On relativistic scalar fields and the quasi-static approximation

(Based on JN, Francesca von Braun-Bates, Pedro Ferreira, arXiv: 1310.xxxx)

Johannes Noller, University of Oxford 7th Aegean Summer School, Paros, 27/09/2013

Interacting spin-2 fields in the Stückelberg picture – Part II

(Based on work in collaboration with James Scargill and Pedro Ferreira,)

Johannes Noller, University of Oxford 7th Aegean Summer School, Paros, 27/09/2013

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The quasi static approximation

$$\nabla^2|X| \gg \mathcal{H}^2 X \quad \text{and} \quad |\dot{X}| \leq \mathcal{H}|X|$$

- A subhorizon approximation $k^2 \gg \mathcal{H}^2$
- The relative suppression of time derivatives of metric/field perturbations compared with their spatial derivatives

$$\ddot{\tilde{\delta}} + \tilde{\mathcal{H}}\dot{\tilde{\delta}} - \frac{3}{2}\tilde{\mathcal{H}}^2\tilde{\Omega}_m(\tilde{\delta} - \frac{\beta\chi}{2}) - 2\dot{\phi}\dot{\chi} + V_\phi\chi = 0$$

$$\ddot{\chi} + 2\tilde{\mathcal{H}}\dot{\chi} + k^2\chi + \tilde{a}^2 V_{,\phi\phi}\,\chi - \dot{\phi}\dot{\tilde{\delta}} - \frac{3\beta}{2}\tilde{\mathcal{H}}^2\tilde{\Omega}_m(\tilde{\delta} - \frac{1}{2}\beta\chi) = 0$$

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Why care?

Let's consider pressureless, non-relativistic dark matter and scalar field 'dark energy' minimally interacting through gravity. In order to N-body simulate structure formation we need to:

- Solve the relativistic Newton-Poisson equation

$$-k^2\Phi = 4\pi G a^2 \rho \delta_{gi} + \mathcal{F}(\phi, \chi).$$

- For a set of N particles with positions $\vec{x}_a$ $(a = 1 \ldots N)$ solve the non-relativistic geodesic equation

$$\frac{d^2\vec{x}_a}{d\tau^2} + \mathcal{H}\frac{d\vec{x}_a}{d\tau} = -\nabla\Phi(\vec{x}_a).$$

- Solve the second order evolution equations for ϕ and χ.

Why care?

Let's consider p... k
energy' minimally ... e
structure formatio...

- Solve the rel...

- For a set of ... n-
relativistic g...

On subhorizon scales and in a chameleon modified gravity setting, Brax et al. (1303.0007) find

$$\frac{m_0}{H_0} \sim 10^3, \qquad \frac{m_0}{H_0} \sim 10^6$$

for the background evolution and *mildly high density regions* respectively.

The quasi-static approximation helps with solving this, but how accurate is it? And for which scales?

- Solve the second order evolution equations for ϕ and χ.

Models and Aims

Models

Pressureless, non-relativistic dark matter + scalar field 'dark energy' ϕ with perturbations χ and with an exponential potential $V \sim e^{\lambda\phi}$:

1. A quintessence-like model with no direct coupling between dark matter and ϕ.
2. A generic $f(R)$ model.
3. A chameleon ($f(R)$ with screening).

$$S_J = \frac{1}{2}\int d^4x\sqrt{-g}\,[R + f(R)] + \int d^4x\sqrt{-g}\,\mathcal{L}_{\rm m}[\Phi_i, g_{\mu\nu}]$$

$$S_E = \frac{1}{2}\int d^4x\sqrt{-\tilde{g}}\,\tilde{R} + \int d^4x\sqrt{-\tilde{g}}\left[-\frac{1}{2}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\phi\tilde{\nabla}_\nu\phi - V(\phi)\right] + S_{matter}[\Phi_i, e^{-\beta\phi}\tilde{g}_{\mu\nu}]$$

