots,\infty$
- **3D case: special and simple**
  - Consistent truncation to $s = 2,3$
  - Standard field theory (CS action)
  - Black hole solutions endowed with spin-3 fields (Higher spin black holes)

Outline

- **Higher spin gravity in 3D**
  - GR with $\Lambda < 0$ as a Chern-Simons theory: sl(2,R) x sl(2,R)
  - CS theory for sl(3,R) x sl(3,R): Interacting massless fields of spins 2,3
  - Generalization of the Bekenstein-Hawking formula
  - Spin-3 analogue of the horizon area, bounds
- **Black holes**
  - BTZ described by gauge fields
  - Higher spin black holes
  - Quick derivation of black hole entropy in the canonical formalism
- **Comparison with different approaches**
  - Agreements & discrepancies (Controversy)
- **Solving puzzles**
  - Asymptotic behavior & hs ext. of conformal symmetry
  - Global charges (W-algebra)

Gravity in three dimensions

Einstein-Hilbert action:

$$I = \frac{1}{16\pi G}\int dx^3 \sqrt{-g}\,(R - 2\Lambda) \qquad \Lambda = -\frac{1}{l^2}$$

$$I = I_{CS}\left[A^+\right] - I_{CS}\left[A^-\right]$$

(up to a b.t.)

$$I_{CS}[A] = \frac{k}{4\pi}\int_M \left\langle AdA + \frac{2}{3}A^3 \right\rangle$$

Level : $k = \frac{l}{4G}$

Gravity in three dimensions

$$I = I_{CS}\left[A^{+}\right] - I_{CS}\left[A^{-}\right]$$

$$I_{CS}\left[A\right] = \frac{k}{4\pi}\int_{M}\left\langle AdA + \frac{2}{3}A^{3}\right\rangle \qquad k = \frac{l}{4G}$$

$$G = SL\left(2,\mathbb{R}\right)\otimes SL\left(2,\mathbb{R}\right)$$

$$L_{i}^{\pm} : i = -1, 0, 1$$

$$\langle\cdots\rangle = \frac{1}{4}\mathrm{tr}\left(\cdots\right) \qquad g_{\mu\nu} = \frac{1}{2}\mathrm{tr}\left(e_{\mu}e_{\nu}\right) : s = 2$$

$$A^{\pm} = \omega \pm \frac{e}{l}$$

Higher spin gravity in three dimensions

$$I = I_{CS}\left[A^+\right] - I_{CS}\left[A^-\right]$$

$$I_{CS}[A] = \frac{k}{4\pi}\int_M \left\langle AdA + \frac{2}{3}A^3 \right\rangle \qquad k = \frac{l}{4G}$$

$$G = SL(2,\mathbb{R}) \otimes SL(2,\mathbb{R}) \rightarrow G = SL(3,\mathbb{R}) \otimes SL(3,\mathbb{R})$$

$$L_i^{\pm} : i = -1, 0, 1 \quad \bigoplus \quad W_m^{\pm} : m = -2, -1, 0, 1, 2$$

$$\langle \cdots \rangle = \frac{1}{4}\mathrm{tr}\left(\cdots\right) \qquad g_{\mu\nu} = \frac{1}{2}\mathrm{tr}\left(e_\mu e_\nu\right) : s = 2$$

Ppal. embedding:

$$A^{\pm} = \omega \pm \frac{e}{l} \qquad \varphi_{\mu\nu\rho} = \frac{1}{3!}\mathrm{tr}\left(e_{(\mu}e_\nu e_{\rho)}\right) : s = 3$$

Consistent truncation of massless interacting (nonpropagating) fields of spin 2 and 3

Higher spin black hole entropy

$$S = \frac{A}{4G} \cos\left[\frac{1}{3}\arcsin\left(3^{3/2}\frac{\varphi_+}{A^3}\right)\right]$$

Reparametrization invariant integrals of the induced fields at the spacelike section of the horizon:

horizon area: $$A = \int_{\partial\Sigma_+} \left(g_{\mu\nu}\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\sigma}\right)^{1/2} d\sigma$$

spin-3 analogue: $$\varphi_+^{1/3} := \int_{\partial\Sigma_+} \left(\varphi_{\mu\nu\rho}\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\sigma}\frac{dx^\rho}{d\sigma}\right)^{1/3} d\sigma$$

