

# Gravitational Particle Production in Gravity Theories with Non-minimal Derivative Couplings

George Koutsoumbas  
Konstantinos Ntrekis  
Eleftherios Papantonopoulos

Department of Physics, National Technical University of Athens

*Seventh Aegean Summer School: Beyond Einstein's theory of Gravity*

September 23, 2013

The Lagrangian density of a scalar field in **curved spacetime**

$$\mathcal{L} = \sqrt{-g} \left\{ \frac{1}{2} [g^{\kappa\lambda} \partial_\kappa \phi \partial_\lambda \phi] - V(\phi) \right\}$$

leads to the equation of motion **Euler-Lagrange**

$$\square \phi + V_\phi = 0 \quad \text{where} \quad \square \phi = (-g)^{-1/2} \partial_\mu [(-g)^{1/2} g^{\mu\nu} \partial_\nu \phi] \quad (1)$$

We define the **inner product** of the solutions of (1) as

$$(\phi_1, \phi_2) \equiv i \int (-g)^{1/2} g^{0\nu} (\phi_1^*(x) \partial_\nu \phi_2(x) - \phi_2(x) \partial_\nu \phi_1^*(x)) d^3x$$

We expand the field  $\phi$  in terms a complete set of modes  $\chi_{\mathbf{k}}$

$$\phi(x) = \sum_{\mathbf{k}} (\hat{a}_{\mathbf{k}} \chi_{\mathbf{k}}(x) + \hat{a}_{\mathbf{k}}^\dagger \chi_{\mathbf{k}}^*(x))$$

with  $\chi_{\mathbf{k}}$  satisfying

$$(\chi_{\mathbf{k}}, \chi_{\mathbf{k}'}') = \delta_{\mathbf{k}\mathbf{k}'} \quad (\chi_{\mathbf{k}}^*, \chi_{\mathbf{k}'}^*) = -\delta_{\mathbf{k}\mathbf{k}'} \quad (\chi_{\mathbf{k}}, \chi_{\mathbf{k}'}^*) = 0$$

The  $n_{\mathbf{k}}$  excitations are given by

$$|n_{\mathbf{k}}\rangle = \frac{1}{\sqrt{n_{\mathbf{k}}!}} (\hat{a}_{\mathbf{k}}^\dagger)^{n_{\mathbf{k}}} |0_\chi\rangle$$

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The transformation connecting two different sets of complete modes  $\chi_{\mathbf{k}}$ ,  $\psi_{\mathbf{k}}$  is the **Bogolyubov transformation**

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The scalar field  $\phi$  can be expanded in terms of any of the two sets

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One can prove that

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An observer in  $|0_\psi\rangle$  vacuum using the  $\chi_{\mathbf{k}}$  modes can count the **average number** of  $\chi$  particles

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- **Inflation** is an **exponential expansion** of the **universe** taking place after the **Big Bang**
- It seems **necessary** in order to understand **observables** that cannot be explained by the Big Bang model.
- The scalar field  $\phi$  which drives the inflation is called **inflaton**. It is **homogeneous** and only **time-dependent**

We consider a FRW Universe with a metric  $ds^2 = dt^2 - a^2(t)d\mathbf{x}^2$  and a scalar field with the action

$$S_\phi = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] \quad \text{with} \quad V(\phi) = \frac{1}{2} M_\phi^2 \phi^2$$

Einstein equations

$$G_{\mu\nu} = -8\pi G T_{\mu\nu}$$

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$$H^2(t) \equiv \frac{\dot{a}^2(t)}{a^2(t)} = \frac{8\pi}{3M_{pl}^2} \left[ \frac{\dot{\phi}^2}{2} + V(\phi) \right]$$

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$$\textit{inflation} \equiv \ddot{a}(t) > 0 \equiv \frac{d}{dt} \left( \frac{H^{-1}(t)}{a(t)} \right) < 0$$

We define the dimensionless time  $\tau = M_\phi t$  and the dimensionless field  $\psi(\tau) = \phi(\tau)/M_\phi$