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Models

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Benchmark

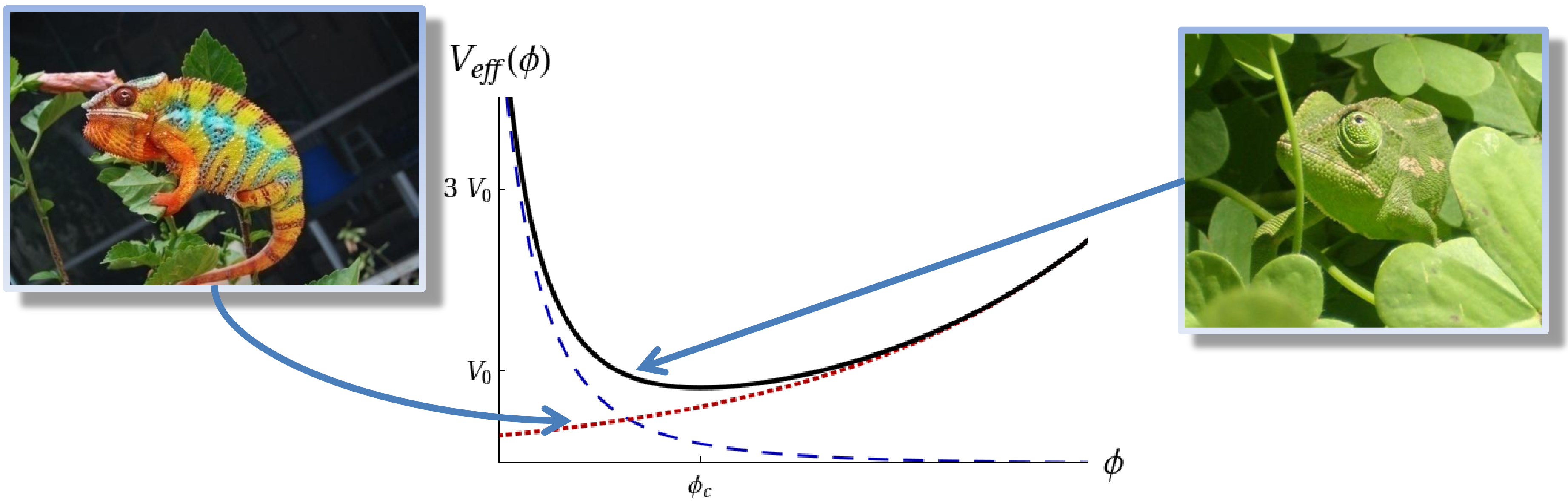
Euclid Science Objective is to determine PS of δ to 1% accuracy.

Aim

Obtain a quantitative understanding of errors introduced by the QSA.

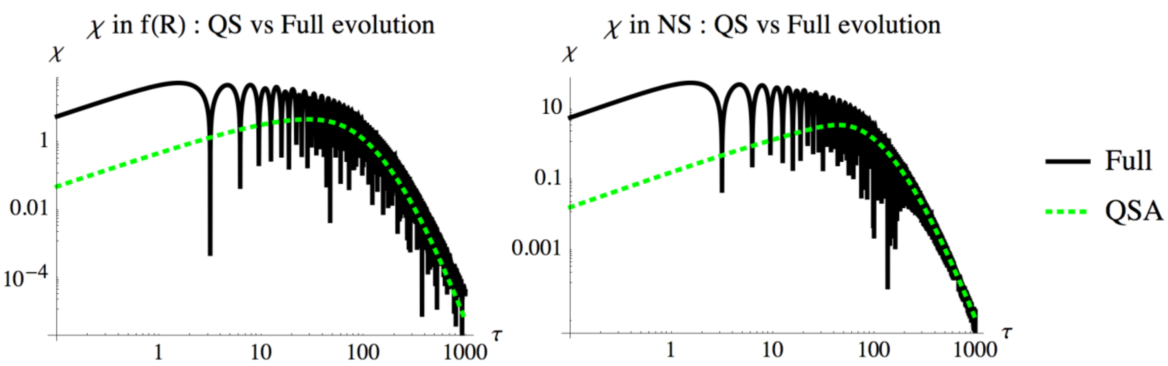
The chameleon mechanism

$$S_E = \frac{1}{2}\int d^4x\sqrt{-\tilde{g}}\,\tilde{R} + \int d^4x\sqrt{-\tilde{g}}\left[-\frac{1}{2}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\phi\tilde{\nabla}_\nu\phi - V(\phi)\right] + S_{matter}[\Phi_i, e^{-\beta\phi}\tilde{g}_{\mu\nu}]$$



$$V_{\rm eff}(\phi) = V(\phi) + \hat{\rho}A(\phi) \qquad m_{\rm min} \equiv \sqrt{V_{eff,\phi\phi}(\phi_{\rm min})}$$

Why and when does the QSA do well?



