Cosmology with nonminimal kinetic coupling and a power-law potential

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Introduction

- Theories with **nonminimal kinetic coupling** of a scalar field and gravity belong to a class of the most general scalar-tensor theories. In the context of inflationary cosmology these models were proposed to use by **Amendola in 1993.**
- In theories of this type in the cosmological action there are combinations of various curvature tensor componets and derivatives of a scalar field $R\varphi_{,\mu}\varphi^{,\mu}$, $R_{\mu\nu}\varphi^{,\mu}\varphi^{,\nu}$ and others.
- The Lagrangian giving second-order equations of motion was dirived by **Horndesky in 1974.**
- The models with nonminimal kinetic coupling can discribe **the inflationary stage** in the early Universe and its **late time accelerated expansion**.

Purpose of the work

The purpose of this work was the investigation of cosmological dynamics in the model with nonminimal kinetic coupling of a scalar field and gravity with the Lagrangian of the form

$$L=\frac{1}{2}\sqrt{-g}\left(m_{Pl}R-(g^{\mu\nu}-\kappa G^{\mu\nu})\varphi_{,\mu}\varphi_{,\nu}-2V(\varphi)\right),$$

where $G^{\mu\nu}=R^{\mu\nu}-\frac{1}{2}Rg^{\mu\nu}$ – the Einstein tensor,

$$V(\varphi)=V_0\varphi^N,$$

N – real number.

Method of the investigation

For the derivation of exact cosmological solutions and the investigation of stability of them **the dynamical systems theory** can effectivly help that was written in the book «Dynamical systems in cosmology» of Wainwright and Ellis, 1997.

Scheme of the method

- Introduction of new variables $(H, \varphi, \ldots) \rightarrow (x, y, \ldots)$

$$\begin{cases} x' = f_1(x, y, \ldots) \\ y' = f_2(x, y, \ldots) \\ \ldots\ldots\ldots\ldots\ldots\ldots \end{cases}$$

- Finding of stationary point

$$\begin{cases} f_1 = 0 \\ f_2 = 0 \\ \ldots\ldots \end{cases} \Rightarrow \begin{matrix} (x_{stat1}, y_{stat1}, \ldots) \\ (x_{stat2}, y_{stat2}, \ldots) \\ \ldots\ldots\ldots\ldots\ldots \end{matrix}$$

- Investigation of a stability of stationary points

$$\begin{pmatrix} (\delta x)' \\ (\delta y)' \\ \ldots \end{pmatrix} = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & ,,, \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & ,,, \\ \ldots & \ldots & \ldots \end{pmatrix} \begin{pmatrix} \delta x \\ \delta y \\ \ldots \end{pmatrix} \Rightarrow \lambda_1, \lambda_2, \lambda_3$$

eigenvalues

The metrics was used

$$ds^2 = -dt^2 + a^2(t)\,d^2 l \quad ,$$

then Ricci scalar

$$R = 6\left(2\mathrm{H}^2 + \dot{H}\right) \quad .$$

Planck units were chosen

$$\mathrm{h} = \mathrm{c} = 8\Pi G = 1.$$

Main equations

By varying the action with the named Lagrangian the equations of gravitational and scalar fields are got

$$3H^2 m_{Pl} = \frac{1}{2}\dot{\varphi}^2(1-9\kappa H^2)+V(\varphi) \quad ,(1)$$

$$R\, m_{Pl} = -\dot{\varphi}^2+4V(\varphi)-\kappa\left(3\dot{\varphi}^2 H^2+\frac{\dot{\varphi}^2}{2}R+6H\dot{\varphi}\ddot{\varphi}\right),(2)$$

$$(\ddot{\varphi}+3H\dot{\varphi})(1-3\kappa H^2)-6\kappa H\dot{H}\dot{\varphi}+V'(\varphi)=0 \quad ,(3)$$

where $m_{Pl}=\frac{1}{8\Pi G}=1$.

