Interacting Spin-2 Fields in the Stückelberg Picture

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Why consider multiple, interacting spin-2 fields?

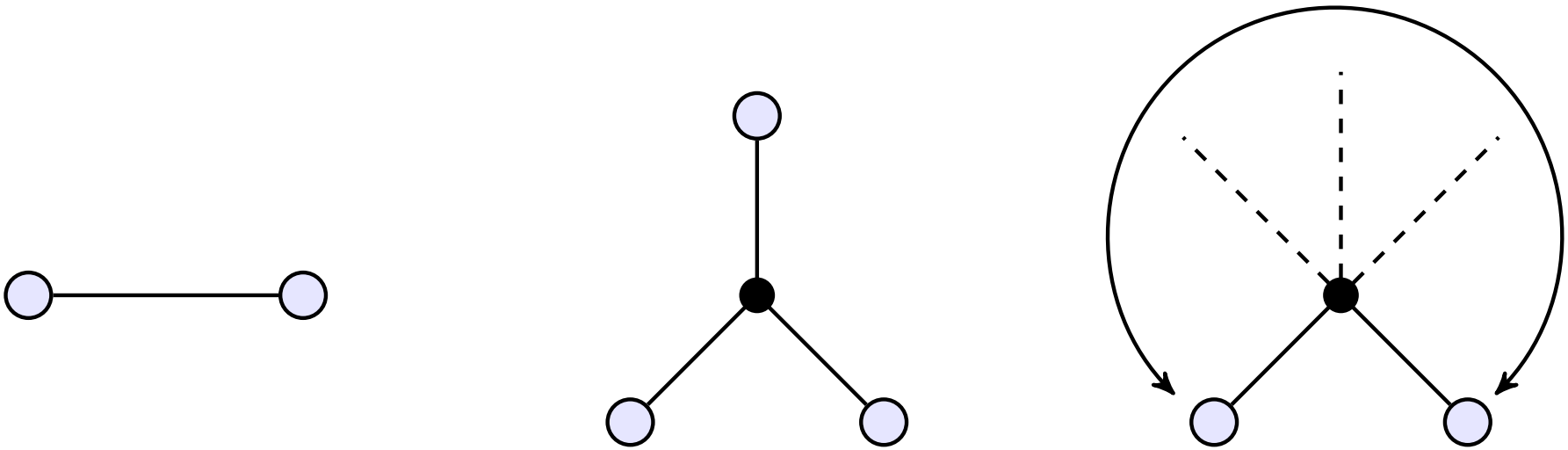
- Theoretical
  - What interactions of spin-2 fields are consistent?
  - Natural extension of massive gravity models
  - Deconstructing dimensions [Arkani-Hamed & Schwartz Phys.Rev.D69:104001 (2004), de Rham, Matas & Tolley 1308.4136]
- Phenomenological
  - Multi-gravity models can give self-accelerating cosmological solutions.

Gravity in Theory Space

Interactions can be represented using [Hinterbichler & Rosen JHEP 07(2012)047]

sites: graviton self interactions, e.g. $\int d^D x\, R[g^{(i)}]$

links: interactions between different gravitons, e.g. $\int d^D x\, e_n(\sqrt{g^{-1}f})$, $\int d^D x\, g^{(1)\mu\nu} g^{(2)}_{\nu\alpha} g^{(3)\alpha\beta} g^{(4)}_{\beta\mu}$.



(a) Bimetric (b) Trimetric (c) N-metric

- Each site comes with a GC invariant kinetic term.
- When sites are connected $GC \times GC$ breaks to the diagonal subgroup.

The Stückelberg Trick

- New fields (re)introduce (broken) gauge invariance.
- Introduction is patterned after gauge transformation

$$G^{(i,j)}_{\mu\nu}(x) = \partial_\mu Y^{\alpha}_{(j,i)} \partial_\nu Y^{\beta}_{(j,i)} g^{(i)}_{\alpha\beta}(Y(x)) \qquad \text{or} \qquad \mathbf{G}^{(i,j)} = \mathbf{g}^{(i)} \circ Y_{(j,i)}.$$

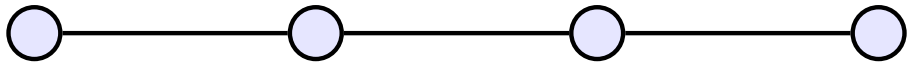
- Transformation properties of Y determine those of G:

$$GC_i: \quad Y_{(j,i)} \to f^{-1}_{(i)} \circ Y_{(j,i)} \implies \mathbf{G}^{(i,j)} \to \mathbf{G}^{(i,j)}$$
$$GC_{j\neq i}: \quad Y_{(j,i)} \to Y_{(j,i)} \circ f_{(j)} \implies \mathbf{G}^{(i,j)} \to \mathbf{G}^{(i,j)} \circ f_{(j)}.$$

- Thus Stückelberg fields act to pull back metric from one site, to another. [Arkani-Hamed, Georgi & Schwartz Ann.Phys.305:96-118 (2003)]
- If need be multiple Stückelberg fields can be applied.

