Astrophysical solutions in Randall-Sundrum gravity

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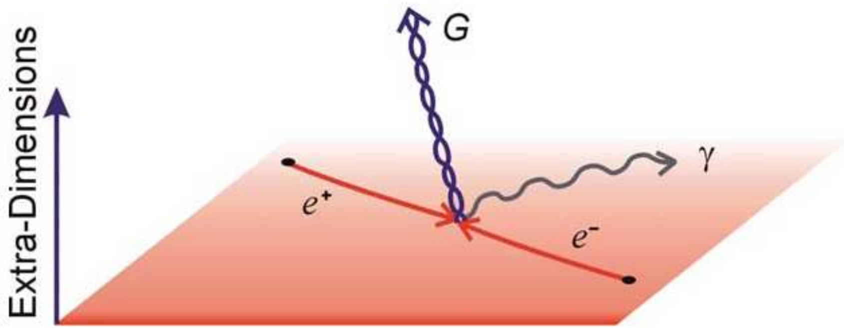
Seventh Aegean Summer School

Beyond Einstein...

- Motivation
  - The standard model and the general relativity represents the two great theories in fundamental physics. The success of general relativity is beyond any doubt, however due to its inconsistency with quantum mechanics, it is not possible to ensure that this theory keeps its original structure at high energies.
  - One of the goals of the current study is to see what features of theories beyond Einstein could lead to an answer to any of the open problems in astrophysics (dark matter) or cosmology (dark energy)
  - In this talk: Astrophysics in the Braneworld
  - In this talk: Micro Black Holes in the Braneworld

