Cosmological Applications of Massive Gravity

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Overview

Prehistory and Motivations

Decoupling Limit Cosmology

Cosmology in Massive Gravity, Bigravity + Alternatives

Prehistory

Most modifications of gravity change physics at high energies UV
- e.g. string theory, Kaluza-Klein theory

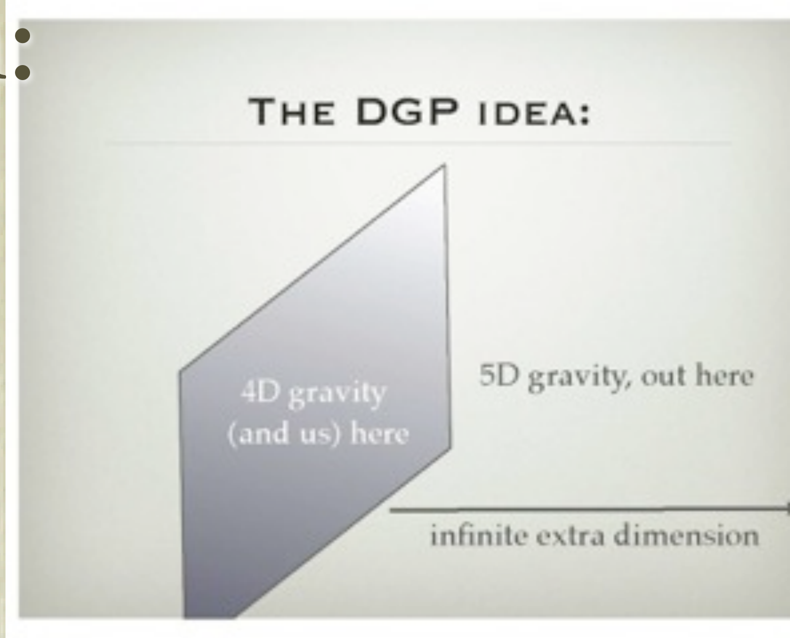
In gravity, ***high energy*** means **high curvatures** which means **early times**

Thus string theory/KK modifications have little impact on late-time cosmology

But its late time cosmology that we least understand - Cosmic Acceleration

What if we have a modification of gravity at low energies IR?

The quintessential IR modification: Dvali-Gabadadze-Porrati model



Imagine a brane in infinite 5 dimensions with a localized Einstein-Hilbert term on the brane

More irrelevant

More relevant

$$S = \int d^4x\sqrt{-g_4}\frac{M_4^2}{2}R_4 + \int d^4x\sqrt{-g_4}\mathcal{L}_M + \int d^5x\sqrt{-g_5}\frac{M_5^3}{2}R_5$$

Dominates in UV

Dominates in IR

At low energies, we feel all 5 dimensions and so force of gravity is 5d $1/r^3$

At high energies, brane kinetic term dominates, and so force of gravity is 4d $1/r^2$

Cosmology of DGP

Gravitons can condense to form a condensate whose energy density **sources** self-acceleration

$\rho_{\text{matter}} \sim 0$ $\qquad\qquad$ $H \sim m \neq 0$

DGP exhibits self-accelerating cosmological solutions

Deffayet 2000

$$H^2 \mp mH = \frac{1}{3M_{\text{pl}}^2}\rho$$

Two signs correspond two two embeddings of the brane

c.f. Cedric's talk

Cosmology of DGP

Bad news: DGP has a ghost

Koyama et al 2005
Charmousis et al 2006

Solution also sits at strong coupling threshold, quantum stability?

c.f. Cedric's talk

One of the motivations for Galileon models was to find self-acceleration without the ghost

Nicolis, Ratazzi, Trincherini 2008

Not necessarily Galileons as scalar fields (covariant Galileon) but also has remnants of higher spin fields (DGP, cascading gravity, massive gravity) - in this latter case Galileon symmetry is exact with gravity!

Cosmology of IR modifications

Infrared modifications can be used to weaken the strength of gravity at large (cosmological) distances

But thats not all!

Self-acceleration?

Screening/Self-tuning mechanism

Degravitation mechanism?

Cosmology of IR modifications

Gravitons can condense to form a condensate whose energy density **compensates** the cosmological constant

Screening or self-tuning mechanism - The Cosmological Constant can be LARGE with the cosmic acceleration SMALL

c.f. Christos's talk - Fab Four self-tuning

No degravitation/screening for DGP

FRW is completely local relation between energy density and Hubble rate

$$H^2 \mp mH = \frac{1}{3M_{\rm pl}^2}\rho$$

As long as the FRW equation is local we can never use IR modification to resolve the OLD cosmological constant problem

In higher than 5D full evolution is expected to be non-local in 4D

$$H^2 + F(H) \sim \frac{8\pi G}{3} G(\Box)\rho$$

Screening/Self-tuning in massive gravity

mass term

$$G_{\mu\nu} + m^2 \frac{\partial L_M}{\partial g_{\mu\nu}} = -\Lambda g_{\mu\nu}$$

Graviton condensate:

Spacetime is Minkowski in presence of an arbitrary large Λ

$$g_{\mu\nu} = \left(1 + f\left(\frac{\Lambda}{m^2}\right)\right)\eta_{\mu\nu} \qquad G_{\mu\nu} = 0 \qquad m^2 \frac{\partial L_M}{\partial g_{\mu\nu}} = -\Lambda g_{\mu\nu}$$

Equivalent Statement: The cosmological constant can be reabsorbed into a **redefinition** of the metric and coupling constants - and is hence a **redundant** operator

Degravitation = Dynamical Relaxation

Dynamical evolution to screened solution

Arkani-Hamed, Dimopoulos, Dvali, Gabadadze 2002
Dvali, Hofmann, Khoury 2007

- Can we modify gravity in the IR such that low energy sources couple more weakly to gravity?
- A cosmological constant is the most low energy thing we can write as $\partial_\mu \Lambda = 0$
- DGP is not sufficiently IR modified, need Friedman equation which is more non-local
- Possible solution - generalize DGP to higher dimensions - Cascading Gravity

Gravity in Higher Dimensions

In *4+n* dimensional spacetime, gravitational potential scales as

$$V(r) \sim \frac{1}{r^{1+n}}$$

weaker gravity

we want to achieve this in the IR

$$V(r) \sim \frac{1}{r} \longrightarrow V(r) \sim \frac{1}{r^{1+n}}$$

UV, small r

IR, large r

Gravity in Higher Dimensions

Form of gravity potential
Kallen-Lehman spectral representation

$$V(r) = \frac{Z}{r} + \int_0^\infty ds^2 \rho(s^2) \frac{e^{-sr}}{r}$$

corresponds to propagator

$$G_F(k) = \frac{Z}{k^2 - i\epsilon} + \int_0^\infty ds^2 \rho(s^2) \frac{1}{k^2 + s^2 - i\epsilon}$$
$$= \frac{Z}{k^2 - i\epsilon} + \frac{(1 - Z)}{k^2 + m^2(k) - i\epsilon}$$

we can interpret $m^2(k)$ as an effective mass for the graviton

N.B. I have neglected the tensor structure but all the massive modes in the spectral rep have the Fierz-Pauli tensor structure and the massless have the Einstein tensor structure

Gravity in Higher Dimensions

$$G_F(k) = \frac{Z}{k^2 - i\epsilon} + \int_0^\infty ds^2 \rho(s^2) \frac{1}{k^2 + s^2 - i\epsilon}$$
$$= \frac{Z}{k^2 - i\epsilon} + \frac{(1-Z)}{k^2 + m^2(k) - i\epsilon}$$

$$m^2(k) \sim m_0^2(-k^2L^2)^\alpha$$

$\alpha = 1/2$	5d
$\alpha \sim 0$	6d
$\alpha = 0$	$\geq 7d$

de Rham, Hofmann, Khoury, AJT (2008)

led to 6 or higher dim's to provide Degravitation

$Z = 0$ for infinite extra dimension

How can we achieve 4D to 6D transition?

Answer: Cascading Gravity!

More irrelevant ⟵⟶ More relevant

$$S=\int d^4x\sqrt{-g_4}\left(\frac{M_4^2}{2}R_4+\mathcal{L}_M\right)+\int d^5x\sqrt{-g_5}\left(\frac{M_5^2}{2}\right)+\int d^6x\sqrt{-g_6}\left(\frac{M_6^2}{2}\right)$$

Dominates in UV ⟵⟶ Dominates in IR

Gravity transitions from 4D to 5D to 6D

Brane on Brane solves UV divergence problem of codimension 2 branes (c.f. Christos' talk)

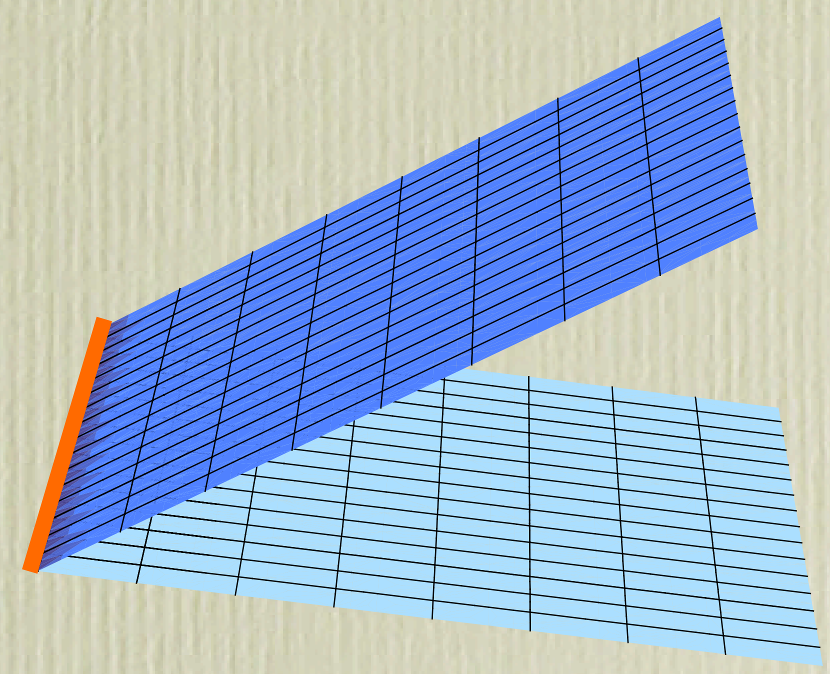
de Rham, Dvali, Hofmann, Khoury, Pujolas, Redi, AJT 2008

Does cascading model realize dynamical relaxation?

