ASPECT OF NON-MINIMAL DERIVATIVE COUPLING

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INTRODUCTION

- There are many models have been proposed to describe the accelerated expansion of the universe.
- The derivative coupling of a scalar filed to the curvature can be considered as a generalized scalar-tensor.
- It can be extending a scalar-tensor theories by allowing the non-minimal couplings between the derivatives of the scalar fields and the Einstein tensor which giving more interesting behaviors.
- The power-law expansion, $a \propto t^{\alpha}$, is a model that can remove flatness and horizon problems in a simplest inflationary model and it is also considering in specific gravity or dark energy model i.e. *f*(*R*) and *f*(*G*) gravities. It is also considering in a context of scalar field cosmology, phantom scalar field cosmology.

INTRODUCTION

- In this work, it is also assumed flat FLRW universe filled with a perfect fluid and scalar field.
- We use a power-law expansion to constrain the couplings constant in non-minimal derivative scalar field which there is non-minimal coupling between derivative of the scalar field and the Einstein tensor.
- We use the observational data from a combined WMAP9 data set (WMAP9+eCMB+BAO+H_0) to estimate (approximately) the value of the cosmological parameters and use them to find the value of the coupling constants.
- The signature using in this work is most positive and we set a speed of light $c = 1$.

NMDC AND COSMOLOGICAL EQUATIONS

- Consider the theory of gravity with the action

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{8\pi G} - \left[\varepsilon g_{\mu\nu} + \kappa G_{\mu\nu} \right] \phi^{,\mu} \phi^{,\nu} - 2V(\phi) \right\} + S_m \qquad (1)$$

- In the flat FLRW universe, we can write

$$3H^2 = 4\pi G \dot{\phi}^2 \left(\varepsilon - 9\kappa H^2 \right) + 8\pi G V + 8\pi G \rho \qquad (2)$$

$$2\dot{H} + 3H^2 = -4\pi G \left[\varepsilon + \kappa \left(2\dot{H} + +3H^2 + 4H\ddot{\phi}\dot{\phi}^{-1} \right) \right] + 8\pi G V - 8\pi G p \qquad (3)$$

NMDC AND COSMOLOGICAL EQUATIONS

$$\left(\varepsilon - 3\kappa H^2\right)\ddot{\phi} + 3H\left(\varepsilon - 3\kappa H^2 - 2\kappa\dot{H}\right)\dot{\phi} = -V_{,\phi} \qquad (4)$$

where $V_{,\phi}$ (= $dV/d\phi$)is a derivative with respect to scalar field ϕ.

- From above equations, we can extract the energy density and pressure of scalar field as follow,

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2\left(\varepsilon - 9\kappa H^2\right) + V \qquad (5)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2\left(\varepsilon + 2\kappa\dot{H} + 3\kappa H^2 + 4\kappa H\ddot{\phi}\dot{\phi}^{-1}\right) - V \qquad (6)$$

NMDC AND COSMOLOGICAL EQUATIONS

- Considering the continuity equation of scalar filed

$$\dot{\rho}_\phi = -3H\rho_\phi(1+w_\phi) \qquad (7)$$

- Substituting the energy density from (5) on the LHS of (7)

$$\dot{\rho}_\phi = -3H\left[\frac{1}{2}\dot{\phi}^2\left(\varepsilon - 9\kappa H^2\right) + V\right](1+w_\phi) \qquad (8)$$

NMDC AND COSMOLOGICAL EQUATIONS

- Comparing with the time derivative of the energy density from (5),

$$\dot{\rho}_{\phi} = \left(\varepsilon - 9\kappa H^{2}\right)\dot{\phi}\ddot{\phi} - 9\kappa H\dot{H}\dot{\phi}^{2} + V_{,\phi}\dot{\phi} \qquad (9)$$

- We can obtain the equation of state (eos) parameter form

$$w_{\phi} = -\frac{\left(\varepsilon - 9\kappa H^{2}\right)\dot{\phi}\ddot{\phi} - 9\kappa H\dot{H}\dot{\phi}^{2} + V_{,\phi}\dot{\phi}}{3H\left(\frac{1}{2}\dot{\phi}^{2}\left(\varepsilon - 9\kappa H^{2}\right) + V\right)} - 1 \qquad (10)$$

NMDC AND COSMOLOGICAL EQUATIONS

- By using Klein-Gordon (KG) equation (4), we have

$$\ddot{\phi} = -3H\dot{\phi} + \frac{6\kappa H\dot{H}}{\varepsilon - 3\kappa H^2}\dot{\phi} - \frac{V_{,\phi}}{\varepsilon - 3\kappa H^2} \qquad (11)$$

