Geons in quadratic Palatini gravity

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in collaboration with

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Motivations

- **Quadratic gravity** has been thoroughly studied in the literature:
  - $R \Rightarrow R + l_P^2 \left(aR^2 + bR_{\mu\nu}R^{\mu\nu}\right)$
  - Renormalizability of QFT in curved spaces requires such terms.
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  - Massive spin-2 gravitons and a massive spin-0 ghost propagate in vacuum.
  - Perturbative methods are generally required to explore deviations from GR.

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- **HOWEVER**, the **Palatini** version of the theory is completely different.
  - Second-order field equations govern the dynamics $\Rightarrow$ exact solutions.
  - Only massless spin-2 gravitons propagate in vacuum $\Rightarrow$ GR+Λ recovered.
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- We will see that in the **quadratic Palatini theory**:
  - The central singularity of charged BHs is replaced by a Wormhole.
  - Reissner-Nordstrom solutions turn into geons.
  - Stable remnants arise in the lowest part of the mass and charge spectrum.

Field Equations in Palatini Theories

Palatini -Vs- Metric: generalities

- In the Palatini formalism, one assumes that $g_{\mu\nu}$ and $\Gamma^{\alpha}_{\beta\gamma}$ are independent entities: $S=\int d^n x\sqrt{-g}L[g_{\mu\nu},\Gamma^{\alpha}_{\beta\gamma}]+S_{matter}[g_{\mu\nu},\psi_m]$

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- The field equations follow from variation of the action:
  - Palatini approach:

$$\delta S=\int d^n x\left[\sqrt{-g}\left(\frac{\delta L}{\delta g^{\mu\nu}}-\frac{L}{2}g_{\mu\nu}\right)\delta g^{\mu\nu}+\sqrt{-g}\frac{\delta L}{\delta\Gamma^{\alpha}_{\beta\gamma}}\delta\Gamma^{\alpha}_{\beta\gamma}\right]+\delta S_{matter}$$

$$\delta g^{\mu\nu}\Rightarrow\frac{\delta L}{\delta g^{\mu\nu}}-\frac{L}{2}g_{\mu\nu}=8\pi G T_{\mu\nu}$$

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  - Metric approach:

The relation $\delta\Gamma^{\alpha}_{\beta\gamma}=\frac{g^{\alpha\rho}}{2}\left[\nabla_\beta\delta g_{\rho\gamma}+\nabla_\gamma\delta g_{\rho\beta}-\nabla_\rho\delta g_{\beta\gamma}\right]$ implies

$$\frac{\delta L}{\delta\Gamma^{\alpha}_{\beta\gamma}}\delta\Gamma^{\alpha}_{\beta\gamma}=\left\{g^{\alpha\mu}\frac{\delta L}{\delta\Gamma^{\alpha}_{\lambda\nu}}-\frac{g^{\alpha\lambda}}{2}\frac{\delta L}{\delta\Gamma^{\alpha}_{\mu\nu}}\right\}\nabla_\lambda\delta g_{\mu\nu}$$

and leads to

$$\delta g^{\mu\nu}\Rightarrow\left(\frac{\delta L}{\delta g^{\mu\nu}}-\frac{L}{2}g_{\mu\nu}\right)+\nabla_\lambda\left[g_{\gamma\nu}\frac{\delta L}{\delta\Gamma^{\mu}_{\lambda\gamma}}-g_{\beta\mu}g_{\gamma\nu}g^{\alpha\lambda}\frac{\delta L}{\delta\Gamma^{\alpha}_{\beta\gamma}}\right]=8\pi G T_{\mu\nu}$$

- Metric and **Palatini** variations generally lead to different field equations.

Dynamics of Palatini $f(R,Q)$ gravity

- Consider the **Palatini** theory $S[g,\Gamma,\psi_m]=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}f(R,Q)+S_m[g,\psi_m]$

  with $Q\equiv R_{\mu\nu}R^{\mu\nu}$, , $R_{\mu\nu}=R^{\rho}{}_{\mu\rho\nu}$, and $R^{\alpha}{}_{\beta\mu\nu}=\partial_\mu\Gamma^\alpha_{\nu\beta}-\partial_\nu\Gamma^\alpha_{\mu\beta}+\Gamma^\alpha_{\mu\lambda}\Gamma^\lambda_{\nu\beta}-\Gamma^\alpha_{\nu\lambda}\Gamma^\lambda_{\mu\beta}$

