Super-renormalizable or Finite Quantum Gravity

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Quantum Gravity Field Theory or
Quantum Gravity for Particle Physicists

Efimov ,
Krasnikov ,
Tomboulis ,
Moffat, Cornish ,
Namsrai , Buchbinder, Odintsov, Shapiro,
Barvinsky, Vilkovisky, Gusev , Anselmi.

References

String Inspired
Siegel,
Brandenberger,
Mazumdar, Biswas.

1. Super-renormalizable quantum gravity, L.M., Phys. Rev. D 86, 0440055 (2012) .
2. Finite quantum gravity, L.M., [arXiv:1305.6741[hep-th]].
3. Multidimensional Super-renormalizable Gravity: Theory and Applications, L.M., Atronomical. Rev.
3. The hidden quantum groups symmetry of super-renormalizable gravity, S. Alexander, A. Marciano', L.M., Phys. Rev. D 85 124030 (2012).
4. Inflation in (Super-)renormalizable Gravity. F. Briscese, A. Marciano, L. Modesto, E. N. Saridakis, Phys.Rev. D87 (2013) 083507 [arXiv:1212.3611 [hep-th]].
5. Black holes in an ultraviolet complete quantum gravity, L. Modesto, J. W. Moffat, P. Nicolini Phys.Lett. B695 (2011) 397-400
6. Towards a finite quantum supergravity, L.M., Arxiv.
7. Towards singularity and ghost free theories of gravity, T. Biswas, E. Gerwick, T. Koivisto, A. Mazumdar, Phys.Rev.Lett. 108 (2012).
8. Modified non-local gravity, A. S. Koshelev [arXiv:1112.6410].

Outline

- Non-polynomial Higher Derivative Gravity.
- Super-renormalizable & Unitary (no ghost) Quantum Gravity.
- Finite Quantum Gravity.
- Quantum Supergravity & M-Theory.
- Super-renormamizable Completion of the Starobinsky Theory.
- Non-local Massive Gravity.

Applications and implications

- Regular (multi-horizon-)black holes.

  J. Moffat, L.M., P. Nicolini.

- Terminating black holes.

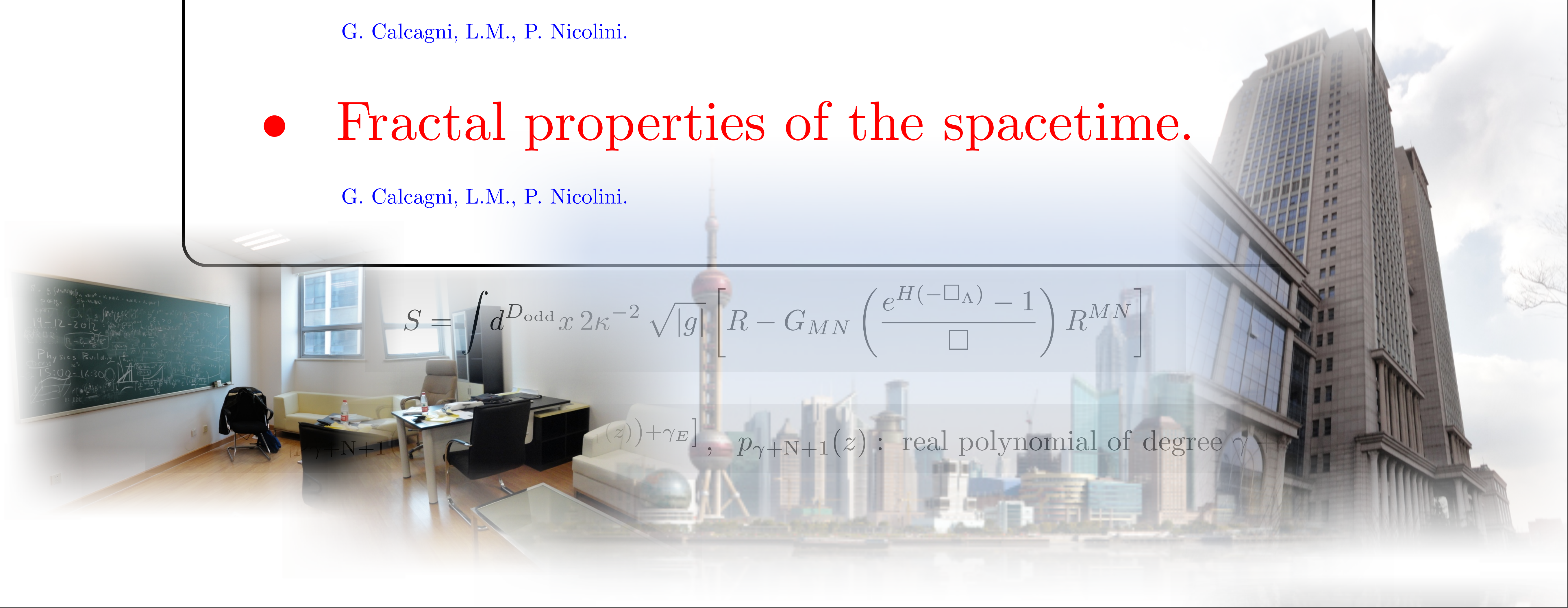
  C. Bambi, D. Malafarina, L.M..

- Super-accelerating bouncing cosmology.

  G. Calcagni, L.M., P. Nicolini.

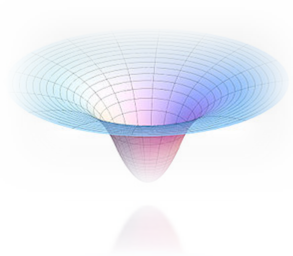
- Fractal properties of the spacetime.

  G. Calcagni, L.M., P. Nicolini.



Einstein Gravity completion

Classical Gravity



- General covariance ,
- $\nabla^a G_{ab} = 0$
- GRAVITY = CURVATURE ,
- low energy limit $(M_P \to +\infty) \;\to\;$ Einstein's gravity,

G. R.

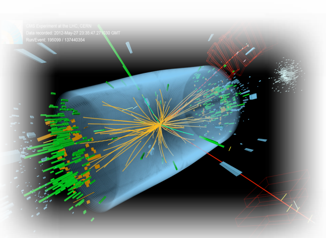
- Second order differential equations,
- General covariance,
- $\nabla^a T_{ab} = 0$,
- GRAVITY = CURVATURE .

Guiding Principle: Desingularization

- non singular classical solutions. Example: black holes $(r \sim 0)$:

$$ds^2 \approx -(1 - r^\alpha)dt^2 + \frac{dr^2}{1 - r^\alpha} + r^2 d\Omega^{(2)} \;,\; \alpha > 0.$$

Quantum Gravity



- Renormalizability and/or super-renormalizability,

$$G(p) \to \frac{1}{p^n} \,,\;\; n \geqslant 4 \;\text{ in the UV,}$$

- Unitary quantum theory (ghosts free),

$$G(p) \propto \frac{f(p)}{p^2} \;,\;\; f(p) \text{ does not have poles.}$$

- Decreasing of the "spacetime dimension" at small distances :

$$d_s \leqslant 2.$$

Classical Guiding Principle

Desingularization

Quantum Guiding Principle

Super-renormalizability or Finiteness

Complete Quantum Gravity

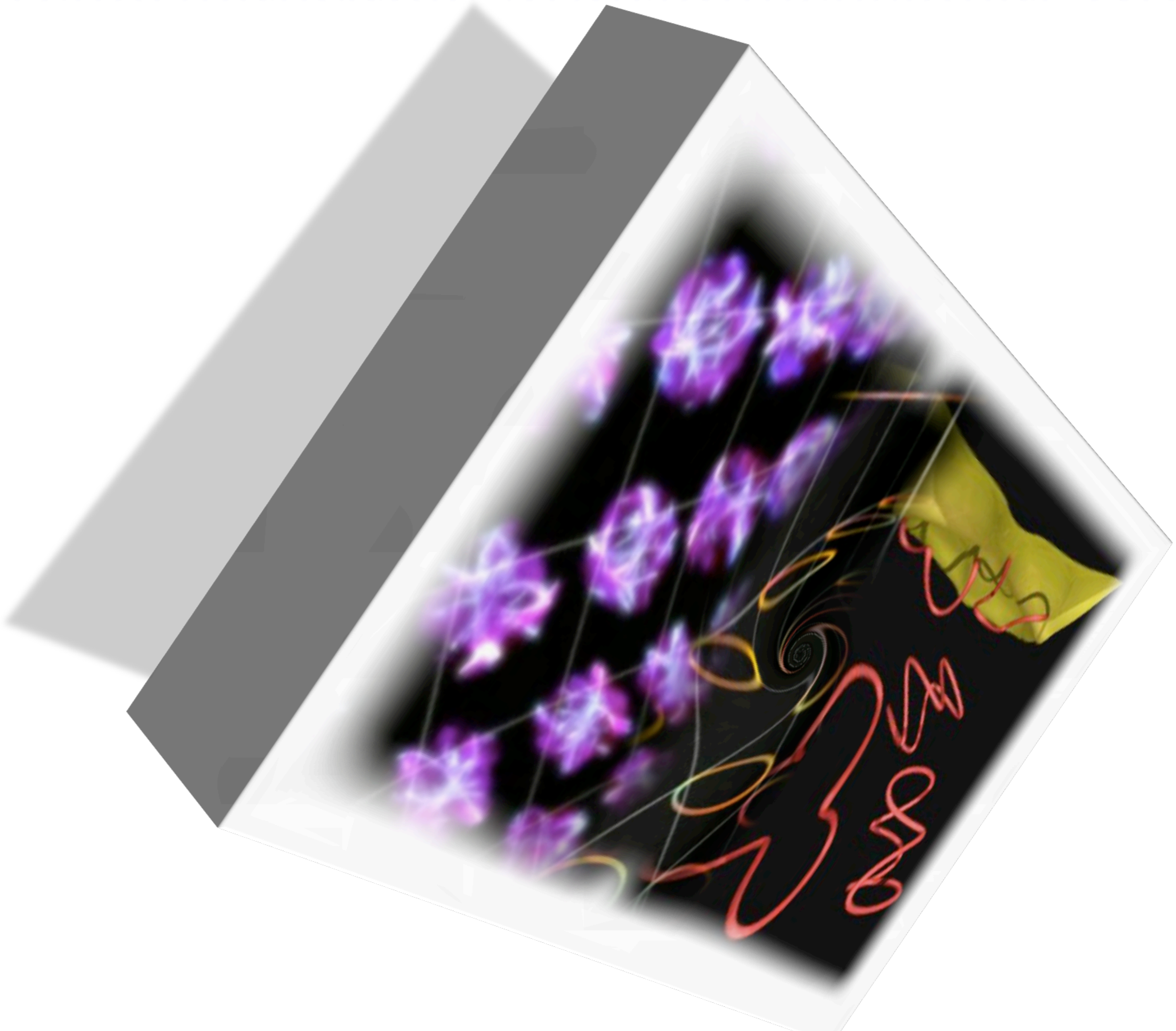
Stelle Theory

$$S = \int d^4x \sqrt{-g} \left[\frac{2}{\kappa^2} R + \frac{\alpha}{2} R^2 + \frac{\beta}{2} R_{\mu\nu} R^{\mu\nu} \right]$$

Krasnikov, Tomboulis, M.