Criterion 1: Screening/Self-tuning

Existence of a Minkowski vacuum solution in the presence of a cosmological constant on the 3-brane

Tension creates deficit angle in bulk



similar properties found in 7 dimensional cascading model = 3-Brane on a 4-Brane on a 5-Brane in 7d

de Rham, Khoury, AJT 2009

Does cascading model realize degravitation?

Criterion 2:

Dynamical and causal process by which we can relax to this solution

At linearized level it works - Dvali, Hofmann, Khoury 2007

Nonlinearly: much harder to check! As yet no one has demonstrated this mainly due to complexity of problem

One strong motivation for considering **Massive Gravity** is as a toy model of higher dimensional gravity models (eg Cascading Gravity) that potentially exhibit degravitation

Why Massive Gravity?

Simpler! Departure from GR is governed by essentially a *single* parameter - Graviton Mass

GAIN: Nonlinear theory easier than Higher dimensional framework

LOSS: Diff invariance

Vainshtein Screening mechanism ensures recovery of GR in limit $m \to 0$

This ensures massive gravity can be easily made to be consistent with most tests of GR (effectively placing an upper bound on m) without spoiling its role as an IR modification

Ghost-free Massive Gravity

c.f. Claudia and Eugeny's talks

$$\mathcal{L} = M_{\rm Pl}^2 \sqrt{-\,^{(4)}g}\left(\,^{(4)}R + 2m^2\mathcal{U}(g,f)\right) + \mathcal{L}_M$$

$$\mathcal{K}^\mu_\nu(g,f) = \delta^\mu_\nu - \sqrt{g^{\mu\alpha} f_{\alpha\nu}} \qquad \mathcal{U}(g,H) = \mathcal{U}_2 + \alpha_3\mathcal{U}_3 + \alpha_4\mathcal{U}_4$$

$$\begin{aligned}
\mathcal{U}_2 &= \left([\mathcal{K}]^2 - [\mathcal{K}^2]\right), \\
\mathcal{U}_3 &= \left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right), \\
\mathcal{U}_4 &= \left([\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]\right)
\end{aligned}$$

de Rham, Gabadadze, AJT 2011

Properties of Mass terms

Equivalent representation:

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}\left(M_P^2 R[g] - m^2 \sum_{n=0}^{4} \beta_n \mathcal{U}_n\right) + \mathcal{L}_M$$

Mass terms are ***characteristic polynomials***

$$\mathrm{Det}[1+\lambda X] = \sum_{n=0}^{D} \lambda^n \mathcal{U}_n(X)$$

Finite number of allowed interactions in any dimension

$$X^\mu_\nu = \sqrt{g^{\mu\alpha} f_{\alpha\nu}}$$

Interactions protected by a Non-renormalization theorem
de Rham et al 2012/2013

Generalized to arbitrary (dynamical - bigravity) reference metrics by Hassan, Rosen 2011

Cosmology of massive gravity: A basic tension

dRGT removes the Boulware-Deser ghost (6th), but it does not guarantee that all 5 remaining degrees of freedom are ghost free

Representation theory of de Sitter group gives the Higuchi bound for massive spin 2 reps

$m^2 = 0$ G.R. 2 d.o.f.

$m^2 = 2H^2$ Partially massless theory 4 d.o.f.

$m^2 > 2H^2$ Massive 5 d.o.f.

c.f. Paul Townsend talk

For every cosmological solution we need to check carefully if helicity-zero mode is unitary or not since it is not guaranteed

Not guaranteed a problem,
e.g. in DGP bound always satisfied

Koyama et al 2005
Charmousis et al 2006

Not obviously relevant for Minkowski reference metric which breaks de Sitter symmetry

Decoupling limit cosmology

c.f. Claudias talk

We can take a decoupling limit of massive gravity (and bigravity see later) where after diagonalization massive gravity is equivalent to a free helicity-2 particle, and a helicity-1 coupled to a helicity-0 particle

de Rham et al 2010

$$M_{\text{Planck}} \to \infty \qquad m \to 0 \qquad \Lambda^3 = (M_{\text{Planck}} m^2)^{1/3}$$

Helicity-0 interactions are **true** Galileons i.e. preserve Galileon symmetry

Since Galileon symmetry is EXACT we only require that $\partial_\mu \partial_\nu \pi$ is homogeneous and isotropic to describe FRW