- We use the **slow-roll approximation**  $\ddot{\phi} \ll 3H\dot{\phi}$  and  $\dot{\phi}^2 \ll V(\phi)$

$$\text{Friedmann equation} \rightarrow H^2(\tau) \simeq \frac{4\pi}{3} \psi^2(\tau)$$

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Inflation ends at

$$\tau_f = 2\sqrt{3\pi}\psi_0 - 1$$

- We solve the **initial equations numerically**

$$\text{Friedmann equation} \rightarrow \frac{\dot{a}^2(\tau)}{a^2(\tau)} = \frac{4\pi}{3} (\dot{\psi}(\tau)^2 + \psi^2(\tau))$$

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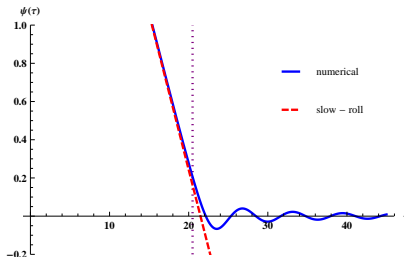
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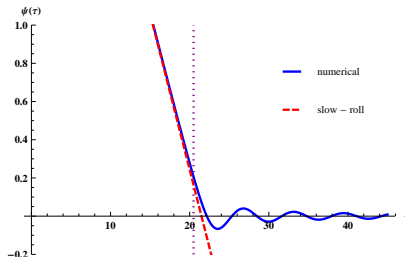
$$\text{equation of motion} \rightarrow \ddot{\psi}(\tau) + 3H(\tau)\dot{\psi}(\tau) + \psi(\tau) = 0$$

Dimensionless inflaton  $\psi$  in terms of the dimensionless time  $\tau$



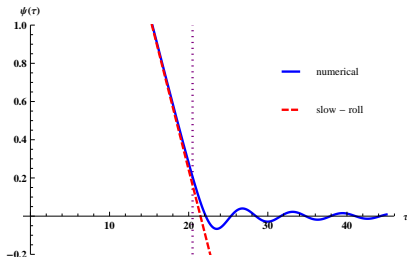
- Until the end of inflation the numerical and the approximated solution coincide
- After the end of inflation the field  $\psi$  oscillates at the bottom of the potential producing particles until it discharges

Dimensionless inflaton  $\psi$  in terms of the dimensionless time  $\tau$

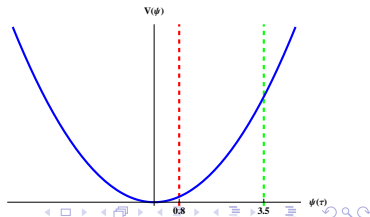


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We consider the **action**

$$S_\phi = \int d^4x \sqrt{-g} \left\{ \frac{R}{16\pi G} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} \lambda_1 G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right\}$$

**Friedmann equation**

$$H^2(\tau) = \frac{4\pi}{3} \left[ \dot{\psi}^2(\tau) \left( 1 - 9\bar{\lambda}_1 H^2(\tau) \right) + \psi^2(\tau) \right] \quad (\bar{\lambda}_1 = \lambda_1 M_\phi^2)$$

**equation of motion**

$$\left( 1 - 3\bar{\lambda}_1 H(\tau)^2 \right) \ddot{\psi}(\tau) + \left( 3H(\tau) - 3\bar{\lambda}_1 H(\tau) (2\dot{H}(\tau) + 3H^2(\tau)) \right) \dot{\psi}(\tau) + \psi(\tau) = 0$$