Higher spin black hole entropy

$$S = \frac{A}{4G} \cos\left[\frac{1}{3} \arcsin\left(3^{3/2} \frac{\varphi_+}{A^3}\right)\right]$$

Bounds:

$$S \leq \frac{A}{4G}$$

$$|\varphi_+|^{1/3} \leq \frac{A}{\sqrt{3}}$$

BTZ Black hole

Spacetime metric (normal coordinates):

$$ds^2 = l^2 \left[d\rho^2 + \frac{2\pi}{k} \left(\mathcal{L}^+ (dx^+)^2 + \mathcal{L}^- (dx^-)^2 \right) - \left(e^{2\rho} + \frac{4\pi^2}{k^2} \mathcal{L}^+ \mathcal{L}^- e^{-2\rho} \right) dx^+ dx^- \right]$$

$$x^\pm = \frac{t}{l} \pm \theta$$

Event horizon: $$e^{2\rho_+} = \frac{2\pi}{k} \sqrt{\mathcal{L}^+ \mathcal{L}^-}$$

Mass & angular momentum: $$\mathcal{L}^\pm = \frac{1}{4\pi} (Ml \pm J)$$

Euclidean Black hole

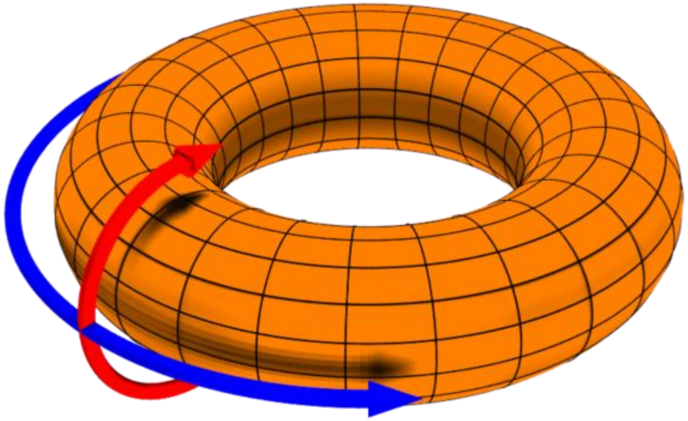
$$t \to i\tau$$

$$0 \leq \theta < 2\pi$$

$$0 \leq \tau < \beta$$

Conformal compactification:

solid torus



Noncontractible cycle (Blue: angle)

Contractible cycle (thermal) (Red: Euclidean time)

Smothness of the metric at ρ_+ **:**

Fixes the temperature β **and the angular velocity of the horizon** Ω
(Modular parameter of the torus) (Chemical potentials)

$$\delta S = \beta \delta M - \beta \Omega \delta J$$

$$S = \frac{A}{4G}$$

Black holes described by gauge fields

Locally flat connections : $F^{\pm} = 0$

Suitable gauge choice : $A^{\pm} = g_{\pm}^{-1} a^{\pm} g_{\pm} + g_{\pm}^{-1} d g_{\pm}$

Radial dependence : $g_{\pm} = e^{\pm \rho L_0^{\pm}}$

$$a^{\pm} = \pm \left(L_{\pm 1}^{\pm} - \frac{2\pi}{k} \mathcal{L}^{\pm} L_{\mp 1}^{\pm} \right) dx^{\pm}$$

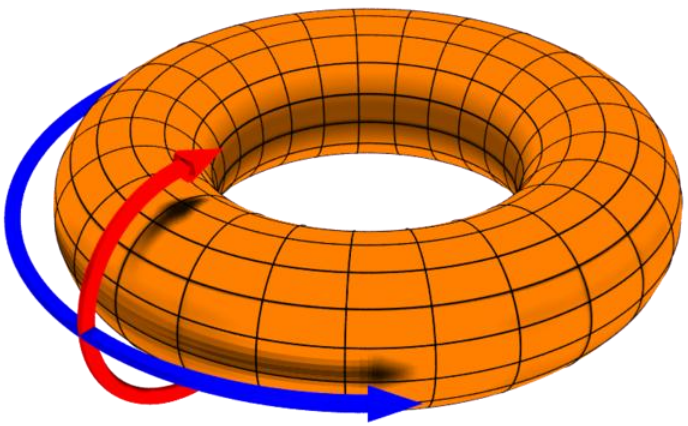
$$A^{\pm} = \omega \pm \frac{e}{l} \quad \longrightarrow \quad g_{\mu\nu} = \frac{1}{2} \mathrm{tr} \left(e_{\mu} e_{\nu} \right) : s = 2$$

How to extract the relevant information directly from the connection?