Decoupling limit cosmology

de Rham, Gabadadze, Heisenberg, Pirtzkhalava 2010

The generic solution form for the helicity zero mode near x=0 which is isotropic in this limit is

$$\pi \sim A(t) + B(t)\mathbf{x}^2$$

N.B. there is NO equivalent of this in covariant Galileon/Hordenski because there symmetry is broken

$$\mathrm{d}s^2 = -[1-(\dot{H}+H^2)\mathbf{x}^2]\mathrm{d}t^2 + \left[1-\frac{1}{2}H^2\mathbf{x}^2\right]\mathrm{d}\mathbf{x}^2 = \left(\eta_{\mu\nu}+h^{\mathrm{FRW}}_{\mu\nu}\right)\mathrm{d}x^\mu \mathrm{d}x^\nu$$

Equations of motion fix A and B - for example for pure cc source B=constant

$$A = -Bt^2$$

Decoupling limit cosmology

de Rham et al 2010

$$\mathcal{L} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + h^{\mu\nu}\sum_{n=1}^{3}\frac{a_n}{\Lambda_3^{3(n-1)}}X^{(n)}_{\mu\nu}[\Pi] + \frac{1}{M_{\rm Pl}}h^{\mu\nu}T_{\mu\nu}$$

$$\begin{aligned} \pi &= \frac{1}{2}q_{\rm dS}\,\Lambda_3^3\,x^2 + \phi\,, \\ h_{\mu\nu} &= -\frac{1}{2}H_{\rm dS}^2\,x^2\,\eta_{\mu\nu} + \chi_{\mu\nu} \\ T_{\mu\nu} &= -\lambda\eta_{\mu\nu} + \tau_{\mu\nu}\,. \end{aligned}$$

Self-accelerating Branch

$$a_1 + 2a_2q_{\rm dS} + 3a_3q_{\rm dS}^2 = 0\,,$$

$$H_{\rm dS}^2 = \frac{\lambda}{3M_{\rm Pl}^2} + \frac{2\Lambda_3^3}{M_{\rm Pl}}\left(a_1q_{\rm dS} + a_2q_{\rm dS}^2 + a_3q_{\rm dS}^3\right)$$

background plus perturbations split

$$\mathcal{L}^{(2)} = -\frac{1}{2}\chi^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\chi_{\alpha\beta} + \frac{6H_{\rm dS}^2M_{\rm Pl}}{\Lambda_3^3}(a_2 + 3a_3q_{\rm dS})\phi\Box\phi + \frac{1}{M_{\rm Pl}}\chi^{\mu\nu}\tau_{\mu\nu}$$

coefficient of helicity zero simple function of α_3 α_4

Decoupling limit cosmology

$$\mathcal{L}^{(2)} = -\frac{1}{2}\chi^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\chi_{\alpha\beta} + \frac{6H^2_{\mathrm{dS}}M_{\mathrm{Pl}}}{\Lambda_3^3}(a_2 + 3a_3 q_{\mathrm{dS}})\phi\Box\phi + \frac{1}{M_{\mathrm{Pl}}}\chi^{\mu\nu}\tau_{\mu\nu}$$

In S.A. branch helicity zero does not couple to matter perts
- no vDVZ discontinuity - no Vainshtein

Screening/Self-tuning
(Degravitating) Branch

$$\mathcal{L} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + h^{\mu\nu}\sum_{n=1}^{3}\frac{a_n}{\Lambda_3^{3(n-1)}}X^{(n)}_{\mu\nu}[\Pi] + \frac{1}{M_{\mathrm{Pl}}}h^{\mu\nu}T_{\mu\nu}$$

$$\pi = \frac{1}{2}q_{\mathrm{dS}}\Lambda_3^3 x^2 + \phi$$

$$h_{\mu\nu} = 0 + \chi_{\mu\nu}$$

$$T_{\mu\nu} = -\lambda\eta_{\mu\nu} + \tau_{\mu\nu}$$

Minkowski solutions for any value of the cosmological constant!!!!!

Perturbations stable and Vainshtein mechanism present

$$\mathcal{L}^{(2)} = -\frac{1}{2}\bar{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\bar{h}_{\alpha\beta} + \frac{3}{2}\phi\Box\phi + \frac{1}{M_{\mathrm{Pl}}}\left(\bar{h}^{\mu\nu} + \phi\,\eta^{\mu\nu}\right)\tau_{\mu\nu}$$

Decoupling limit cosmology

More generally - Decoupling limit implies existence of isotropic and **inhomogeneous** cosmological solutions for massive gravity which for suitable range of parameters are free from Higuchi bound (no-ghost in helicity zero sector)

Absence of Higuchi bound frees up possibility for background Vainshtein effect - consistency with standard expansion history at early times

All these solutions are in the Decoupling Limit - they all map to solutions in the full nonlinear theory - however very hard to find!