Dimensionless variables

We introduce new variables and the parameter:

$$x = \frac{\dot{\varphi}^2}{6H^2(1+\kappa\dot{\varphi}^2)} \quad ,$$

$$y = -\frac{\kappa\dot{\varphi}^2}{2(1+\kappa\dot{\varphi}^2)} \quad ,$$

$$z = \frac{V(\varphi)}{3H^2(1+\kappa\dot{\varphi}^2)} \quad ,$$

$$m = \frac{\dot{\varphi}}{\varphi H} \quad .$$

$$b = \frac{V'(\varphi)\varphi}{V(\varphi)} = \mathrm{N}$$

Taking derivatives of the introduced variables with respect to **ln(a)**, the first-order system of differential equation is found:

$$\frac{\dot{x}}{H}=\frac{dx}{d(\ln(a))}=x'=2x(X(1+2\mathrm{y})-Y) \quad ,$$

$$\frac{\dot{y}}{H}=\frac{dy}{d(\ln(a))}=y'=2Xy(1+2\mathrm{y}) \quad ,$$

$$\frac{\dot{z}}{H}=\frac{dz}{d(\ln(a))}=z'=z(bm-2Y+4Xy) \quad , \tag{4}$$

$$\frac{\dot{m}}{H}=\frac{dm}{d(\ln(a))}=m'=m(X-m-Y) \quad ,$$

where $X = \frac{\ddot{\varphi}}{\dot{\varphi} H}$, $Y = \frac{\dot{H}}{H^2}$.

