Holographic perfect fluidity, TMG and gravitational duality

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Highlights

Framework: holographic applications

- Holographic fluids: hydrodynamic approximation of finite-T and finite-μ states of a boundary CFT
  - universal bound $\eta/s \geq \hbar/4\pi k_B$ [1, 2]
  - general fluid/gravity correspondence [3, 4]
- Reminder: bulk geometries solve (in lowest order in α') Einstein's Eqs. – boundaries have a priori no gravitational dofs
- Still: "gravitational dynamics" of the boundary is relevant
  - as an external input for controlling the bulk
  - in HS AdS_4/CFT_3 [see comments by Misha Vasiliev]
- Guideline: bulk gravitational duality

Many aspects rooted in (modified) gravity [subject of the school]

More specifically

$3+1$ bulk $\to$ $2+1$ boundary

- Describe non-trivial global equilibrium on non-trivial boundary geometries $\to$ *perfect fluidity*
- Set conditions on the boundary geometry for equilibrium states to exist $\to$ *perfect geometry: TMG and beyond*
- Determine holographically classes of transport coefficients $\to$ *bonus* (usually corr. functs. in $\omega, k \to 0$ [see Diana Vaman's lectures])

Note: not to be confused with recent activity on $2+1$ bulk $\to$ $1+1$ boundary AdS_3/CFT_2 with TMG, EWG, NMG, ZDG solutions in the bulk [see Paul Townsend's lectures]

Highlights

On vector-field congruences [5]

Vector field u *with* $u_\mu u^\mu = -1$ *and space–time variation* $\nabla_\mu u_\nu$

$$\nabla_\mu u_\nu = -u_\mu a_\nu + \sigma_{\mu\nu} + \frac{1}{D-1}\Theta h_{\mu\nu} + \omega_{\mu\nu}$$

- $h_{\mu\nu} = u_\mu u_\nu + g_{\mu\nu}$: projector/metric on the orthogonal space
- $a_\mu = u^\nu \nabla_\nu u_\mu$: acceleration
- $\sigma_{\mu\nu}$: symmetric traceless part – shear
- $\Theta = \nabla_\mu u^\mu$: trace – expansion
- $\omega_{\mu\nu}$: antisymmetric part – vorticity

$$\omega = \frac{1}{2}\omega_{\mu\nu}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu = \frac{1}{2}(\mathrm{du} + \mathrm{u} \wedge \mathrm{a})$$

Fluid dynamics

Fluids in gravitational backgrounds $g_{\mu\nu}$ *described in terms of* $\mathsf{u}, \varepsilon, p$ *all inside* $T_{\mu\nu}$ *satisfying Euler equations*

$$\nabla_\mu T^{\mu\nu} = 0$$

plus equation of state (involving T, s)

Perfect fluids: $T^{\mu\nu}_{\text{perf}} = \varepsilon u^\mu u^\nu + p h^{\mu\nu}$

$$\begin{cases} \nabla_{\mathsf{u}}\varepsilon + (\varepsilon + p)\Theta = 0 \\ \nabla_\perp p - (\varepsilon + p)\mathsf{a} = 0 \end{cases}$$

General fluids: viscous part – in $2+1$ *dim & at* $\mathrm{O}(\nabla \mathrm{u})$ [6]

$$T^{\mu\nu}_{\mathsf{visc}} = -\left(2\eta\sigma^{\mu\nu} + \zeta h^{\mu\nu}\Theta + \zeta_{\mathsf{H}}\epsilon^{\rho\lambda(\mu}u_\rho\sigma_\lambda{}^{\nu)}\right)$$

Landau frame: all corrections are transverse

Conformal fluids: $\varepsilon = 2p, \zeta = 0, \ldots$

Transverse, traceless and conformal corrections

Transport coefficients: dissipative and non-dissipative

Zero frequency & momentum limit of correlation functions

Equilibrium – backgrounds with a time-like Killing field

Fluids at equilibrium: stationary states without external forces – evolution without dissipation