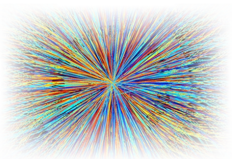
$$S = \int d^4x \sqrt{-g} \left[\frac{2}{\kappa^2} R + \frac{1}{2} R \, \alpha(-\Box_\Lambda) R + \frac{1}{2} R_{\mu\nu} \, \beta(-\Box_\Lambda) R_{\mu\nu} \right]$$

Super-renormalizable Multidimensional Gravity

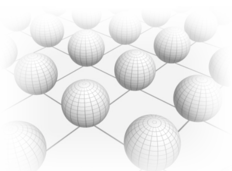


$$\mathcal{L} = R + R_{\mu\nu}\gamma_2(\Box)R^{\mu\nu} + R\,\gamma_0(\Box)R + R_{\mu\nu\rho\sigma}\gamma_4(\Box)R^{\mu\nu\rho\sigma} + \underbrace{O(R^3)\ldots\ldots\cdots + R^{D/2}}_{\text{finite number of terms}}\,.$$

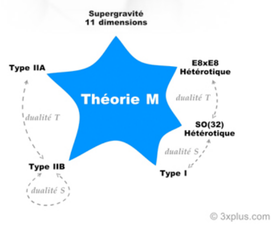
Motivations for Extra-dimensions



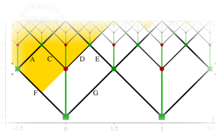
1. Finite quantum (super)gravity;



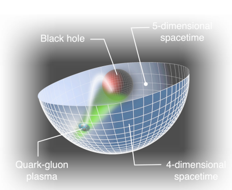
2. well defined "Kaluza-Klein grand-unification";



3. Completion of 11-dimensional supergravity and "M-Theory";



4. universality of the quantization procedure;



5. gauge/gravity correspondence, ADS/CFT.

Multidimensional Renormalizable Gravity

L.M. (2012).

$$\mathcal{L}_{D-\mathrm{Ren}} = a_1 R + a_2 R^2 + b_2 R_{\mu\nu}^2 + \cdots + a_X R^{X/2} + b_X R_{\mu\nu}^{X/2} + c_X R_{\mu\nu\rho\sigma}^{X/2} + d_X R \Box^{\frac{X}{2}-2} R \ldots$$

For $X = D$ this theory is renormalizable but not unitary.

The Action

L.M. (2012).

$$S = \int d^D x \sqrt{|g|} \Big[2\,\kappa^{-2} R + \bar{\lambda} + \sum_{n=0}^{N} \Big(a_n\, R\, (-\Box_\Lambda)^n\, R + b_n\, R_{\mu\nu}\, (-\Box_\Lambda)^n\, R^{\mu\nu} \Big) + R\, h_0(-\Box_\Lambda)\, R + R_{\mu\nu}\, h_2(-\Box_\Lambda)\, R^{\mu\nu} \Big] + \underbrace{O(R^3) \cdots + R^{N+2}}_{\text{Finite number of terms},} \,.$$

$$N = \frac{D-4}{2} \,.$$

$$h_2(z) = \frac{V(z)^{-1} - 1 - \frac{\kappa^2 \Lambda^2}{2}\, z \sum_{n=0}^{N} \tilde{b}_n\, z^n}{\frac{\kappa^2 \Lambda^2}{2}\, z} \,,$$

$$h_0(z) = -\frac{V(z)^{-1} - 1 + \kappa^2 \Lambda^2\, z \sum_{n=0}^{N} \tilde{a}_n\, z^n}{\kappa^2 \Lambda^2\, z} \,.$$

The Lagrangian

N. V. Krasnikov (1988), A.T. Tomboulis (1997), L.M. (2011)

$$\mathcal{L} = \sqrt{|g|}\Bigg[2\kappa^{-2}\left(R - G_{\mu\nu}\,\frac{V^{-1}(-\Box_\Lambda) - 1}{\Box}\,R^{\mu\nu}\right) + c_1^{(1)} R^3 + \cdots + c_1^{(\mathrm{N})} R^{\mathrm{N}+2}$$
$$+ \sum_{n=0}^{\mathrm{N}}\Big((a_n - \tilde{a}_n)\,R\,(-\Box_\Lambda)^n\,R + (b_n - \tilde{b}_n)\,R_{\mu\nu}\,(-\Box_\Lambda)^n\,R^{\mu\nu}\Bigg]$$

Coupling constants :

$$\alpha_i \in \{\kappa_D^{-2}, \bar{\lambda}, a_n, b_n, c_1^{(1)}, \cdots, c_1^{(\mathrm{N})}\} \equiv \{\kappa_D^{-2}, \bar{\lambda}, \tilde{\alpha}_n\}.$$

The Lagrangian

N. V. Krasnikov (1988), A.T. Tomboulis (1997), L.M. (2011)

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Coupling constants :

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$$D = 4$$

$$\mathcal{L} = \sqrt{|g|}\left[2\kappa^{-2}\left(R - G_{\mu\nu}\,\frac{V^{-1}(-\Box_\Lambda) - 1}{\Box}\,R^{\mu\nu}\right) + (a - \tilde{a})\,R^2 + (b - \tilde{b})\,R_{\mu\nu}R^{\mu\nu}\right)\right]$$

$$D = 4$$

$$\mathcal{L} = \sqrt{|g|}\;\; 2\kappa^{-2} \left(R - G_{\mu\nu} \frac{V^{-1}(-\Box_\Lambda) - 1}{\Box} R^{\mu\nu} \right)$$

Transcendental Entire functions $h_i(z)$

1. $V(z)^{-1}$ is real and positive on the real axis and it has no zeroes on the whole complex plane $|z| < +\infty$. This requirement implies that there are no gauge-invariant poles other then the transverse massless physical graviton pole.

2. There exists $\Theta > 0$ such that

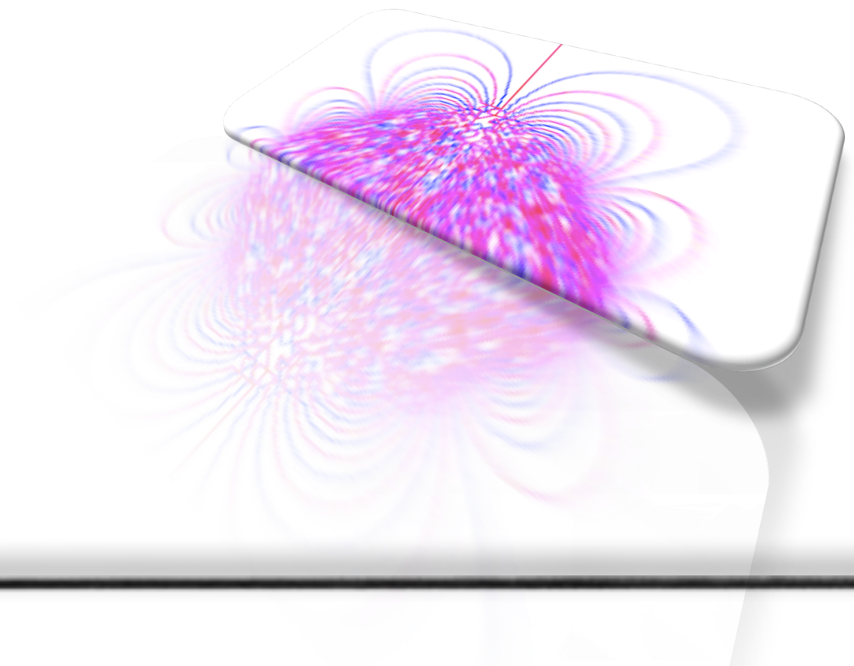
$$\lim_{|z|\to+\infty} |h_i(z)| \to |z|^{\gamma+\mathrm{N}} \quad \text{or} \quad \lim_{|z|\to+\infty} |V(z)^{-1}| \propto |z|^{\gamma+\mathrm{N}+1} \,,$$

$$\gamma \geqslant D/2 \ \text{ for } \ D = D_{\mathrm{even}} \ \text{ and } \ \gamma \geqslant (D-1)/2 \ \text{ for } \ D = D_{\mathrm{odd}} \,,$$

for the argument of z in the following conical regions

$$C = \Big\{ z \,|\, -\Theta < \arg z < +\Theta \,,\ \pi - \Theta < \arg z < \pi + \Theta \Big\}, \ \text{ for } \ 0 < \Theta < \pi/2 \,.$$

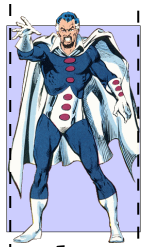
The Form Factor



$$V^{-1}(z) = \left|p_{\gamma+\mathrm{N}+1}\right| \, e^{\frac{1}{2}\left[\Gamma\left(0, p^2_{\gamma+\mathrm{N}+1}(z)\right)+\gamma_E\right]},$$

$$\lim_{z\to+\infty} e^{\frac{1}{2}\left[\Gamma\left(0, p^2_{\gamma+\mathrm{N}+1}(z)\right)+\gamma_E\right]} = e^{\frac{1}{2}\gamma_E}\,,$$

$$\lim_{z\to+\infty}\left(\frac{V(z)^{-1}}{e^{\frac{\gamma_E}{2}}\,|z|^{\gamma+\mathrm{N}+1}} - 1\right) z^n = 0 \quad \forall n \in \mathbb{N}\,.$$



$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$

Propagator and Unitarity

$$\mathcal{O}^{-1}(k) = \frac{P^{(2)}}{k^2\left[1 + \frac{k^2\kappa^2}{2}\left(\sum_{n=0}^{N} b_n(-\Box_\Lambda)^n + h_2(-\Box_\Lambda)\right)\right]}$$

$$-\frac{P^{(0)}}{2k^2\left\{\frac{D-2}{2} - k^2\kappa^2\left[\frac{D}{4}\left(\sum_{n=0}^{N} b_n(-\Box_\Lambda)^n + h_2(-\Box_\Lambda)\right) + (D-1)\left(\sum_{n=0}^{N} a_n(-\Box_\Lambda)^n + h_0(-\Box_\Lambda)\right)\right]\right\}}.$$

We assume that the theory is renormalized at some scale μ_0 and we set :

$$\tilde{a}_n = a_n(\mu_0)\ , \quad \tilde{b}_n = b_n(\mu_0),$$

$$\mathcal{O}^{-1}(k) = \frac{V(k^2/\Lambda^2)}{k^2}\left(P^{(2)} - \frac{P^{(0)}}{D-2}\right).$$

Coincidence limit : $\mathcal{O}^{-1}(x,y)_{\mathrm{SRQG}} \propto |x-y|^{2\gamma+2N}$ in the UV ,

$$\mathcal{O}^{-1}(x,y)_{\mathrm{Einstein}} \propto \frac{1}{|x-y|^2}.$$

Power Counting

Propagator

$$G(k) \sim \frac{1}{k^{2\gamma+2N+4}} \,.$$

Leading Interactions

$$\mathcal{L}_{\rm int} \sim R_{\dots} h_i(-\Box) R_{\dots} \quad \rightarrow \quad k^2 \, k^{2\gamma+2N} \, k^2 \,.$$

$$\mathcal{A}^{(L)} \sim \int (d^D k)^L \left(\frac{1}{k^{2\gamma+2\mathrm{N}+4}} \right)^I \left(k^{2\gamma+2\mathrm{N}+4}\right)^V$$
$$= \int (dk)^{DL} \left(\frac{1}{k^{2\gamma+2\mathrm{N}+4}} \right)^{L-1} \,.$$