Full theory - A No-Go

The simplest model (dRGT model - Massive Gravity in Minkowski) does not support spatially flat (or closed) cosmological solutions which are FRW meaning homogeneous and isotropic

Argument is simple: as in GR we have Friedman equation and Raychaudhuri equation - the 2nd follows from 1st by diff invariance

But in MG diff invariance is broken and so 2nd does not follow from 1st - consistency of two imposes condition on scale factor

$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\mathrm{d}\vec{x}^2$$

$$\mathcal{L} = 3M_{\rm Pl}^2\left(-\frac{a\dot{a}^2}{N} - m^2(a^3 - a^2) + m^2 N(2a^3 - 3a^2 + a)\right)$$

D'Amico, de Rham, Dubovsky, Gabadadze, Pirtskhalava, AJT 2011

$$m^2\partial_0(a^3 - a^2) = 0$$

Resolution 1

Accept inhomogeneities:

D'Amico, de Rham, Dubovsky, Gabadadze, Pirtskhalava, AJT
`Massive Cosmologies' 2011

Not as bad as it sounds! Vainshtein mechanism should guarantee inhomogeneities unobservable before late times

c.f. Eugeny's talk

Inhomogenities only appear on scale set by inverse graviton mass

Observational constraints on inhomgeneities as current Hubble scale are actually very weak

Resolution 1

Inhomogeneities/Anisotropies can be hidden inside Stueckelberg fields which do not directly couple to matter, only indirectly through Mplanck suppressed terms

Even if metric is perfectly homogeneous+isotropic, inhomogeneities show up in cosmological perturbations, but can easily be small

Volkov 2011/2012/2013,
Koyama 2011,
Gumrukcuoglu et al 2011,
Gratia, Hu, Wyman 2012,
Kobayashi et al 2012,
DeFelice 2011/2013,
Gumrukcuoglu 2012,
Tasinato et al :2012.2013,
Maeda + Volkov 2013

Resolution II

Or modify assumptions to allow FRW:

Open Universe solutions: Gumrukcuogli et al 2011 (however unstable) Vakili et al 2012

* Make reference metric **de Sitter** - AJT and Fasiello - 2012 (for decoupling limit see de Rham, Renaux-Petel 2012) (also unstable - see later)

* Make reference metric **dynamical** - Bigravity/Bimetric

von Strauss et al 2011
Comelli et al 2011, 2012
Volkov 2011
Akrami et al 2012
Koshravi et 2012
Berg et al 2012

no-ghost instabilities - see later

see Marco Crisostomi's talk

Resolution II

Or significant modifications of theory to allow for FRW:

Quasi-dilaton massive gravity (unstable)

D'Amico, Gabadadze, Hui, Pirtskhalava 2012

Generalized quasi-dilaton massive gravity (stable!)

De Felice, Gumrukcuoglu, Mukohyama 2013

Lorentz violating massive gravity

Comelli, Nesti, Pilo 2013

Varying Mass

Wu et al 2013, Leon et al 2013

Multi-vierbeins

Tamanini, Saridakis, Koivisto 2013

Extended massive gravity

Hinterbichler, Stokes, Trodden 2013

Nonlocal massive gravity

Jaccard, Maggiore, Mitsou 2013
Modesto, Tsujikawa 2013

Massive gravity with FRW reference metric

Deriving Friedman equation

Fasiello and AJT - 2012

Nice approach is with Stuckelberg fields

$$ds^2 = -N^2 dt^2 + a(t)^2 d\vec{x}^2$$

$$ds^2 = -\dot{\phi^0}^2 dt^2 + b^2(\phi^0) d\vec{x}^2$$

$$\mathcal{K}^\mu_\nu(g,f) = \delta^\mu_\nu - \sqrt{g^{\mu\alpha} f_{\alpha\nu}}$$

eg in de Sitter $b(\phi^0) = e^{H_b \phi^0}$

$$g^{-1}f = \begin{pmatrix} \frac{\dot{\phi^0}^2}{N^2} & 0_j \\ 0_i & \frac{b(\phi^0)^2}{a^2}\delta_{ij} \end{pmatrix}$$

$$\sqrt{g^{-1}f} = \begin{pmatrix} \frac{\dot{\phi^0}}{N} & 0_j \\ 0_i & \frac{b(\phi^0)}{a}\delta_{ij} \end{pmatrix}$$

If metric transits from acceleration to decceleration we need $\dot{\phi^0}$ to change sign

At this point one of the eigenvalues of $\sqrt{g^{-1}f}$ vanishes

Problem? Answer - vierbein formulation can accommodate changes of sign

Vierbein formulation

Nibbelink et al
Chamseddine, Mukhanov, Volkov
Hinterbichler and Rosen

The vierbein formulation is *analytic* in the ϕ^a

reference vierbein

Mass term is $Det\left[e^a_\mu + \lambda \Lambda^a_b f^b_c \partial_\mu \phi^c\right]$

As long as it is possible to solve the equation for the Lorentz Stuckelberg fields Λ^a_b $\quad \Lambda \eta \Lambda^T = \eta$

$$e^{\mu a} \Lambda^b_c f^c_d \partial_\mu \phi_d - e^{\mu b} \Lambda^b_c f^c_d \partial_\mu \phi_d = 0$$

6 equations for 6 unknown Lorentz transformations

Even when $\dot{\phi}^0 = 0$ we can solve for $\delta \Lambda^a_b = \ldots \partial \delta \phi_c$

Recently emphasized explicitly by Gratia, Hu, Wyman 2013

Dressed Mass and Higuchi

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}\left[M_{\rm p}^2 R - m^2 \sum_{n=0}^{4} \beta_n \mathcal{U}_n(X) \right] + \mathcal{L}_M \qquad X^\mu_\nu = \sqrt{g^{\mu\alpha} f_{\alpha\nu}}$$

$$\left(\sum_{n=0}^{2} \frac{\hat{\beta}_{n+1}}{(2-n)!n!}\left(\frac{b}{a}\right)^{n+1}\right)\left(\frac{H}{b} - \frac{H_f}{a}\right) = 0$$

