

The background features a complex pattern of overlapping lines and circular motifs, resembling mechanical gears or a technical drawing. The colors are muted, with shades of grey, black, and a light beige. The overall aesthetic is technical and precise.

# On relativistic scalar fields and the quasi-static approximation

(Based on JN, Francesca von Braun-Bates, Pedro Ferreira, arXiv: 1310.xxxx)

The background features a complex, layered design. On the left side, there are several overlapping circular gears of various sizes, some with teeth and others as solid discs. These are set against a background of thin, parallel lines that create a grid-like pattern. The overall color palette is a mix of light and dark greys, with some white highlights on the gears and lines.

# Interacting spin-2 fields in the Stückelberg picture – Part II

(Based on work in collaboration with James Scargill and Pedro Ferreira,)

The background features a complex pattern of overlapping lines and circular motifs, resembling mechanical gears or a technical drawing. The colors are muted, with shades of grey, black, and a light beige. The overall aesthetic is technical and scientific.

# On relativistic scalar fields and the quasi-static approximation

(Based on JN, Francesca von Braun-Bates, Pedro Ferreira, arXiv: 1310.xxxx)

# The quasi static approximation

$$\nabla^2 |X| \gg \mathcal{H}^2 X \quad \text{and} \quad |\dot{X}| \leq \mathcal{H} |X|$$

- A subhorizon approximation  $k^2 \gg \mathcal{H}^2$
- The relative suppression of time derivatives of metric/field perturbations compared with their spatial derivatives

$$\begin{aligned} \ddot{\delta} + \tilde{\mathcal{H}} \dot{\delta} - \frac{3}{2} \tilde{\mathcal{H}}^2 \tilde{\Omega}_m \left( \tilde{\delta} - \frac{\beta \chi}{2} \right) - 2 \dot{\phi} \dot{\chi} + V_{\phi} \chi &= 0 \\ \ddot{\chi} + 2 \tilde{\mathcal{H}} \dot{\chi} + k^2 \chi + \tilde{a}^2 V_{,\phi\phi} \chi - \dot{\phi} \tilde{\delta} - \frac{3\beta}{2} \tilde{\mathcal{H}}^2 \tilde{\Omega}_m \left( \tilde{\delta} - \frac{1}{2} \beta \chi \right) &= 0 \end{aligned}$$

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~~$$\ddot{\chi} + 2\mathcal{H} \dot{\chi} + k^2 \chi + \tilde{a}^2 V_{,\phi\phi} \chi - \dot{\phi} \tilde{\delta} - \frac{3\beta}{2} \tilde{\mathcal{H}}^2 \tilde{\Omega}_m \left( \tilde{\delta} - \frac{1}{2} \beta \chi \right) = 0$$~~

# Why care?

Let's consider pressureless, non-relativistic dark matter and scalar field 'dark energy' minimally interacting through gravity. In order to N-body simulate structure formation we need to:

- Solve the relativistic Newton-Poisson equation

$$-k^2\Phi = 4\pi G a^2 \rho \delta_{gi} + \mathcal{F}(\phi, \chi).$$

- For a set of N particles with positions  $\vec{x}_a$  ( $a = 1 \dots N$ ) solve the non-relativistic geodesic equation

$$\frac{d^2 \vec{x}_a}{d\tau^2} + \mathcal{H} \frac{d\vec{x}_a}{d\tau} = -\nabla \Phi(\vec{x}_a).$$

- Solve the second order evolution equations for  $\phi$  and  $\chi$ .

# Why care?

Let's consider perturbations of the 'energy' minimally coupled to the metric for structure formation

- Solve the relativistic equations

On subhorizon scales and in a chameleon modified gravity setting, Brax et al. (1303.0007) find

$$\frac{m_0}{H_0} \sim 10^3, \quad \frac{m_0}{H_0} \sim 10^6$$

for the background evolution and *mildly high density regions* respectively.

- For a set of relativistic geodesics

The quasi-static approximation helps with solving this, but how accurate is it? And for which scales?

- Solve the second order evolution equations for  $\phi$  and  $\chi$ .