Entire function :

$$\lim_{|z|\to+\infty} |h_i(z)| \to |z|^{\gamma+N},$$
$$\gamma \geqslant D/2 \quad \text{for} \quad D = D_{\rm even} \,,$$
$$\gamma \geqslant (D-1)/2 \quad \text{for} \quad D = D_{\rm odd} \,,$$

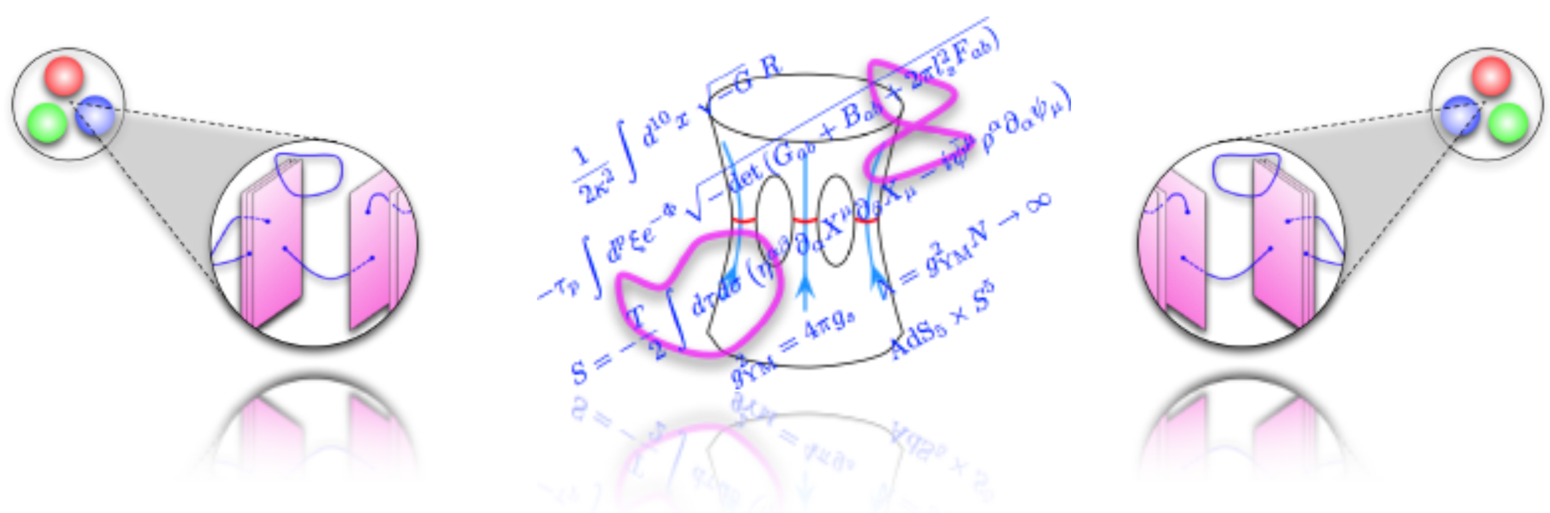
$$V(z) := e^{-H(z)}$$
$$H(z) = \frac{1}{2}\left[\gamma_E + \Gamma\left(0, z^{2\gamma+2N+2}\right)\right] + \log|z^{\gamma+N+1}| \,.$$

Power counting renormalizability :

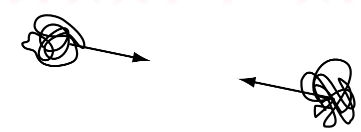
$$\mathcal{O}^{-1}(k) \sim 1/k^{2\gamma+2N+4} \quad \text{for large} \quad k^2 \,,$$

$$\begin{aligned}\delta_{\rm even} &= D_{\rm even} L - (2\gamma+2N+4)I + (2\gamma+2N+4)V \\ &= D_{\rm even} L - (2\gamma+D_{\rm even})I + (2\gamma+D_{\rm even})V \\ &= D_{\rm even} - 2\gamma(L-1).\end{aligned}$$
$$\delta_{\rm odd} = D_{\rm odd} - (2\gamma+1)(L-1) \,.$$

String Field Theory



String field theory effective action



$$S_{\rm SFT} = \int d^D x \left(\frac{1}{2} \phi_i K_{ij}(\Box) \phi_j - v_{ijk} \tilde{\phi}_i \tilde{\phi}_j \tilde{\phi}_k \right) .$$

$$\text{Field redefinition}: \quad \tilde{\phi}_i \equiv e^{\alpha' \frac{\ln(3 \frac{\sqrt{3}}{4})}{2} \Box} \phi_i := e^{\tilde{\alpha}' \Box} \phi_i \,,$$

$$S_{\rm SFT} = \int d^D x \Big(\frac{1}{2} \phi_i \, \Box \, e^{-\tilde{\alpha}' \Box} \phi_i - v_{ijk} \phi_i \, \phi_j \, \phi_k \Big) .$$

$$\text{Propagator}: \quad D(k) \propto \frac{e^{-\tilde{\alpha}' k^2}}{k^2} .$$

Stringy Renormalizable Gravity

(Krasnikov 1988)

$$V(z) = e^{-(-z)^n} \quad n = 1 \implies V(-\Box_\Lambda) = e^{-\Box/\Lambda^2} \, ,$$

$$\mathcal{O}^{-1}(k) = \frac{e^{-(k^2/\Lambda^2)^n}}{k^2} \, ,$$

$$\mathcal{A}^{(L)} \leqslant \int (d^D k)^L \left(\frac{e^{-k^{2n}/\Lambda^{2n}}}{k^2} \right)^I \left(e^{k^{2n}/\Lambda^{2n}} k^2 \right)^V$$

$$= \int (dk)^{DL} \left(\frac{e^{-k^{2n}/\Lambda^{2n}}}{k^2} \right)^{I-V} \longleftarrow I = V + L - 1$$

$$= \int (dk)^{DL} \left(\frac{e^{-k^{2n}/\Lambda^{2n}}}{k^2} \right)^{L-1} .$$

$A(L)$ in convergent for $L > 1$.

Unitarity

$$\Downarrow$$

Form factor $V(z)$.

Origin of the Form Factor

- Underling extended objects
  (spectral dimension can be a useful tool (G. Calcagni, L.M..)

- Wetterich-Reuter renormalization group equation.

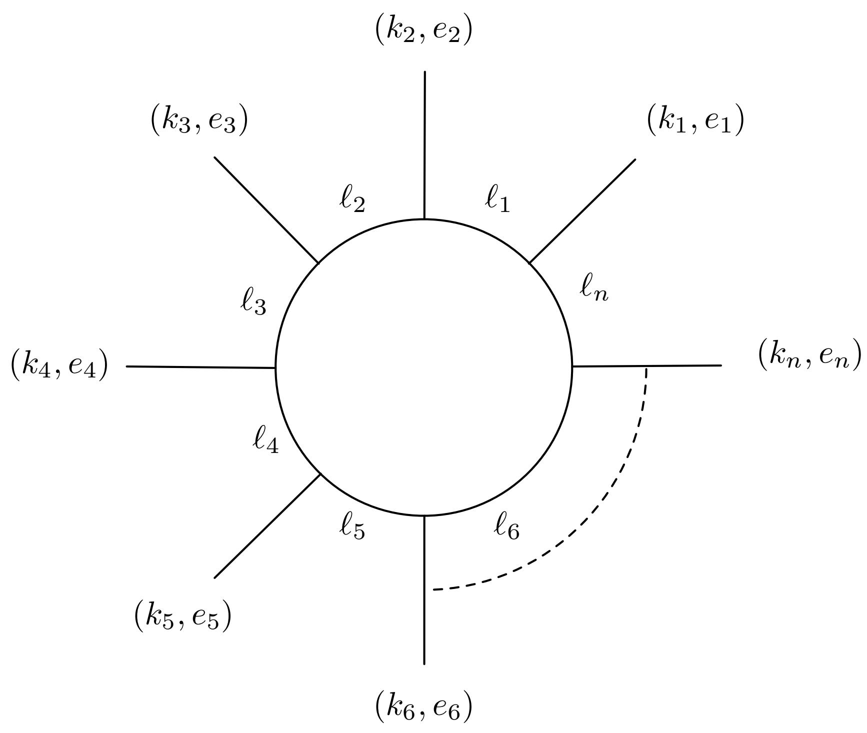
Renormalization

Vertexes

$$\text{set 1.} \quad R,\, R^2,\, R^3, \ldots,\, R^{\mathrm{N}+2} \quad \sim \quad h^n(\partial^2 h),\, h^n(\partial^2 h)^2,\, h^n(\partial^2 h)^3, \ldots, h^n(\partial^2 h)^{\mathrm{N}+2}\,,$$

$$\text{set 2.} \quad R_{\ldots}\, h_i(-\Box_\Lambda) R_{\ldots} \quad \sim \quad h^n(\partial^2 h) h_i(-\Box_\Lambda)\, h^m\,(\partial^2 h) = h^n(\partial^2 h) \left[\sum_{r=0}^{+\infty} c_r(-\Box_\Lambda)^r \right] h^m\,(\partial^2 h)$$