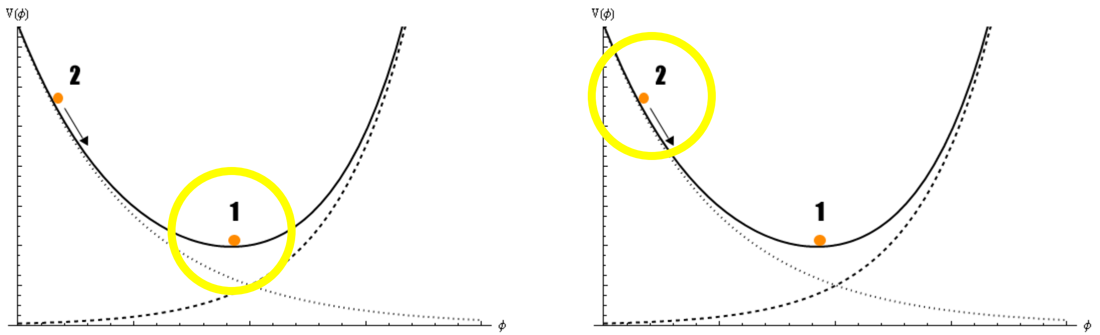
$$\chi^{NS}_{QSA} = \frac{\dot\phi\dot\delta}{k^2 + a^2 V_{,\phi\phi}},$$

$$\chi^{f(R)}_{QSA} = \frac{\dot\phi\dot\delta + \frac{3}{2}\beta\mathcal{H}^2\Omega_m\delta}{k^2 + a^2 V_{,\phi\phi} + \frac{3}{4}\beta^2\mathcal{H}^2\Omega_m}$$

$$\ddot{\tilde\delta} + \tilde{\mathcal{H}}\dot{\tilde\delta} - \frac{3}{2}\tilde{\mathcal{H}}^2\tilde\Omega_m\tilde\delta - 2\dot\phi\dot\chi + \left(V_\phi + \frac{3}{2}\tilde{\mathcal{H}}^2\tilde\Omega_m\frac{\beta}{2}\right)\chi = 0$$

$$\ddot\chi + 2\tilde{\mathcal{H}}\dot\chi + k^2\chi + \tilde a^2 V_{,\phi\phi}\chi - \dot\phi\tilde\delta - \frac{3\beta}{2}\tilde{\mathcal{H}}^2\tilde\Omega_m(\tilde\delta - \frac{1}{2}\beta\chi) = 0$$