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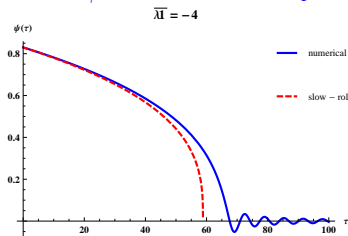
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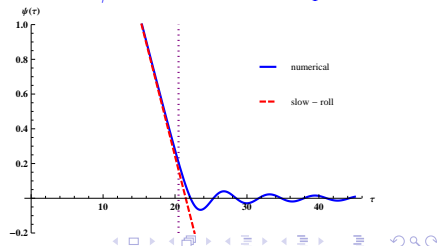
**equation of motion**

$$\left( 1 - 3\bar{\lambda}_1 H(\tau)^2 \right) \ddot{\psi}(\tau) + \left( 3H(\tau) - 3\bar{\lambda}_1 H(\tau) (2\dot{H}(\tau) + 3H^2(\tau)) \right) \dot{\psi}(\tau) + \psi(\tau) = 0$$

inflaton  $\psi$  in terms of  $\tau$  for  $\bar{\lambda}_1 = -4$



inflaton  $\psi$  in terms of  $\tau$  for  $\bar{\lambda}_1 = 0$



We consider the **action**

$$S_\phi = \int d^4x \sqrt{-g} \left\{ \frac{R}{16\pi G} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} \lambda_1 G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right\}$$

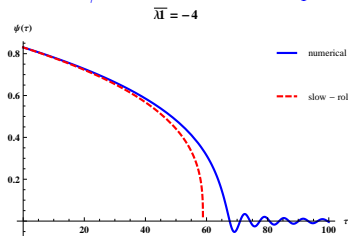
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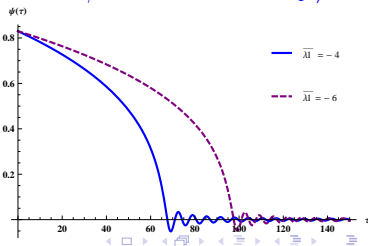
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inflaton  $\psi$  in terms of  $\tau$  for  $\bar{\lambda}_1 = -4$



inflaton  $\psi$  in terms of  $\tau$  for  $\bar{\lambda}_1 \neq 0$



The complete action of the theory is

$$S = \int d^4x \sqrt{-g} \frac{M_{pl}^2 R}{16\pi} + \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \left[ (g^{\mu\nu} + \lambda_1 G^{\mu\nu}) \partial_\mu \phi \partial_\nu \phi - M_\phi^2 \phi^2 \right] \right. \\ \left. + \frac{1}{2} \left[ (g^{\mu\nu} + \lambda_2 G^{\mu\nu}) \partial_\mu X \partial_\nu X - (M_X^2 + \zeta R(t) + g^2 \phi^2(t)) X^2 \right] \right\}$$

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Equation of motion for the quantum field  $X$

$$\left[ 1 - 3\lambda_2 M_\phi^2 \frac{\dot{a}^2(\tau)}{a^2(\tau)} \right] \ddot{\chi}_{\mathbf{k}}(\tau) + 3 \left[ \frac{\dot{a}(\tau)}{a(\tau)} - \lambda_2 M_\phi^2 \left( \frac{\dot{a}^3(\tau)}{a^3(\tau)} + \frac{2\dot{a}(\tau)\ddot{a}(\tau)}{a^2(\tau)} \right) \right] \dot{\chi}_{\mathbf{k}}(\tau) + \\ \left[ \frac{k^2}{M_\phi^2 a^2(\tau)} - \lambda_2 k^2 \frac{2\ddot{a}(\tau)a(\tau) + \dot{a}^2(\tau)}{a^4(\tau)} + \frac{m_X^2}{M_\phi^2} + \frac{\zeta R(\tau)}{M_\phi^2} + \frac{g^2 \psi^2(\tau) M_{pl}^2}{M_\phi^2} \right] \chi_{\mathbf{k}}(\tau) = 0$$