- Then substituting back to obtain the eos parameter (10) becomes

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2\left(\varepsilon - 9\kappa H^2\right)\left[2\kappa\dot{H}\left(\frac{\varepsilon + 9\kappa H^2}{\left(\varepsilon - 9\kappa H^2\right)\left(\varepsilon - 3\kappa H^2\right)}\right) + 1\right] - \frac{2\kappa H\dot{\phi}V_{,\phi}}{\left(\varepsilon - 3\kappa H^2\right)} - V}{\frac{1}{2}\dot{\phi}^2\left(\varepsilon - 9\kappa H^2\right) + V} \qquad (12)$$

NMDC AND COSMOLOGICAL EQUATIONS

- From the eos parameter, we obtain the scalar field pressure in a form

$$p_\phi = \frac{1}{2}\dot{\phi}^2\left(\varepsilon - 9\kappa H^2\right)\left[2\kappa\dot{H}\left(\frac{\varepsilon + 9\kappa H^2}{\left(\varepsilon - 9\kappa H^2\right)\left(\varepsilon - 3\kappa H^2\right)}\right) + 1\right] - \frac{2\kappa H\dot{\phi}V_{,\phi}}{\left(\varepsilon - 3\kappa H^2\right)} - V \qquad (13)$$

- Taking time derivative to Friedmann equation (2), we have

$$\dot{H} = -\frac{4\pi G}{3H}\dot{\phi}\left[3\kappa H\dot{H}\dot{\phi} - (\varepsilon - 9\kappa H^2)\ddot{\phi} + V_{,\phi}\right] + \frac{4\pi G}{3H}\dot{\rho} \qquad (14)$$

NMDC AND COSMOLOGICAL EQUATIONS

- By using a continuity of pressure-less matter and using (11), we obtain

$$\dot{\phi}^2 = \frac{\left(\varepsilon - 9\kappa H^2\right)V_{,\phi}\dot{\phi} + \frac{\left(\varepsilon - 3\kappa H^2\right)}{3H}V_{,\phi} + \frac{\left(\varepsilon - 3\kappa H^2\right)}{4\pi G}\left(-\dot{H} + 4\pi G\rho\right)}{3\kappa\dot{H}\left(\varepsilon - 3\kappa H^2\right) + \left(\varepsilon - 3\kappa H^2 - 2\kappa\dot{H}\right)\left(\varepsilon - 9\kappa H^2\right)} \tag{15}$$

THE CONSTANT POTENTIAL

- We are interesting in the case of the scalar field's potential plays a role of a cosmological constant in a form

$$V \equiv \frac{\Lambda}{8\pi G} \tag{16}$$

where $\Lambda \geq 0$ and $V_{,\phi} = 0$ in this case.

- Then the kinetic term of scalar field (15) becomes

$$\dot{\phi}^2 = \frac{\frac{1}{4\pi G}\left(\varepsilon - 3\kappa H^2\right)\left(-\dot{H} + 4\pi G\rho\right)}{3\kappa\dot{H}\left(\varepsilon - 3\kappa H^2\right) + \left(\varepsilon - 3\kappa H^2 - 2\kappa\dot{H}\right)\left(\varepsilon - 9\kappa H^2\right)} \tag{17}$$

THE CONSTANT POTENTIAL

- The eos parameter becomes

$$w_\phi = \frac{\begin{array}{l}\{(-\dot{H}+4\pi G\rho)\left[2\kappa\dot{\mathrm{H}}\left(\varepsilon+9\kappa H^2\right)+\left(\varepsilon-9\kappa H^2\right)\left(\varepsilon-3\kappa H^2\right)\right]\\ -3\kappa\dot{H}\left(\varepsilon-3\kappa H^2\right)\Lambda-\left(\varepsilon-3\kappa H^2-2\kappa\dot{H}\right)\left(\varepsilon-9\kappa H^2\right)\Lambda\}\end{array}}{\begin{array}{l}\{(-\dot{H}+4\pi G\rho)\left(\varepsilon-3\kappa H^2\right)\left(\varepsilon-9\kappa H^2\right)+3\kappa\dot{H}\left(\varepsilon-3\kappa H^2\right)\Lambda\\ +\left(\varepsilon-3\kappa H^2-2\kappa\dot{H}\right)\left(\varepsilon-9\kappa H^2\right)\Lambda\}\end{array}} \tag{18}$$

POWER-LAW WITH NMDC

- Now we consider the power-law expansion model

$$a \equiv a_0 \left(\frac{t}{t_0} \right)^{\alpha} \tag{19}$$

where α is a constant in a range $0 \le \alpha \le \infty$, a is scale factor at a cosmic time t and a_0 is a scale factor at a present time t_0 and set to unity in FLRW universe.