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- In general $f(R,Q)$ **Palatini** theories we find that:
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Solving for $\Gamma^{\alpha}_{\mu\nu}$

- Recall the field equations: $f_X \equiv \partial_X f$
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- Step 1: define $P_\mu{}^\nu \equiv R_{\mu\alpha} g^{\alpha\nu}$, with $R = \mathrm{Tr}[\hat{P}]$ and $Q = \mathrm{Tr}[\hat{P}^2]$, and write (1) as

$$f_R P_\mu{}^\nu - \frac{f}{2} \delta_\mu{}^\nu + 2 f_Q P_\mu{}^\alpha P_\alpha{}^\nu = \kappa^2 T_\mu{}^\nu \Leftrightarrow 2 f_Q \left(\hat{P} + \frac{f_R}{4 f_Q} \hat{I} \right)^2 = \kappa^2 \hat{T} + \frac{1}{2} \left(f + \frac{f_R^2}{4 f_Q} \right) \hat{I}$$

  - This establishes an algebraic relation between $P_\mu{}^\nu$ and $T_\mu{}^\nu$: $\hat{P} = \hat{P}(\hat{T})$

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$$\left(f_R g^{\mu\nu} + 2 f_Q R^{\mu\nu}\right) \Rightarrow g^{\mu\alpha}\left(f_R \delta_\alpha{}^\nu + 2 f_Q P_\alpha{}^\nu\right) \equiv g^{\mu\alpha}\Sigma_\alpha{}^\nu(\hat{T}).$$

  - One finds that (2) $\rightarrow$ $\nabla_\beta\left[\sqrt{-g}\, g^{\mu\alpha}\Sigma_\alpha{}^\nu\right] = 0 \Leftrightarrow \nabla_\beta\left[\sqrt{-h}\, h^{\mu\nu}\right] = 0$
  - with $h^{\mu\nu} = \frac{g^{\mu\alpha}\Sigma_\alpha{}^\nu}{\sqrt{\det \Sigma}}$, $h_{\mu\nu} = \left(\sqrt{\det \Sigma}\right)\left[\Sigma^{-1}\right]_\mu{}^\alpha g_{\alpha\nu}$

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- $\Gamma^{\alpha}_{\mu\nu}$ turns out to be the Levi-Civita connection of $h_{\mu\nu}$.

Metric field equations

- From the definition of $P_\mu{}^\nu \equiv R_{\mu\alpha} g^{\alpha\nu}$ and $\Sigma_\alpha{}^\nu = \left(f_R \delta_\alpha{}^\nu + 2 f_Q P_\alpha{}^\nu\right)$, we have:
  - Step 1:Metric variation $\Rightarrow$ $f_R P_\mu{}^\nu - \frac{f}{2}\delta_\mu{}^\nu + 2 f_Q P_\mu{}^\alpha P_\alpha{}^\nu = \kappa^2 T_\mu{}^\nu$
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- In vacuum: $\hat{P}_v = A(R_v, Q_v)\hat{I}$ $\Rightarrow$ $R_v = 4A(R_v, Q_v)\ ,\ Q_v = 4A^2(R_v, Q_v)$.
  - Solutions: R_v and Q_v are constant $\Rightarrow$ de Sitter space-times: $Q_v = R_v^2/4$

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  - Solutions: R_v and Q_v are constant $\Rightarrow$ de Sitter space-times: $Q_v = R_v^2/4$
  - In vacuum, the theory boils down to GR+Λ, with $g_{\mu\nu} = \text{constant} \times h_{\mu\nu}$.
  - No massive spin-2 gravitons. No ghost-like instabilities.
  - Matter-induced nonlinearities instead of new d.o.f.

Spherically symmetric charged solutions

Electrovacuum geometry

- In GR, the RN solution is characterized by: ($r_S = 2M_0$ and $r_q^2 = 2Gq^2$)

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- In the quadratic Palatini theory $R + l_P^2\left(aR^2 + R_{\mu\nu}R^{\mu\nu}\right)$ with an electric field, a new scale characterized by $r_c \equiv \sqrt{r_q l_P}$ arises.