- Kinematics & pressure/density distributions tight to the geometry
- Hard to achieve/study because of transport (dissipative and non-dissipative)

Special situation: perfect equilibrium

Stationary states with conspiring kinematics and transport coefs.

$$T^{\mu\nu} = T^{\mu\nu}_{\text{perf}}$$

The fluid is not perfect – the equilibrium is perfect

Simple example: Minkowski background

Fluid

- in inertial motion – no $a^\mu, \Theta, \sigma^{\mu\nu}, \omega^{\mu\nu}$
- with constant T and p

is in global equilibrium

The fluid is not assumed perfect – even for viscous fluids kinematics implies $T^{\mu\nu} = T^{\mu\nu}_{\text{perf}}$ and Euler Eqs. are satisfied

Reason: all rank-2 tensors built out of $\nabla^n u$ vanish in this background

$\exists$ less trivial backgrounds allowing for less trivial perfect equilibrium states?

Specific situation (motivated by holography)

- Conformal fluids – reduces the set of allowed tensors
- Backgrounds with a normalized time-like Killing vector as e.g. Papapetrou–Randers

Killing vector field of constant norm: a remarkable congruence

$$\nabla_{(\mu}\xi_{\nu)} = 0 \quad \xi_\mu \xi^\mu = -1$$

- accelerationless – geodesic
- shearless – $\sigma = 0$
- expansionless – $\Theta = 0$
- carries vorticity

$$\omega = \frac{1}{2}\mathrm{d}\xi \Leftrightarrow \omega_{\mu\nu} = \nabla_\mu \xi_\nu$$

Perfect equilibrium $\leftrightarrow$ *alignement of the velocity* u *with the Killing* ξ

$$\nabla p = 0 \begin{cases} \varepsilon = 2p \\ \nabla_{\mathsf{u}}\varepsilon = 0 \\ \nabla_{\perp} p = 0 \end{cases}$$

geodesic, shearless & expansionless fluid with vorticity at constant pressure/density

Non-perfect equilibrium

$$\begin{cases} \mathsf{u} = \xi + \delta\mathsf{u}(x) \\ p = p_0 + \delta\mathsf{p}(x) \end{cases}$$

out of our scope

Conditions for perfect equilibrium

Interplay between the geometry and the fluid transport properties

- *Dangerous tensors:* rank-2, transverse, traceless, conformal tensors built out of $\nabla^n \xi$ and geometrical tensors (e.g. Ricci, Cotton–York) with non-zero divergence – geometry
- *Conditions:* vanishing of transport coefficients coupled to dangerous tensors – statement about the microscopic theory

Realized in a wide class of holographic fluids

- *Specific boundary geometries:* perfect-Cotton geometries
- *Handle on microscopic properties:* infinite sequence of vanishing transport coefficients

Highlights

Holography in a nutshell

Applied beyond the original framework – type IIB string on $AdS_5 \times S^5 \to$ maximal susy YM in $D = 4$ – usually in the classical gravity approximation without backreaction

- Bulk with $\Lambda = -3k^2$: asymptotically AdS $d = D + 1$-dim $\mathcal{M}$
- Boundary at $r \to \infty$: asymptotic coframe E^μ $\mu = 0, \ldots, D - 1$

$$\mathrm{d}s^2_{\mathrm{bulk}} \approx \frac{\mathrm{d}r^2}{k^2 r^2} + k^2 r^2 \eta_{\mu\nu} E^\mu E^\nu = \frac{\mathrm{d}r^2}{k^2 r^2} + k^2 r^2 g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu$$

Where is the fluid and where are its data?