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Writing  $\chi_{\mathbf{k}}(\tau) = f(\tau) h_{\mathbf{k}}(\tau)$  we get a more “convenient” form

$$\ddot{h}_{\mathbf{k}}(\tau) + \left[ B(\tau) - \frac{\dot{A}(\tau)}{2} - \frac{A^2(\tau)}{4} \right] h_{\mathbf{k}}(\tau) = 0$$

where

$$A(\tau) = 3 \frac{\frac{\dot{a}(\tau)}{a(\tau)} - \overline{\lambda_2} \left( \frac{\dot{a}^3(\tau)}{a^3(\tau)} + 2 \frac{\dot{a}(\tau) \ddot{a}(\tau)}{a^2(\tau)} \right)}{1 - 3 \overline{\lambda_2} \frac{\dot{a}^2(\tau)}{a^2(\tau)}} \quad (\overline{\lambda_2} = \lambda_2 M_\phi^2)$$

$$B(\tau) = \frac{\frac{k^2}{M_\phi^2 a^2(\tau)} - \lambda_2 k^2 \frac{2a(\tau) \ddot{a}(\tau) + \dot{a}^2(\tau)}{a^4(\tau)} + \frac{M_X^2}{M_\phi^2} + \frac{\zeta R(\tau)}{M_\phi^2} + \frac{g^2 \psi^2(\tau) M_{pl}^2}{M_\phi^2}}{1 - 3 \overline{\lambda_2} \frac{\dot{a}^2(\tau)}{a^2(\tau)}}$$

The complete action of the theory is

$$S = \int d^4x \sqrt{-g} \frac{M_{pl}^2 R}{16\pi} + \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \left[ (g^{\mu\nu} + \lambda_1 G^{\mu\nu}) \partial_\mu \phi \partial_\nu \phi - M_\phi^2 \phi^2 \right] + \frac{1}{2} \left[ (g^{\mu\nu} + \lambda_2 G^{\mu\nu}) \partial_\mu X \partial_\nu X - (M_X^2 + \zeta R(t) + g^2 \phi^2(t)) X^2 \right] \right\}$$

For the differential equation

$$\ddot{h}_{\mathbf{k}}(\tau) + \Omega^2(\tau) h_{\mathbf{k}}(\tau) = 0$$

there is a solution

$$h_{\mathbf{k}}(\tau) = \frac{\alpha_{\mathbf{k}}(\tau)}{\sqrt{2\Omega(\tau)}} e^{-i \int \Omega(\tau') d\tau'} + \frac{\beta_{\mathbf{k}}(\tau)}{\sqrt{2\Omega(\tau)}} e^{i \int \Omega(\tau') d\tau'}$$

if we define the Bogolyubov coefficients through a system of differential equations

$$\dot{\alpha}_{\mathbf{k}}(\tau) = \frac{\dot{\Omega}_{\mathbf{k}}(\tau)}{2\Omega_{\mathbf{k}}(\tau)} \exp \left[ 2i \int \Omega_{\mathbf{k}}(\tau') d\tau' \right] \beta_{\mathbf{k}}(\tau)$$

$$\dot{\beta}_{\mathbf{k}}(\tau) = \frac{\dot{\Omega}_{\mathbf{k}}(\tau)}{2\Omega_{\mathbf{k}}(\tau)} \exp \left[ -2i \int \Omega_{\mathbf{k}}(\tau') d\tau' \right] \alpha_{\mathbf{k}}(\tau)$$

- The quantity  $|\beta_{\mathbf{k}}|^2$  is the **mean number of particles** in state  $k$ .
- The system should obey the relations  $\Omega^2 > 0$  and  $\frac{\dot{\Omega}}{\Omega} \ll 1$  (**adiabatic approximation**)
- **After the end of inflation** we enter the phase of **matter domination**,  
 $(a(\tau) = \tau^{2/3})$
- There are no particles before the end of inflation  
 $(\beta_{\mathbf{k}}(\tau_f) = 0, \alpha_{\mathbf{k}}(\tau_f) = 1)$
- We measure the  $X$  field **production** at the time  $\tau_{flat}$  when the **Bogolyubov** coefficients have become time-independent
- We set the value of the **inflaton** mass  $M_\phi = 10^{-6} M_{pl}$
- $\bar{\lambda}_1$  and  $\bar{\lambda}_2$  are the **couplings** of the **Einstein tensor** with the **kinetic terms** of the fields  $\phi$  and  $X$  respectively
- We study the quantity  $|\beta_{\mathbf{k}}|^2$  in terms of the mass  $M_X$  for different values of the **couplings**