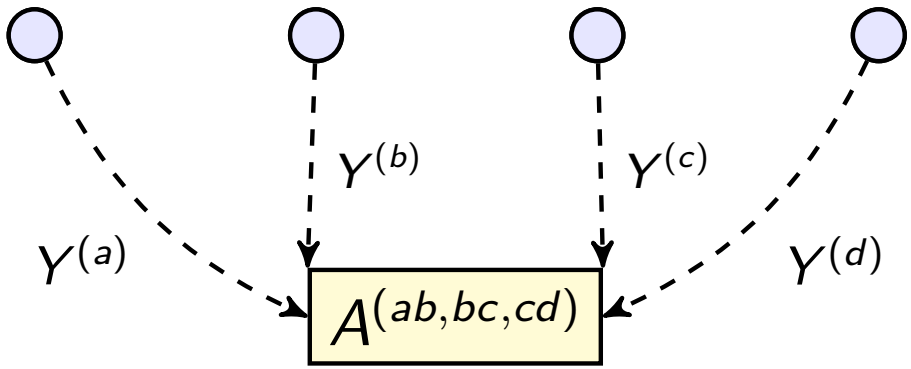
Three ways to Stückelberg

Consider a tetra-metric theory, with bimetric interactions:



There are three ways one can introduce Stückelberg fields.

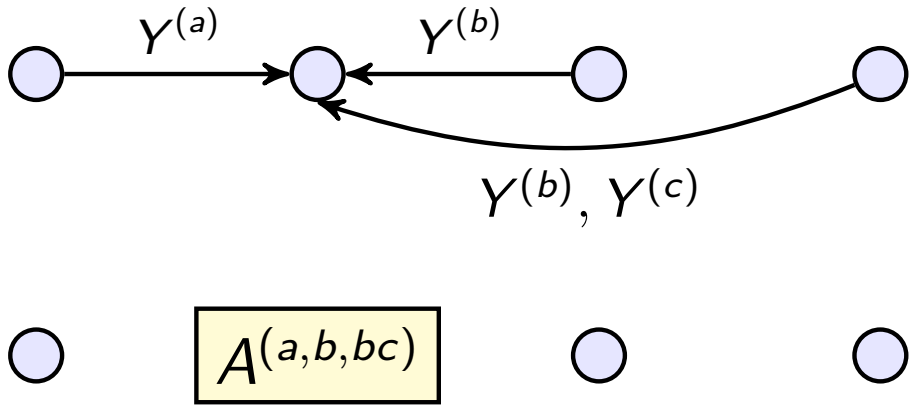
1. Pull all sites together



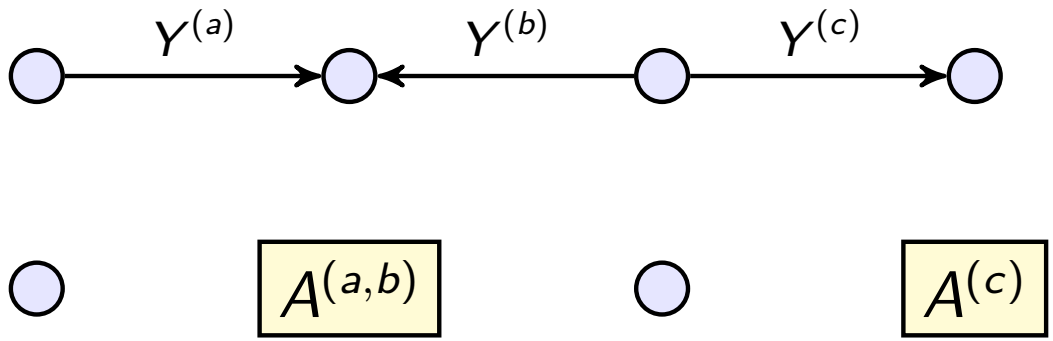
We also need some sort of constraint since we are introducing one too many Stückelberg fields for the number of broken gauge invariances.