Holonomies

Gauge invariant quantities:

$$H_{\mathcal{C}} = P \exp\left(\int_{\mathcal{C}} A_\mu dx^\mu \right)$$

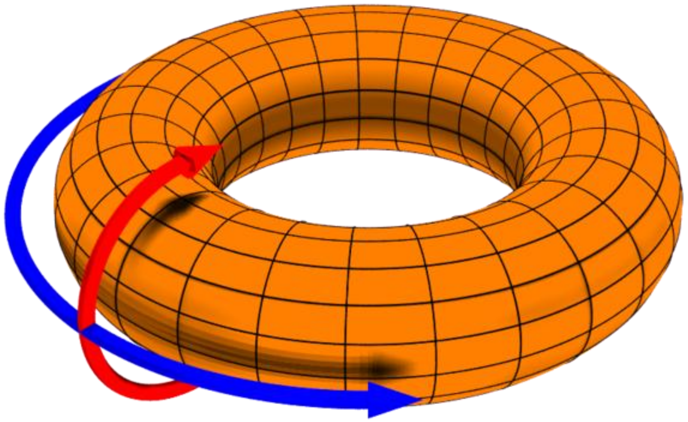


- **Noncontractible cycle: nontrivial holonomy (Event horizon)**

- **Thermal cycle: trivial holonomy (Smooth metric at the horizon)**

Noncontractible cycle

- **Blue: nontrivial holonomy**
  Captures the size of the event horizon



$$H_\theta = e^{2\pi A_\theta} = g^{-1} e^{2\pi a_\theta} g$$

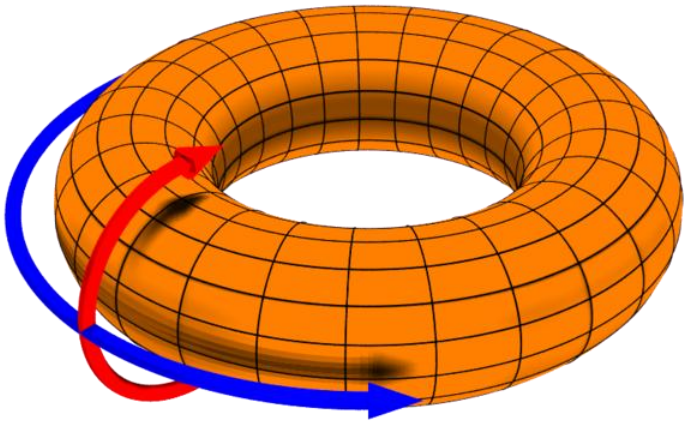
Holonomy is characterized by the eigenvalues of : $2\pi a_\theta \in sl(2,R)$

$$\lambda^2 = 2\pi^2 Tr(a_\theta^2)$$

Simplicity: Static case $\mathcal{L}^+ = \mathcal{L}^-$ $\quad Tr(a_\theta^2) = \mathcal{L}$

$$g_{\theta\theta}\big|_{\rho_+} = \left(\frac{A}{2\pi}\right)^2 = 2l^2 \mathrm{tr}\left(A_\theta^{+2}\right) = \frac{8\pi l^2 \mathcal{L}}{k}$$

Contractible (thermal) cycle



- **Red: trivial holonomy**

$$H_\tau = g^{-1} e^{i\beta a_\tau} g$$

$$H_\tau = -\mathbf{1} \to e^{i\beta a_t} = -\mathbf{1}$$

In the center of $sl(2,R)$

$$\beta^2 Tr(a_t^2) = 2n^2\pi^2$$

cycle winds once: $n = 1$

$$Tr(a_t^2) = \frac{2\pi^2}{\beta^2}$$

$$\beta = l\sqrt{\frac{\pi k}{2\mathcal{L}}}$$

Higher spin black holes

- **Exact higher spin black hole solution**

Gutperle, Kraus, arXiv: 1103.4304
Ammon, Gutperle, Kraus, Perlmutter, arXiv:1106.4788

$$A^{\pm} = g_{\pm}^{-1} a^{\pm} g_{\pm} + g_{\pm}^{-1} d g_{\pm} \ , \quad g_{\pm} = g_{\pm}(\rho) \in SL(3, \mathbb{R})_{\pm}$$