# Models and Aims

## Models

Pressureless, non-relativistic dark matter + scalar field 'dark energy'  $\phi$  with perturbations  $\chi$  and with an exponential potential  $V \sim e^{\lambda\phi}$ :

1. A quintessence-like model with no direct coupling between dark matter and  $\phi$ .
2. A generic  $f(R)$  model.
3. A chameleon ( $f(R)$  with screening).

$$S_J = \frac{1}{2} \int d^4x \sqrt{-g} [R + f(R)] + \int d^4x \sqrt{-g} \mathcal{L}_m[\Phi_i, g_{\mu\nu}]$$
$$S_E = \frac{1}{2} \int d^4x \sqrt{-\tilde{g}} \tilde{R} + \int d^4x \sqrt{-\tilde{g}} \left[ -\frac{1}{2} \tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \phi \tilde{\nabla}_\nu \phi - V(\phi) \right]$$
$$+ S_{matter}[\Phi_i, e^{-\beta\phi} \tilde{g}_{\mu\nu}]$$



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## Benchmark

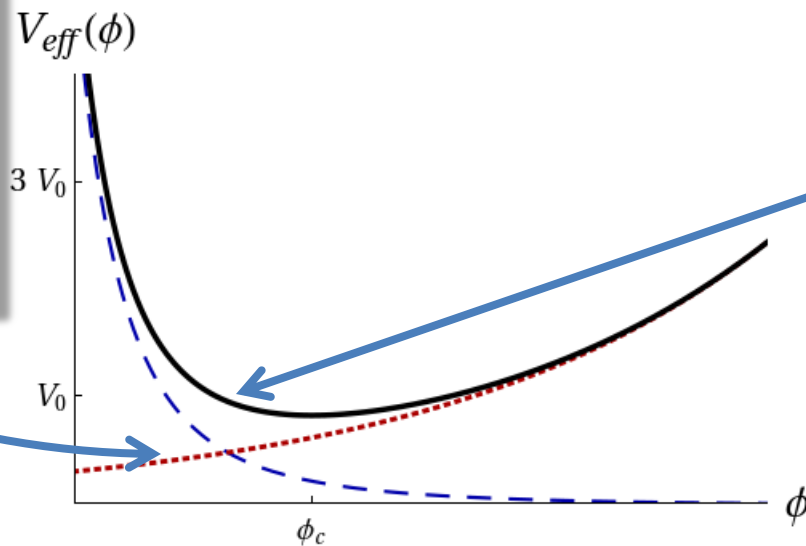
Euclid Science Objective is to determine PS of  $\delta$  to 1% accuracy.

## Aim

Obtain a quantitative understanding of errors introduced by the QSA.

# The chameleon mechanism

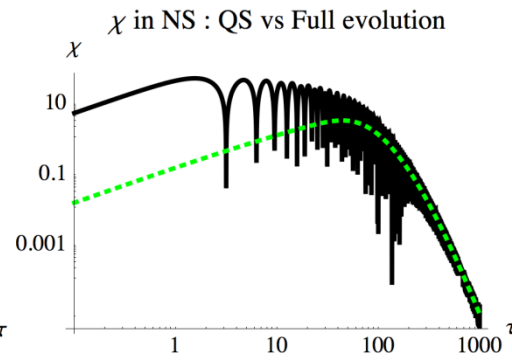
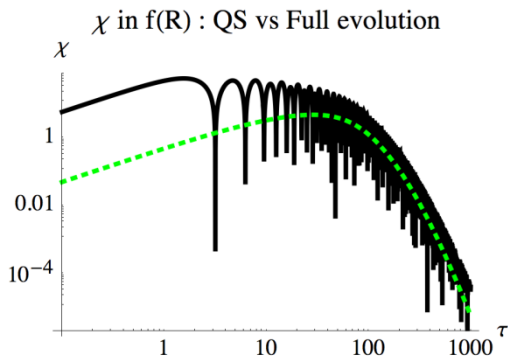
$$S_E = \frac{1}{2} \int d^4x \sqrt{-\tilde{g}} \tilde{R} + \int d^4x \sqrt{-\tilde{g}} \left[ -\frac{1}{2} \tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \phi \tilde{\nabla}_\nu \phi - V(\phi) \right] \\ + S_{matter}[\Phi_i, e^{-\beta\phi} \tilde{g}_{\mu\nu}]$$



