Born-Infeld extension of Lovelock brane gravity

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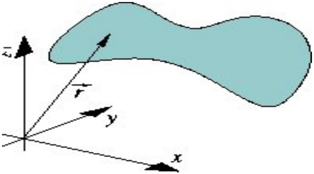
Outline

Quick kinematic description for branes

- Mechanical information is contained in the brane trajectory described by $y^{\mu} = X^{\mu}(x^{a})$ (m worldvolume)



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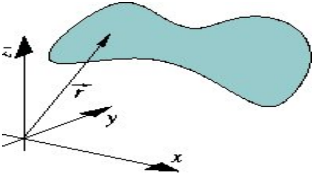

First fundamental form (induced metric on m)

$$g_{ab} := e_a \cdot e_b = \eta_{\mu\nu} e^\mu{}_a e^\nu{}_b$$

Second fundamental form (extrinsic curvature of m)

$$K_{ab} := -n \cdot \nabla_a \nabla_b X$$

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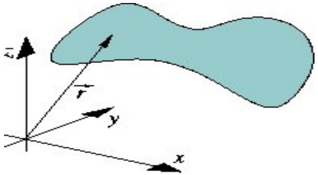

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Typical actions

$$S[X] = \int_m d^{p+1}x \sqrt{-g}\, L(g_{ab}, K_{ab}) \quad \Longrightarrow \quad \text{fourth-order EOM}$$

(Carter, Vilenkin, Garriga, Guven, Capovilla)

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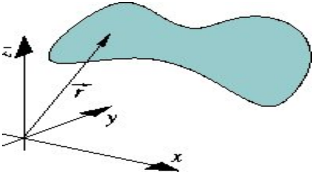

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Typical actions

$$S[X] = C \int_m d^3x \sqrt{g}\, K^2 \qquad \Longrightarrow \qquad \nabla^2 K - 2K_{ab}K^{ab}K = 0$$

(Carter, Vilenkin, Garriga, Guven, Capovilla)

Lovelock brane gravity

Lovelock brane theory

It is possible to extend the original Lovelock theory to the case of branes

(R Reilly (1973), C de Rham, Tolley, Deffayet, M Trodden, and many others)

$$S[X] = \int_m d^{p+1}x \sqrt{-g} \sum_{n=0}^{t} \alpha_n L_n(g_{ab}, K_{ab})$$

$$L_n(g_{ab}, K_{ab}) = \delta^{a_1 a_2 a_3 \cdots a_n}_{b_1 b_2 b_3 \cdots b_n} K^{b_1}{}_{a_1} K^{b_2}{}_{a_2} \cdots K^{b_n}{}_{a_n}$$

$a, b = 0, 1, \ldots, p$

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$a, b = 0, 1, \ldots, p$

$$\begin{aligned} \mathcal{L} &= \sqrt{-g}\Big\{ \underbrace{\alpha_0}_{\text{Nambu-Goto}} + \alpha_1 \underbrace{K}_{\text{Gibbons-Hawking-York}} + \alpha_2 \underbrace{\mathcal{R}}_{\text{Regge-Teitelboim}} \\ &+ \alpha_3 \underbrace{\left(-\frac{1}{3}K^3 + KK^a{}_bK^b{}_a - \frac{2}{3}K^a{}_bK^b{}_cK^c{}_a - 2G_{ab}K^{ab}\right)}_{\text{GHY-Myers}} \\ &+ \alpha_4 \underbrace{\left(\mathcal{R}^2 - 4\mathcal{R}_{ab}\mathcal{R}^{ab} + \mathcal{R}_{abcd}\mathcal{R}^{abcd}\right)}_{\text{Gauss-Bonnet}} + \mathcal{O}(K^5)\Big\} \end{aligned}$$

Lovelock brane theory

Second-order EOM in X

$$J_n^{ab} K_{ab} = L_{n+1} = 0$$

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$$J_n^{ab} K_{ab} = L_{n+1} = 0$$

- $J_0^{ab} = g^{ab}$
- $J_1^{ab} = g^{ab} K - K^{ab} = g^{ab} L_1 - K^{ab}$
- $J_2^{ab} = G^{ab} = \mathcal{R}^{ab} - \frac{1}{2} g^{ab} \mathcal{R}$
- $J_3^{ab} = g^{ab} L_3 - 3\mathcal{R} K^{ab} + 6 K K^a{}_c K^{cb} - 6 K^a{}_c K^c{}_d K^{db}$

Lovelock brane theory

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Utility

- Cosmology in extra dimensions
- Galileons
- Standard GR. Well defined variational principles
- Extensible models for the electron (Dirac)
- Lipid membranes. Corrections to the Helfrich's energy