Three ways to Stückelberg

2. Pull all (but one) back to one site



3. Treat each interaction term differently: pull to adjacent sites



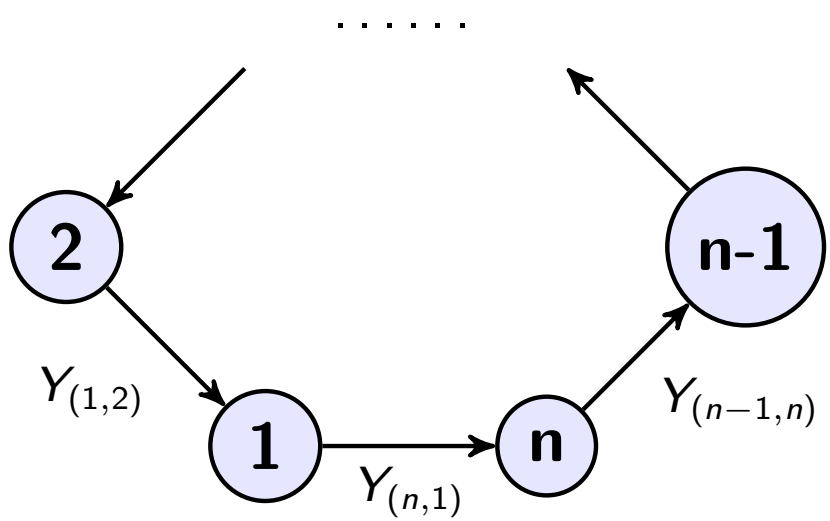
In what follows the third method will be used as it is the most simple.

Too Many Links!

This corresponds to the case of loops in the theory graph.

- Could treat 'closing link' like others,

$$\int d^D x\, f(\mathbf{g}_{(n)}, \mathbf{g}_{(1)}) \to \int d^D x\, f(\mathbf{g}_{(n)}, \mathbf{g}_{(1)} \circ Y_{(n,1)}),$$

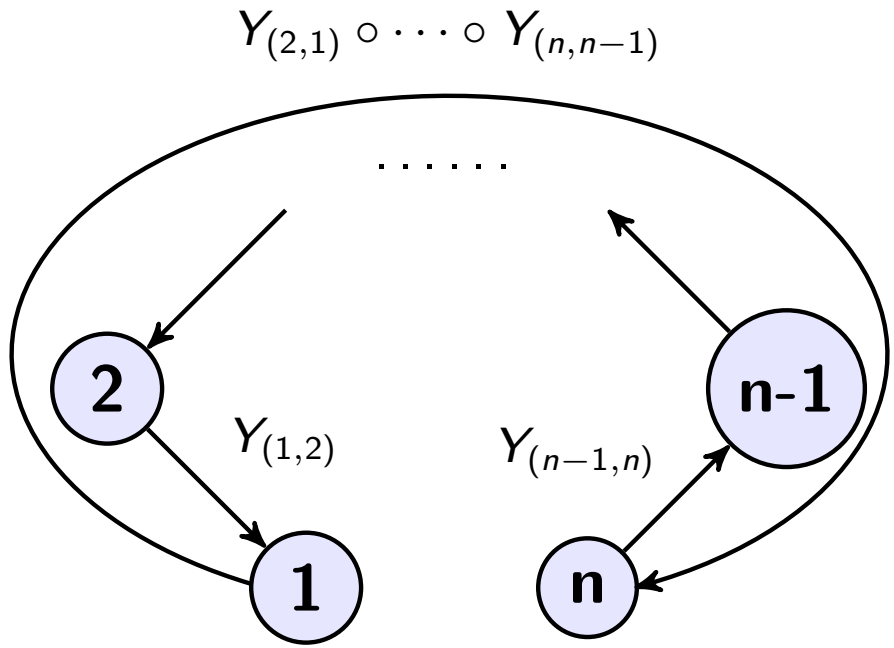


but then encounter the same problem as approach 1 mentioned above.

Too Many Links!

- Thus may be preferable to 'bring it all the way around the loop':

$$\int d^D x\, f(\mathbf{g}_{(n)}, \mathbf{g}_{(1)}) \to \int d^D x\, f(\mathbf{g}_{(n)}, \mathbf{g}_{(1)} \circ Y_{(2,1)} \circ \cdots \circ Y_{(n,n-1)}).$$



- This introduces interactions between all the Stückelberg fields.

Canonical Normalisation and Decoupling Limit

- In massive/bi-gravity scalar kinetic term gets introduced with coefficient $\sim m^4 M^2$;
- bimetric interactions without closed loops lead to the same;
- higher order interactions generally do not change this.
- Loops lead to interactions between many Stückelberg fields;
- scaling changes from $\Lambda_\lambda^{4-n_h-2n_A-3n_\phi} = m^{2-n_A-2n_\phi} M_{\rm P}^{2-n_h-n_A-n_\phi}$ to $m_{\rm loop}^2 \left(\prod_i m_{{\rm eff},(i)}^{-n_A^{(i)}-2n_\phi^{(i)}} \right) M_{\rm P}^{2-n_h-n_A-n_\phi}$, where $m_{{\rm eff},i}^4 = m_i^4 + m_{\rm loop}^4$.
- Possibility of hierarchy of scales.

Conclusions

- Have looked at how interacting spin-2 fields may be analysed in the Stückelberg picture.
- In a multi metric context there are several ways in which the Stückelberg fields may be introduced.
- Loops in the theory graph introduce interactions between many Stückelberg fields.
- This will also change the normalisation of the fields, and leads to the possibility of another hierarchy of scales.