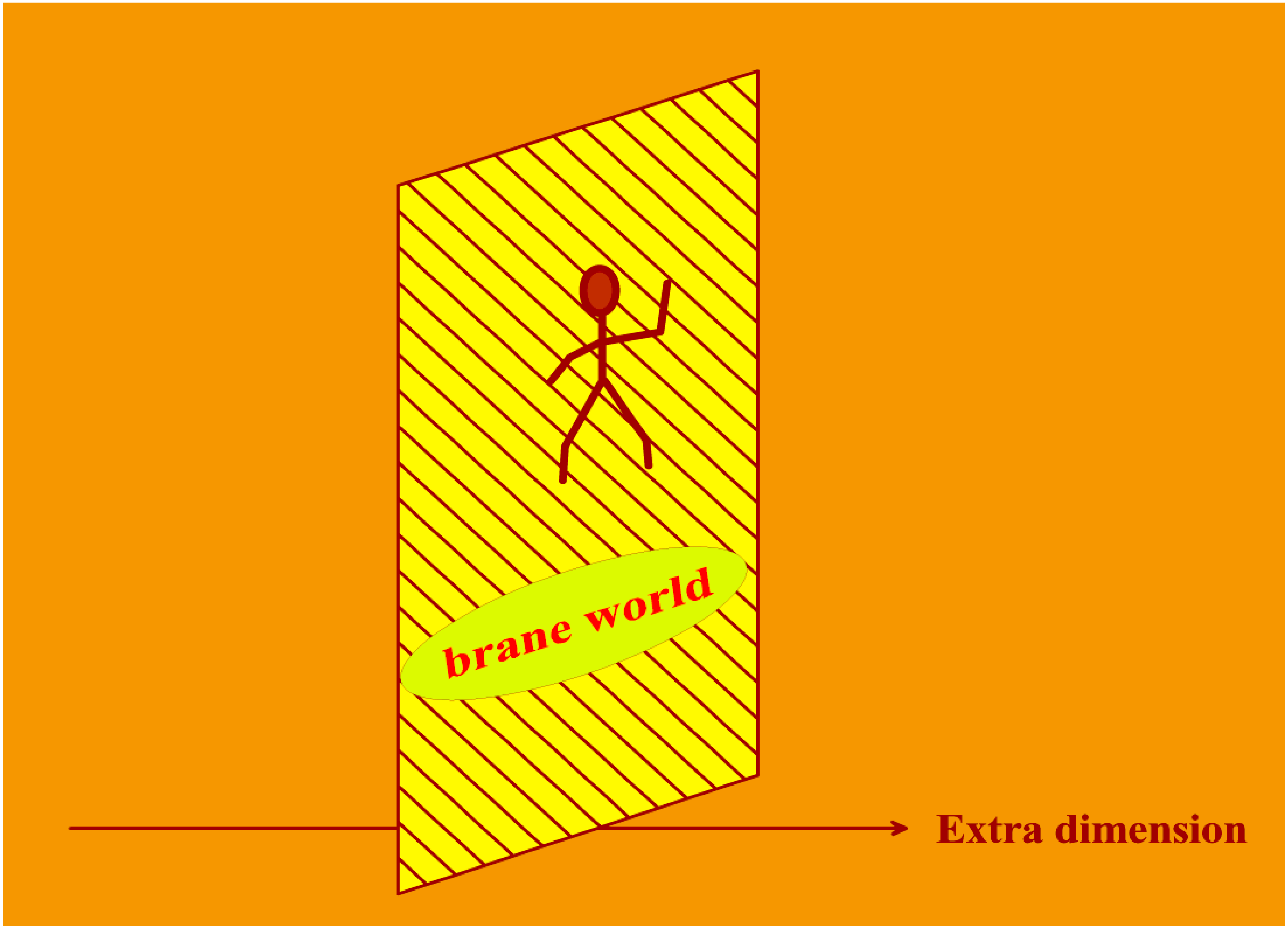
Black holes, neutron stars, quark stars

Extra dimension



- **Large Extra Dimension** (ADD theory) Arkani-Hamed, Dimopoulos, Dvali (1998)
- **Braneworld** (RS theory) L. Randall and R. Sundrum (1999)
  - Both models explain the hierarchy problem
  - ADD: **Many flat** extra dimensions
  - Braneworld: **Only one** extra dimension with a **warped geometry**.
- **No experimental evidence for extra dimensions so far:**
  - **LEP:** LEP Exotica Working Group, LEP Exotica WG 2004-03;
  - **Tevatron:** CDF Collaboration, Phys. Rev. Lett. 101 (2008) 181602; D0 Collaboration, Phys. Rev. Lett. 101 (2008) 011601.
  - **LHC:** ATLAS Collaboration, Phys. Lett. B 705 (2011) 294; Phys. Lett. B 709 (2012) 322.
  - **LHC:** CMS Collaboration, Phys. Rev. Lett. 107 (2011) 201804.
  - Recently: **LHC:** ATLAS collaboration, arXiv:1204.4646v2[hep-ex] Sep.2012.

The Braneworld



Einstein field equations on the brane

The Einstein field equations on the brane may be written as a modification of the standard field equations [Shiromizu et al 2002]

5D Einstein equations:

$$G_{ab} + \Lambda_5 g_{ab} = \kappa_5^2 T_{ab}; \quad \kappa_5 = 8\pi G_5 \quad a = 0, ...4 \quad (Bulk)$$

$$G_{\mu\nu} = -8\pi T^T_{\mu\nu} - \Lambda g_{\mu\nu}, \quad \mu = 0, ...3 \quad (Brane)$$

where the energy-momentum tensor has new terms carrying bulk effects onto the brane:

$$T_{\mu\nu} \rightarrow T_{\mu\nu}{}^T = T_{\mu\nu} + \frac{6}{\sigma} S_{\mu\nu} + \frac{1}{8\pi} \mathcal{E}_{\mu\nu}$$

Here σ is the brane tension

The new terms and are the high-energy corrections $S_{\mu\nu}$ and the projection of the bulk Weyl tensor on the brane $\mathcal{E}_{\mu\nu}$

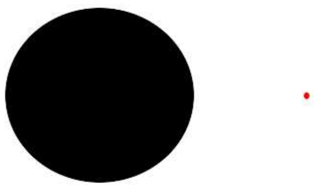
$$S_{\mu\nu} = \frac{1}{12} T_{\alpha}^{\ \alpha} T_{\mu\nu} - \frac{1}{4} T_{\mu\alpha} T_{\nu}^{\alpha} + \frac{1}{24} g_{\mu\nu} \left[3 T_{\alpha\beta} T^{\alpha\beta} - (T_{\alpha}^{\ \alpha})^2 \right]$$

$$-8\pi \mathcal{E}_{\mu\nu} = -\frac{6}{\sigma} \left[\mathcal{U}(u_\mu u_\nu + \frac{1}{3} h_{\mu\nu}) + \mathcal{P}_{\mu\nu} + \mathcal{Q}_{(\mu} u_{\nu)} \right]$$