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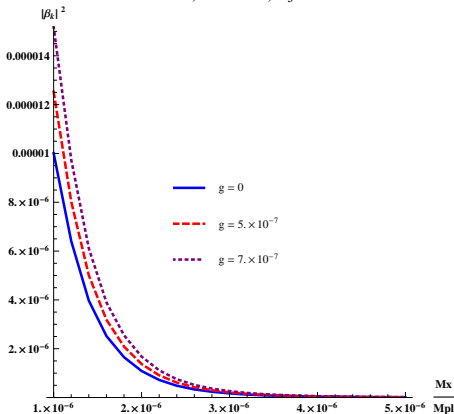
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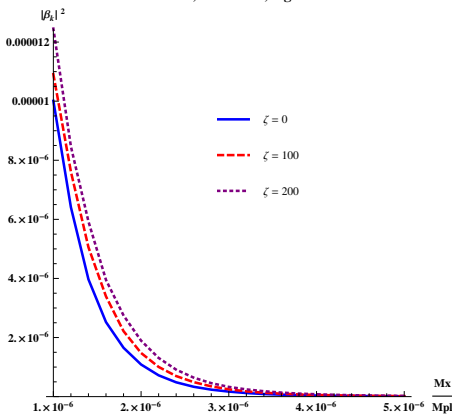
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Fields  $\phi$  and  $X$  without non-minimal kinetic terms

$\bar{\lambda}_1 = 0$  ,  $\bar{\lambda}_2 = 0$  ,  $\zeta = 0$



$\bar{\lambda}_1 = 0$  ,  $\bar{\lambda}_2 = 0$  ,  $g = 0$



- Enhancement of the particle production as the couplings  $g$  ,  $\zeta$  are increased
- Possible correlation of the results