Via holography: boundary field theory at finite T and μ

Fluid $\equiv$ hydrodynamic approximation of the boundary state

Fluid: described in terms of $T_{\mu\nu}$ in a background $g_{\mu\nu}$, – data read off from the large-r expansion of θ^μ [7, 8]

θ^a: bulk orthonormal coframe ($\eta : + - ++$, $\Lambda = -3k^2$)

$$ds^2_{\text{bulk}} = \eta_{ab}\theta^a\theta^b$$

with a gauge choice s.t. $\theta^r = \frac{dr}{kr}, \quad \theta^\mu = \theta^\mu{}_\nu dx^\nu, \quad \mu = 0, 1, 2$

Holography: Hamiltonian evolution from data on the boundary subject to a regularity condition on the horizon – captured in Fefferman–Graham expansion for large r [9]

$$\theta^\mu(r,x) = kr\, E^\mu(x) + \frac{1}{kr} F^\mu_{[2]}(x) + \frac{1}{k^2 r^2} F^\mu(x) + \cdots$$

Independent $2+1$ boundary data: vector-valued 1-forms E^μ and F^μ

- E^μ: boundary orthonormal coframe – allows to determine

$$\mathrm{d}s^2 = \eta_{\mu\nu} E^\mu E^\nu = g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu$$

- F^μ: stress current one-form – allows to construct the vev of the boundary stress tensor ($\kappa = {}^{3k}/_{8\pi G}$)

$$\mathrm{T} = \kappa F^\mu e_\mu = T^\mu{}_\nu E^\nu \otimes e_\mu$$

The rest $F^\mu_{[2]}, B^\mu_{[2]}, \ldots$: Schouten, Cotton, …

Back to the original question

- $\exists\ 2+1$ *boundary geometries* $\mathrm{d}s^2$ *s.t.*

$$\mathrm{d}s^2 \ \& \ T_{\mu\nu}^{\mathrm{perf}}$$
$$\downarrow FG$$
$$\textit{exact regular } 3+1 \textit{ Einstein geometry?}$$

- *If yes:*
  - *the holographic boundary fluid behaves as a perfect fluid*
  - *information on its transport properties is made available*
- *Answer: perfect-Cotton boundary geometries*

Highlights

Fluids in Papapetrou–Randers backgrounds

Stationary geometries with unique normalized time-like Killing congruence – fluid lines

$$\mathrm{d}s^2 = -\left(\mathrm{d}t - b_i \mathrm{d}x^i\right)^2 + a_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

- Killing field: $\partial_t \leftrightarrow \mathrm{u} = -\mathrm{d}t + \mathrm{b}$ with vorticity $\omega = \frac{1}{2}\mathrm{db}$
- Perfect fluidity: comoving fluid – aligned with ∂_t

Analysing perfect fluidity: tensors in $2+1$ dimensions

- Vorticity: $\omega_{\mu\nu} = -\frac{q}{2}\eta_{\mu\nu\rho}u^\rho$, q scalar field
- Curvature: $R_{\mu\nu}$
- Cotton–York: $C^{\mu\nu} = \eta^{\mu\rho\sigma}\nabla_\rho\left(R^\nu{}_\sigma - \frac{1}{4}R\delta^\nu_\sigma\right)$

In summary

$$\mathrm{d}s^2 = -\mathrm{u}^2 + \mathrm{d}\ell^2 \quad \mathrm{u} = -\mathrm{d}t + \mathrm{b} \quad R = \hat{R} + \frac{q^2}{2}$$

$$R_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \frac{q^2}{2}\mathrm{u}^2 + \frac{\hat{R} + q^2}{2}\mathrm{d}\ell^2 - \mathrm{u}\mathrm{d}x^\rho u^\sigma \eta_{\rho\sigma\mu}\nabla^\mu q$$

$$\begin{aligned} C_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu &= \frac{1}{2}\left(\hat{\nabla}^2 q + \frac{q}{2}(\hat{R} + 2q^2)\right)\left(2\mathrm{u}^2 + \mathrm{d}\ell^2\right) \\ &\quad - \frac{1}{2}\left(\hat{\nabla}_i \hat{\nabla}_j q \mathrm{d}x^i \mathrm{d}x^j + \hat{\nabla}^2 q \mathrm{u}^2\right) \\ &\quad - \frac{\mathrm{u}}{2}\mathrm{d}x^\rho u^\sigma \eta_{\rho\sigma\mu}\nabla^\mu(\hat{R} + 3q^2) \end{aligned}$$

with

$$T^{\text{perf}}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = p\left(2\mathrm{u}^2 + \mathrm{d}\ell^2\right)$$