$$V_{\text{eff}}(\phi) = V(\phi) + \hat{\rho}A(\phi)$$

$$m_{\text{min}} \equiv \sqrt{V_{\text{eff},\phi\phi}(\phi_{\text{min}})}$$

# Why and when does the QSA do well?



— Full  
- - - QSA

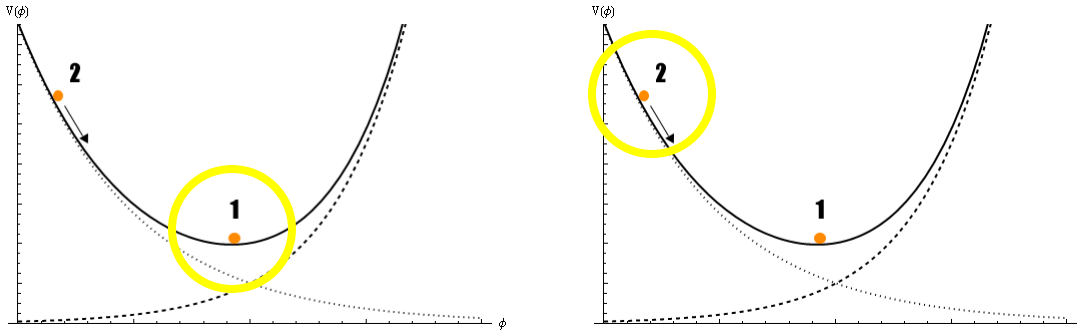
$$\chi_{QSA}^{NS} = \frac{\dot{\phi}\dot{\delta}}{k^2 + a^2 V_{,\phi\phi}},$$

$$\chi_{QSA}^{f(R)} = \frac{\dot{\phi}\dot{\delta} + \frac{3}{2}\beta\mathcal{H}^2\Omega_m\delta}{k^2 + a^2 V_{,\phi\phi} + \frac{3}{4}\beta^2\mathcal{H}^2\Omega_m}$$

$$\ddot{\delta} + \tilde{\mathcal{H}}\dot{\delta} - \frac{3}{2}\tilde{\mathcal{H}}^2\tilde{\Omega}_m\delta - 2\dot{\phi}\dot{\chi} + \left( V_\phi + \frac{3}{2}\tilde{\mathcal{H}}^2\tilde{\Omega}_m\frac{\beta}{2} \right) \chi = 0$$

~~$$\ddot{\chi} + 2\mathcal{H}\dot{\chi} + k^2\chi + \tilde{a}^2 V_{,\phi\phi}\chi - \dot{\phi}\dot{\delta} - \frac{3\beta}{2}\tilde{\mathcal{H}}^2\tilde{\Omega}_m\left(\delta - \frac{1}{2}\beta\chi\right) = 0$$~~

# Slow-roll vs. Fast-roll

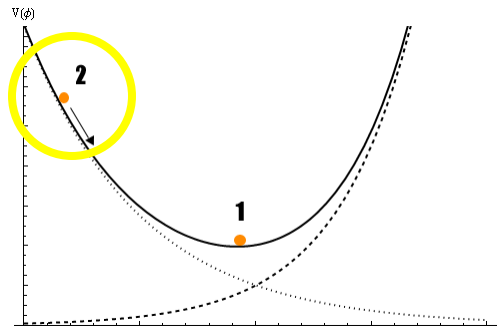
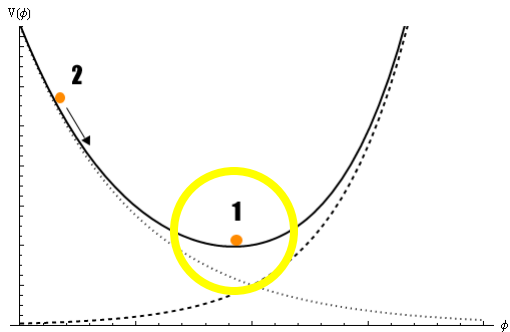


$$-2\dot{\phi}\dot{\chi} + \left( V_{\phi} + \frac{3}{2}\tilde{\mathcal{H}}^2\tilde{\Omega}_m\frac{\beta}{2} \right) \chi$$