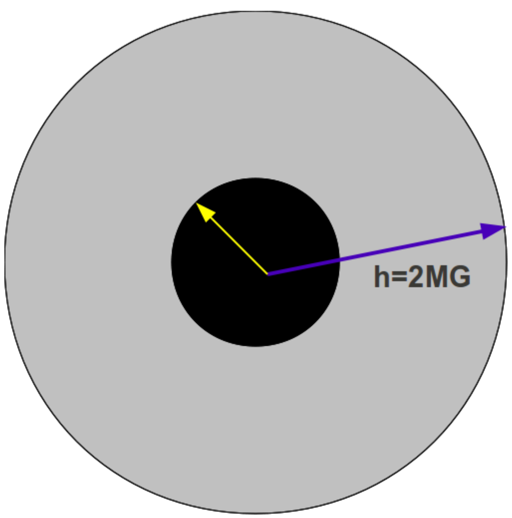
$$\begin{aligned} \mathcal{U} &\rightarrow Dark\ \ radiation \\ \mathcal{P}_{\mu\nu} &\rightarrow Anisotropic\ \ stress \\ \mathcal{Q}_{\mu} &\rightarrow Energy\ \ flux \end{aligned}$$

Black Holes in 4D

Schwarzschild-like coordinates $ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$

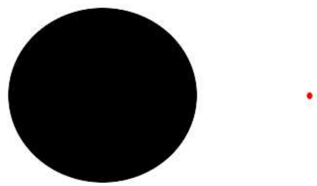


$$e^{\nu} = e^{-\lambda} = 1 - \frac{2\,G\,M}{r} \Rightarrow h = 2\,G\,M$$



$$R < 2\,M\,G = 2\,M\,\frac{l_p}{M_p}$$

Black Holes in the Braneworld



$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

- Dadhich,Maartens,Papadopoulos and Rezania (DMPR Solution):

$$e^{\nu^+} = e^{-\lambda^+} = 1 - \frac{2\mathcal{M}}{r} - \frac{q}{r^2}, \quad \mathcal{U}^+ = -\frac{\mathcal{P}^+}{2} = \frac{4}{3}\pi q \sigma \frac{1}{r^4},$$

- Casadio, Fabbri and Mazzacurati (CFM Solution)

$$e^{\nu^+} = \left[\frac{\eta + \sqrt{1 - \frac{2\mathcal{M}}{r}(1+\eta)}}{1+\eta}\right]^2, \quad e^{\lambda^+} = \left[1 - \frac{2\mathcal{M}}{r}(1+\eta)\right]^{-1},$$

$$\frac{16\pi\mathcal{P}^+}{k^4\sigma} = -\frac{\mathcal{M}(1+\eta)\eta}{\eta + \sqrt{1 - \frac{2\mathcal{M}}{r}(1+\eta)}}\frac{1}{r^3}, \quad \mathcal{U}^+ = 0,$$

Finding the tidal charge q

$$e^{\nu} = e^{-\lambda} = 1 - \frac{2\,\ell_{\rm P}\,\mathcal{M}}{M_{\rm P}\,r} - \frac{q}{r^2}$$

- What is the relationship between $\mathcal{M}$ and q?
- We need the complete 5D solution (unknown).
- We have to consider an alternative way: the $Minimal\ Geometric\ Deformation$
  - In the GR limit $\sigma^{-1} \to 0$, the tidal charge q must vanish.
  - We expect $\mathcal{M} = 0 \implies q = 0$

$$\text{Hence } \mathrm{q} = \mathrm{q}(\mathcal{M}, \sigma)$$

Minimal geometric deformation

Let us see the "solution" for the geometric function

$$e^{-\lambda} = 1 - \frac{8\pi}{r}\int_0^r r^2\left[\rho + \frac{1}{\sigma}\left(\frac{\rho^2}{2} + \frac{6}{k^4}\mathcal{U}\right)\right]dr,$$