Special PR backgrounds

Critical points of topologically massive gravity (TMG)

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \lambda g_{\mu\nu} = \frac{1}{\mu} C_{\mu\nu}$$

- Absence of dangerous tensors
- Perfect equilibrium guaranteed for *any comoving conformal fluid* – no constraint on transport coefficients

Next non-trivial generalization of Minkowski space–time

Solving TMG with PR *[see Philippe Spindel's talk]*

- $q = {}^{2\mu}/{}_{3}$: constant vorticity
- $\hat{R} = 6\lambda - {}^{2\mu^2}/{}_{9}$: $\mathrm{d}\ell^2$ is $S^2, \mathbb{R}^2$ or H_2

Squashed 3-dim maximally symmetric space–times with $\mathbb{R} \times SU(2), \mathbb{R} \times H(2), \mathbb{R} \times SL(2, \mathbb{R})$ *isometry*

Petrov type $\mathrm{D_t}$ homogeneous spaces: TMG monopoles

Example: Gödel

A concrete example

Warped AdS$_3$: fibration over H_2

$$\begin{aligned}\mathrm{d}s^2 &= -\left(\mathrm{d}t - 2n(\cosh\sigma - 1)\mathrm{d}\varphi\right)^2 + \tfrac{1}{k^2}\left(\mathrm{d}\sigma^2 + \sinh^2\sigma\,\mathrm{d}\varphi^2\right) = \\ &= -\left(\mathrm{d}t - \mathrm{b}\right)^2 + \left(E^\sigma\right)^2 + \left(E^\varphi\right)^2 = -\left(E^t\right)^2 + \left(E^\sigma\right)^2 + \left(E^\varphi\right)^2\end{aligned}$$

$$b = 2nk\frac{\cosh\sigma - 1}{\sinh\sigma}E^\varphi \quad \omega = k^2 n E^\sigma \wedge E^\varphi$$

Dirac-monopole-like *vortex* (“hedgehog” or homogeneous) on H_2 with “magnetic charge” $q = {}^{2\mu}/{}_{3} = 2k^2 n$ $\left(\lambda = {}^{k^2}/{}_{3}(k^2n^2 - 1)\right)$

Perfect fluidity

Determining the dangerous tensors

- $C_{\mu\nu} = \mu \left(\lambda + \mu^2/9\right) \left(3u_\mu u_\nu + g_{\mu\nu}\right)$
- $R_{\mu\nu} = \left(3\lambda + \mu^2/3\right) \; u_\mu u_\nu + \left(3\lambda + \mu^2/9\right) g_{\mu\nu}$
- Homogeneity $\Rightarrow$ $\nabla_\mu \nabla_\nu \ldots u_\rho$ algebraic $\sim u_\rho$

No higher-derivative transverse, traceless rank-2 tensor (building blocks: $u_\mu u_\nu, g_{\mu\nu}$) – consequence of homogeneity as in Minkowski

Remarks

- Any comoving fluid is at perfect equilibrium & carries vorticity
- The Cotton tensor is responsible for the vorticity: $q = 2\mu/3$

Next

So far: used gravitational dynamics to shape boundary geometry in order to make possible perfect equilibrium

Questions at hand: holographic realization

- *4-dim Einstein space-times as bulk geometries?*
- *Bulk source for the boundary Cotton tensor – and vorticity?*
- *Other PR boundary backgrounds*
  - *have holographic bulk realization with perfect energy–momentum tensor?*
  - *probe transport coefficients?*