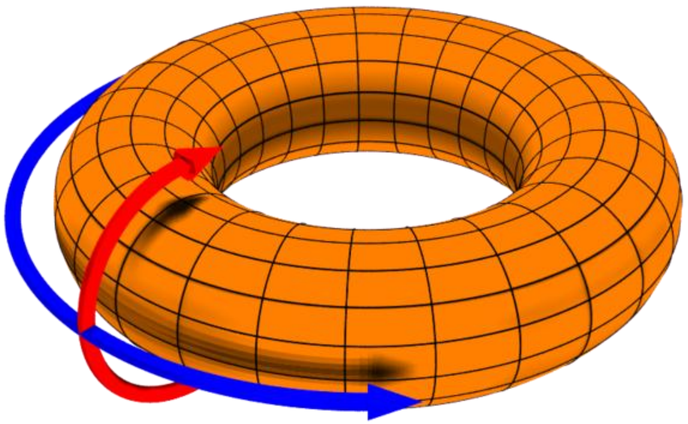
$$a^{\pm} = \pm \left(L_{\pm 1}^{\pm} - \frac{2\pi}{k} \mathcal{L} L_{\mp 1}^{\pm} \mp \frac{\pi}{2k} \mathcal{W} W_{\mp 2}^{\pm} \right) dx^{\pm}$$
$$+ \mu \left(W_{\pm 2}^{\pm} - \frac{4\pi}{k} \mathcal{L} W_{0}^{\pm} + \frac{4\pi^2}{k^2} \mathcal{L}^2 W_{\mp 2}^{\pm} \pm \frac{4\pi}{k} \mathcal{W} L_{\mp 1}^{\pm} \right) dx^{\mp}$$

asymptotically AdS_3 of radius $\tilde{l} = l/2$ $\qquad x^{\pm} = \frac{1}{\tilde{l}} t \pm \theta$

Contractible (thermal) cycle

$$H_\tau = g^{-1} e^{X} g$$

$$X = i\beta a_t$$



- **Trivial holonomy:**

$$\mathrm{tr}\left[X^3\right] = 0 \qquad \mathrm{tr}\left[X^2\right] + 8\pi^2 = 0$$

Fix the "chemical potential" μ , and the temperature

$$\beta = \frac{l}{4}\sqrt{\frac{\pi k}{2\mathcal{L}}\frac{(2C-3)}{(C-3)}}\left(1-\frac{3}{4C}\right)^{-\frac{1}{2}}$$

$$\frac{C-1}{C^{3/2}} = \sqrt{\frac{k}{32\pi\mathcal{L}^3}}\mathcal{W}$$

Higher spin black hole entropy

- **Strategy: working in the canonical ensemble**

$$\delta S = \beta \delta E$$

Just the variation of the total energy and the temperature

(Avoids the explicit computation of higher spin charges and their chemical potentials)

↕

Grand canonical:

$$\delta S = \beta(\delta M + \mu_1 \delta Q_1 + \mu_2 \delta Q_2 + \cdots)$$

Strategy

- **Canonical ensemble:** $$\delta S = \beta \delta E$$

Variation of the total energy is obtained from the canonical (Hamiltonian) approach:

$$\delta E : \quad H_T = N^{\perp} H_{\perp} + N^{i} H_{\iota} + \lambda^{I} \Phi_{I}$$

↓

$$\delta E = \delta Q\left(\partial_t\right) = \frac{k}{2\pi} \int_{\partial\Sigma} \left(\langle A_t^{+} \delta A_\theta^{+} \rangle - \langle A_t^{-} \delta A_\theta^{-} \rangle \right) d\theta$$