One loop amplitudes



$$A^{(L=1)} \sim \int d^D\ell\, \frac{\mathcal{V}_1(\ell_1, k_1, e_1)\, \mathcal{V}_2(\ell_2, k_2, e_2) \ldots \mathcal{V}_n(\ell_n, k_n, e_n)}{\ell_1^2\, V^{-1}(\ell_1)\, \ell_2^2\, V^{-1}(\ell_2) \ldots\, \ell_n^2\, V^{-1}(\ell_n)}\,,$$

$$\text{divergence}\ : \quad k^{2\mathrm{N}+4} \int d^D\ell\, \frac{1}{\ell^{\mathrm{N}+2}(\ell - k)^{\mathrm{N}+2}} \sim \frac{1}{\epsilon} k^{2\mathrm{N}+4}\,,$$

$$\text{counterterms}\ : \quad \frac{1}{\epsilon}\, R_{\ldots}^{\mathrm{N}+2}\,, \quad \frac{1}{\epsilon} R_{\ldots} \Box^{\mathrm{N}} R_{\ldots}\,, \quad \frac{1}{\epsilon} R_{\ldots} R_{\ldots} \Box^{\mathrm{N}-1} R_{\ldots}\ \ , \ldots\,.$$

$$\alpha_i \in \{\kappa_D^{-2}, \bar{\lambda}, a_n, b_n, c_1^{(1)}, \cdots, c_1^{(\mathrm{N})}\} \equiv \{\kappa_D^{-2}, \bar{\lambda}, \tilde{\alpha}_n\}.$$

$$\begin{aligned}\mathcal{L}_{\mathrm{ren}} &= \mathcal{L} + \mathcal{L}_{\mathrm{ct}} \\ &= \mathcal{L} + 2(Z_\kappa - 1)\kappa^{-2} R + (Z_{\bar{\lambda}} - 1)\bar{\lambda} + (Z_{c_1^{(1)}} - 1)c_1^{(1)} R^3 + \cdots + (Z_{c_1^{(\mathrm{N})}} - 1)c_1^{(\mathrm{N})} R^{\mathrm{N}+2} \\ &\quad + \sum_{n=0}^{\mathrm{N}} \Big((Z_{a_n} - 1)a_n R\,(-\Box_\Lambda)^n R + (Z_{b_n} - 1)b_n R_{MN}\,(-\Box_\Lambda)^n R^{MN} \Big),\end{aligned}$$

$$\mathcal{L}_{\mathrm{ct}} = \frac{1}{\epsilon}\left[\beta_\kappa R + \beta_{\bar{\lambda}} + \sum_{n=0}^{\mathrm{N}} \Big(\beta_{a_n} R\,(-\Box_\Lambda)^n R + \beta_{b_n} R_{MN}\,(-\Box_\Lambda)^n R^{MN}\Big) + \beta_{c_1^{(1)}} R^3 + \cdots + \beta_{c_1^{(\mathrm{N})}} R^{\mathrm{N}+2}\right],$$

$$\text{Beta functions :} \quad \beta_\kappa, \beta_{\bar{\lambda}}, \beta_{a_n}, \beta_{b_n}, \beta_{c_1^{(1)}}, \cdots, \beta_{c_1^{(\mathrm{N})}},$$

$$(Z_{\alpha_i} - 1)\alpha_i = \frac{1}{\epsilon}\beta_{\alpha_i} \implies Z_{\alpha_i} = 1 + \frac{1}{\epsilon}\beta_{\alpha_i}\frac{1}{\alpha_i},$$

$$\alpha_i(\mu) \sim \alpha_i(\mu_0) + \beta_i \log\left(\frac{\mu}{\mu_0}\right).$$

Yang-Mills & Gravity

Asymptotic Freedom

$$\mathcal{L}_{\rm YM} = \frac{1}{2g^2(t)}{\rm Tr}F^2 \sim dA\, dA + g(t)A^2 dA + g^2(t)A^4\,,$$

$$g(t)^{-2} = g_o^{-2} + \beta_g t\,,$$

$$\mathcal{L}_{\rm G} = \frac{2}{\kappa_D^2(t)}\left(R - G_{\mu\nu}\frac{e^{H(-\Box_\Lambda)}-1}{\Box}R^{\mu\nu}\right) \sim dh\, dh + \kappa_D\, hdh\, dh + O(\kappa_D^2)\,,$$

$$\kappa_D^{-2}(t) = \kappa_{D\,o}^{-2} + \beta_{\kappa_D} t\,.$$

All the beta-functions do not depend on the gauge fixing condition.

Quantum Corrections to the Propagator

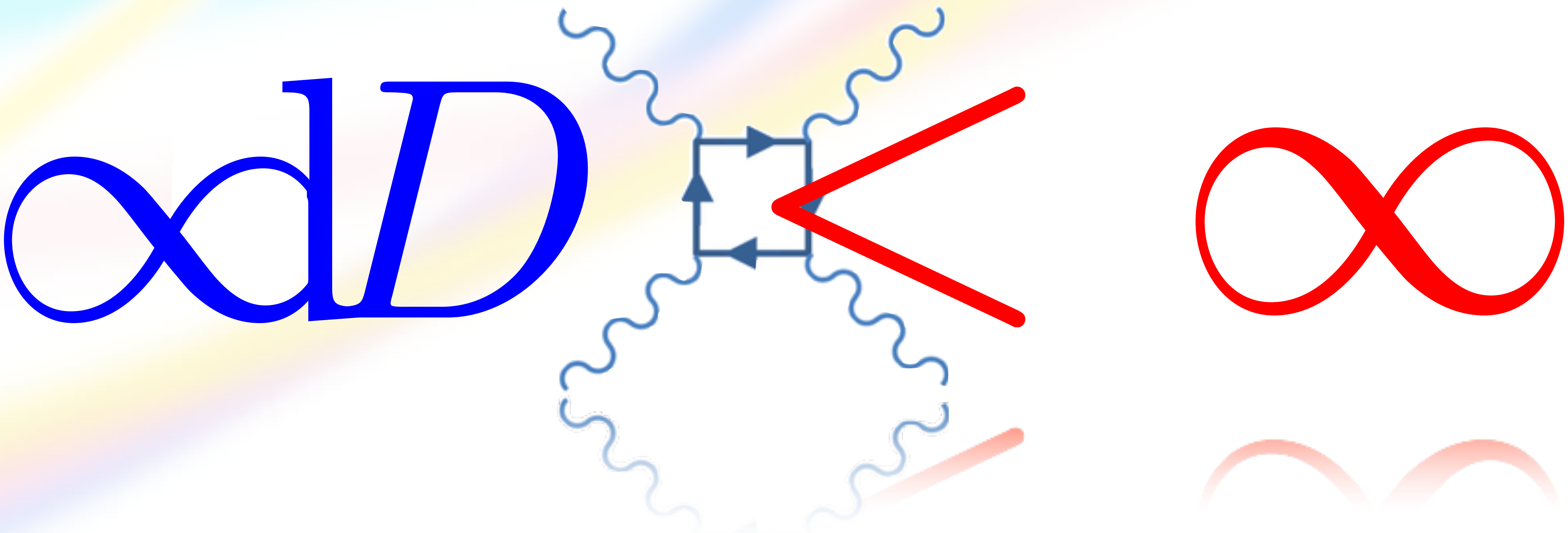
$$\mathcal{O}_R^{-1}(k) \sim \frac{V\left(k^2/\Lambda^2\right)}{k^2\left\{1 + V(k^2/\Lambda^2)\,\left[c_0 k^2 + \cdots + c_{\mathrm{N}} k^{2\mathrm{N}+2}\,\right]\,\log\left(\frac{k^2}{\mu^2}\right)\right\}}\,.$$

Could we make quantum gravity finite?

$$\beta_{\alpha_i} \equiv 0$$

- Supergravity ,
- Higher-dimensional gravity.

Finite Quantum Gravity



$1 - \text{Loop}$

No counter terms in Odd dimension

Finite Quantum Gravity in Odd dimension

$$\beta_i = 0$$

One loop effective action:

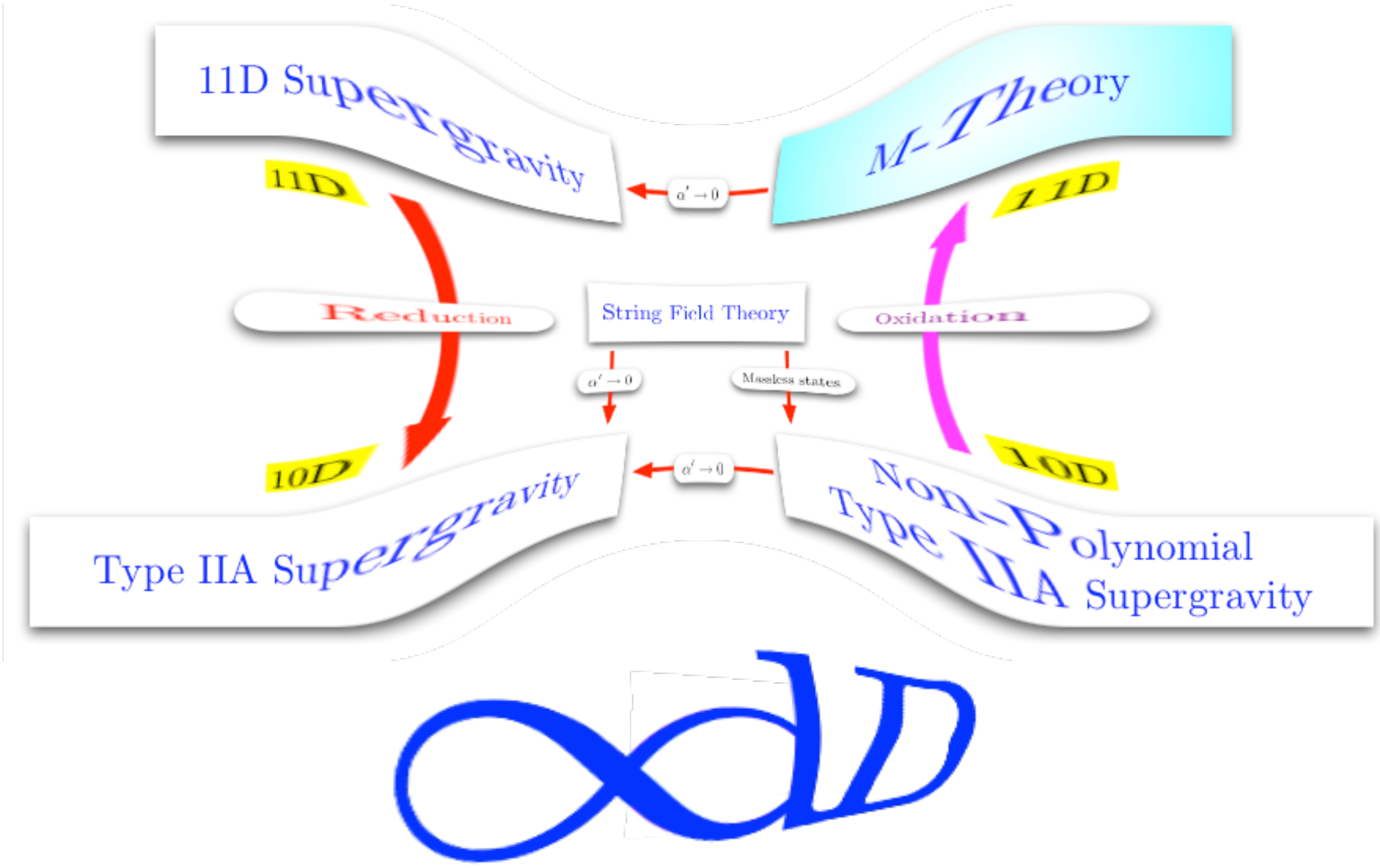
$$W_{\rm div} = -\frac{1}{\epsilon}\left[c_D\,A_D(\Delta) + c_{D-2}\,A_{D-2}(\Delta) + c_{D-4}\,A_{D-4}(\Delta) + \cdots + c_0\,A_0(\Delta)\right] = 0\,,$$

$$A_l(\Delta) \propto \int d^D x R^{l/2}_{\mu\nu\rho\sigma}.$$

$$\beta_{a_n} = \beta_{b_n} = \beta_{c_i^{(n)}} = 0\,, \quad i \in 1,\ldots,(\text{number} \quad \text{of} \quad \text{invariants of order } N)\,, \quad n = 1,\ldots,N.$$

$$\mathcal{L}_{\rm odd} = \sqrt{|g|}\,2\kappa^{-2}\Big(R - G_{\mu\nu}\,\gamma(\Box)\,R^{\mu\nu}\Big)\,, \quad \gamma(\Box) := \frac{V(-\Box_\Lambda)^{-1} - 1}{\Box}\,.$$