It can be written as

$$e^{-\lambda} = \underbrace{1 - \frac{8\pi}{r}\int_0^r r^2\rho dr}_{General\ Relativity}$$ **+ "DEFORMATIONS"**

The deformation undergone by the geometric function λ produces anisotropic consequences, as can be seen through

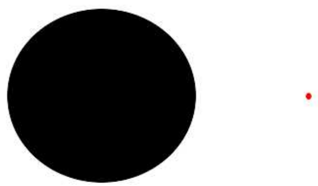
$$\frac{8\pi}{k^4}\frac{\mathcal{P}}{\sigma} = \frac{1}{6}\left(G_1^1 - G_2^2\right),$$

Minimal geometric deformation

- THE MGD WORKS!
- When a solution of the four-dimensional Einstein equations is considered as a possible solution of the BW system, the geometric deformation produced by extra-dimensional effects is minimized, and the open system of effective BW equations is automatically satisfied JO, F. Linares, A. Pascua, A. Sotomayor Class. Quant. Grav. **30** 175019 (2013).
- This approach was successfully used to generate physically acceptable interior solutions for stellar systems and even exact solutions were found:
  - JO Int. J. Mod. Phys. D **18**, 837 (2009);
  - Also: JO + F. Linares (Guanajuato University) “Exact Tolman IV Braneworld Solution” (2013)

Finding the tidal charge q

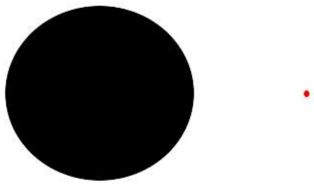
- Exterior



$$e^{\nu^+} = e^{-\lambda^+} = 1 - \frac{2\,\ell_{\rm P}\,\mathcal{M}}{M_{\rm P}\,r} - \frac{q}{r^2}$$

Finding the tidal charge q

- Exterior



$$e^{\nu^+} = e^{-\lambda^+} = 1 - \frac{2\,\ell_{\rm P}\,\mathcal{M}}{M_{\rm P}\,r} - \frac{q}{r^2}$$

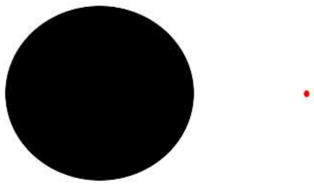
- Interior

$$e^{-\lambda^-} = 1 - \frac{2\,\tilde{m}(r)}{r}$$

where the interior mass function $\tilde{m}$ is given by $\tilde{m}(r) = m(r) - \frac{r}{2}\,f^*(r)$, with $f^*(r)$ $the\ minimal\ geometric\ deformation.$

Finding the tidal charge q

- Exterior



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- Matching conditions at $r = R$

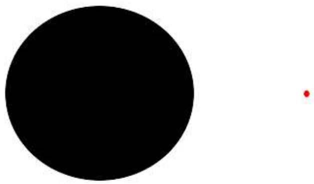
$$e^{\nu_R} = 1 - \frac{2\,\ell_{\rm P}\,\mathcal{M}}{M_{\rm P}\,R} - \frac{q}{R^2}$$

$$\frac{2\,\mathcal{M}}{R} = \frac{2\,M}{R} - \frac{M_{\rm P}}{\ell_{\rm P}}\left(f^* + \frac{q}{R^2}\right)$$

$$\frac{q}{R^4} = \left(\frac{\nu'_R}{R} + \frac{1}{R^2}\right) f^* + 8\,\pi\,\frac{\ell_{\rm P}}{M_{\rm P}}\,p_R$$

Finding the tidal charge q

- Exterior



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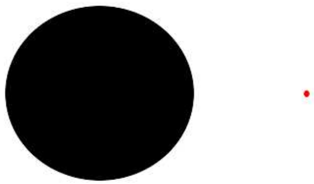
- Matching conditions at $r = R$
  - We then ontain the tidal charge as

$$\frac{M_{\rm P}}{\ell_{\rm P}}\,q = \left(\frac{R\,\nu'_R + 1}{R\,\nu'_R + 2}\right)\left(\frac{2\,M}{R} - \frac{2\,\mathcal{M}}{R}\right)R^2 + \frac{8\,\pi\,p_R\,R^4}{2 + R\,\nu'_R}$$