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$$R_{GR} = 0\,, \qquad Q_{GR} \equiv R_{\mu\nu}R^{\mu\nu} = \frac{r_q^4}{r^8}, \qquad K_{GR} \equiv R^{\alpha}{}_{\beta\mu\nu}R_{\alpha}{}^{\beta\mu\nu} = \frac{12r_S^2}{r^6} - \frac{24r_S r_q^2}{r^7} + \frac{14r_q^4}{r^8}$$

- In the quadratic Palatini theory $R + l_P^2\left(aR^2 + R_{\mu\nu}R^{\mu\nu}\right)$ with an electric field, a new scale characterized by $r_c \equiv \sqrt{r_q l_P}$ arises.
- When $r \gg r_c$:

$$R(g) \approx -\frac{48r_c^8}{r^{10}} + O\left(\frac{r_c^9}{r^{11}}\right), \quad Q(g) \approx \frac{r_q^4}{r^8}\left(1 - \frac{16l_P^2}{r^2} + \ldots\right), \quad K(g) \approx K_{GR} + \frac{144r_S r_q^2 l_P^2}{r^9} + \ldots$$

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- But when $z \equiv r/r_c \to 1^+$: $\left[\ \delta_1 = \frac{1}{2}\sqrt{\frac{r_q^3}{r_S^2 l_P}}\ ,\ \delta_2 = \frac{r_c}{r_S},\ \text{and}\ \ \delta_1^* \approx 0.572\ \right]$

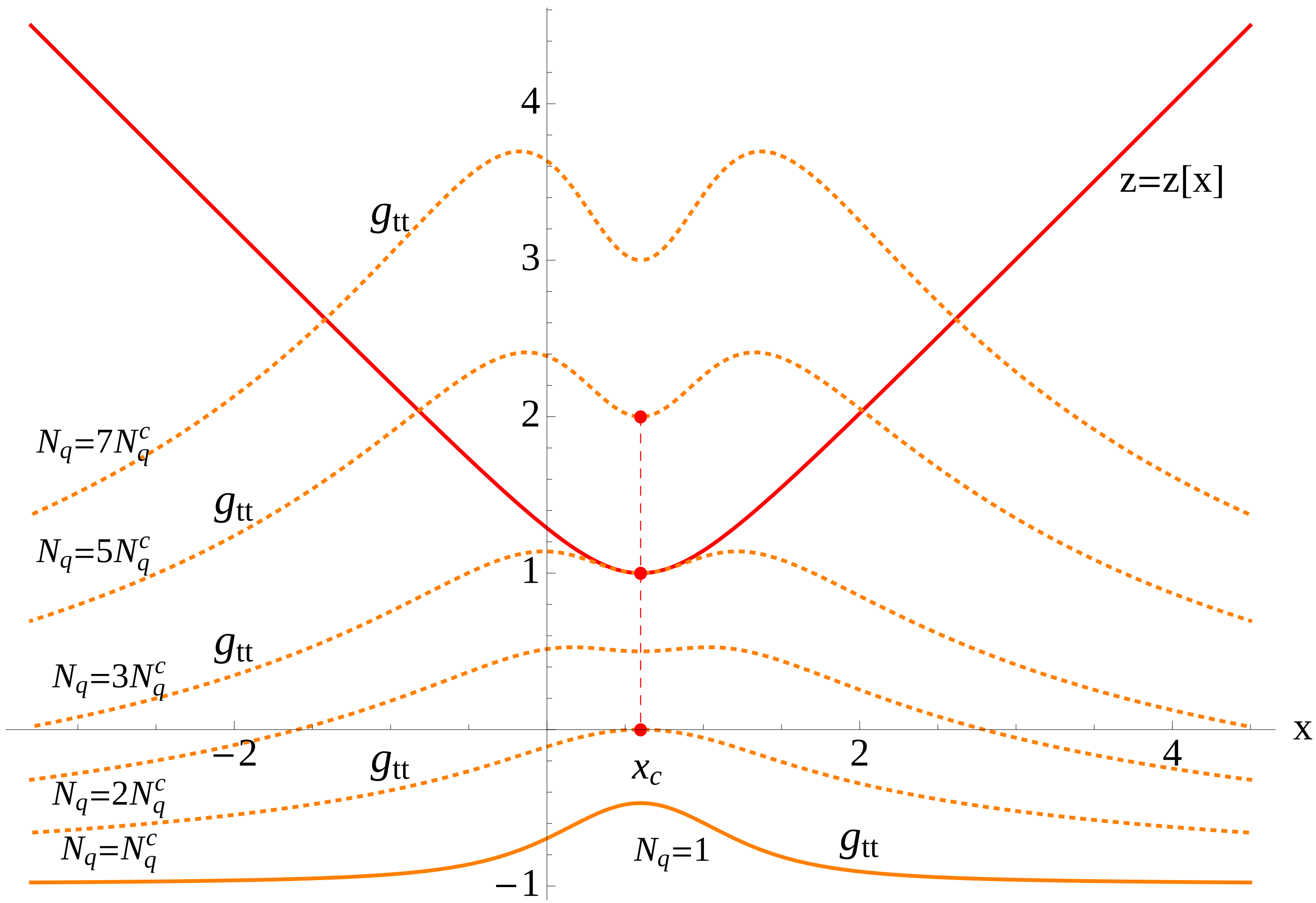
$$r_c^2 R(g) \approx \left(-4 + \frac{16\delta_1^*}{3\delta_2}\right) + O(z-1) + \ldots - \frac{1}{2\delta_2}\left(1 - \frac{\delta_1^*}{\delta_1}\right)\left[\frac{1}{(z-1)^{3/2}} - O\left(\frac{1}{\sqrt{z-1}}\right)\right]$$