Quantum M-Theory



Non-polynomial Type IIA Supergravity

- General Covariance,
- String Field Theory Form Factor : $V^{-1}(\Box_\Lambda) = e^{-\Box_\Lambda}$,

$$S_{10}^{\text{SugraIIA}} = \boxed{\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\Bigg(R + G_{MN}\frac{e^{H(-\Box_\Lambda)}-1}{\Box}R^{MN}} + 4\partial_M\Phi\, e^{H(-\Box_\Lambda)}\,\partial^M\Phi - \frac{1}{2\,3!}H_{MNP}\,e^{H(-\Box_\Lambda)}\,H^{MNP}\Bigg)$$

$$+\frac{1}{4\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left(\frac{1}{2!}F_{MN}\,e^{H(-\Box_\Lambda)}\,F^{MN} + \frac{1}{4!}\tilde{F}_{MNPQ}\,e^{H(-\Box_\Lambda)}\,\tilde{F}^{MNPQ}\right) - \frac{1}{4\kappa_{10}^2}\int B_2\wedge F_4\wedge F_4 + \dots\,.$$

Finite Quantum Supergravity in 11D and 10D

G. Calcagni, L.M.

$$S_{11}^{\text{Finite Sugra}} = \frac{1}{2\kappa_{11}} \int d^{11}x \sqrt{-g} \left(R + G_{MN} \frac{e^{H(-\Box_\Lambda)} - 1}{\Box} R^{MN} - \frac{1}{4!} F_{MNPQ}\, e^{H(-\Box_\Lambda)}\, F^{MNPQ} \right)$$
$$- \frac{1}{12\kappa_{11}} \int A_3 \wedge F_4 \wedge F_4 + \dots .$$

Oxidation ↑ ↓ Redation

$$S_{10}^{\text{SugraIIA}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left(R + G_{MN} \frac{e^{H(-\Box_\Lambda)} - 1}{\Box} R^{MN} + 4\partial_M \Phi\, e^{H(-\Box_\Lambda)}\, \partial^M \Phi - \frac{1}{2\,3!} H_{MNP}\, e^{H(-\Box_\Lambda)}\, H^{MNP} \right)$$
$$+ \frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left(\frac{1}{2!} F_{MN}\, e^{H(-\Box_\Lambda)}\, F^{MN} + \frac{1}{4!} \tilde{F}_{MNPQ}\, e^{H(-\Box_\Lambda)}\, \tilde{F}^{MNPQ} \right) - \frac{1}{4\kappa_{10}^2} \int B_2 \wedge F_4 \wedge F_4 + \dots .$$

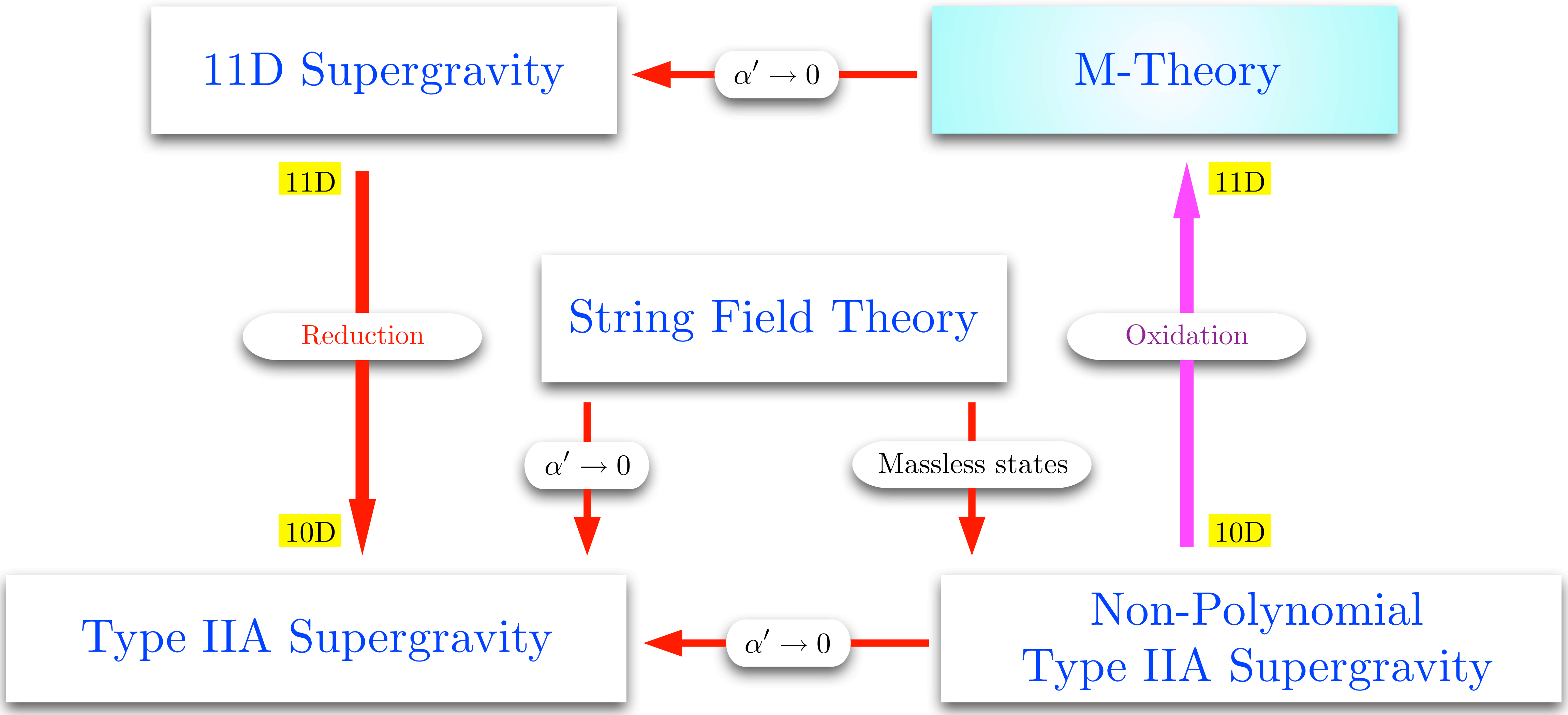
Check of the conjecture (Gross & Witten)

$$S_{\mathrm{IIA,IIB}} = \int d^{10}x \left[R + R^4_{\mu\nu\rho\sigma} \right]$$

$$S_{\mathrm{IIA,IIB}} = \int d^{10}x \left[R - G_{\mu\nu} R^{\mu\nu} + R^4_{\mu\nu\rho\sigma} \right]$$

$$\frac{e^{-\Box} - 1}{\Box} \approx -1 \,, \quad R + G_{\mu\nu} \frac{e^{-\Box} - 1}{\Box} R^{\mu\nu} \quad \to \quad R - G_{\mu\nu} R^{\mu\nu} \,.$$