Variation of the total energy

$$A^{\pm} = g_{\pm}^{-1} a^{\pm} g_{\pm} + g_{\pm}^{-1} d g_{\pm} \ , \quad g_{\pm} = g_{\pm}(\rho) \in SL(3,\mathbb{R})_{\pm}$$

$$\delta E = \frac{k}{2\pi} \int_{\partial\Sigma} \left(\langle A_t^{+} \delta A_\theta^{+} \rangle - \langle A_t^{-} \delta A_\theta^{-} \rangle \right) d\theta$$
$$= \frac{k}{2\pi} \int_{\partial\Sigma} \left(\langle a_t^{+} \delta a_\theta^{+} \rangle - \langle a_t^{-} \delta a_\theta^{-} \rangle \right) d\theta$$

$$\delta E = \frac{8\pi}{l} \left[\delta\mathcal{L} - \frac{32\pi}{3k} \delta(\mathcal{L}^2 \mu^2) + \mu \delta\mathcal{W} + 3\mathcal{W} \delta\mu \right]$$

(Not an exact differential)

Pérez, Tempo, Troncoso, arXiv:1207.2844

Higher spin black hole entropy

$$\delta E = \frac{8\pi}{l}\frac{(C-3)}{(2C-3)^2}\left[(4C-3)\,\delta\mathcal{L} - \frac{9}{C}\frac{(2C-1)}{(2C-3)}\mathcal{L}\delta C\right]$$

(Not an exact differential)

$$\beta = \frac{l}{4}\sqrt{\frac{\pi k}{2\mathcal{L}}\frac{(2C-3)}{(C-3)}}\left(1-\frac{3}{4C}\right)^{-\frac{1}{2}}$$

$$\delta S = \beta\delta E = \delta\left[4\pi\sqrt{2\pi k\mathcal{L}}\left(1-\frac{3}{2C}\right)^{-1}\sqrt{1-\frac{3}{4C}}\right]$$

↓

- **Entropy**

$$S = 4\pi\sqrt{2\pi k\mathcal{L}}\left(1-\frac{3}{2C}\right)^{-1}\sqrt{1-\frac{3}{4C}}$$

Generalized Bekenstein-Haking formula

$$S = 4\pi\sqrt{2\pi k\mathcal{L}}\left(1-\frac{3}{2C}\right)^{-1}\sqrt{1-\frac{3}{4C}}$$

$\Updownarrow$

$$g_{\theta\theta}\big|_{\rho_+} = \left(\frac{A}{2\pi}\right)^2 = \frac{l^2}{2}\mathrm{tr}\left(A_\theta^{+2}\right) = \frac{8\pi l^2}{k}\frac{4C^3+9C^2-36C+27}{C\left(2C-3\right)^2}\mathcal{L}$$

$$\varphi_{\theta\theta\theta}\big|_{\rho_+} = \frac{\varphi_+}{(2\pi)^3} = \frac{l^3}{6}\mathrm{tr}\left(A_\theta^{+3}\right) = -\left(\frac{8\pi l^2\mathcal{L}}{kC}\right)^{3/2}\frac{(C-1)\left(4C^2-3C+9\right)}{2C-3}$$

$\Updownarrow$

$$S = \frac{A}{4G}\cos\left[\frac{1}{3}\arcsin\left(3^{3/2}\frac{\varphi_+}{A^3}\right)\right]$$

Remarks

$$S = \frac{A}{4G} \cos\left[\frac{1}{3}\arcsin\left(3^{3/2}\frac{\varphi_+}{A^3}\right)\right]$$

Invariant under proper gauge transformations $\delta S = \beta \delta E$

Static case: both,

$$A^2 = (2\pi)^2 g_{\theta\theta}\big|_{\rho_+} = 8\pi^2 l^2 \mathrm{tr}\left(A_\theta^2\right)$$

$$\varphi_+ = (2\pi)^3 \varphi_{\theta\theta\theta}\big|_{\rho_+} = \frac{32}{3}\pi^3 l^3 \mathrm{tr}\left(A_\theta^3\right)$$

are manifestly invariant

(invariants that characterize the holonomy around the noncontractible cycle)

Pérez, Tempo, Troncoso, arXiv:1301.0847

Comparison with different approaches

- **Agreements**

$$S = 4\pi\sqrt{2\pi k\mathcal{L}}\left(1-\frac{3}{2C}\right)^{-1}\sqrt{1-\frac{3}{4C}}$$

Weak spin-3 field expansion,
in terms of the original variables:

$$S = 4\pi\sqrt{2\pi k\mathcal{L}}\left[1+\frac{9k}{256\pi}\left(\frac{\mathcal{W}^2}{\mathcal{L}^3}\right)+\mathcal{O}\left(\frac{\mathcal{W}^2}{\mathcal{L}^3}\right)^2\right]$$