$$r_c^4 Q(g) \approx \left(10 + \frac{86\delta_1^2}{9\delta_2^2} - \frac{52\delta_1}{3\delta_2}\right) + \ldots + \left(1 - \frac{\delta_1^*}{\delta_1}\right)\left[\frac{6\delta_2 - 5\delta_1}{3\delta_2^2(z-1)^{3/2}} + \ldots\right] + \left(1 - \frac{\delta_1^*}{\delta_1}\right)^2\left[\frac{1}{8\delta_2^2(z-1)^3} - \ldots\right.$$

$$r_c^4 K(g) \approx \left(16 + \frac{88\delta_1^2}{9\delta_2^2} - \frac{64\delta_1}{3\delta_2}\right) + \ldots + \left(1 - \frac{\delta_1^*}{\delta_1}\right)\left[\frac{2(2\delta_1 - 3\delta_2)}{3\delta_2^2(z-1)^{3/2}} + \ldots\right] + \left(1 - \frac{\delta_1^*}{\delta_1}\right)^2\left[\frac{1}{4\delta_2^2(z-1)^3} + \ldots\right.$$

Smooth wormhole geometries

- When $\delta_1 = \delta_1^*$ a wormhole arises at $r = r_c \Leftrightarrow z = 1$:



- $ds^2 = g_{tt}dt^2 - \frac{1}{g_{tt}}dx^2 + r^2(x)d\Omega^2$.