Finding the tidal charge q

- Exterior



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  - We need an interior solution to evaluate ν'_R and then to find $q = q(\mathcal{M}, \sigma)$

Finding the tidal charge q

Taking $p_R = 0$ and imposing the boundary constraint

$$R\,\nu'_R = -\frac{(M-\mathcal{M}) - \frac{2\,\mathcal{M}\,K\,M_{\rm P}}{\sigma\,R^2\,\ell_{\rm P}}}{(M-\mathcal{M}) - \frac{\mathcal{M}\,K\,M_{\rm P}}{\sigma\,R^2\,\ell_{\rm P}}}$$

where K is a (dimensionful) constant we can fix later, we obtain a simple relation between q and $\mathcal{M}$ given by (R. Casadio, JO, Phys. Lett. B, 715, 251-255 (2012)).

$$q = \frac{2\,K\,\mathcal{M}}{\sigma\,R}$$

Finding the tidal charge q

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$$q = \frac{2\,K\,\mathcal{M}}{\sigma\,R}$$

- it vanishes for $\mathcal{M} \to 0$ and for $\sigma^{-1} \to 0$, and
- it vanishes for very small star density, that is for $R \to \infty$ at fixed $\mathcal{M}$ and σ.

Finding the tidal charge q

Taking $p_R = 0$ and imposing the boundary constraint

$$R\,\nu'_R = -\frac{(M-\mathcal{M}) - \frac{2\,\mathcal{M}\,K\,M_{\rm P}}{\sigma\,R^2\,\ell_{\rm P}}}{(M-\mathcal{M}) - \frac{\mathcal{M}\,K\,M_{\rm P}}{\sigma\,R^2\,\ell_{\rm P}}}$$

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As the pressure does not need to vanish at the surface in the BW, we can get the same simple $q = q(\mathcal{M}, \sigma)$ solution by

$$4\,\pi\,R^3\,p_R = \frac{M_{\rm P}\,\mathcal{M}\,K}{\ell_{\rm P}\,\sigma\,R^2}\left(2 + R\,\nu'_R\right) - (M-\mathcal{M})\left(1 + R\,\nu'_R\right)$$

Finding the tidal charge q

Taking $p_R = 0$ and imposing the boundary constraint

$$R\,\nu'_R = -\frac{(M-\mathcal{M}) - \frac{2\,\mathcal{M}\,K\,M_{\rm P}}{\sigma\,R^2\,\ell_{\rm P}}}{(M-\mathcal{M}) - \frac{\mathcal{M}\,K\,M_{\rm P}}{\sigma\,R^2\,\ell_{\rm P}}}$$

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In our solution $(*)$ R is still a free parameter. We need an interior solution to fix it!

Spherically symmetric static distribution

Schwarzschild-like coordinates

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right)$$

A perfect fluid (General Relativity)+high energy corrections+Weyl functions

$$-8\pi \left(\rho + \frac{1}{\sigma} \left(\frac{\rho^2}{2} + 6\mathcal{U} \right) \right) = -\frac{1}{r^2} + e^{-\lambda} \left(\frac{1}{r^2} - \frac{\lambda'}{r} \right),$$

$$-8\pi \left(-p - \frac{1}{\sigma} \left(\frac{\rho^2}{2} + \rho p + 2\mathcal{U} \right) + \frac{\mathcal{P}}{\sigma} \right) = -\frac{1}{r^2} + e^{-\lambda} \left(\frac{1}{r^2} + \frac{\nu'}{r} \right),$$

$$-8\pi \left(-p - \frac{1}{\sigma} \left(\frac{\rho^2}{2} + \rho p + 2\mathcal{U} \right) - \frac{\mathcal{P}}{2\sigma} \right) = \frac{1}{4} e^{-\lambda} \left[2\nu'' + \nu'^2 - \lambda'\nu' + 2\frac{(\nu' - \lambda')}{r} \right],$$

$$p' = -\frac{\nu'}{2}(\rho + p).$$

Finding the tidal charge q

Let us consider the exact interior solution (JO Int. J. Mod. Phys. D **18**, 837 (2009))

$$e^{\nu} = A\left(1 + C\,r^2\right)^4$$

$$\rho = C_\rho \left(\frac{M_{\rm P}}{\ell_{\rm P}}\right) \frac{C\left(9 + 2\,C\,r^2 + C^2\,r^4\right)}{7\,\pi\,(1 + C\,r^2)^3}$$

where $C_\rho = C_\rho(K)$ is a constant to be determined for consistency, and

$$p_R = \left(\frac{M_{\rm P}}{\ell_{\rm P}}\right) \frac{2\,C\left(2 - 7\,C\,R^2 - C^2\,R^4\right)}{7\,\pi\,(1 + C\,R^2)^3} = 0.$$