From the system (4) we exclude y, X, Y using equations following from the initial system (1)-(3)

$$y = 1 - x - z \quad ,$$

$$X = \frac{-6(x+z)(1-z) + b(x+z-2)mz}{2(5z^2 + 4x^2 - 9x(1-z)y^2 - 11z + 6)} \, ,$$

$$Y = \frac{3(1-z)(2x+3z-3) + b(x+z-1)mz}{5z^2 + 4x^2 - 9x(1-z) - 11z + 6} \, .$$

.

Time dependencese **a(t)** is found from a stationary value **Y** and $\varphi(t)$ – from one of coordinates of a stationary point, which isn't equal to zero, for example,m:

as $Y = \frac{\dot{H}}{H^2}$, then $\frac{dH}{H^2} = Y_{stat}\, dt$,

$$a(t) = a_0 |t - t_0|^{\frac{-1}{Y_{stat}}} ,$$

$$m_{stat} = \frac{\dot{\varphi}}{\varphi H} = \frac{\dot{\varphi} a}{\varphi \dot{a}} , \qquad \frac{\dot{\varphi}}{\varphi} = m_{stat} \frac{\dot{a}}{a} ,$$

$$\varphi(t) = \varphi_0 |t - t_0|^{-\frac{m_{stat}}{Y_{stat}}} .$$

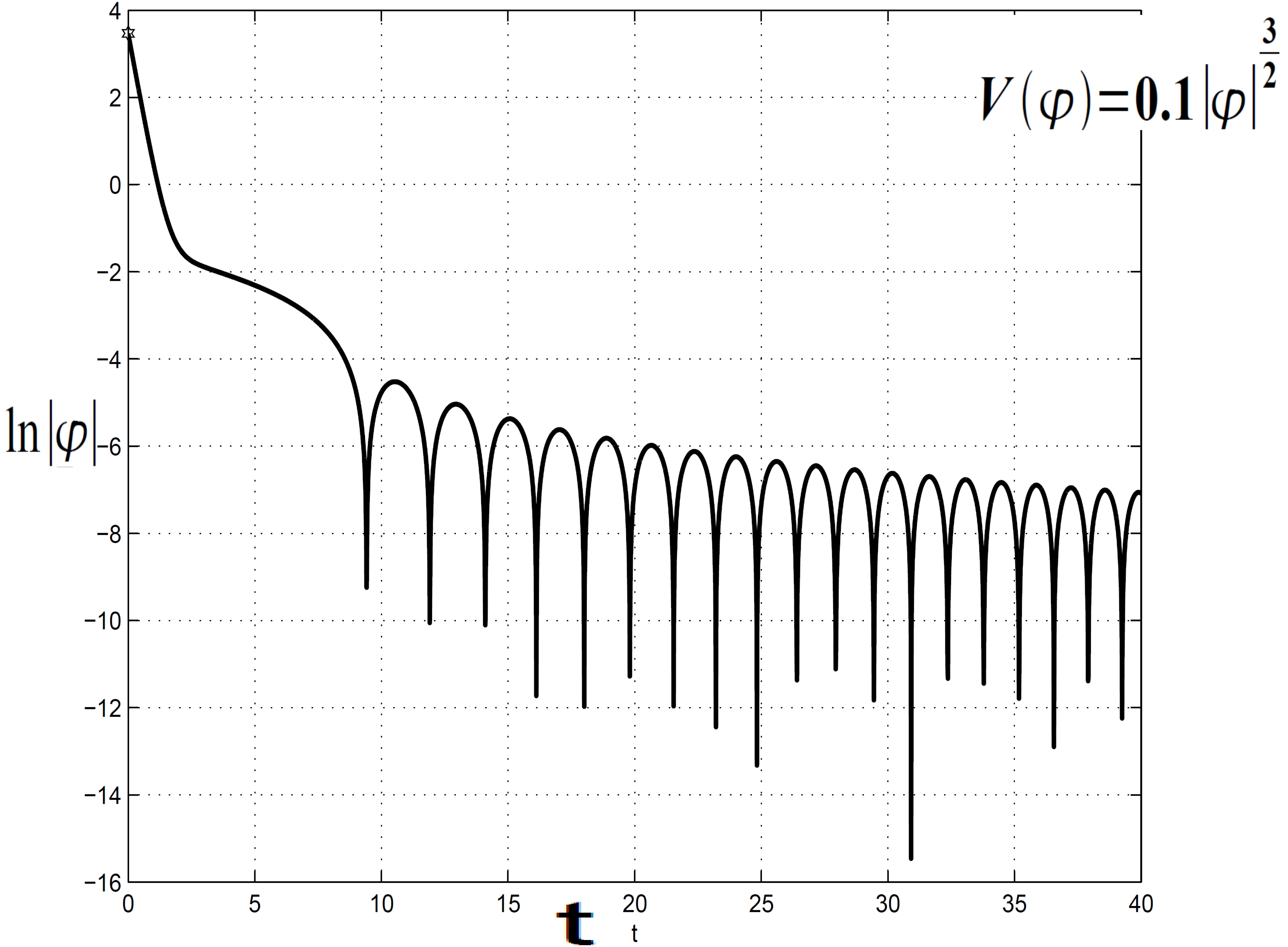
Stationary point, their character of stability and corresponding solutions a(t), $\varphi(t)$ for $N \neq 2$.