Born-Infeld type model for surfaces

Born-Infeld-Lovelock action for branes

$$
\begin{aligned}
S[X] &= \Lambda \int_m d^{p+1}x \, \sqrt{-\det(g_{ab} + X_{ab})} \\
X^a{}_b &:= 2\alpha K^a{}_b + \alpha^2 K^{ac} K_{cb}
\end{aligned}
$$

M. Cruz & E. Rojas, CQG, 2013

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Geometrically, it describes to parallel surfaces, $\bar{X}^\mu = X^\mu + \alpha\, n^\mu$.

$$
\begin{aligned}
S[X] &= \Lambda \int_m d^{p+1}x \sqrt{-g} \sum_{n=0}^{t} \left(\frac{\alpha_n}{n!}\right) L_n(g_{ab}, K_{ab}) \\
&= \Lambda \int_m d^{p+1}\xi \sqrt{-g} \left\{ 1 + \alpha\, K + \frac{\alpha^2}{2} \mathcal{R} \right. \\
&+ \frac{\alpha^3}{6} \left(K^3 - 3KK^a{}_b K^b{}_a + 2K^a{}_b K^b{}_c K^c{}_a \right) \\
&+ \left. \frac{\alpha^4}{24} \left(\mathcal{R}^2 - 4\mathcal{R}_{ab}\mathcal{R}^{ab} + \mathcal{R}_{abcd}\mathcal{R}^{abcd} \right) + \mathcal{O}(K^5) \right\}
\end{aligned}
$$

Born-Infeld type model for branes

Some comments

- Cosmology in extra dimensions
- Charged membrane models. Extensible models for the electron
- Equivalent actions with Weyl invariance
- Different point of view to study scalar field theories, Galileons
- Inclusion of matter

This effective theory is another geometrical alternative to understand the mechanical content of the branes

Modified Regge-Teitelboim model

Example

$$S[X] = \int_m d^4x \sqrt{-g}\,(\alpha' \mathcal{R} + \beta\, K - \Lambda) + S_{\text{matt}}$$

R Cordero, M Cruz, A Molgado, & E Rojas, CQG (2012)

- Second-order EOM $T^{ab} K_{ab} = 0$
- $T^{ab} = \alpha G^{ab} + \beta\, S^{ab} + \Lambda\, g^{ab} + T^{ab}_{\text{matt}}$ stress tensor
- $G_{ab} = \mathcal{R}_{ab} - \frac{1}{2}\mathcal{R}\, g_{ab}$ Einstein tensor on m
- $S_{ab} = K_{ab} - K\, g_{ab}$ conserved tensor
- This brane-like universe shows a self-(non-self-) acceleration behaviour

Modified Regge-Teitelboim model. FRW cosmology

Within a FRW scenario, a 3-brane in a fixed Minkowski bulk, then
$ds_4^2 = N^2 d\tau^2 + a^2 d\Omega_3^2$, (with $N^2 = \dot{t}^2 - \dot{a}^2$, and $\rho = \rho_0/a^3$).

Friedmann type equation

$$-\frac{E}{a^4} = \frac{(\dot{a}^2 + k)^{1/2}}{a}\left[\frac{(\dot{a}^2 + k)}{a^2} - (\bar{\Lambda} + \bar{\rho})\right] + 3\bar{\beta}\frac{(\dot{a}^2 + k)}{a^2}$$

Classical analysis

$$\dot{a}^2 + U(a, E) = 0$$

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Classical analysis

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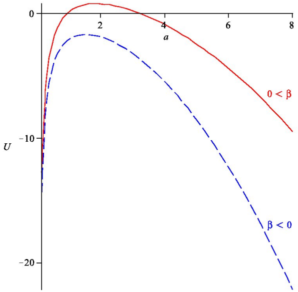
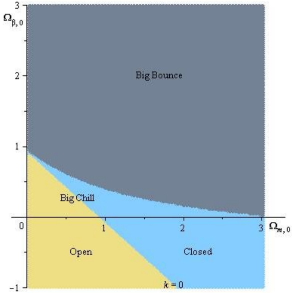
The potential

$$\frac{U(a,E)}{H_0^2} = -\Omega_{k,0} - \frac{a^2}{9}\left\{2\left[\Omega_{\beta,0}^2 + 3\left(\Omega_{\Lambda,0} + \frac{\Omega_{m,0}}{a^3}\right)\right]^{1/2} \times F\left[\frac{1}{3}F^{-1}\left(\frac{\Omega_{\beta,0}\left[\Omega_{\beta,0}^2 + \frac{9}{2}\left(\Omega_{\Lambda,0} + \frac{\Omega_{m,0}}{a^3}\right)\right] - \frac{27\Omega_{dr}}{2a^4}}{\left[\Omega_{\beta,0}^2 + 3\left(\Omega_{\Lambda,0} + \frac{\Omega_{m,0}}{a^3}\right)\right]^{3/2}}\right)\right] - \Omega_{\beta,0}\right\}^2$$

where $F(x) = \cosh x, \cos x$ Energy density parameters, $\Omega_k, \Omega_\Lambda, \Omega_m, \Omega_\beta, \Omega_{dr}$