Full agreement with the result of
Campoleoni, Fredenhagen, Pfenninger, Thiesen, arXiv: 1208.1851

Remarks

- **Static circularly symmetric black holes**

Event horizon at $\rho = \rho_+$ **:** $A = 2\pi\sqrt{g_{\theta\theta}}\Big|_{\rho_+}$

$$\varphi_+ = (2\pi)^3 \, \varphi_{\theta\theta\theta}\Big|_{\rho_+}$$

$$S = \frac{A}{4G}\cos\left[\frac{1}{3}\arcsin\left(3^{3/2}\frac{\varphi_+}{A^3}\right)\right]$$

↓

$$S = \frac{A}{4G}\cos\left[\frac{1}{3}\arcsin\left(3^{3/2}\frac{\varphi_{\theta\theta\theta}}{g_{\theta\theta}^{3/2}}\right)\right]\Bigg|_{\rho_+}$$

Remarks

- **Static circularly symmetric black holes**

$$S = \frac{A}{4G} \cos\left[\frac{1}{3}\arcsin\left(3^{3/2}\frac{\varphi_{\theta\theta\theta}}{g_{\theta\theta}^{3/2}}\right)\right]\Bigg|_{\rho_+}$$

Weak spin-3 field limit,
S expands as :

$$S = \frac{A}{4G}\left(1 - \frac{3}{2}\left(g^{\theta\theta}\right)^{3}\varphi_{\theta\theta\theta}^{2} + \mathcal{O}\left(\varphi^{4}\right)\right)\Bigg|_{\rho_+}$$

Full agreement with
Campoleoni, Fredenhagen, Pfenninger, Thiesen, arXiv: 1208.1851

Action expressed by the metric and the perturbative expansion of the spin-3 field up to quadratic order. S was found through Wald's formula

Agreement with nonperturbative approaches

$$S = 4\pi\sqrt{2\pi k \mathcal{L}}\left(1 - \frac{3}{2C}\right)^{-1}\sqrt{1 - \frac{3}{4C}}$$

- de Boer, Jottar, arXiv:1302.0816

  Euclidean action, entropy in terms of eigenvalues A (H)

- Compère, Song, arXiv:1306.0014
- Compère, Jottar, Song, arXiv:1308.2175

  Asymptotic symmetries & global charges (Agreement with E)

- de Boer, Jottar, arXiv:1306.4347
- Ammon, Castro, Iqbal, arXiv:1306.4338

  Different approaches based on entanglement entropy

- Li, Lin, Wang, arXiv:1308.2959

  Modular invariance of the partition function

Comparison with different approaches

- **Disagreements**

$$S = 4\pi\sqrt{2\pi k \mathcal{L}}\left(1 - \frac{3}{2C}\right)^{-1}\sqrt{1 - \frac{3}{4C}}$$

- **Holographic & CFT (holomorphic approaches)**

$$\tilde{S} = 4\pi\sqrt{2\pi k \mathcal{L}}\sqrt{1 - \frac{3}{4C}}$$

Disagreement :

$$S = \tilde{S}\left(1 - \frac{3}{2C}\right)^{-1}$$

Comparison with different approaches

$$\tilde{S} = 4\pi\sqrt{2\pi k\mathcal{L}}\sqrt{1-\frac{3}{4C}}$$

- Gutperle, Kraus, arXiv: 1103.4304
- Ammon, Gutperle, Kraus, Perlmutter, arXiv:1106.4788

Holographic computation: Field eqs. seen as Ward Identities, global charges identified with L,W,
Entropy: thermodynamic integrability conds. coincide with trivial holonomy conditions

- Gaberdiel, Hartman, Jin, arXiv:1203.0015

Computation directly performed in the dual theory (extended conformal symmetry) in 2D
Modular invariance, no ref. to holonomies

- Kraus, Ugajin, arXiv:1302.1583

Conical singularity approach (along a non contractible cycle).
Evaluating its contribution to the action

Discrepancy

Different approaches, but a common root:

mismatch in δE :

inherited by the entropy through $\delta E = T\delta S$

Higher spin black hole:
Relaxed asymptotic behavior as compared with

Henneaux, Rey, arXiv:1008.4579
Campoleoni, Fredenhagen, Pfenninger, Thiesen, arXiv:1008.4744

$$a^{\pm} = \pm\left(L_{\pm1}^{\pm} - \frac{2\pi}{k}\mathcal{L}^{\pm}L_{\mp1}^{\pm} \mp \frac{\pi}{2k}\mathcal{W}^{\pm}W_{\mp2}^{\pm}\right)dx^{\pm}$$