Slow-roll vs. Fast-roll

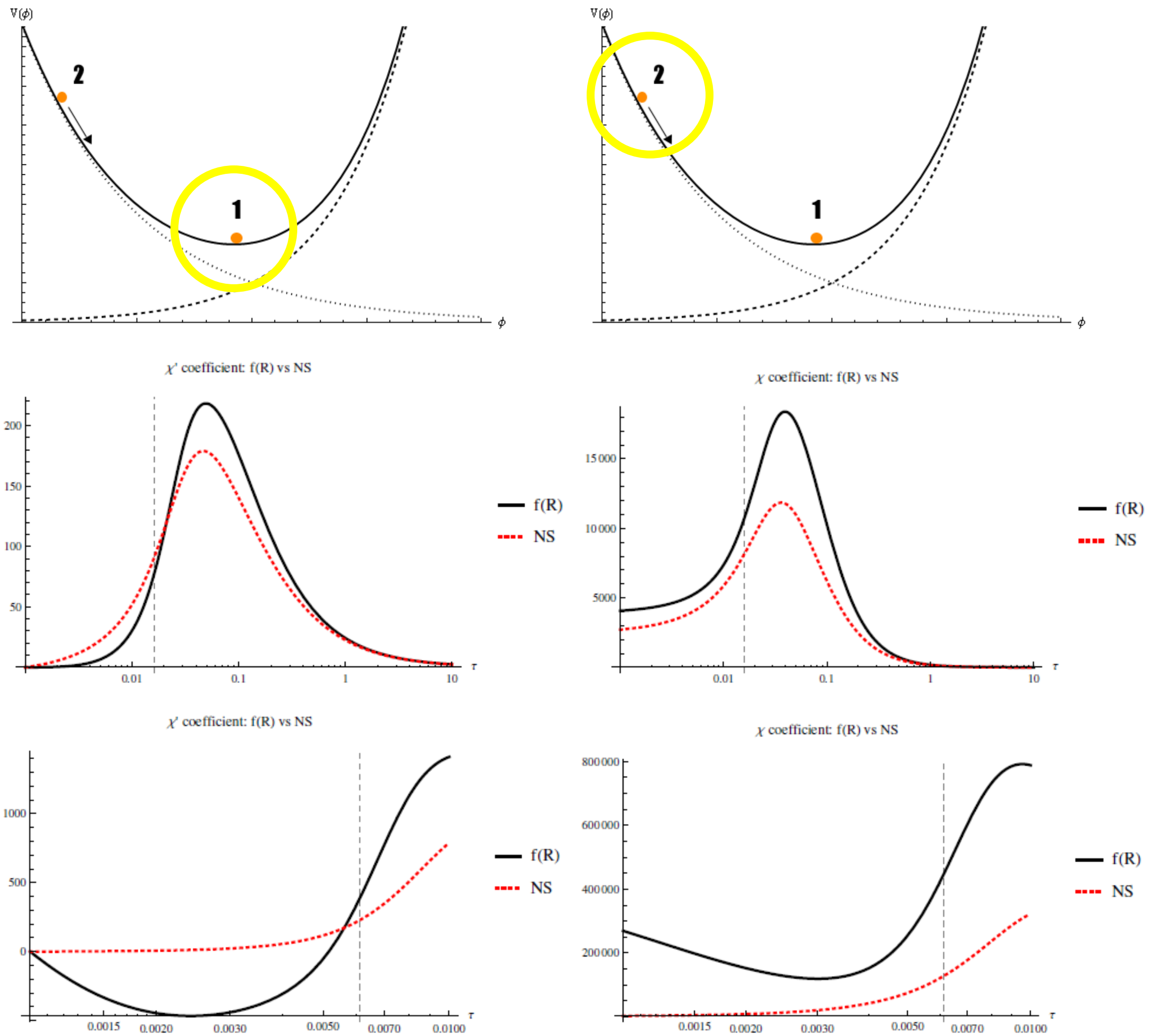


$$-2\dot{\phi}\dot{\chi} + \left(V_\phi + \frac{3}{2}\tilde{\mathcal{H}}^2\tilde{\Omega}_m\frac{\beta}{2} \right)\chi$$