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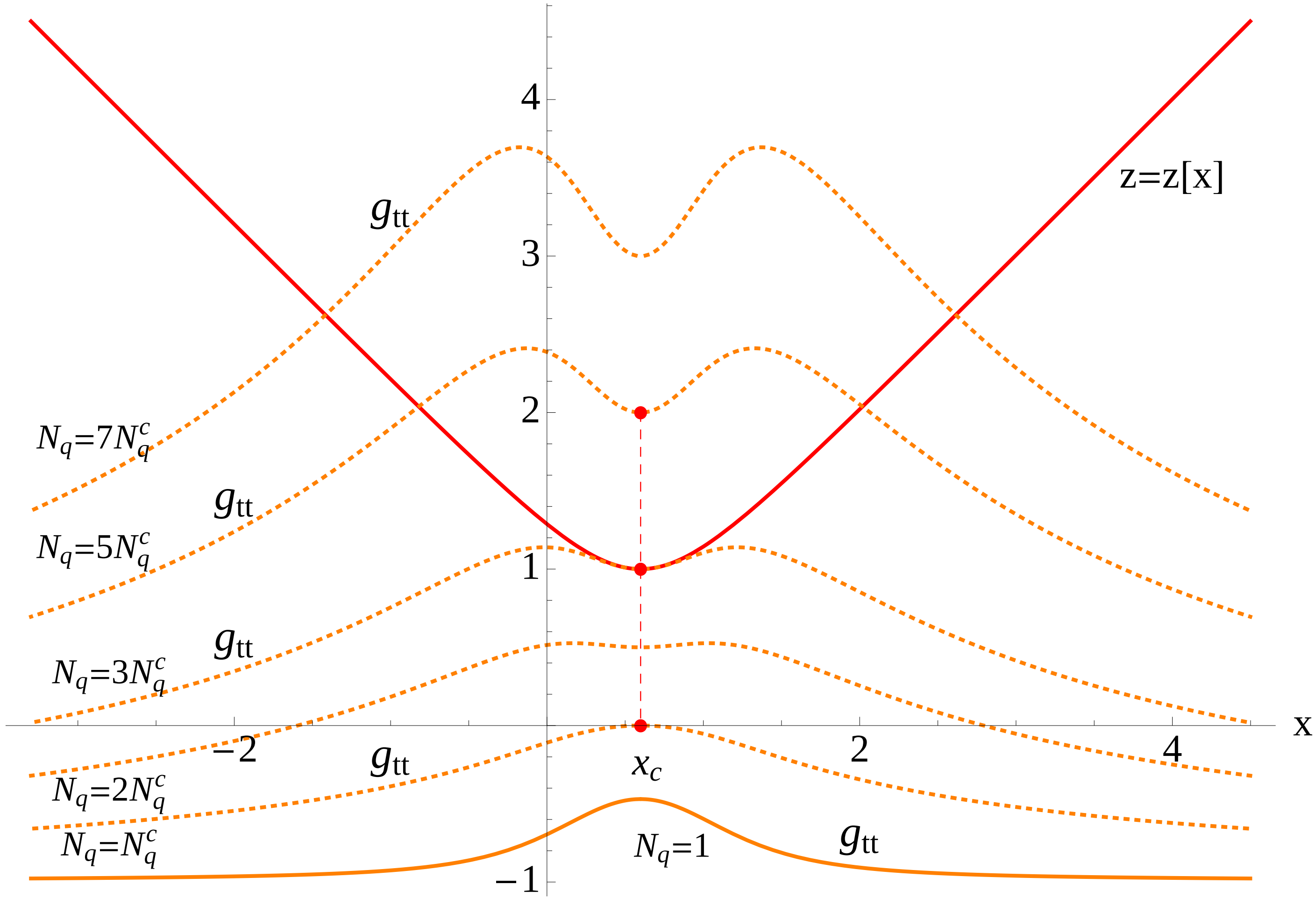
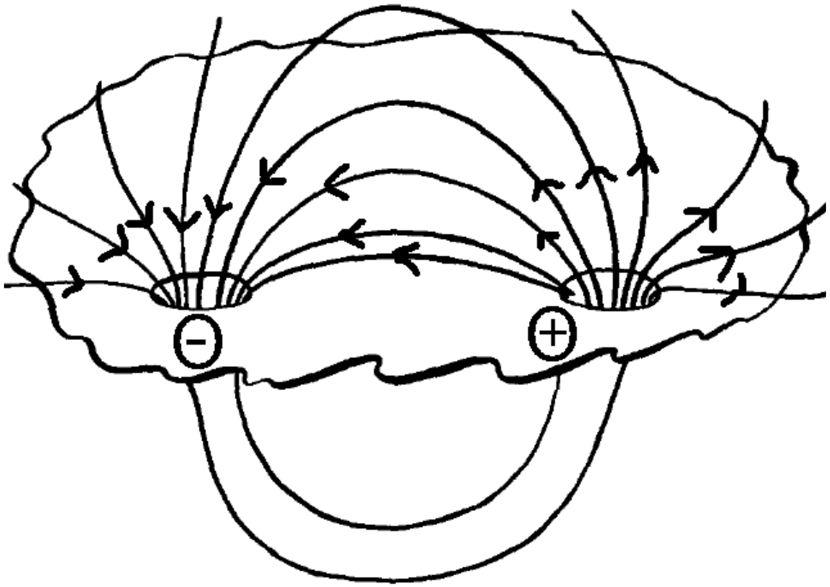


- Note that for $N_q \leq N_q^c \approx 16.55$ charges there is no event horizon.

Palatini Geons

- Electric charge of **geons** (≡self-gravitating electromagnetic entitites):
  - The lines of force of the electric field enter through one of the wormhole mouths and exit through the other creating the illusion of a negatively charged object on one side and a positively charged object on the other.



  - The locally measured electric charge is defined by the flux $\Phi \equiv \int_S *F = 4\pi q$ through any 2−surface S enclosing a wormhole mouth.