Completion of 11-Dimensional Supergravity



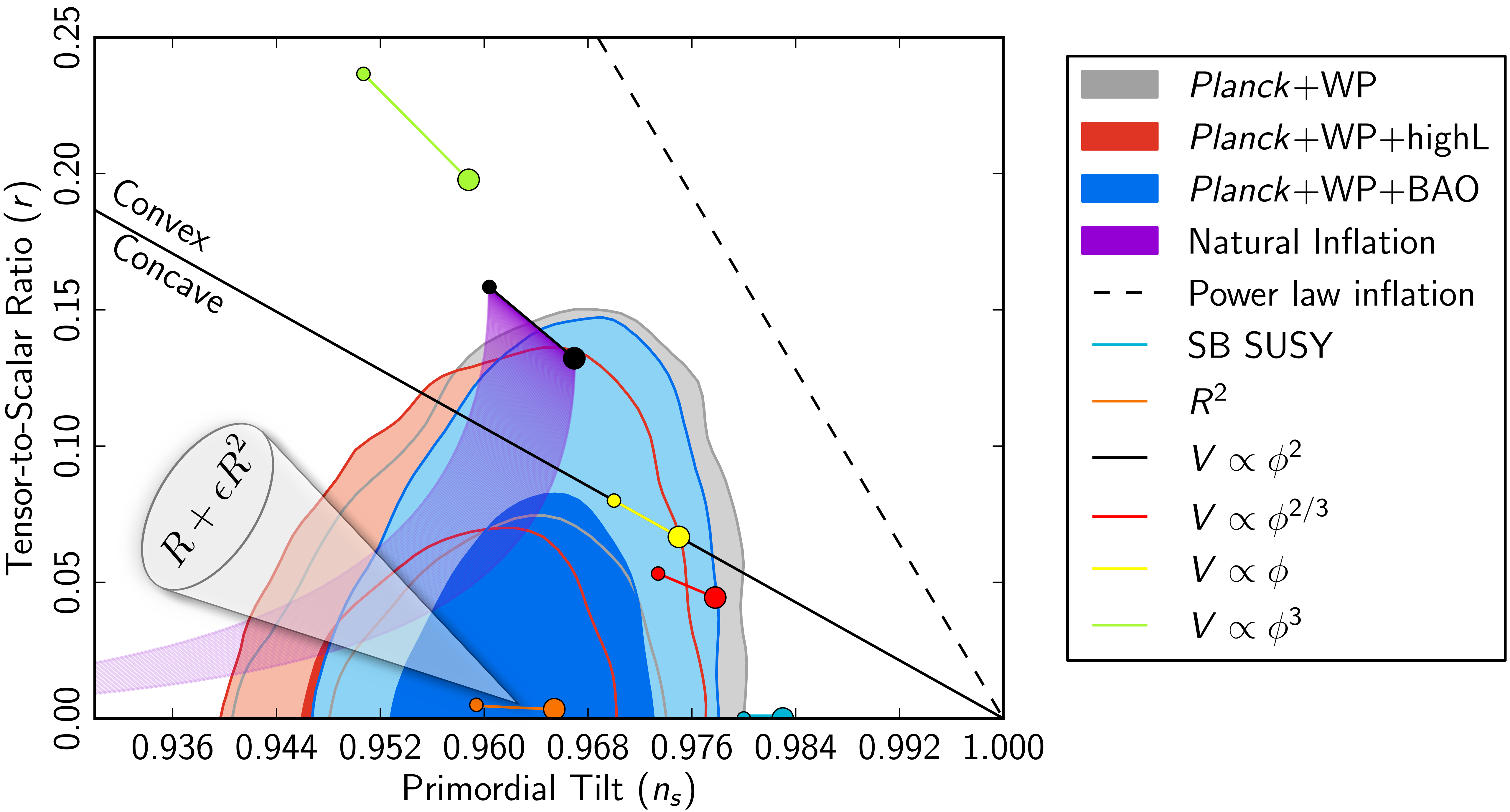
The **FIELD** Theory of Everything

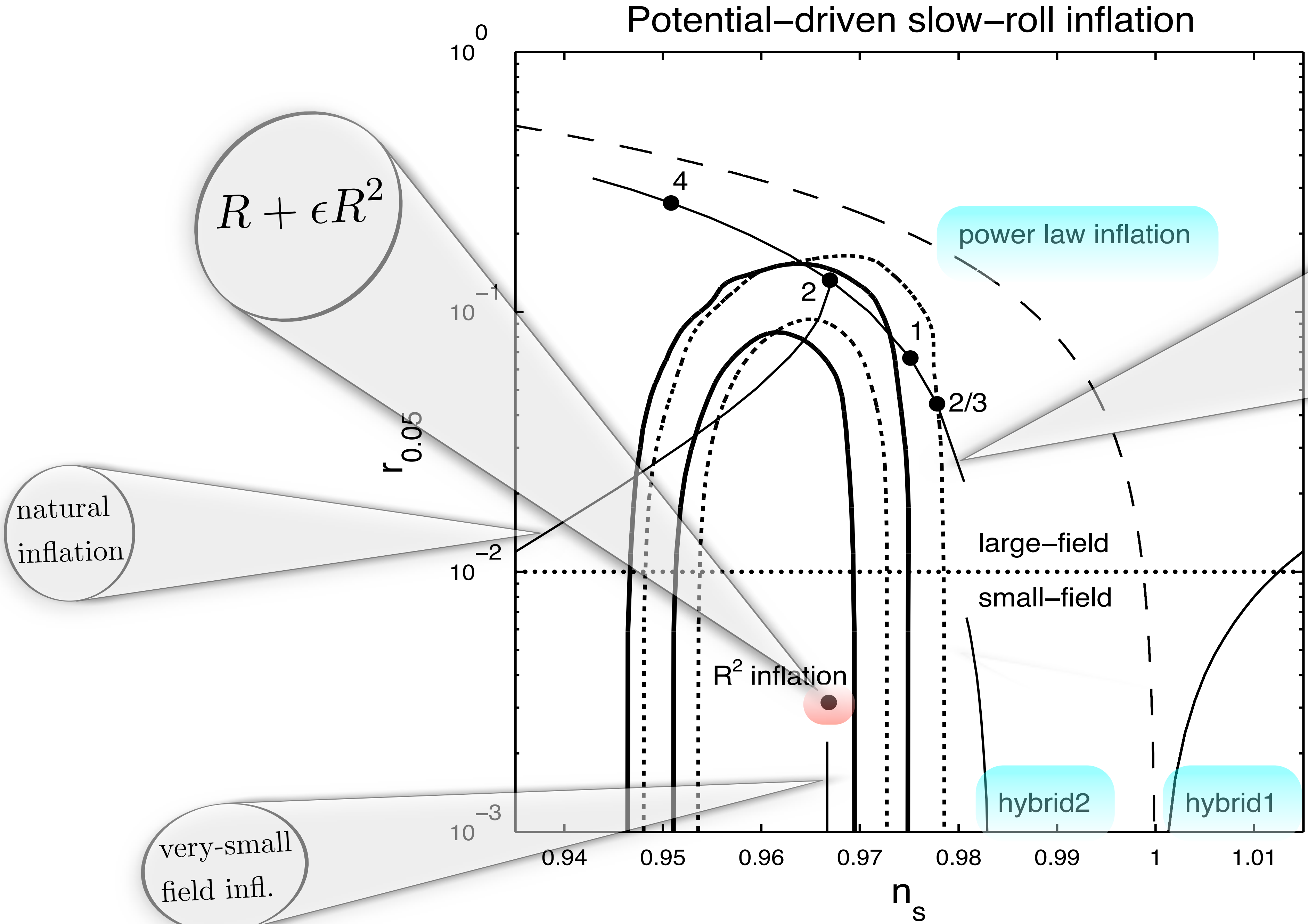
Inflation

Starobinsky Inflation

$$R + \epsilon R^2$$

Planck Results





Spectral index :

$$n_s - 1 \simeq \frac{8(4N_e + 9)}{(4N_e + 3)} .$$

Tensor-to-scalar ratio :

$$r = \frac{192}{(4N_e + 3)^2} ,$$

$$\frac{m}{M_{Pl}} \simeq 1.5 \times 10^{-5} \times \left(\frac{50}{N_e} \right) .$$

Shinji Tsujikawa, Junko Ohashi, Sachiko Kuroyanagi, Antonio De Felice

Figure 1: 2-dimensional observational constraints on potential-driven slow-roll inflation in the (n_s, r) plane with the number of e-foldings $N = 60$ and the pivot wave number $k_0 = 0.05\ \mathrm{Mpc}^{-1}$. The bold solid curves represent the 68 % CL (inside) and 95 % CL (outside) boundaries derived by the joint data analysis of Planck+WP+BAO+high-ℓ, whereas the dotted curves correspond to the 68 % and 95 % contours constrained by Planck+WP+BAO. In both cases the consistency relation $r(k_0) = -8n_t(k_0)$ is used. We show the theoretical predictions for the models: (i) chaotic inflation with the potential $V(\phi) = \lambda\phi^n/n$ for general n (thin solid curve) and for $n = 4, 2, 1, 2/3$ (denoted as black circles), (ii) natural inflation with the potential $V(\phi) = \Lambda^4[1 + \cos(\phi/f)]$ for general f, (iii) hybrid inflation with the potentials $V(\phi) = \Lambda^4 + m^2\phi^2/2$ ("hybrid1") and $V(\phi) = \Lambda^4[1 + c\ln(\phi/\mu)]$ ("hybrid2"), (iv) very small-field inflation with the potential $V(\phi) = \Lambda^4(1 - e^{-\phi/M})$ in the regime $M < M_{\mathrm{pl}}$, and (v) power-law inflation with the exponential potential $V(\phi) = V_0 e^{-\gamma\phi/M_{\mathrm{pl}}}$. The dotted line ($r = 10^{-2}$) corresponds to the boundary between "large-field" and "small-field" models. For comparison, we also show the theoretical prediction of the Starobinsky's model $f(R) = R + R^2/(6M^2)$ (denoted as "R^2 inflation").

Super-renormalizable or Finite completion of the Starobinsky theory

F. Briscese, L.M., S. Tsujikawa

- Lorentz invariance .
- At least quadratic in the curvature .
- Unitarity (no ghosts) .
- (Super-)renormalizable or finite ,
- Particle content: graviton & gravi-scalar .
- No energy conditions violation from the matter side .

Multidimensional Lagrangian

F. Briscese, L.M., S. Tsujikawa

$$S = \int d^D x \sqrt{|g|} \Big[2\,\kappa^{-2}\, R + \bar{\lambda} + \sum_{n=0}^{N} \Big(a_n\, R\, (-\Box_\Lambda)^n\, R + b_n\, R_{\mu\nu}\, (-\Box_\Lambda)^n\, R^{\mu\nu} \Big)$$

$$+ R\, h_0(-\Box_\Lambda)\, R + R_{\mu\nu}\, h_2(-\Box_\Lambda)\, R^{\mu\nu} \Big] + \underbrace{O(R^3) \cdots + R^{N+2}}_{\text{Finite number of terms}} .$$

$$h_2(z) = \frac{V_2(z)^{-1} - 1}{\frac{\kappa_D^2 \Lambda^2}{2}\, z} - \sum_{n=0}^{\mathrm{N}} \tilde{b}_n\, z^n ,$$

$$h_0(z) = -\frac{V_0(z)^{-1} - 1}{\kappa_D^2 \Lambda^2\, z} - \sum_{n=0}^{\mathrm{N}} \tilde{a}_n\, z^n .$$

$$V_2(z)^{-1} = e^{H_2(z)} ,$$

$$V_0(z)^{-1} = \frac{(D-2)}{2(D-1)} e^{H_0(z)} \left(1 - \frac{\Lambda^2 z}{m^2} \right) + \frac{D}{2(D-1)} e^{H_2(z)} .$$

$$2N + 4 = D_{\rm even} ,$$

$$2N + 4 = D_{\rm odd} + 1 .$$

Lagrangian

$$\mathcal{L} = \frac{2}{\kappa_D^2}\left(R - G_{\mu\nu}\frac{V_2^{-1} - 1}{\Box}R^{\mu\nu} + \frac{1}{2}R\frac{V_0^{-1} - V_2^{-1}}{\Box}R\right),$$

$$V_0^{-1} - V_2^{-1} = \frac{D-2}{2(D-1)}\left[e^{H_0}\left(1 + \frac{\Box}{m^2}\right) - e^{H_2}\right].$$

$$V_2(z)^{-1} = e^{H_2(z)},$$

$$V_0(z)^{-1} = \frac{(D-2)}{2(D-1)}e^{H_0(z)}\left(1 - \frac{\Lambda^2 z}{m^2}\right) + \frac{D}{2(D-1)}e^{H_2(z)}.$$

Propagator

$$\mathcal{O}^{-1}(k) = \frac{P^{(2)}}{k^2 e^{H_2}} + \frac{m^2 P^{(0)}}{k^2 e^{H_0}(k^2 - m^2)(D-2)}$$

$$= \frac{P^{(2)}}{k^2 e^{H_2}} - \frac{P^{(0)}}{(D-2)k^2 e^{H_0}} + \frac{P^{(0)}}{(D-2)e^{H_0}(k^2 - m^2)}\,.$$

$$e^{H_2} \;\rightarrow\; z^{\gamma_2+\mathrm{N}+1}\,,$$

$$e^{H_0}\left(1 - \frac{\Lambda^2 z}{m^2}\right) \;\rightarrow\; z^{\gamma_0+\mathrm{N}+2}\,,$$

$$H_i(z) = \frac{1}{2}\left[\gamma_E + \Gamma\left(0, p^2_{\gamma_i+\mathrm{N}+1}(z)\right) + \log\left(p^2_{\gamma_i+\mathrm{N}+1}(z)\right)\right]\,,$$

$$\gamma_2 = \gamma\,,$$

$$\gamma_0 = \gamma - 1\,.$$

Unitarity

M. Veltman, P.v. Nieuwenhuizen, A. Acciolly, A. Azeredo,
F.S. Bemfica, M. Gomes, B. Pereira-Dias, J.A. Helayël-Neto.

$$T^{\mu\nu} = ak^\mu k^\nu + b\tilde{k}^\mu \tilde{k}^\nu + c^{ij}\epsilon_i^{(\mu}\epsilon_j^{\nu)} + d\,k^{(\mu}\tilde{k}^{\nu)} + e^i k^{(\mu}\epsilon_i^{\nu)} + f^i \tilde{k}^{(\mu}\epsilon_i^{\nu)}.$$

$$A = g^2 T^{\mu\nu}\Big\{\frac{P^{(2)}_{\mu\nu,\rho\sigma}}{k^2 e^{H_2} p^{(n_2)}} - \frac{P^{(0)}_{\mu\nu,\rho\sigma}}{(D-2)k^2 e^{H_0} p^{(n_0)}}\Big\}T^{\rho\sigma}$$

$$= g^2\left\{\frac{T_{\mu\nu}T^{\mu\nu} - \frac{T^2}{D-1}}{k^2 e^{H_2}} - \frac{\frac{T^2}{(D-1)}}{(D-2)k^2 e^{H_0}\left(1-\frac{k^2}{m^2}\right)}\right\}.$$