Normal branch $\quad \dfrac{b}{a} = \dfrac{H}{H_f}$

$$H^2 = \sum_{n=0}^{3} \frac{3m^2\beta_n}{(3-n)!n!}\left(\frac{H}{H_f}\right)^n + \frac{\rho}{3M_{\rm Planck}^2}$$

$$\tilde{m}^2(H) = \frac{m^2}{2M_{\rm p}^2}\frac{H}{H_f}\left[\beta_1 + 2\beta_2\frac{H}{H_f} + \beta_3\frac{H^2}{H_f^2}\right]$$

Generalized Higuchi bound is $\tilde{m}^2(H) > 2H^2$

arises from coefficient of kinetic term for helicity zero mode

$$\mathcal{L}_{\rm helicityzero} \propto -\tilde{m}^2(H)(\tilde{m}^2(H) - 2H^2)(\partial\pi)^2$$

Partially Massless Gravity

Coefficient of kinetic term in general is proportional to

$$\tilde{m}^2(H) = \frac{m^2}{2M_{\rm p}^2}\frac{H}{H_f}\left[\beta_1 + 2\beta_2\frac{H}{H_f} + \beta_3\frac{H^2}{H_f^2}\right]$$

If we make the special choice

de Rham and Renaux-Petel 2012

$$\beta_1 = \beta_3 = 0 \quad \beta_2 = 1$$

$$\tilde{m}^2(H) - 2H^2 = \frac{H^2}{H_f^2}(m^2 - 2H_f^2)$$

and so if we choose

$$m^2 = 2H_f^2 \longrightarrow \tilde{m}^2(H) = 2H^2$$

Kinetic term vanishes regardless of matter source!!!

Unfortunately partially massless (bi)gravity does not work for vectors!

Higuchi versus Vainshtein

Higuchi $$\tilde{m}^2(H) = \frac{m^2}{2M_{\rm p}^2}\frac{H}{H_f}\left[\beta_1 + 2\beta_2\frac{H}{H_f} + \beta_3\frac{H^2}{H_f^2}\right] \geq 2H^2$$

Vainshtein $$\frac{m^2}{2M_{\rm p}^2}\left[3\beta_1\frac{H}{H_f} + 3\beta_2\frac{H^2}{H_f^2} + \beta_3\frac{H^3}{H_f^3}\right] \ll 3H^2$$

$$\left[\frac{3}{2}\beta_1\frac{H}{H_f} + 3\beta_2\frac{H^2}{H_f^2} + \frac{3}{2}\beta_3\frac{H^3}{H_f^3}\right] \gg \left[3\beta_1\frac{H}{H_f} + 3\beta_2\frac{H^2}{H_f^2} + \beta_3\frac{H^3}{H_f^3}\right]$$

Impossible to satisfy!

Bigravity resolves Higuchi

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}\left[M_P^2\, R(g) - m^2 \sum_{n=0}^{4} \beta_n\, \mathcal{U}_n\left(g^{-1}f\right)\right] + \frac{1}{2}\sqrt{-f}M_f^2\, R(f) + \mathcal{L}_M$$

<u>Stability bound</u>

$$\boxed{\tilde{m}^2\left[1 + \left(\frac{H_f/M_f}{H/M_P}\right)^2\right] \geq 2H^2}$$

Massive gravity bound in the limit $M_f \to \infty,\ \ M_P, H_f$ finite.

Not possible before: $\boxed{\dfrac{H_f}{M_f} \gg \dfrac{H}{M_P}}$ not directly invoking m

Friedman side

$$H^2 = \frac{1}{3M_P^2}\left[\rho(a) + \sum_{n=0}^{3} \frac{3m^2\beta_n}{(3-n)!n!}\left(\frac{H}{H_f}\right)^n\right] \quad ; \quad H_f^2 = \frac{1}{3M_f^2}\left[\sum_{n=0}^{3} \frac{3\beta_{n+1}}{(3-n)!n!}\left(\frac{H}{H_f}\right)^{(n-3)}\right]$$

$m^2 \times \Theta(1) \ll H^2$ it's the <u>only direct requirement</u> on m, but now:

In the $\frac{H_f}{M_f} \gg \frac{H}{M_P}$ region with $\beta_1 \neq 0$ solve for $\tilde{m}^2$, H_f, bound reads:

$$3H^2 > 2H^2 \checkmark$$

The stability vs observations tension is resolved in bigravity !

Stable Self-accelerating Solution

Set: $\beta_2 = 0 = \beta_3; \;\; \beta_1 = 2M_P^2$

$$H^2 = \frac{1}{6M_P^2}\left(\rho(a) + \sqrt{\rho(a)^2 + \frac{12\, m^4 M_P^6}{M_f^2}}\right)$$

Model	$\mathbf{B_0}$	$\mathbf{B_1}$	$\mathbf{B_2}$	$\mathbf{B_3}$	$\mathbf{B_4}$	$\mathbf{\Omega_m}$	$\chi^2_{\rm min}$	p-value	log-evidence
ΛCDM	free	0	0	0	0	free	546.54	0.8709	-278.50
$(\mathbf{B_1}, \mathbf{\Omega_m^0})$	0	free	0	0	0	free	551.60	0.8355	-281.73

Observationally viable!