- From a definition of Hubble parameter $H = \dot{a}/a$, then

$$H = \frac{\alpha}{t}, \quad \dot{H} = -\frac{\alpha}{t^2} \tag{20}$$

POWER-LAW WITH NMDC

- Substituting these parameter into eos parameter (18),

$$w_\phi = \frac{\left\{\left(\frac{\alpha t^{3\alpha} + 4\pi G\rho_0 t_0^{3\alpha} t^2}{t^{2+3\alpha}}\right)\left[-2\alpha\kappa\left(\varepsilon t^2 + 9\kappa\alpha^2\right) + \left(\varepsilon t^2 - 9\kappa\alpha^2\right)\left(\varepsilon t^2 - 3\kappa\alpha^2\right)\right] + 3\alpha\kappa\left(\varepsilon t^2 - 3\kappa\alpha^2\right)\Lambda - \left(\varepsilon t^2 - 3\kappa\alpha^2 + 2\kappa\alpha\right)\left(\varepsilon t^2 - 9\kappa\alpha^2\right)\Lambda\right\}}{\left\{\left(\frac{\alpha t^{3\alpha} + 4\pi G\rho_0 t_0^{3\alpha} t^2}{t^{2+3\alpha}}\right)\left(\varepsilon t^2 - 3\kappa\alpha^2\right)\left(\varepsilon t^2 - 9\kappa\alpha^2\right) - 3\kappa\alpha^2\left(\varepsilon t^2 - 3\kappa\alpha^2\right)\Lambda + \left(\varepsilon t^2 - 3\kappa\alpha^2 + 2\kappa\alpha\right)\left(\varepsilon t^2 - 9\kappa\alpha^2\right)\Lambda\right\}} \qquad (21)$$

POWER-LAW WITH NMDC

- Consider a cosmic time $t \rightarrow t_0$ then $w_\phi \rightarrow w_{\phi,0}$, $\Lambda \rightarrow \Lambda_0$ and $\alpha \rightarrow \alpha_0$ and the eos parameter becomes

$$w_{\phi,0} = \frac{\begin{gathered}\left\{A_0\left[-2\alpha_0 t_0^2 \varepsilon\kappa\left(1+6\alpha_0\right)-9\alpha_0^3\kappa^2\left(2-3\alpha_0\right)+\varepsilon^2 t_0^4\right]+3\alpha_0\kappa\left(\varepsilon t_0^2-3\kappa\alpha_0^2\right)\Lambda_0\right. \\ \left.-\left[\varepsilon^2 t_0^4+2\varepsilon\kappa\alpha_0 t_0^2\left(1-6\alpha_0\right)-9\alpha_0^3\kappa^2\left(2-3\alpha_0\right)\right]\Lambda_0\right\}\end{gathered}}{\begin{gathered}\left\{A_0\left[\varepsilon^2 t_0^4-12\varepsilon\kappa\alpha_0^2 t_0^2+27\alpha_0^4\kappa^2\right]-3\kappa\alpha_0^2\left(\varepsilon t_0^2-3\kappa\alpha_0^2\right)\right. \\ \left.+\left[\varepsilon^2 t_0^4+2\varepsilon\kappa\alpha_0 t_0^2\left(1-6\alpha_0\right)-9\kappa^2\alpha_0^3\left(2-3\alpha_0\right)\right]\Lambda_0\right\}\end{gathered}} \tag{22}$$

POWER-LAW WITH NMDC

where A_0 is constant and

$$A_0 = \left(\frac{\alpha_0}{t_0^2} + 4\pi G \rho_0 \right) \tag{23}$$

- To derive the coupling constant, κ, we use the ΛCDM data set derived from a combined WMAP9 data set (WMAP9+eCMB+BAO+H_0) as shown in Table-I.
- We are considering into 2 cases,
  - $w_{\phi,0} = w_{\mathrm{DE}} = -1$ and
  - $w_{\phi,0} = w_{\mathrm{DE}} = -1.073$ (derived from a combined WMAP9 data set)

POWER-LAW WITH NMDC

Table-I: Cosmological parameters from a combined WMAP9 (WMAP9+eCMB+BAO+H_0) and derived parameters (in a lower part)