BBN constraints on time-variation of particle masses

cf. Brax et al., astro-ph/0408415

Slow-roll vs. Fast-roll



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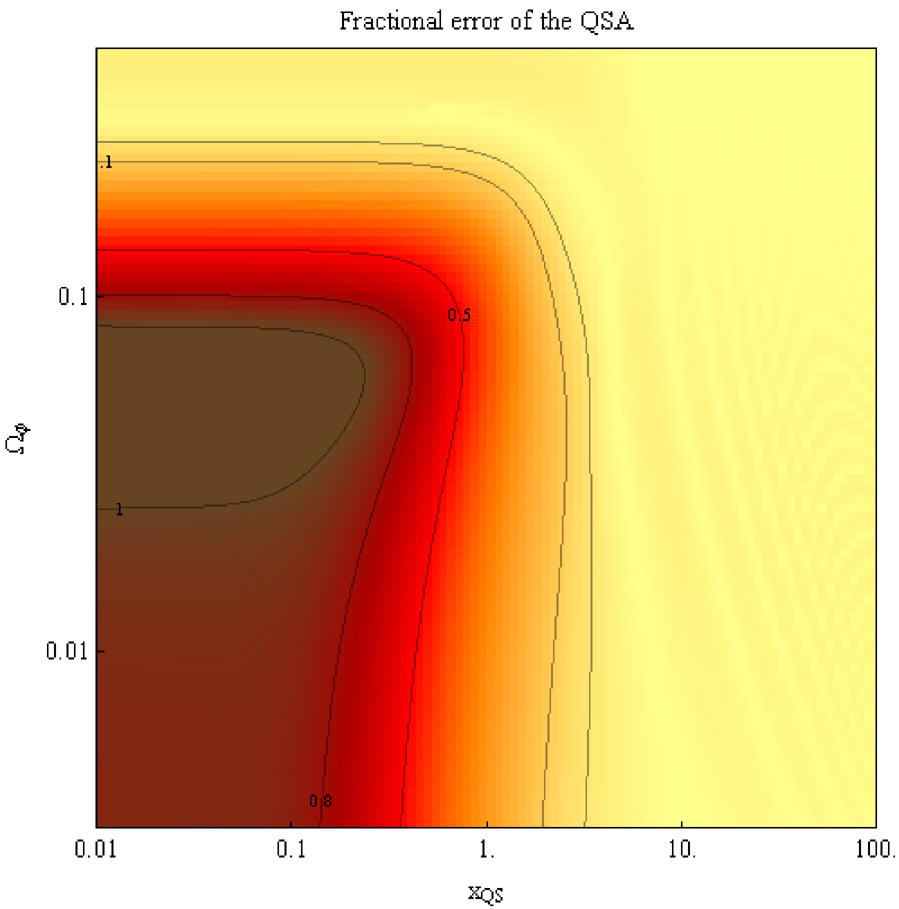
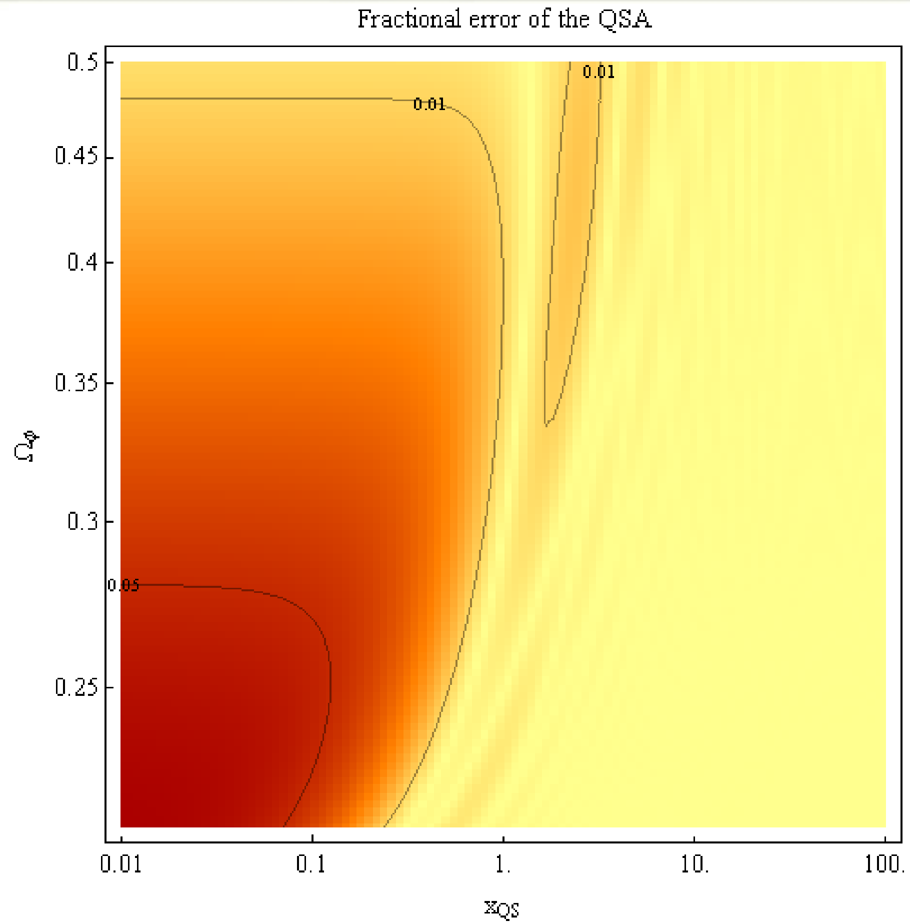
$\sim 30\% \quad vs. \quad > 1000\%$

The chameleon mechanism





Fast-roll $\frac{\delta_{QS}}{\delta_{full}} - 1$ Slow-roll

Summary

QSA analysis : $f(R)+$ chameleons in the linearised regime

Observational viability:
Screening + BBN constraints

→

QSA does well,
even on super-horizon scales

However, even in 'best case' scenarios we obtain $\sim 5\%$ errors on superhorizon scales still.

→

Good enough?

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Thank you!

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