The uplift of the TMG monopoles

The backgrounds leading to a holographic fluid on TMG monopoles in perfect equilibrium are the Taub–NUT AdS black-hole geometries

- nut charge n source for the Cotton

$$nk^4 \left(\nu + 4n^2k^2\right)\left(2\mathrm{u}^2 + \mathrm{d}\ell^2\right)$$

  related to the vorticity q: $q = 2k^2 n$

- mass M source for the energy–momentum tensor

$$\kappa Mk/2 \left(2\mathrm{u}^2 + \mathrm{d}\ell^2\right)$$

  related to the energy density ε: $3\varepsilon = 2\kappa Mk$

with $\lambda = 1/3\left(k^2\nu + q^2/4\right)$, $\nu = 1, 0, -1$ for $S^2, \mathbb{R}^2, H_2$

Taub–NUT geometries in AdS

The hyperbolic case

$$\begin{aligned} ds^2_{\text{bulk}} &= \frac{d\tilde{r}^2}{V(\tilde{r})} - V(\tilde{r}) \left[dt - 4n \sinh^2 \frac{\sigma}{2}\, d\varphi \right]^2 + \rho^2 \left[d\sigma^2 + \sinh^2 \sigma\, d\varphi \right]^2 \\ &= (\theta^r)^2 - (\theta^t)^2 + (\theta^\sigma)^2 + (\theta^\varphi)^2 \end{aligned}$$

$V(\tilde{r}) = {}^{\Delta}/_{\rho^2}$ with

$$\begin{aligned} \Delta &= \left(n^2 - \tilde{r}^2\right)\left(1 - k^2\left(\tilde{r}^2 + 3n^2\right)\right) + 4k^2 n^2 \tilde{r}^2 - 2M\tilde{r} \\ \rho^2 &= \tilde{r}^2 + n^2 \end{aligned}$$

Hyperbolic horizon, no rigid angular velocity but nut charge n – one of the most peculiar solutions to Einstein's Eqs. *[Misner '63]*

FG expansion reproduces squashed AdS_3 with perfect e–m tensor

Following FG $\to$ boundary metric and stress tensor [10, 11, 12]

- Boundary geometry: warped AdS_3

$$
\begin{aligned}
\mathrm{d}s^2 &= g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \eta_{\mu\nu}E^\mu E^\nu \\
&= -\left(\mathrm{d}t - 2n(\cosh\sigma - 1)\mathrm{d}\varphi\right)^2 + \tfrac{1}{k^2}\left(\mathrm{d}\sigma^2 + \sinh^2\sigma\,\mathrm{d}\varphi^2\right)
\end{aligned}
$$

- Fluid: perfect-like stress tensor

$$
T_{\mu\nu}E^\mu E^\nu = \frac{\kappa M k}{3}\left(2(E^t)^2 + (E^\sigma)^2 + (E^\varphi)^2\right)
$$

  - conformal: $\varepsilon = 2p = {}^{2\kappa Mk}/_3$
  - comoving: $\mathrm{u} = \partial_t$

The stationary fluid aligned with the Killing of const. norm: inertial, no expansion, no shear but *uniform vorticity sourced by the nut*

Highlights

PR backgrounds $\mathrm{d}s^2 = -\mathrm{u}^2 + \mathrm{d}\ell^2$ *solving TMG provide exact bulk Einstein spaces*

- $C_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = {}^c\!/_2\left(2\mathrm{u}^2 + \mathrm{d}\ell^2\right)$ $(\nabla^\mu C_{\mu\nu} = 0 \Rightarrow c$ constant $)$
- $T^{\mathrm{perf}}_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = {}^\varepsilon\!/_2\left(2\mathrm{u}^2 + \mathrm{d}\ell^2\right)$

Can one generalize that beyond TMG?