Finding the tidal charge q

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$$R = 2\,n\left(\frac{\ell_{\rm P}}{M_{\rm P}}\right)\frac{M}{C_\rho}; \qquad K = \left(\frac{M_{\rm P}}{M_{\rm G}}\right)^2\frac{\ell_{\rm G}}{M_{\rm G}}; \qquad C_\rho = (M_{\rm G}/M_{\rm P})^4$$

Finding the tidal charge q

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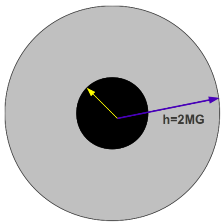
$$R = 2\,n\left(\frac{\ell_{\rm P}}{M_{\rm P}}\right)\frac{M}{C_\rho}; \qquad K = \left(\frac{M_{\rm P}}{M_{\rm G}}\right)^2\frac{\ell_{\rm G}}{M_{\rm G}}; \qquad C_\rho = (M_{\rm G}/M_{\rm P})^4$$

$$\mathcal{M} = \frac{M^3}{M^2 + n_1\,M_{\rm G}^2} \qquad q = \frac{\ell_{\rm G}^2\,M^2}{n\,\left(M^2 + n_1\,M_{\rm G}^2\right)}$$

where we used $\sigma \simeq \ell_{\rm G}^{-2}$.