BBN constraints on  
time-variation of  
particle masses

cf. Brax et al., astro-ph/0408415

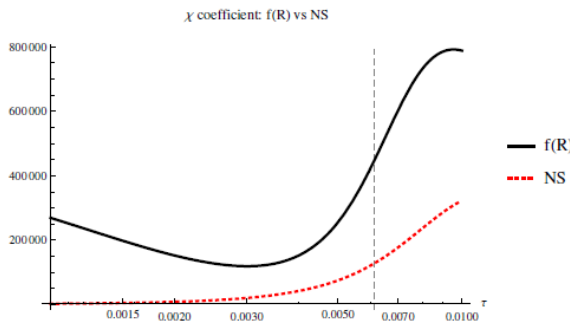
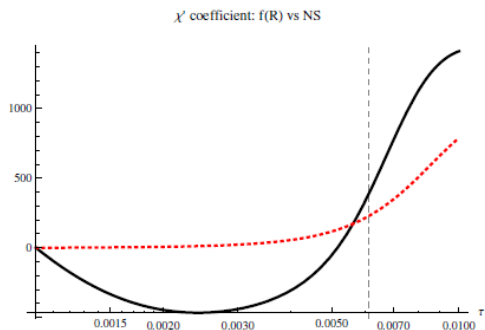
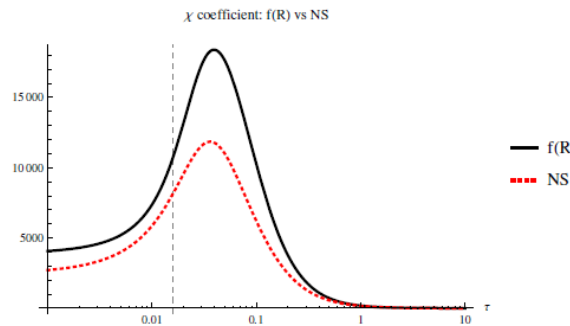
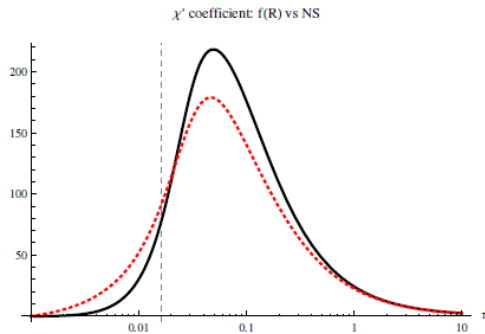
# Slow-roll vs. Fast-roll



$$-2\dot{\phi}\dot{\chi} + \left( V_{\phi} + \frac{3}{2}\tilde{\mathcal{H}}^2\tilde{\Omega}_m\frac{\beta}{2} \right) \chi$$