Akrami, Koivisto, Sandstad (2012,2013)

Stability bound? It reduces to

$$\left(\frac{1}{M_P^2} + \frac{12 M_f^2}{m^4 \beta_1^2} H^4\right) > 0 \quad \checkmark$$

Stable as well.

see Marco Crisostomi's talk for more discussion

Decoupling limit of Bigravity

Fasiello, AJT 2013

In **Massive Gravity** - Mass term breaks a single copy of local Diffeomorphism Group down to a global Lorentz group

$$Diff(M) \to \text{Global Lorentz}$$

In **Bigravity** - Mass term breaks two copies of local Diffeomorphism Group down to a single copy of Diff group

$$Diff(M) \times Diff(M) \to Diff(M)_{\text{diagonal}}$$

Decoupling limit of Bigravity

$$Diff(M) \times Diff(M) \rightarrow Diff(M)_{\text{diagonal}}$$

Thus Bigravity also is best understood with Stueckelberg fields for broken diffs which in turn lead to a Galileon field in its decoupling limit - dominates interaction in bigravity model

Dynamical metric I

$$g_{\mu\nu}(x)$$

Dynamical metric II

$$F_{\mu\nu} = f_{AB}(\phi)\partial_\mu \phi^A \partial_\nu \phi^B$$

$$\phi^A = x^A + \frac{1}{mM_{\text{Planck}}}\partial^A \pi(x)$$

Decoupling limit of Bigravity

Fasiello, AJT 2013

$$S_{\mathrm{helicity}-2/0} = \int \mathrm{d}^4x \left[-\frac{1}{4} h^{\mu\nu} \hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} - \frac{1}{4} v^{\mu\nu} \hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu} v_{\alpha\beta} \right.$$
$$\left. + \frac{\Lambda_3^3}{2} h^{\mu\nu}(x) X^{\mu\nu} + \frac{M_{\mathrm{p}}\Lambda_3^3}{2M_{\mathrm{f}}} v_{\mu A}[x^a + \Lambda_3^{-3}\partial^a \pi](\eta^A_\nu + \Pi^A_\nu) Y^{\mu\nu} \right]$$

$$X^{\mu\nu} = -\frac{1}{2} \sum_{n=0}^{4} \frac{\hat{\beta}_n}{(3-n)!n!} \varepsilon^{\mu\ldots}\varepsilon^{\nu\ldots}(\eta + \Pi)^n \eta^{3-n}$$

$$\Pi_{ab} = \frac{\partial_a \partial_b \pi}{\Lambda_3^3}$$

$$Y^{\mu\nu} = -\frac{1}{2} \sum_{n=0}^{4} \frac{\hat{\beta}_n}{(4-n)!(n-1)!} \varepsilon^{\mu\ldots}\varepsilon^{\nu\ldots}(\eta + \Pi)^{(n-1)} \eta^{4-n}$$

Two massless spin-two fields coupled to a Galileon in a highly non-minimal way!

No Partially Massless (Bi)gravity

de Rham, Hinterbichler, Rosen, AJT 2013
Fasiello, AJT 2013

The decoupling limit of bigravity and massive gravity gives the following helicity-1/helicity-0 interactions

$$\Pi_{ab} = \frac{\partial_a \partial_b \pi}{\Lambda_3^3}$$

Ondo, AJT 2013

$$S_{\mathrm{helicity}-1/0} = -\frac{1}{8}\delta^{\mu\nu\rho\sigma}_{abcd}\left(2G^a_\mu(\delta+\Pi)^b_\nu {\omega^c}_\rho \delta^d_\sigma + (\delta+\Pi)^a_\mu(\delta+\Pi)^b_\nu[{\omega^c}_\rho {\omega^d}_\sigma + \delta^d_\sigma {\omega^c}_\alpha {\omega^\alpha}_\rho]\right)$$

$$\omega_{ab} = \int_0^\infty \mathrm{d}u \;\; e^{-2u} e^{-u{\Pi_a}^{a'}} G_{a'b'} e^{-u{\Pi^{b'}}_b} \qquad G_{ab} = \partial_a B_b - \partial_b B_a = \omega_{ac}{(\delta+\Pi)^c}_b + {(\delta+\Pi)_a}^c \omega_{cb}$$

helicity-1

Partially massless (bi)gravity should have only (7) 4 propagating degrees of freedom - helicity-0 must be pure gauge - above action shows that is **not** the case!

Two metrics means two ways to introduce Galileon!

Fasiello, AJT 2013

But there are two ways to introduce Stuckelberg fields!