Remarks

$$a^{\pm} = \pm \left(L^{\pm}_{\pm 1} - \frac{2\pi}{k} \mathcal{L}^{\pm} L^{\pm}_{\mp 1} \mp \frac{\pi}{2k} \mathcal{W}^{\pm} W^{\pm}_{\mp 2} \right) dx^{\pm}$$

$$L^{\pm}_{0} = Q\left(\partial_{\pm}\right) = \int \mathcal{L}^{\pm} d\phi$$

$$E = Q\left(\partial_{t}\right) = \frac{1}{l} \int \left(\mathcal{L}^{+} + \mathcal{L}^{-}\right) d\phi$$

These expressions do not apply for the higher spin black hole:

$$a^{\pm}_{bh} = \pm \left(L^{\pm}_{\pm 1} - \frac{2\pi}{k} \mathcal{L} L^{\pm}_{\mp 1} \mp \frac{\pi}{2k} \mathcal{W} W^{\pm}_{\mp 2} \right) dx^{\pm}$$

$$+ \mu \left(W^{\pm}_{\pm 2} - \frac{4\pi}{k} \mathcal{L} W^{\pm}_{0} + \frac{4\pi^{2}}{k^{2}} \mathcal{L}^{2} W^{\pm}_{\mp 2} \pm \frac{4\pi}{k} \mathcal{W} L^{\pm}_{\mp 1} \right) dx^{\mp}$$

Energy has to be computed from scratch

Remarks

For the higher spin black hole:

$$\delta E = \frac{8\pi}{l}\left[\delta\mathcal{L} - \frac{32\pi}{3k}\delta(\mathcal{L}^2\mu^2) + \mu\delta\mathcal{W} + 3\mathcal{W}\delta\mu\right]$$

do not depend linearly on the deviation of the fields w.r.t. to the reference background

Additional nonlinear contributions cannot be neglected even in the weak spin-3 field limit.

$$\delta E \neq \frac{8\pi}{l}\left[\delta\mathcal{L} + \mu\delta\mathcal{W}\right]$$

$$T\delta S \neq T\delta\tilde{S}$$

$$\delta S \neq \delta\tilde{S}$$

L and W are not "the real" global charges

Solving the puzzle

- **Chemical potentials & compatibility with W-algebra**

Standard boundary conditions (No chemical potentials included)

$$a^{\pm} = \pm \left(L^{\pm}_{\pm 1} - \frac{2\pi}{k} \mathcal{L}^{\pm} L^{\pm}_{\mp 1} \mp \frac{\pi}{2k} \mathcal{W}^{\pm} W^{\pm}_{\mp 2} \right) dx^{\pm}$$

Fixed t slice (dynamical fields):

$$a^{\pm}(t = \text{const}) = \left(L^{\pm}_{\pm 1} - \frac{2\pi}{k} \mathcal{L}_{\pm} L^{\pm}_{\mp 1} - \frac{\pi}{2k} \mathcal{W}_{\pm} W^{\pm}_{\mp 2} \right) d\theta$$

$$\begin{aligned}\Lambda^{\pm}(\varepsilon_{\pm}, \chi_{\pm}) = {} & \varepsilon_{\pm} L^{\pm}_{\pm 1} + \chi_{\pm} W^{\pm}_{\pm 2} \mp \varepsilon'_{\pm} L^{\pm}_{0} \mp \chi'_{\pm} W^{\pm}_{\pm 1} + \frac{1}{2}\left(\varepsilon''_{\pm} - \frac{4\pi}{k} \varepsilon_{\pm} \mathcal{L}_{\pm} + \frac{8\pi}{k} \mathcal{W}_{\pm} \chi_{\pm} \right) L^{\pm}_{\mp 1} \\ & - \left(\frac{\pi}{2k} \mathcal{W}_{\pm} \varepsilon_{\pm} + \frac{7\pi}{6k} \mathcal{L}'_{\pm} \chi'_{\pm} + \frac{\pi}{3k} \chi_{\pm} \mathcal{L}''_{\pm} + \frac{4\pi}{3k} \mathcal{L}_{\pm} \chi''_{\pm} - \frac{4\pi^2}{k^2} \mathcal{L}^2_{\pm} \chi_{\pm} - \frac{1}{24} \chi''''_{\pm} \right) W^{\pm}_{\mp 2} \\ & + \frac{1}{2}\left(\chi''_{\pm} - \frac{8\pi}{k} \mathcal{L}_{\pm} \chi_{\pm} \right) W^{\pm}_{0} \mp \frac{1}{6}\left(\chi'''_{\pm} - \frac{8\pi}{k} \chi_{\pm} \mathcal{L}'_{\pm} - \frac{20\pi}{k} \mathcal{L}_{\pm} \chi'_{\pm} \right) W^{\pm}_{\mp 1} \,,\end{aligned}$$