Black Holes in 4D



$$R < 2\,M\,G = 2\,M\,\frac{l_p}{M_p}$$

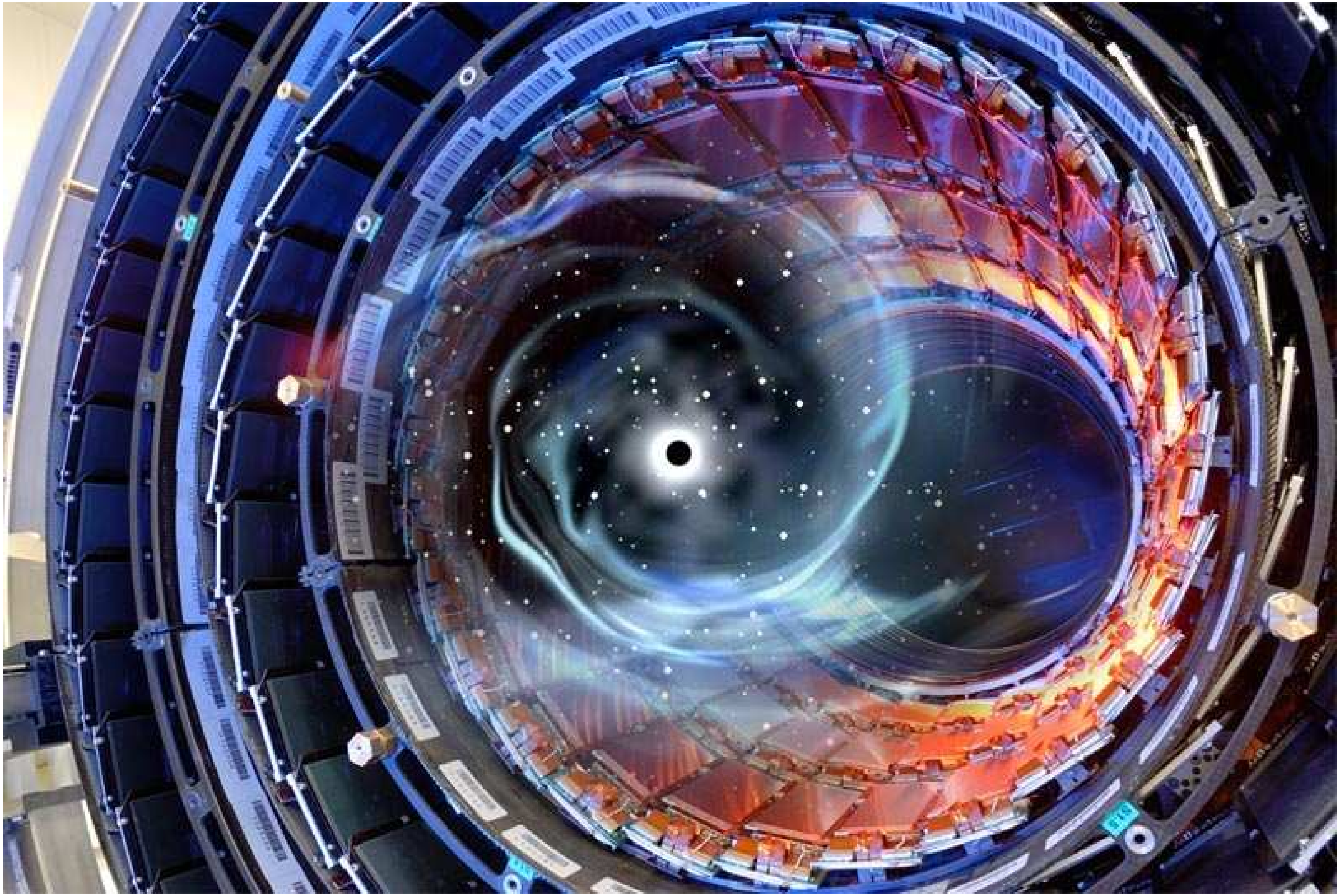
- **Micro Black Holes in 4D**

$$R \Rightarrow \lambda_C \qquad \lambda_C \simeq \frac{\hbar}{M} = \frac{l_p\,M_p}{M}$$

$$\frac{l_p\,M_p}{M} \lesssim 2\,M\,\frac{l_p}{M_p} \Rightarrow M_c \approx M_p \quad \text{(Minimum possible mass)}$$

According to 4D gravity...

Black Holes in the Braneworld



Micro Black Holes in the BW

- The tidally charged metric

$$ds^2 = e^{\nu}\, dt^2 - e^{\lambda}\, dr^2 - r^2(d\theta^2 + sin^2\theta\, d\phi^2); \qquad e^{\nu} = e^{-\lambda} = 1 - \frac{2\,\ell_{\rm P}\,\mathcal{M}}{M_{\rm P}\,r} - \frac{q}{r^2}$$

$$\text{Its horizon} \quad h = \ell_{\rm P}\left[\frac{\mathcal{M}}{M_{\rm P}} + \sqrt{\frac{\mathcal{M}^2}{M_{\rm P}^2} + q\,\frac{M_{\rm P}^2}{M_G^2}}\,\right]$$

$$\text{Micro Black Holes :} \quad \lambda_C \lesssim h \;\Rightarrow\; \frac{\ell_{\rm P}\,M_{\rm P}}{M} \lesssim \ell_{\rm P}\left[\frac{\mathcal{M}}{M_{\rm P}} + \sqrt{\frac{\mathcal{M}^2}{M_{\rm P}^2} + q\,\frac{M_{\rm P}^2}{M_G^2}}\,\right]$$

$$\text{We consider black holes near their minimum possible mass } \mathrm{M} \sim \mathcal{M} \approx \mathrm{M_G} \ll \mathrm{M_P}$$

$$\rightarrow \; M_c \approx \frac{M_G}{\sqrt{q}} \quad G.L.Alberghi, R.Casadio, O.Micu, and A.Orlandi, JHEP\,\mathbf{1109}, 023(2011)$$

But q is unknown!!! We need the complete $5D$ solution, which is unknown so far. However...