Classical potential, ($\beta > 0$, $\beta < 0$)



Types of expansion

- $\beta < 0$ Big Chill
- $\beta > 0$ Big Bounce

In terms of the Hubble parameter $H = \dot{a}/a$

$$\left(\frac{H^2}{H_0^2} - \frac{\Omega_k}{a^2}\right)^{1/2} \left(\frac{H^2}{H_0^2} - \frac{\Omega_k}{a^2} - \frac{\Omega_m}{a^3} - \Omega_\Lambda\right) + \Omega_\beta \left(\frac{H^2}{H_0^2} - \frac{\Omega_k}{a^2}\right) = \frac{\Omega_{dr}}{a^4}$$

Flat spacetime Galileons

Unitary gauge $y^\mu = X^\mu(x^a) = \begin{pmatrix} x^a \\ \phi(x^a) \end{pmatrix}$

Bulk spacetime $ds_5^2 = f^2(\phi)\, q_{ab} dx^a dx^b + d\phi^2$ yields

$$\begin{aligned} g_{ab} &= f^2(\phi)\, q_{ab} + \nabla_a\phi\nabla_b\phi \\ K_{ab} &= \gamma\left(-\nabla_a\nabla_b\phi + 2f^{-1}f'\nabla_a\phi\nabla_b\phi + ff' q_{ab}\right) \end{aligned}$$

where $\gamma = 1/\sqrt{1 + f^{-2}\nabla^a\phi\nabla_a\phi}$

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Born-Infeld-Lovelock action in terms of ϕ

$$\begin{aligned} S[\phi] &= \Lambda \int_m d^{p+1}x\, \sqrt{-g}\, [-\det(g^a{}_b + X^a{}_b)] \\ X^a{}_b &:= f^2 q^a{}_b + \nabla^a\phi\nabla_b\phi + \alpha\gamma f^{-2}\left(-\nabla^a\nabla_b\phi + \gamma^2 f^{-2}\nabla^a\phi\nabla^c\phi\nabla_c\nabla_b\phi\right. \\ &+ \left. f^{-1}f'\gamma^2\nabla^a\phi\nabla_b\phi + ff' q^a{}_b\right) \end{aligned}$$

ϕ brane bending mode. Invariance under $\phi \to \phi + b_a x^a + c$

Flat spacetime Galileons

with $\Pi_{ab} := \nabla_a \nabla_b \phi$ and $[\pi^n] := \nabla\phi \cdot \Pi^{n-2} \cdot \nabla\phi$

$$
\begin{aligned}
S[\phi] &= \Lambda \int_m d^{p+1}x \sqrt{-g} \sum_{n=0}^{p+1} \left(\frac{\alpha_n}{n!}\right) L_n(\phi) \\
\mathcal{L}_1 = \sqrt{-g} &= \sqrt{-q}\,\gamma^{-1} f^{p+1} \\
\mathcal{L}_2 = \sqrt{-g}K &= \sqrt{-q}\, f^{p-1} \left\{ -[\Pi] + \frac{\gamma^2}{f^2}[\pi^3] + ff'(p+2-\gamma^2) \right\} \\
\mathcal{L}_3 = \sqrt{-g}\mathcal{R} &= \sqrt{-q}\,\gamma\, f^{p-3} \left\{ [\Pi]^2 - [\Pi^2] + 2\frac{\gamma^2}{f^2}(-[\Pi][\pi^3] + [\pi^4]) \right. \\
&\quad +2ff' \left[-(p+1)[\Pi] + \frac{\gamma^2}{f^2}(f^2[\Pi] + (p+1)[\pi^3]) \right] \\
&\quad \left. +f^2 f'^2 p(p+3-2\gamma^2) \right\}
\end{aligned}
$$

- $f = 1,\ q_{ab} = \eta_{ab}$ DBI Galileons
- $f = \phi,\ q_{ab} = q_{ab}^{\mathrm{dS}_{(p+1)}}$ type II dS DBI Galileons

Conclusions

- Born-Infeld-Lovelock brane model is under current investigation.
- This effective theory is another geometrical alternative to understand the mechanical content of the branes
- Main applications in cosmology in extra dimensions
- Galileons. Scalar fields + cosmology
- Equivalent Weyl invariant actions