Coordinates	Character of stability	a(t), $\varphi(t)$
$x=0$, $y=1$, $z=0$, $m=0$.	Unstable node Exists for $t \to t_0$	$a(t)=a_0\lvert t-t_0\rvert^{\frac{2}{3}}$ $\varphi(t)=\varphi_0 \pm \sqrt{-\frac{2}{3\kappa}}(t-t_0)$
$x=-y=\frac{1}{2}$, $z=1$, $m=0$.	Complex type Exists for $0<N<2$ $t \to \infty$	$a(t)=a_0 e^{\pm\sqrt{\frac{1}{3\kappa}}(t-t_0)}$ $\lvert\varphi(t)\rvert^{\frac{2-N}{2}}=\varphi_0 \pm \frac{\sqrt{\lvert V_0\rvert}(2-N)}{2}(t-t_0)$

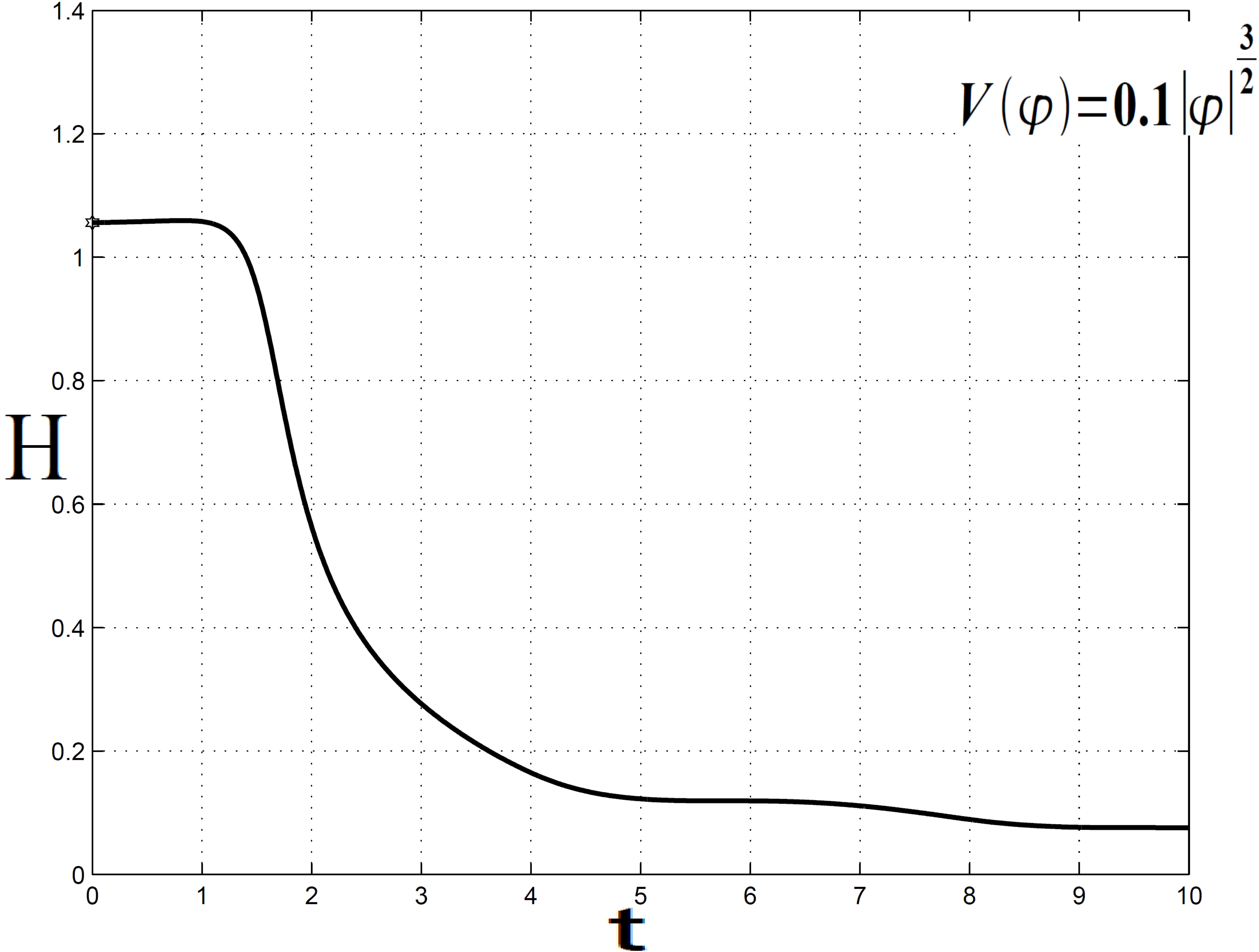
Coordinates	Character of stability	a(t), $\varphi(t)$
$x=1$, $y=0$, $z=0$, $m=0$.	Saddle Exists for $N \leqslant 0,$ $t \to \infty$	$a(t)=a_0\left\|t-t_0\right\|^{\frac{1}{3}}$ $\varphi(t)=\pm 2\ln\left\|\frac{t-t_0}{t'-t'_0}\right\|$
$x=\frac{3}{2}$, $y=-\frac{1}{2}$, $z=0$, $m=-3$.	Unstable node Exists for $0<N<2$	$a(t)=a_0 e^{\pm\frac{1}{3\sqrt{\kappa}}(t-t_0)}$ $\varphi(t)=\varphi_0 e^{\pm\frac{1}{\sqrt{\kappa}}(t-t_0)}$

Coordinates	Character of stability	a(t), $\varphi(t)$
$x=0$, $y=-\frac{1}{2}$, $z=\frac{3}{2}$, $m=\frac{12}{3N+2}$.	**Stable node for N > 2** Exists for N > 2, $t \to t_0$	$a(t)=a_0\left\|t-t_0\right\|^{\frac{3N+2}{3(2-N)}}$ $\varphi(t)=\varphi_0\left\|t-t_0\right\|^{\frac{4}{2-N}}$