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- Evaluating the action on the solutions we find:

$$S_T = S_{Quad.Grav.} + S_{e.m.} = 2M_0 c^2 \frac{\delta_1}{\delta_1^*} \int dt . \; ([S] = [\text{Energy}] \times [\text{time}])$$

  - For $\delta_1 = \delta_1^* \Rightarrow M_0 c^2 =$ e.m. + grav. binding energy!!! .
  - Coincides with the action of a point-like particle at rest !!!??? (geonic soliton): $S_{p.p.} = mc^2 \int dt \sqrt{1 - v^2/c^2}$

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- Density of lines of force crossing the wormhole throat:

$$\mathcal{E} = \frac{\Phi}{4\pi r_c^2} = \frac{q}{r_c^2} = \sqrt{\frac{c^7}{2(\hbar G)^2}} \quad \Rightarrow \textbf{Universal quantity}.$$

  - Since $\mathcal{E}$ is independent of q and M $\Rightarrow$ geon structure even when $\delta_1 \neq \delta_1^*$.
  - WH (topological) structure even if there are (local) curvature divergences.

Stability and quantum properties

- The event horizon is expected to force the decay into $\delta_1 \neq \delta_1^*$ states:
  - $g_{tt} = \frac{(1-\delta_1/\delta_1^*)}{4\delta_2\sqrt{z-1}} - \frac{1}{2}\left(1-\frac{\delta_1}{\delta_2}\right) + O(\sqrt{z-1})$ Recall: $\delta_1 = \frac{1}{2r_S}\sqrt{\frac{r_q^3}{l_P}}$
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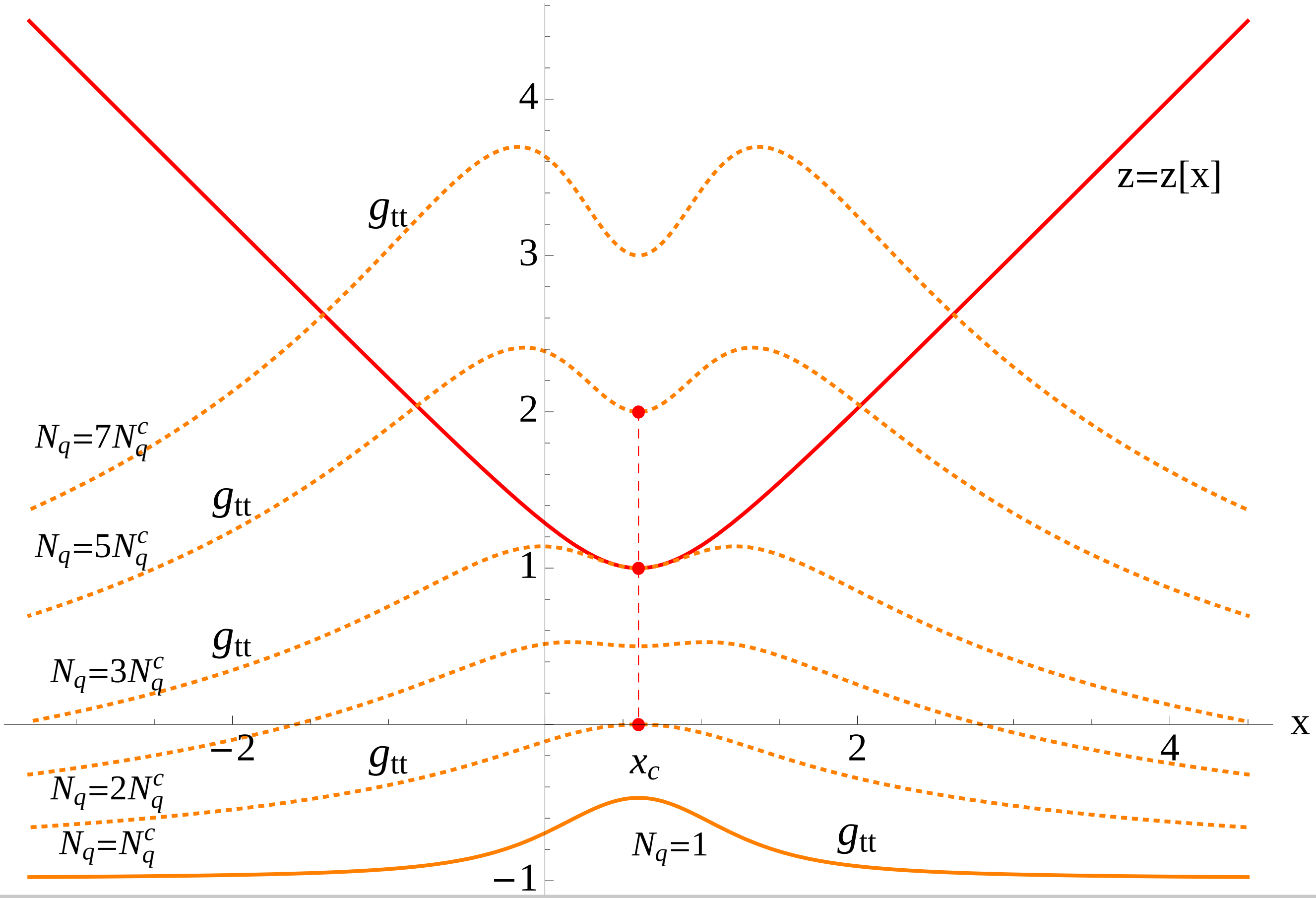