BBN constraints on time-variation of particle masses

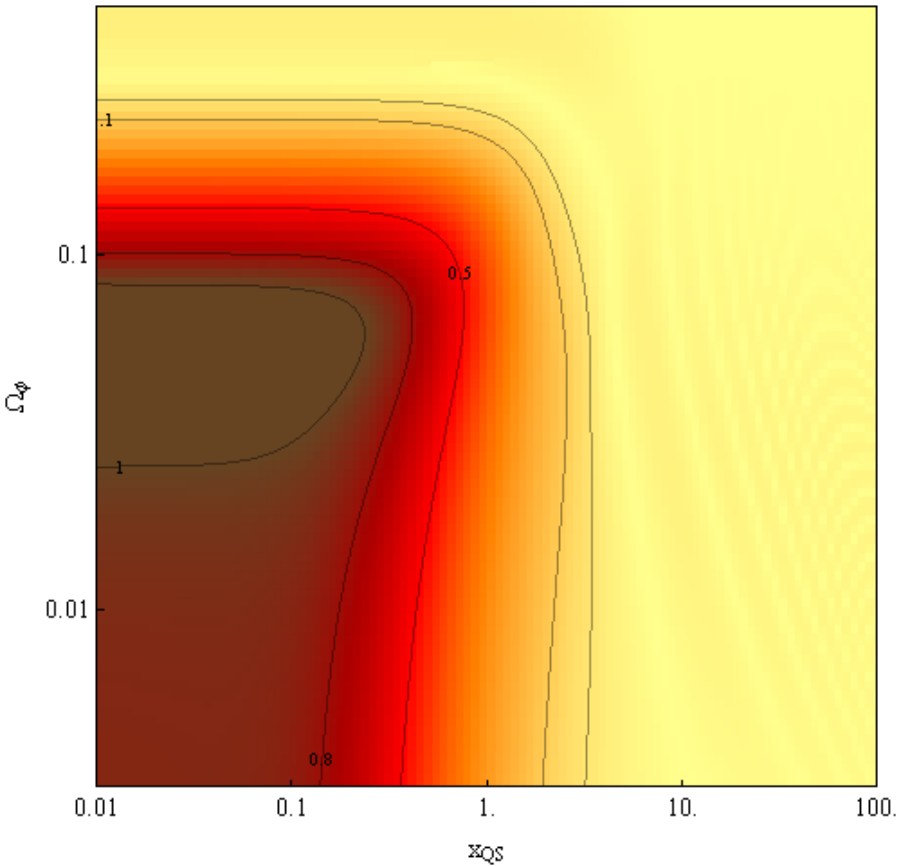
cf. Brax et al., astro-ph/0408415



$\sim 30\%$  vs.  $> 1000\%$

# The chameleon mechanism

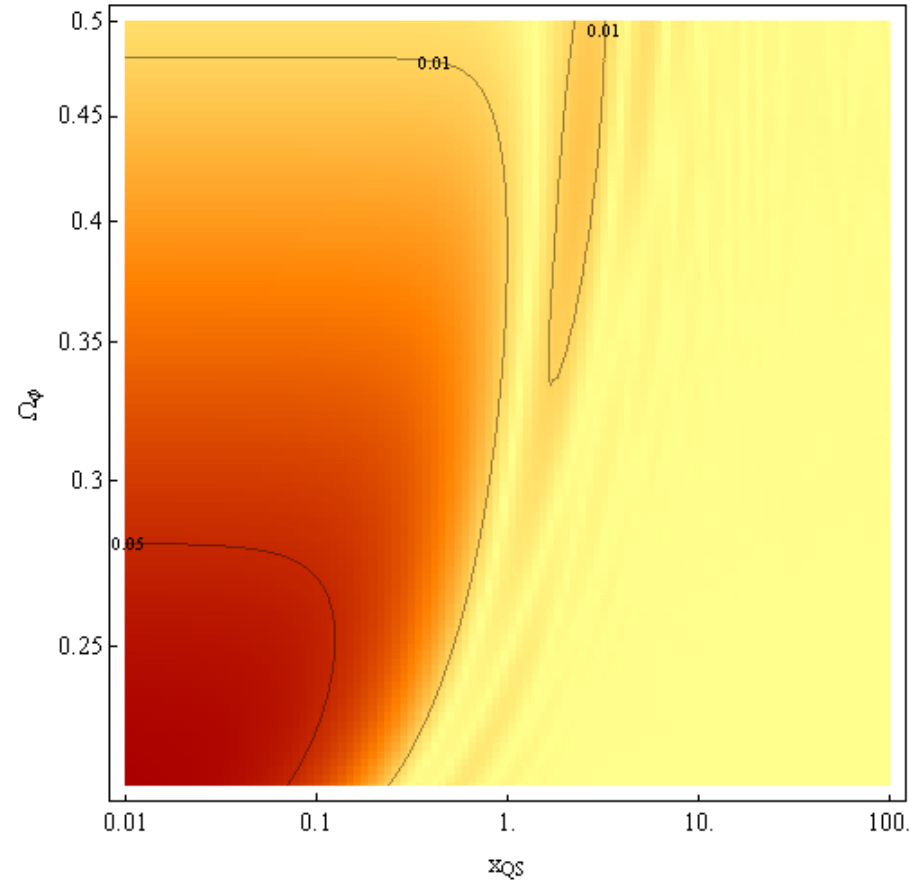
Fractional error of the QSA



Fast-roll

$$\frac{\delta_{QS}}{\delta_{full}} \approx 1$$

Fractional error of the QSA



Slow-roll

# Summary

QSA analysis :  $f(R)$ + chameleons in the linearised regime

Observational viability:  
Screening + BBN constraints



QSA does well,  
even on super-horizon scales

However, even in 'best case' scenarios we obtain  
 $\sim 5\%$  errors on superhorizon scales still.



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## Thank you!