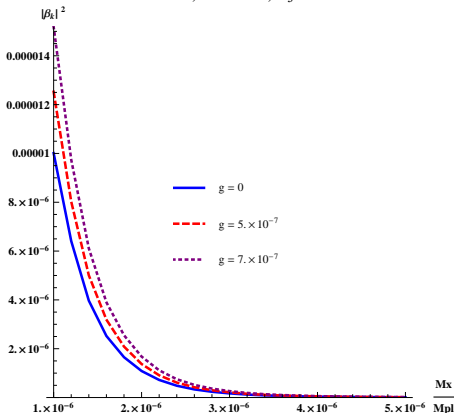
$$(\zeta R(t) + g^2 \phi^2(t)) X^2$$



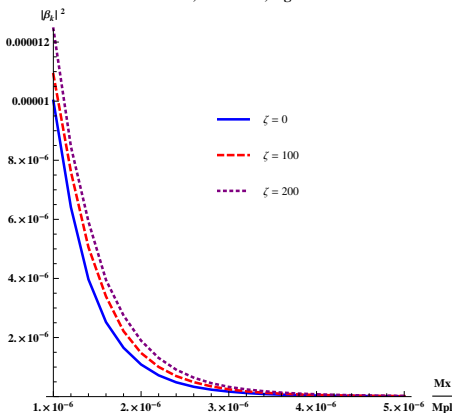


Fields  $\phi$  and  $X$  without non-minimal kinetic terms

$\bar{\mathcal{M}} = 0$  ,  $\bar{\mathcal{L}} = 0$  ,  $\zeta = 0$



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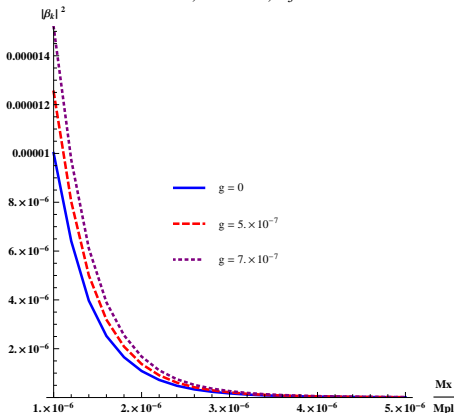
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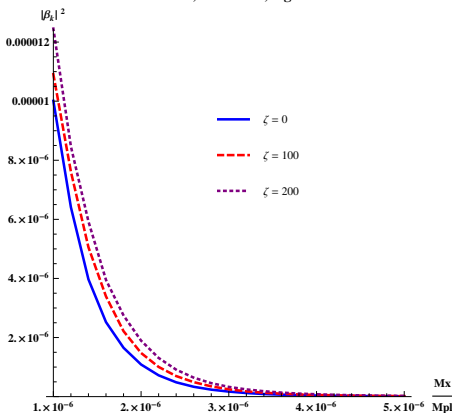


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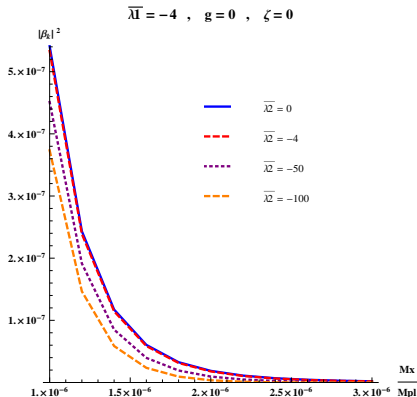


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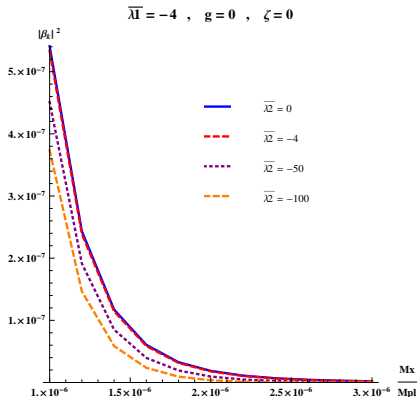


Field  $\phi$  with coupling  $\bar{\lambda}_1 = -4$  and  $X$  with  $\bar{\lambda}_2 \neq 0$



- We need quite larger  $\bar{\lambda}_2$  than  $\bar{\lambda}_1$  in order to have some change in particle production
- After the end of inflation Einstein tensor has been weakened so we need stronger coupling  $\bar{\lambda}_2$

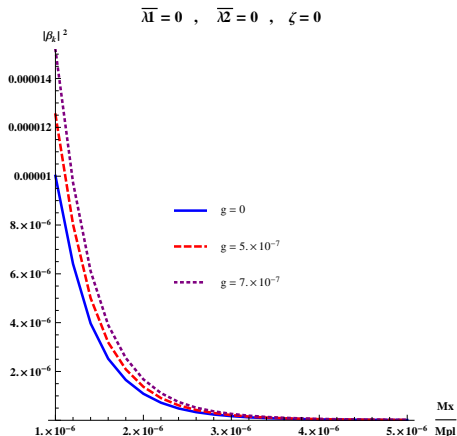
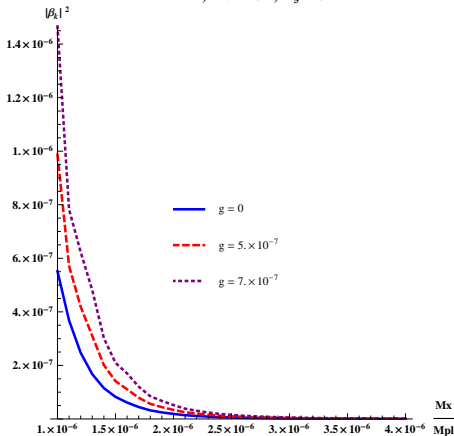
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Field  $\phi$  with coupling  $\overline{\lambda}_1 = -4$ ,  $\overline{\lambda}_2 = 0$  and  $X$  with  $\overline{\lambda}_2 = 0$