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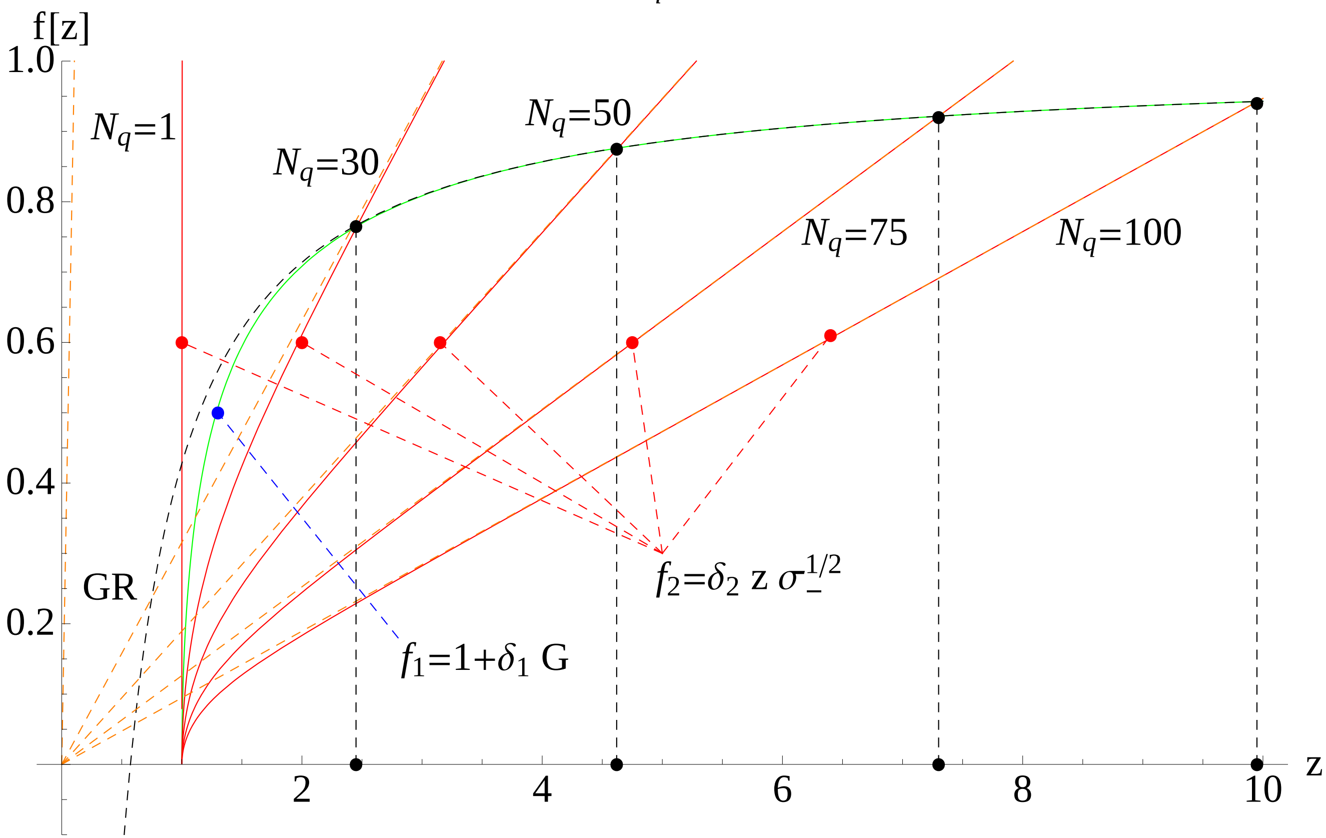
Horizon location as N_q changes when $\delta_1=\delta_1{}^*$