Perfect-Cotton geometries [13]

PR metrics beyond TMG: Petrov type $\mathrm{D_t}$

$$C_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} = \frac{c}{2}\left(2\mathrm{u}^2 + \mathrm{d}\ell^2\right)$$

In other words: $C_{\mu\nu} = \chi\, T^{\mathrm{perf}}_{\mu\nu}$

- FG expansion is resummable with $T^{\mathrm{perf}}_{\mu\nu}$ into exact regular Einstein space–times $\longrightarrow$ comoving holographic fluids at perfect equilibrium
- Petrov type $\mathrm{D_t}$ geometries have dangerous tensors $\longrightarrow$ non-trivial information on the transport coefficients

Why the perfect-Cotton choice?

In the Euclidean $C_{\mu\nu} = \pm 8\pi G k^2 T_{\mu\nu}$ is equivalent to the bulk Weyl (anti-)self-duality leading e.g. to quaternionic Taub–NUT and relating M and n (electric and magnetic gravitational charges) [7, 8, 9, 14, 15]

$$n(\nu - 4k^2 n^2) = \pm M$$

Holography allows for a kind of Lorentzian generalization: perfect-Cotton geometries [13]

$$C_{\mu\nu} = \chi T^{\text{perf}}_{\mu\nu}$$

is a kind of holographic integrability condition

Bulk uplift of PR perfect-Cotton geometries

$$
\begin{aligned}
\mathrm{d}s^2 \quad &= \quad -2\mathrm{u}\left(\mathrm{d}r - \frac{1}{2}\mathrm{d}x^\rho u^\sigma \eta_{\rho\sigma\mu}\nabla^\mu q\right) + \rho^2 \mathrm{d}\ell^2 \\
&\quad - \left(r^2 + \frac{\delta}{2} - \frac{q^2}{4} - \frac{1}{\rho^2}\left(2Mr + \frac{qc}{2}\right)\right)\mathrm{u}^2
\end{aligned}
$$

$$
\delta = \hat{R} + 3q^2 \quad \rho^2 = r^2 + \frac{q^2}{4}
$$

non-constant q, $\hat{R}$ (perfect-Cotton geometry Eqs. for q and $\mathrm{d}\ell^2$)

The landscape of Petrov type $\mathrm{D_t}$ *& bulks*

Solutions with 4 isometries: $\partial_t \oplus$ *Bianchi IX, II, VIII*

- Boundary: TMG monopoles – no dangerous tensors
- Bulk: Taub–NUT black holes

Solutions with 2 isometries: $\partial_t \oplus \partial_\varphi$

- Boundary: monopole plus dipole (cyclonic vorticity – $\mathrm{d}\ell^2$: squashed $S^2, \mathbb{R}^2, H_2$) – infinite sequences of dangerous tensors
- Bulk: general classes of Kerr Taub–NUT black holes with regular horizons [many new solutions in [13] – such as flat-horizon Kerr Taub–NUT]

Solutions with 1 isometry: ∂_t

Uncharted territory (exact and numerical)

- Do these boundary exist?
- Do they define regular bulk geometries?

If yes they will provide a kind of multipolar probe for the transport properties [multipolar solutions exist for flat space–time, Weyl '18]

An example of monopole & dipole

AdS Kerr Taub–NUT geometries in 4 dim

$$ds^2 = \frac{\rho^2}{\Delta_r}dr^2 - \frac{\Delta_r}{\rho^2}\left(dt + \beta d\varphi\right)^2 + \frac{\rho^2}{\Delta_\sigma}d\sigma^2 + \frac{\sinh^2\sigma\Delta_\sigma}{\rho^2}\left(a dt + \alpha d\phi\right)^2$$

with

$$\begin{aligned}
\Delta_r &= k^2 r^4 + r^2(k^2(a^2+6n^2)-1) - 2Mr + (a^2-n^2)(3k^2n^2-1)\\
\rho^2 &= r^2 + (n - a\cosh\sigma)^2\\
\Delta_\sigma &= 1 - ak^2\cosh\sigma(4n - a\cosh\sigma)\\
\alpha &= \tfrac{r^2+(n-a)^2}{Z}\\
\beta &= -\tfrac{2n(\cosh\sigma-1)+a\sinh^2\sigma}{Z}\\
Z &= 1 + k^2a^2
\end{aligned}$$