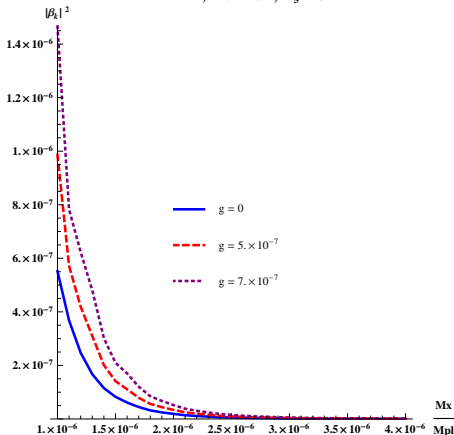
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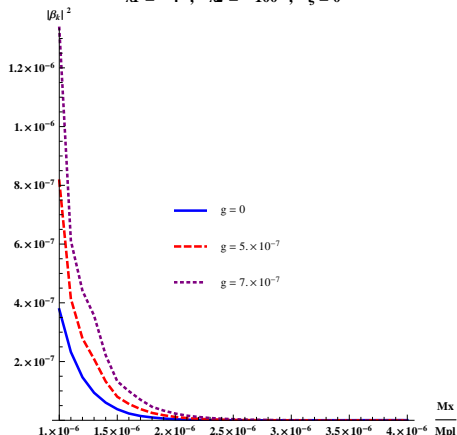
Suppression of particle production with the introduction of non-minimal kinetic terms to the inflaton

Field  $\phi$  with coupling  $\bar{\lambda}_1 = -4$ ,  $\bar{\lambda}_1 = 0$  and  $X$  with  $\bar{\lambda}_2 = 0$

$\bar{\lambda}_1 = -4$ ,  $\bar{\lambda}_2 = 0$ ,  $\zeta = 0$



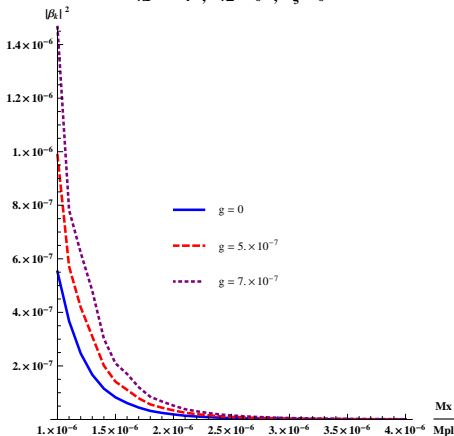
$\bar{\lambda}_1 = -4$ ,  $\bar{\lambda}_2 = -100$ ,  $\zeta = 0$



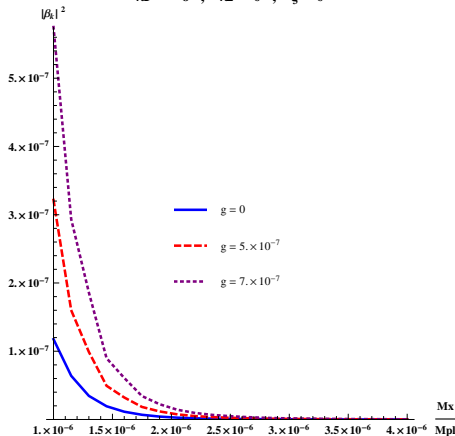
Further suppression of particle production with the introduction of non-minimal kinetic terms to the  $X$  field

Field  $\phi$  with coupling  $\bar{\lambda}_1 = -4$ ,  $\bar{\lambda}_1 = -6$  and  $X$  with  $\bar{\lambda}_2 = 0$