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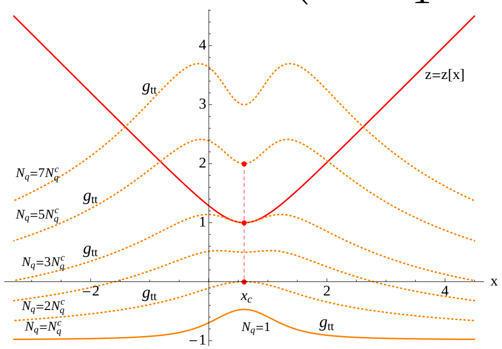

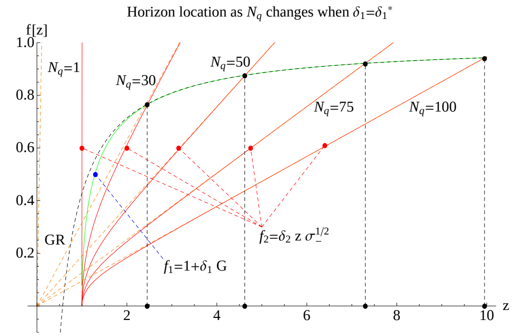

- For $\delta_1 = \delta_1^*$ we find:
  - Mass spectrum: $M \approx 1.23605\left(\frac{N_q}{N_q^c}\right)^{3/2} m_P$, where $N_q^c \equiv \sqrt{2/\alpha_{em}} \approx 16.55$
  - The absence of an event horizon for $N_q < 16.55$ yields quantum mechanically stable objects: NO Hawking decay.
  - The topological nature of their charge makes them stable against arbitrary classical perturbations that preserve the topology.

Hawking estimates and BH remnants

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GRAVITATIONALLY COLLAPSED OBJECTS OF VERY LOW MASS

Stephen Hawking

(Communicated by M. J. Rees)

(Received 1970 November 9)

SUMMARY

It is suggested that there may be a large number of gravitationally collapsed objects of mass 10^{-5} g upwards which were formed as a result of fluctuations in the early Universe. They could carry an electric charge of up to ± 30 electron units. Such objects would produce distinctive tracks in bubble chambers and could form atoms with orbiting electrons or protons. A mass of 10^{17} g of such objects could have accumulated at the centre of a star like the Sun. If such a star later became a neutron star there would be a steady accretion of matter by a central collapsed object which could eventually swallow up the whole star in about ten million years.

Hawking estimates and BH remnants

- Hawking's estimates, based entirely on the process of classical collapse, are in excellent **quantitative** agreement with our results:
  - According to Hawking, a large number of objects with $M \sim m_P$ and $N_q \lesssim 30$ could have been formed in the early universe.
  - A fraction of them could reach the stability conditions found here.
  - They may also arise from the evaporation of more massive objects.

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  - A fraction of them could reach the stability conditions found here.
  - They may also arise from the evaporation of more massive objects.
- The existence of stable solutions in the lowest part of the mass and charge spectrum which can be continuously connected with black hole states, supports the view that these objects can be naturally identified as black hole remnants.
  - Stable remnants implies a maximum temperature in the evaporation process.
  - The lack of observations supporting black hole explosions is compatible with this result.

Summary and Conclusions

- For a given $f(R,Q)$ Lagrangian, the field eqs in metric and **Palatini** differ:
  - Metric formalism: higher-order derivatives due to integration by parts.
  - Palatini formalism: second-order equations and algebraic relations.
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- Geons in **Quadratic Palatini gravity**:
  - The existence of completely regular (non-perturbative) solutions with WH structure put forward the geonic nature of such solutions.
  - Universal properties of the electric flux at $r = r_c$: $\frac{\Phi}{4\pi r_c^2} = \sqrt{\frac{c^7}{2(\hbar G)^2}}$

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- Nontrivial implications for dark matter and the information loss problem.

Thanks !!!