AdS Kerr Taub–NUT and its boundary

- Angular velocity a and nut charge n
- FG expansion: 3-dim PR $\mathrm{d}s^2 = -\mathrm{u}^2 + \mathrm{d}\ell^2$
  - $\mathrm{d}\ell^2 = 1/k^2 \left(\frac{\mathrm{d}\sigma^2}{\Delta_\sigma} + \frac{\sinh^2\sigma\Delta_\sigma}{Z^2}\mathrm{d}\varphi^2\right)$ squashed H_2
  - $\mathrm{u} = -\mathrm{d}t + \frac{2n(\cosh\sigma-1)+a\sinh^2\sigma}{1+k^2a^2}\mathrm{d}\varphi$ monopole & dipole

with $\mathrm{T} = \frac{\kappa M k}{3}\left(2\mathrm{u}^2 + \mathrm{d}\ell^2\right)$

  - stationary perfect-like fluid aligned with the Killing ∂_t
  - n and a are sources for the vorticity

$$q = 2k^2(n - a\cosh\sigma)$$

New features

- PR boundary *not* homogeneous but *only* axisymmetric
- *not* TMG *only* Petrov type $\mathrm{D_t}$
- $\nabla q \neq 0$ – refined probe
  - room for corrections to the perfect-fluid em tensor
  - the em tensor is perfect $\Rightarrow \exists$ set of vanishing transport coefficients for the holographic fluid – major consequence
- nut charge: source for a "perfect-like" Cotton tensor

$$C_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} = nk^4\left(k^2\left(4n^2 - a^2\right) - 1\right)\left(2\mathrm{u}^2 + \mathrm{d}\ell^2\right)$$

Highlights

Summary

Framework: fluids on PR backgrounds $\mathrm{d}s^2 = -(\mathrm{d}t - \mathrm{b})^2 + \mathrm{d}\ell^2$

- *Perfect fluids reach equilibrium by aligning* u *with* ∂_t
- *Perfect equilibrium can be reached by more general fluids under conditions on* $\mathrm{d}s^2$ *and on their microscopic properties*
  - *perfect-Cotton PR geometries*
  - *holographic fluids (→ many vanishing transport coefs.)*

Perfect-Cotton geometries (Petrov type $\mathsf{D_t}$)

$$C_{\mu\nu} = \chi T^{\text{perf}}_{\mu\nu} = {}^{c}/_{2} \left(3u_\mu u_\nu + g_{\mu\nu}\right)$$

- *Principle:*
  condition on boundary data reminiscent of conformal (Weyl) self-duality relating mass and nut i.e. energy and vorticity
- *Practice:*
  - *integrable: FG resummable into exact Einstein*
  - *generalization of TMG: more general 3-dim dynamics with Petrov $\mathsf{D_t}$ solutions (Einstein–Weyl, … [more available: NMG, ZDG])*

Beyond

Some questions in holographic fluid dynamics

- Perfect equilibrium on more general boundaries integrable to bulk with regular horizons: holography as a bottom-up solution-generating technique as Geroch, Kerr-Schild … [also TIFR group for perturbative bottom-up as in e.g. [3, 4, 16, 17]]
- Beyond integrable: response to perturbations for probing the non-vanishing transport coefficients

Important questions in gravity

- Non-isometric cases and rigidity theorem in AdS
- Understand deeper the holographic solution-generating technique and its interplay with generalized TMG
- Shed light on gravitational duality [see e.g. [18]]
- Duality and fluid equilibrium in higher dimensions?

Highlights

Short bibliography I

The list below is not complete – it is meant to help the reader throughout the various subjects mostly with review articles

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