$\bar{\lambda}_1 = -4$ ,  $\bar{\lambda}_2 = 0$ ,  $\zeta = 0$



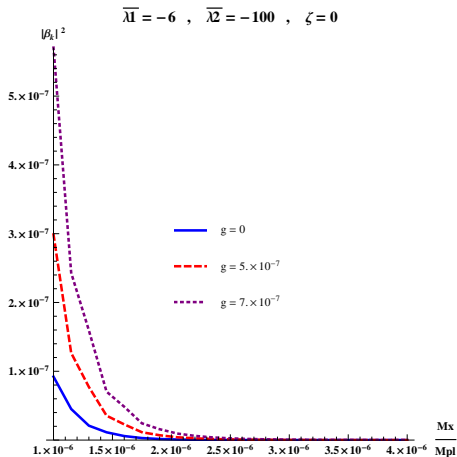
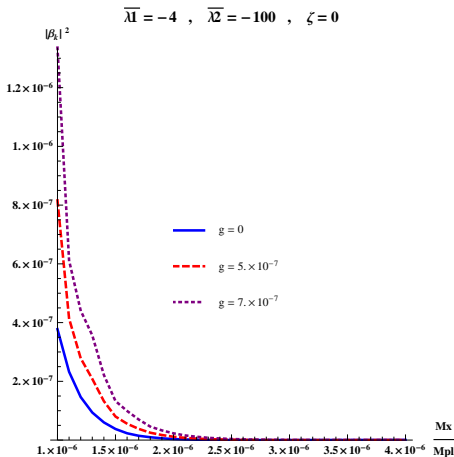
$\bar{\lambda}_1 = -6$ ,  $\bar{\lambda}_2 = 0$ ,  $\zeta = 0$



Suppression of particle production with the increase of the coupling  $\bar{\lambda}_1$

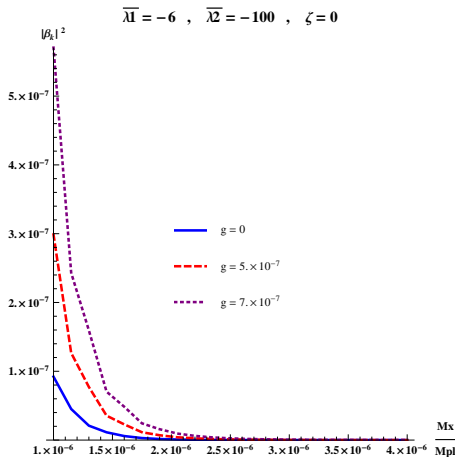
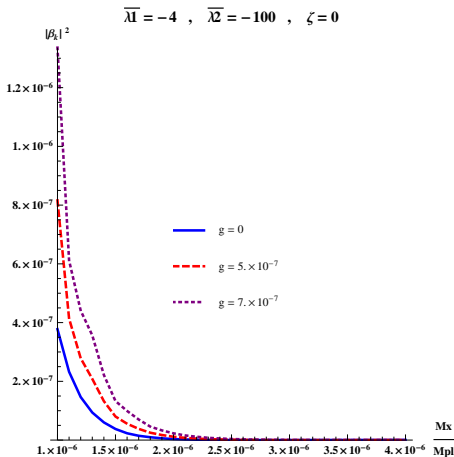


Field  $\phi$  with coupling  $\overline{\lambda}_1 = -4$ ,  $\overline{\lambda}_1 = -6$  and  $X$  with  $\overline{\lambda}_2 = -100$



Suppression of particle production with the increase of the couplings  $\overline{\lambda}_1$  and  $\overline{\lambda}_2$

Field  $\phi$  with coupling  $\bar{\lambda}_1 = -4$ ,  $\bar{\lambda}_1 = -6$  and  $X$  with  $\bar{\lambda}_2 = -100$



Suppression of particle production with the increase of the couplings  $\bar{\lambda}_1$  and  $\bar{\lambda}_2$

which can be reversed with the increase of the coupling  $g$

# Conclusions

- **Enhancement** of particle production with the **increase** of the couplings  $g, \zeta$
- After the end of inflation Einstein tensor has been weakened so we need stronger coupling  $\bar{\lambda}_2$
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