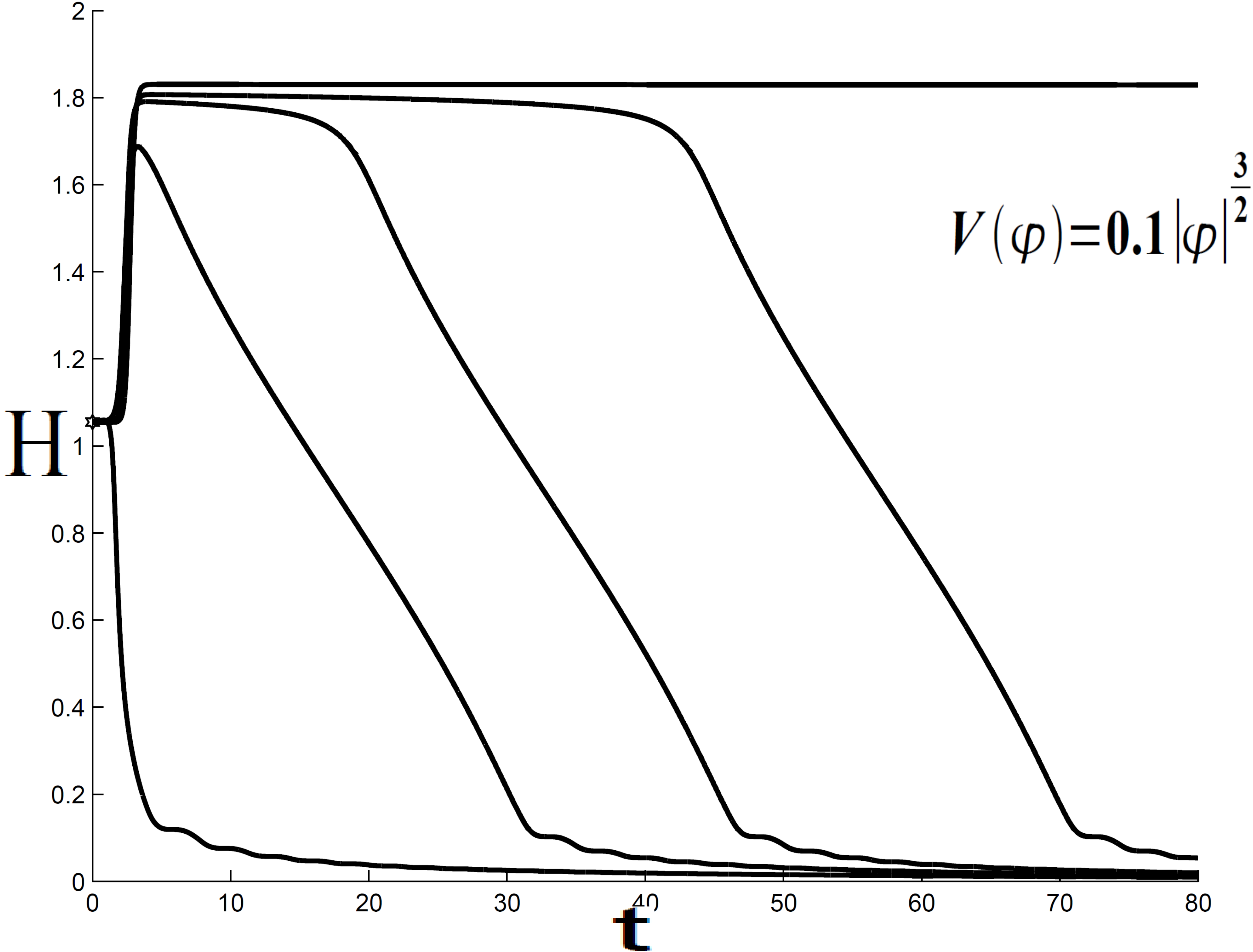
For N > 2 its power index is negative and this solution deverges into «Big Rip» singularity at $t = t_0$.



$V(\varphi)=0.1|\varphi|^{\frac{3}{2}}$
$\ln|\varphi|$
t
t
4
2
0
−2
−4
−6
−8
−10
−12
−14
−16
0
5
10
15
20
25
30
35
40



$V(\varphi) = 0.1|\varphi|^{\frac{3}{2}}$
H
t
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8
9
10



2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
H
0
10
20
30
40
50
60
70
80
t
$V(\varphi)=0.1|\varphi|^{\frac{3}{2}}$

Conclusions

1. In the model of the Universe with nonminimal kinetic coupling whithout matter and the power-low potential $V(\varphi)=V_0\varphi^N$ two asimptotically stable solutions exist:

1). for N > 2 – power-low solution, which have the singularity «Big Rip» and **doesn't depend** on coupling constant $\boldsymbol{K}$.

2). for 0 < N < 2 – solution with $H=\pm\frac{1}{\sqrt{3\kappa}}$ and power-low behaviour of $\varphi(t)$.

2. For 0 < N < 2 unstable exponential solution exists ($H=\pm\frac{1}{3\sqrt{\kappa}}$) and we have the possibility of a transition from it either to ossilations or to stable inflation depending on initial data.

3. The case of **N = 2** is requed of the special research.