Parameter	**WMAP9+eCMB+BAO+H_0**
t_0 (Gyr)	13.772 ± 0.059 $\left[(4.352 \pm 0.019) \times 10^{17}\,\mathrm{s}\right]$
H_0 (km/s/Mpc)	69.32 ± 0.80 $\left[(2.246 \pm 0.026) \times 10^{-18}\,\mathrm{s}^{-1}\right]$
$\Omega_{\mathrm{m},0}$	$0.2865^{+0.0096}_{-0.0095}$
$\Omega_{\Lambda,0}$	$0.7135^{+0.0095}_{-0.0096}$
$w_{\mathrm{DE},0}$	$-1.073^{+0.090}_{-0.089}$
$\rho_{\mathrm{m},0}$ (kg/m^3/s^2)	$\left(2.5868^{+0.1164}_{-0.1148}\right) \times 10^{-27}$
Λ_0 (s^{-2})	$\left(1.0797^{+0.0268}_{-0.0269}\right) \times 10^{-35}$
α_0	0.977 ± 0.016

POWER-LAW WITH NMDC

- Using data from Table-I we can write the eos parameter equation as

$$w_{\phi,0} = \frac{\left(-1.2449\times 10^{35}\right)\varepsilon^2 + 6.8152\varepsilon\kappa - \left(1.1774\times 10^{-34}\right)\kappa^2}{\left(6.5013\times 10^{35}\right)\varepsilon^2 - 41.1782\varepsilon\kappa + \left(3.5314\times 10^{-34}\right)\kappa^2} \quad (24)$$

- In the case of canonical field, $\varepsilon = 1$, $w_{\phi,0}$ becomes

$$w_{\phi,0} = \frac{-1.2449\times 10^{35} + 6.8152\kappa - \left(1.1774\times 10^{-34}\right)\kappa^2}{6.5013\times 10^{35} - 41.1782\kappa + \left(3.5314\times 10^{-34}\right)\kappa^2} \quad (25)$$

POWER-LAW WITH NMDC

- For $w_{\phi,0} = w_{\mathrm{DE}} = -1$, we can solve for the coupling constants and derived

$$\kappa_{c1} = 1.7362 \times 10^{34}$$
$$\kappa_{c2} = 1.2861 \times 10^{35} \quad (26)$$

- For $w_{\phi,0} = w_{\mathrm{DE}} = -1.073$, the coupling constants are

$$\kappa'_{c1} = 1.7469 \times 10^{34}$$
$$\kappa'_{c2} = 1.2561 \times 10^{35} \quad (27)$$

POWER-LAW WITH NMDC

- In the case of phantom field, $\varepsilon = -1$, $w_{\phi,0}$ becomes

$$w_{\phi,0} = \frac{-1.2449\times10^{35} - 6.8152\kappa - \left(1.1774\times10^{-34}\right)\kappa^2}{6.5013\times10^{35} + 41.1782\kappa + \left(3.5314\times10^{-34}\right)\kappa^2} \tag{28}$$

POWER-LAW WITH NMDC

- For $w_{\phi,0} = w_{\mathrm{DE}} = -1$, we can solve for the coupling constants and derived

$$\kappa_{p1} = -1.2861\times10^{35}$$
$$\kappa_{p2} = -1.7362\times10^{34} \qquad (26)$$

- For $w_{\phi,0} = w_{\mathrm{DE}} = -1.073$, the coupling constants are

$$\kappa'_{p1} = -1.2561\times10^{35}$$
$$\kappa'_{p2} = -1.7469\times10^{34} \qquad (27)$$

CONCLUSIONS

- In this work, we using the non-minimal derivative coupling which there is a coupling between the derivative of scalar fields and the Einstein tensor.
- We assumed the flat FLRW universe filled with a perfect fluids and set $c = 1$.
- We used the power-law expansion to constraint the cosmological parameters and try to find the coupling constants.
- To derived the values of the coupling constants, we used the derived parameters from a combined WMAP9 data set (WMAP9+eCMB+BAO+H_0).
- In the case of canonical field ($\varepsilon = 1$) and phantom field ($\varepsilon = -1$) we obtained the coupling constants with the same magnitude but opposite sign for the same value of the eos parameter, $w_{\phi,0} = -1$ and $w_{\phi,0} = -1.073$.
- The coupling constants are shown in Table-II.

CONCLUSIONS

Table-II: The coupling constants derived from the power-law combined with the non-minimal derivative coupling model by using the combined WMAP9 data set

	Canonical ($\varepsilon = 1$)	**Phantom ($\varepsilon = -1$)**
$w_{\phi,0} = -1$	$\kappa_{c1} = 1.7362\times10^{34}$ $\kappa_{c2} = 1.2861\times10^{35}$	$\kappa_{p1} = -1.2861\times10^{35}$ $\kappa_{p2} = -1.7362\times10^{34}$
$w_{\phi,0} = -1.073$	$\kappa'_{c1} = 1.7469\times10^{34}$ $\kappa'_{c2} = 1.2561\times10^{35}$	$\kappa'_{p1} = -1.2561\times10^{35}$ $\kappa'_{p2} = -1.7469\times10^{34}$

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