Dynamical metric I

$$g_{\mu\nu}(x)$$

Dynamical metric II

$$F_{\mu\nu} = f_{AB}(\phi)\partial_\mu \phi^A \partial_\nu \phi^B$$

$$\tilde{x}^A = \phi^A(x) = x^A + \partial^A \pi(x)$$

OR

Dynamical metric I

$$\tilde{G}_{AB}(\tilde{x}) = g_{\mu\nu}(Z)\partial_A Z^\mu \partial_B Z^\nu$$

Dynamical metric II

$$f_{AB}(\tilde{x})$$

$$x^\mu = Z^\mu(\tilde{x}) = \tilde{x}^\mu + \partial^\mu \rho(\tilde{x})$$

Dual Galileons fields

`Galileon Duality' - de Rham, Fasiello, AJT 2013

For every **Galileon** field $\pi(x)$

we define the **Dual Galileon** field via the implicit infinite order in derivatives non-local field redefinition

$$\tilde{x}^A = \phi^A(x) = x^A + \partial^A \pi(x)$$

$$x^\mu = Z^\mu(\tilde{x}) = \tilde{x}^\mu + \partial^\mu \rho(\tilde{x})$$

Explicitly this is

$$\rho(x) = -\pi(x) + \frac{1}{2}(\partial\pi)^2 - \frac{1}{2}\partial^a\pi\partial^b\pi\partial_a\partial_b\pi + \text{infinite number of terms}\ldots$$

or for spherical symmetry

$$\rho(r) = -\pi(r) + \sum_{n=2}^{\infty} \frac{(-1)^n}{n!} \partial_r^{n-2}\left((\partial_r\pi)^n\right)$$

Dual Galileons Lagrangians

Galileon operators: $\mathcal{L}_n(\pi) = \pi\epsilon\epsilon(\partial\partial\pi)^{n-1}\eta^{D-n+1}$

For every **Galileon** field Lagrangian in D spacetime dimensions

$$\mathcal{L}(\pi) = c_2\mathcal{L}_2(\pi) + c_3\mathcal{L}_3(\pi) + c_4\mathcal{L}_4(\pi) + \dots$$

admits a dual formulation as a Galileon

$$\mathcal{L}(\rho) = p_2\mathcal{L}_2(\rho) + p_3\mathcal{L}_3(\rho) + p_4\mathcal{L}_4(\rho) + \dots$$

with distinct coefficients

$$p_n = \frac{1}{n}\sum_{k=2}^{D+1}(-1)^k c_k \frac{k(d-k+1)!}{(n-k)!(d-n+1)!}$$

Quasi-Dilaton Massive Gravity

D'Amico et al 2012

Same argument can be applied to generic cosmological solutions on quasi-dilaton massive gravity

$$S_E = \int \mathrm{d}^4x \frac{M_{\mathrm{Pl}}^2}{2}\sqrt{-g}\left[R - \frac{\omega}{M_{\mathrm{Pl}}^2} g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \frac{m^2}{4}\left(\mathcal{U}_2(\tilde{\mathcal{K}}) + \alpha_3\mathcal{U}_3(\tilde{\mathcal{K}}) + \alpha_4\mathcal{U}_4(\tilde{\mathcal{K}})\right)\right]$$
$$+ \int \mathrm{d}^4x\sqrt{-g}\mathcal{L}_m(g_{\mu\nu},\psi),$$

$$\tilde{\mathcal{K}}^\mu{}_\nu = \delta^\mu_\nu - e^{\sigma/M_{\mathrm{Pl}}}\sqrt{g^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}}$$

Generically we find a nonzero kinetic term for helicity zero mode showing that the general cosmological solutions are healthy!

However has its own instabilites, D'Amico et al 2013

Generalized quasi-dilaton

Previous problems resolved by De Felice, Mukohyama 2013

$$S = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\Big[R - 2\Lambda - \frac{\omega}{M_{\rm Pl}^2}\partial_\mu\sigma\partial^\mu\sigma$$

$$+2m_g^2(\mathcal{L}_2 + \alpha_3\mathcal{L}_3 + \alpha_4\mathcal{L}_4)\Big].$$

$$\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - e^{\sigma/M_{\rm Pl}}\left(\sqrt{g^{-1}\tilde{f}}\right)^\mu{}_\nu$$

$$\tilde{f}_{\mu\nu} \equiv f_{\mu\nu} - \frac{\alpha_\sigma}{M_{\rm Pl}^2 m_g^2}e^{-2\sigma/M_{\rm Pl}}\partial_\mu\sigma\partial_\nu\sigma$$

Massive gravity with dynamical mass and couplings and a dynamical reference metric

Cosmological solutions free of instabilities

Summary

- Massive Gravity is a useful toy model to understand higher dimensional theories

- Potentially exhibit both Self-Acceleration and Self-tuning (Degravitation)

- FRW (fully homogeneous and isotropic) solutions are a problem in Massive Gravity. Inhomogenous/anisotropic solutions do exist - not all stable but some are!

- For Partially Massless Gravity - Higuchi bound is ***automatically satisfied*** for any choice of matter - Unfortunately decoupling limit makes it easy to see absence of partially massless (bi)gravity

- For Massive Gravity on a fixed FRW reference metric, bound is in conflict with Vainshtein mechanism

- For Bigravity, bound is almost always satisfied regardless of the choice of matter as long as $H \ll H_f$