Solving the puzzle

$$a^{\pm} = \pm \left(L^{\pm}_{\pm 1} - \frac{2\pi}{k} \mathcal{L}^{\pm} L^{\pm}_{\mp 1} \mp \frac{\pi}{2k} \mathcal{W}^{\pm} W^{\pm}_{\mp 2} \right) dx^{\pm}$$

Fixed t slice (dynamical fields):

$$a^{\pm}(t = \mathrm{const}) = \left(L^{\pm}_{\pm 1} - \frac{2\pi}{k} \mathcal{L}_{\pm} L^{\pm}_{\mp 1} - \frac{\pi}{2k} \mathcal{W}_{\pm} W^{\pm}_{\mp 2} \right) d\theta$$

$$\delta \mathcal{L}_{\pm} = \varepsilon_{\pm} \mathcal{L}'_{\pm} + 2 \mathcal{L}_{\pm} \varepsilon'_{\pm} - \frac{k}{4\pi} \varepsilon'''_{\pm} - 2 \chi_{\pm} \mathcal{W}'_{\pm} - 3 \mathcal{W}_{\pm} \chi'_{\pm} \ ,$$

$$\delta \mathcal{W}_{\pm} = \varepsilon_{\pm} \mathcal{W}'_{\pm} + 3 \mathcal{W}_{\pm} \varepsilon'_{\pm} - \frac{64\pi}{3k} \mathcal{L}^2_{\pm} \chi'_{\pm} + 3 \chi'_{\pm} \mathcal{L}''_{\pm} + 5 \mathcal{L}'_{\pm} \chi''_{\pm} + \frac{2}{3} \chi_{\pm} \mathcal{L}'''_{\pm} - \frac{k}{12\pi} \chi'''''_{\pm}$$
$$- \frac{64\pi}{3k} \left(\chi_{\pm} \mathcal{L}'_{\pm} - \frac{5k}{32\pi} \chi'''_{\pm} \right) \mathcal{L}_{\pm} \ ,$$

Conserved charges fulfill the W-algebra

$$Q\left[\varepsilon, \chi\right] = -\int \left(\varepsilon \mathcal{L} - \chi \mathcal{W} \right) d\phi$$

Solving the puzzle

- **Including chemical potentials: compatibility with W-algebra**

Henneaux, Pérez, Tempo, Troncoso, arXiv:1309.4362

$$a^{\pm} = \pm\left(L^{\pm}_{\pm 1} - \frac{2\pi}{k}\mathcal{L}_{\pm}L^{\pm}_{\mp 1} - \frac{\pi}{2k}\mathcal{W}_{\pm}W^{\pm}_{\mp 2}\right)dx^{\pm} \pm \frac{1}{l}\Lambda^{\pm}(\nu_{\pm},\mu_{\pm})dt$$

- **This set includes a "W3 black hole": Carrying spin-3 charge**

• Same asymptotic symmerty algebra (W_3) (a_θ : the same in all slices) (Lagrange multipliers belong to the allowed class of gauge parameters

• L and W fulfill the same Poisson-Dirac algebra: same c, indep. of the chemical potentials. Generators depend only on the canonical variables and not on the Lagrange multipliers

• Extension to sl(N,R) or hs(λ) is straightforward

• The previous proposal modifies a_θ in a way that it is incompatible with $W_3{}^2$: Asymptotic symmetries correspond to $W_3{}^2$ (Diagonal embedding of sl(2,R) into sl(3,R): No higher spin charge !)