Residue in $k^2 = 0$:

$$\mathrm{Res}A\Big|_{k^2=0} = g^2\left\{T_{\mu\nu}T^{\mu\nu} - \frac{T^2}{D-2}\right\}\Bigg|_{k^2=0} = g^2\left[(c^{ij})^2 - \frac{(c^{ii})^2}{D-2}\right]\Bigg|_{k^2=0} > 0 \quad \text{for} \quad D > 3.$$

Residue in $k^2 = m^2$:

$$\mathrm{Res}A\Big|_{k^2=m^2} = g^2\frac{T^2 e^{-H_0(m^2/\Lambda^2)}}{(D-1)(D-2)} > 0 \quad \text{for} \quad D > 2\,.$$

Lagrangian

$$\mathcal{L} = \frac{2}{\kappa_D^2}\left(R - G_{\mu\nu}\frac{V_2^{-1}-1}{\Box}R^{\mu\nu} + \frac{1}{2}R\frac{V_0^{-1}-V_2^{-1}}{\Box}R\right),$$

$$V_0^{-1} - V_2^{-1} = \frac{D-2}{2(D-1)}\left[e^{H_0}\left(1+\frac{\Box}{m^2}\right) - e^{H_2}\right].$$

Starobinsky Limit

$$\mathcal{L} = \frac{2}{\kappa_D^2}\left[R + \frac{(D-2)R^2}{4(D-1)m^2} + O\left(R\frac{\Box^{2\gamma+2\mathrm{N}-1}}{\Lambda^{4\gamma+4\mathrm{N}}}R\right)\right].$$

Starobinsky theory from String Theory

F. Briscese, A. Marciano', L.M., E. Saridakis.

- Lorentz invariance ,
- at least quadratic in the curvature ,
- unitarity (no ghosts) ,
- (super-)renormalizable or finite ,
- no energy conditions violation from the matter side.

$$\mathcal{L} = R - G^{\mu\nu} \left(\frac{V(\Box_\Lambda)^{-1} - 1}{\Box} \right) R_{\mu\nu} \;\; \rightarrow \;\; \mathcal{L} = R + \epsilon R^2$$

FRW

$$V(\Box_\Lambda) = e^{-\Box_\Lambda} \,.$$

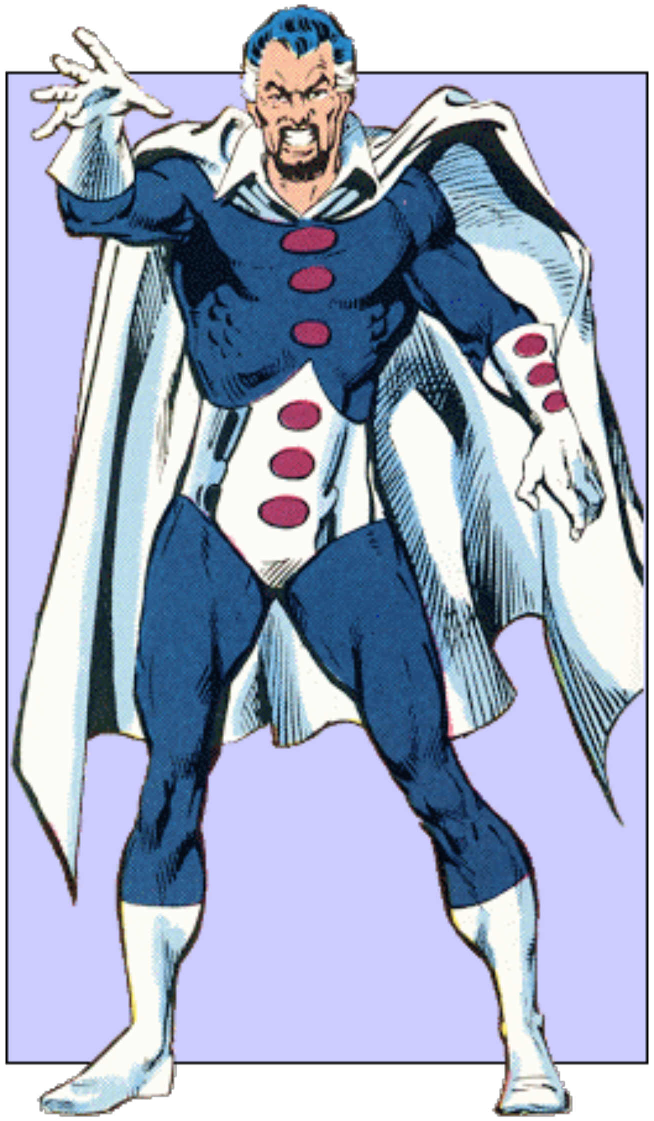
Therefore . . .

If you accept this theory as effective "string field theory"

↓

string theory is in excellent health

Massive Gravity



Lagrangian

M. Jaccard, M. Maggiore, E. Mitsou [arXiv:1305.3034 [hep-th]].

S. Tsujikawa, L.M.

$$\mathcal{L} = \frac{2}{\kappa^2}\sqrt{|g|}\left\{R - G_{\mu\nu}\left[\frac{(\Box + m^2)e^{H(-\Box_\Lambda)} - \Box}{\Box^2}\right]R^{\mu\nu}\right\},$$

$$e^{H(z)} = |p_{\gamma+\mathrm{N}+1}(z)|\; e^{\frac{1}{2}\left[\Gamma\left(0,p^2_{\gamma+\mathrm{N}+1}(z)\right)+\gamma_E\right]}.$$

Propagator

$$\mathcal{O}^{-1}(k) = \frac{e^{-H(k^2/\Lambda^2)}}{k^2 - m^2}\left(P^{(2)} - \frac{P^{(0)}}{(D-2)}\right).$$

Equations of Motion

M. Jaccard, M. Maggiore, E. Mitsou [arXiv:1305.3034 [hep-th]].

$$V^{-1}(\Box)\, G_{\mu\nu} + O(R^2) = 8\pi G_N\, T_{\mu\nu}\,,$$

or

$$G_{\mu\nu} + O(R^2) = 8\pi G_N\, \frac{\Box}{\Box + m^2}\, e^{-H(-\Box_\Lambda)}\, T_{\mu\nu}\,.$$

Invariance : $\quad T_{\mu\nu} \to T_{\mu\nu} + (\text{const.})\, g_{\mu\nu}\,,$

This theory realizes the Afshordi-Smolin idea.

Non-local Gravity from Macro to Micro

S. Tsujikawa, L.M.

$$\mathcal{L} = \frac{2}{\kappa^2}\sqrt{|g|}\left\{R - G_{\mu\nu}\left[\frac{(\Box + m^2)e^{H(-\Box_\Lambda)} - \Box}{\Box^2}\right]R^{\mu\nu}\right\},$$

$$e^{H(z)} = |p_{\gamma+\mathrm{N}+1}(z)|\; e^{\frac{1}{2}\left[\Gamma\left(0, p^2_{\gamma+\mathrm{N}+1}(z)\right)+\gamma_E\right]}.$$

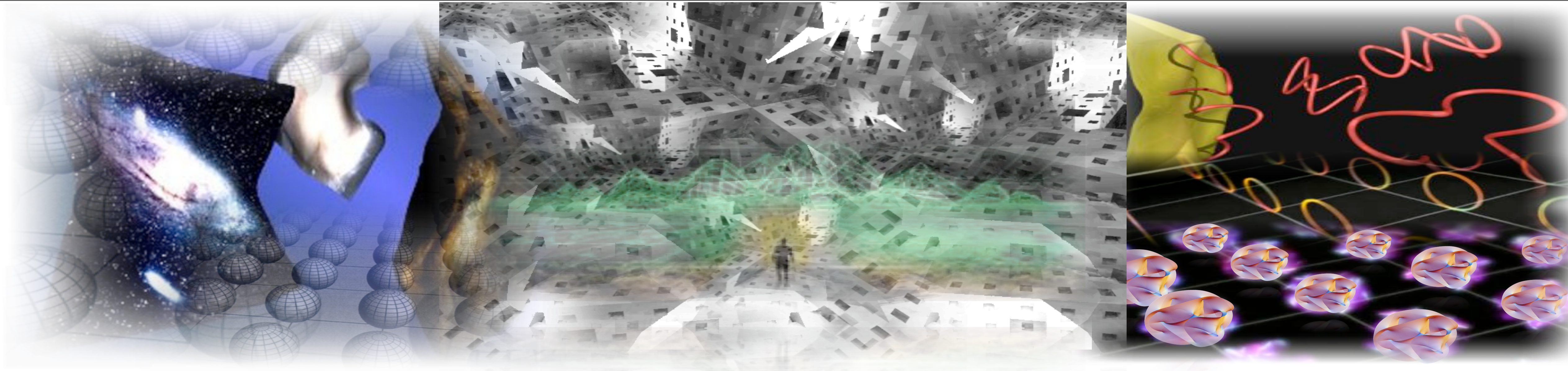
Summary and Conclusions

Gravity Theory

$$S = \int d^{D_{\rm odd}}x \, 2\kappa^{-2} \sqrt{|g|} \left[R - G_{MN} \left(\frac{e^{H(-\Box_\Lambda)} - 1}{\Box} \right) R^{MN} \right]$$

$$e^{H(z)} = |p_{\gamma+\mathrm{N}+1}(z)| \, e^{\frac{1}{2}\left[\Gamma\left(0, p^2_{\gamma+\mathrm{N}+1}(z)\right) + \gamma_E\right]} , \quad p_{\gamma+\mathrm{N}+1}(z) : \text{ real polynomial of degree } \gamma + \mathrm{N} + 1 .$$

- Non-polynomial Higher Derivative Gravity,
- Super-renormalizable & Unitary (no ghosts) Quantum Gravity.
- Finite Quantum Gravity.
- Quantum Supergravity & M-Theory.
- Super-renormalizable completion of the Starobinsky theory.
- New Non-local Massive Gravity.



- Regular (multi-horizon-)black holes,
- Gravitational collapse and terminating black holes,
- Bouncing cosmology,
- Starobinsky inflation,
- Fractal properties of the spacetime,
- Infrared non-polynomial modifications.