Black hole limit and minimum mass

We obtain the horizon radius

$$h = \frac{\ell_{\rm P}}{M_{\rm P}} \left(\mathcal{M} + \sqrt{\mathcal{M}^2 + q\,\frac{M_{\rm P}^2}{\ell_{\rm P}^2}} \right)$$

and the classicality condition $h \gtrsim \lambda_M$ reads

$$\frac{M}{M_{\rm P}^2} \left(\mathcal{M} + \sqrt{\mathcal{M}^2 + q\,\frac{M_{\rm P}^2}{\ell_{\rm P}^2}} \right) \gtrsim 1$$

We expand for $M \sim \mathcal{M} \simeq M_{\rm G} \ll M_{\rm P}$, thus obtaining

$$\frac{h^2}{\lambda_C^2} \simeq \frac{M^2}{M_{\rm P}^2}\,\frac{q}{\ell_{\rm P}^2} \simeq \frac{M_{\rm G}^2}{M_{\rm P}^2}\,\bar{M}^2\,\bar{q}\,\frac{\ell_{\rm G}^2}{\ell_{\rm P}^2} \simeq \bar{M}^2\,\bar{q} \simeq 1$$

Black hole limit and minimum mass

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$$\frac{h^2}{\lambda_C^2} \simeq \frac{M^2}{M_\mathrm{P}^2}\,\frac{q}{\ell_\mathrm{P}^2} \simeq \frac{M_\mathrm{G}^2}{M_\mathrm{P}^2}\,\bar{M}^2\,\bar{q}\,\frac{\ell_\mathrm{G}^2}{\ell_\mathrm{P}^2} \simeq \bar{M}^2\,\bar{q} \simeq 1$$

or $\bar{M}^4 \simeq n\left(n_1 + \bar{M}^2\right)$, which yields

$$M_c \simeq 1.3\,M_\mathrm{G}$$

This can be viewed as the minimum allowed mass for a semiclassical BH in the BW.

Two more BW astrophysical solutions

- A non-uniform BW solution (with $M/R \simeq 0.38\frac{M_{\rm P}}{\ell_{\rm P}}$)

$$e^{-\lambda(r)} = 1 - \frac{3\,C\,r^2}{2\,(1+C\,r^2)} + f^*(r); \qquad e^{\nu(r)} = A(1+Cr^2)^3$$

$$\rho(r) = \frac{3\,C\,\left(3+Cr^2\right)}{2\,k^2\,(1+Cr^2)^2}; \quad p(r) = \frac{9\,C\,\left(1-Cr^2\right)}{2\,k^2\,(1+Cr^2)^2}$$

$$M_c \simeq 1.22\,M_{\rm G}$$

- The Schwarzschild solution (with $M/R \simeq 0.28\frac{M_{\rm P}}{\ell_{\rm P}}$)

$$e^{-\lambda} = 1 - \frac{r^2}{C^2} + f^*; \quad e^{\nu} = \left(A - B\sqrt{1-\frac{r^2}{C^2}}\right)^2$$

$$\rho = \frac{3}{k^2\,C^2}; \quad p(r) = \frac{\rho}{3}\left[\frac{3\,B\,\sqrt{1-\frac{r^2}{C^2}} - A}{A - B\,\sqrt{1-\frac{r^2}{C^2}}}\right]$$

$$M_c \simeq 1.9\,M_{\rm G}$$

Astrophysical consequences for M_c

From $\simeq \lambda_C$, we find

$$\bar{M}^2 \bar{q} \simeq 1$$

which yields

$$M_c^2 \simeq \frac{n}{2} \left(1 + \sqrt{1 + \frac{4\, n_1}{n}} \right) M_G^2$$

Now, from GR we know that the compactness of any stable stellar distribution of mass M and radius R must satisfy the constraint $M/R < 4/9$. This bound leads to $n > 9/8$, and, correspondingly,

$$M_c{}^2 \simeq \frac{n}{2} \underbrace{\left(1 + \sqrt{1 + \frac{4\, n_1}{n}} \right)}_{>2} M_G^2$$

Always a critical mass M_c above M_G JO, R. Casadio, arXiv:1212.0409 [gr-qc] (2012).

Conclusions

- We have analyzed analytical descriptions of stars in the BW, with the tidal charge as an explicit function of the ADM mass and brane tension, which was still an open problem.
- For micro black holes: different astrophysical solutions lead to different critical mass.
- By using the general relativistic constraint $M/R < 4/9$ we found that the minimum mass of a semiclassical microscopic black hole M_c is always above M_G
- A more general solution regarding charged black holes will be considered (in progress).
- The MGD works very well!...and **maybe it could be extended to other beyond Einstein's theories**....?

THANK YOU!