F∞ur

Non-relativistic Quantum Gravity

Non-polynomial Action

$$S = \frac{1}{\kappa^2} \int dt d^3x N \sqrt{g} \left[K_{ab}\, \alpha(\Delta) K^{ab} + K \beta(\Delta) K + a(\Delta) R - 2\lambda \right].$$

One example

$$S = \frac{1}{\kappa^2} \int dt d^3x \sqrt{g} N \left[\left(K_{ab}\, e^{\Delta_\Lambda} K^{ab} - K e^{\Delta_\Lambda} K + e^{\Delta_\Lambda} R \right) - 2\lambda \right]$$

$$\mathcal{O}^{-1} = \frac{(2P^2 - P^0)}{-p^2 e^{-k^2/\Lambda^2} + 2\lambda} = \frac{(2P^2 - P^0)}{(\omega^2 - k^2) e^{-k^2/\Lambda^2} + 2\lambda}$$

$$\mathcal{O}^{-1} \approx \frac{2P^2 - P^0}{2\lambda} \left(1 + \frac{p^2 e^{-k^2}}{2\lambda} \dots \right) \longrightarrow$$

No logarithmic divergences.

$$\mathcal{I}_{k,n} = \int d^D p \frac{(p^2)^k}{(p^2 + C)^n} = i \frac{C^{\frac{D}{2} - (n-k)}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(n - k - D/2)\Gamma(k + D/2)}{\Gamma(D/2)\Gamma(n)},$$

$$\mathcal{I}_{\text{null}} = \int d^D p \frac{1}{p^{2N}} \equiv 0 \;\text{ for }\; N < D/2\,.$$

Summary and Conclusions

Gravity Theory

$$S = \int d^{D_{\rm odd}}x \, 2\kappa^{-2} \sqrt{|g|} \left[R - G_{MN} \left(\frac{e^{H(-\Box_\Lambda)} - 1}{\Box} \right) R^{MN} \right]$$

$$e^{H(z)} = |p_{\gamma+{\rm N}+1}(z)| \, e^{\frac{1}{2}\left[\Gamma\left(0, p^2_{\gamma+{\rm N}+1}(z)\right) + \gamma_E\right]} , \quad p_{\gamma+{\rm N}+1}(z) : \text{ real polynomial of degree } \gamma + {\rm N} + 1 \, .$$

- Non-polynomial Higher Derivative Gravity,
- Super-renormalizable & Unitary (no ghosts) Quantum Gravity.
- Finite Quantum Gravity.
- Quantum Supergravity & M-Theory.
- Super-renormalizable completion of the Starobinsky theory.
- New Non-local Massive Gravity.

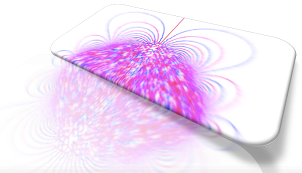
Applications

Black Holes

Modified Einstein Equations

$$\mathcal{L} = \sqrt{|g|}\Big\{2\kappa^{-2}\left[R - G_{\mu\nu}\left(\frac{e^{H(-\Box_\Lambda)}-1}{\Box}\right)R^{\mu\nu}\right],$$

$$G_{\mu\nu} + O(R^2) = 8\pi G_N\, e^{-H(-\Box_\Lambda)}\, T_{\mu\nu}.$$



Multi-horizons Black Holes

$$G_{\mu\nu} = 8\pi G_N \, e^{-H(-\Box_\Lambda)} \, T_{\mu\nu},$$
$$\nabla^\mu (e^{-H} T_{\mu\nu}) = 0 \, .$$

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2\Omega^2,$$
$$F(r) = 1 - \frac{2m(r)}{r}.$$

$$\rho^{\mathrm{e}}(\vec{x}) := -e^{-H(-\Box_\Lambda)} T^0{}_0 = m \, e^{-H(-\Box_\Lambda)} \, \delta(\vec{x}),$$
$$m(r) = 4\pi \int_0^r dr' r'^2 \;\rho^{\mathrm{e}}(r').$$

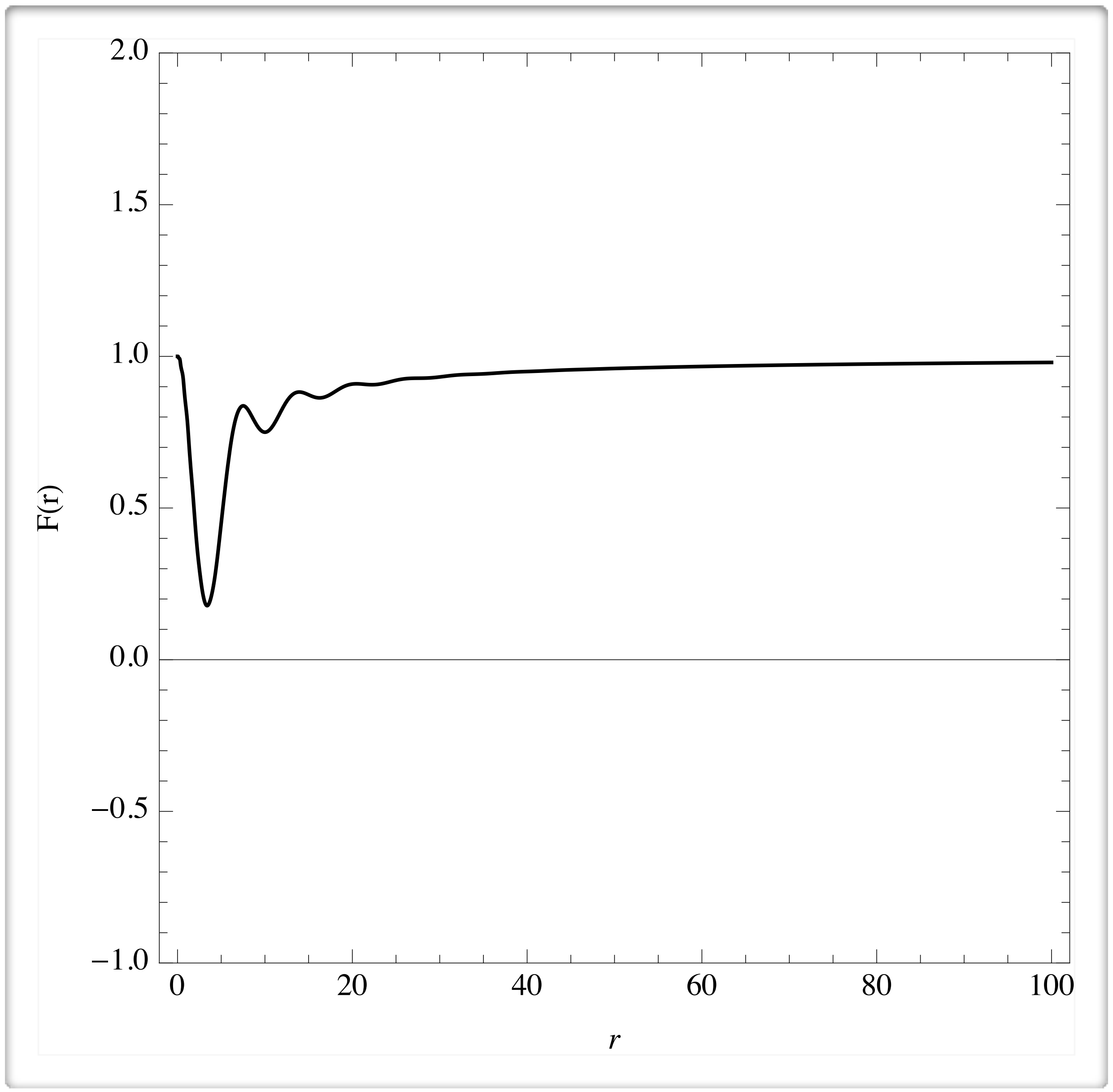
For $r \approx 0$, $F(r) \approx 1 - c\, m\, \Lambda^{2\gamma+2} \, r^{2\gamma+1}$.

$$R = c\, m\, \Lambda^{2\gamma+2} \, (2\gamma+2)(2\gamma+3) \, r^{2\gamma-1},$$
$$R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} =$$
$$= 4\, c^2 \, m^2 \, \Lambda^{4\gamma+4} \, \left(4\gamma^4 + 4\gamma^3 + 5\gamma^2 + 4\gamma + 2\right) r^{4\gamma-2} \, .$$

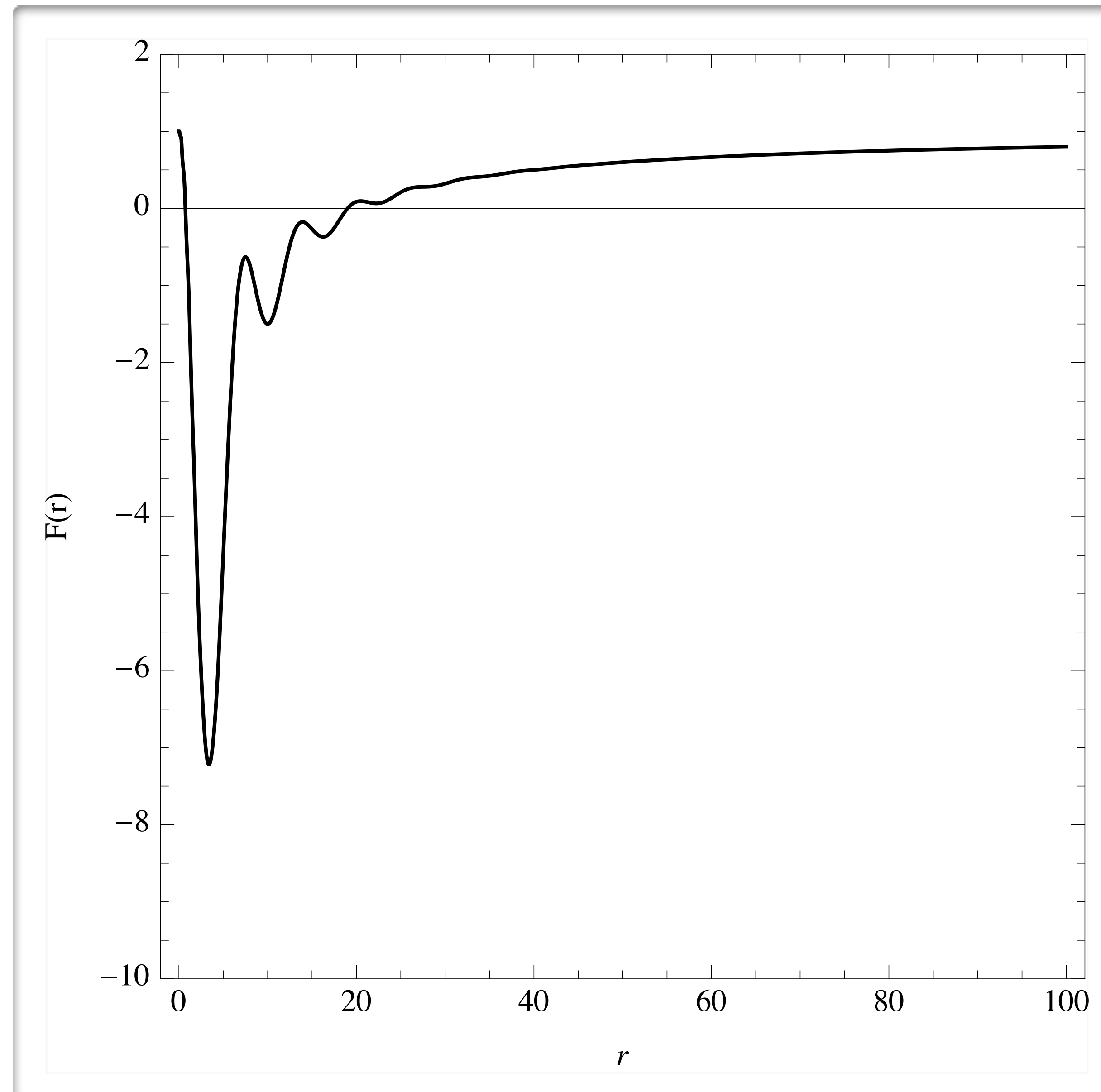
$\gamma \geqslant 3 \;\rightarrow\;$ No Singularity.

Exact Solution

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2 d\Omega^{(2)}.$$

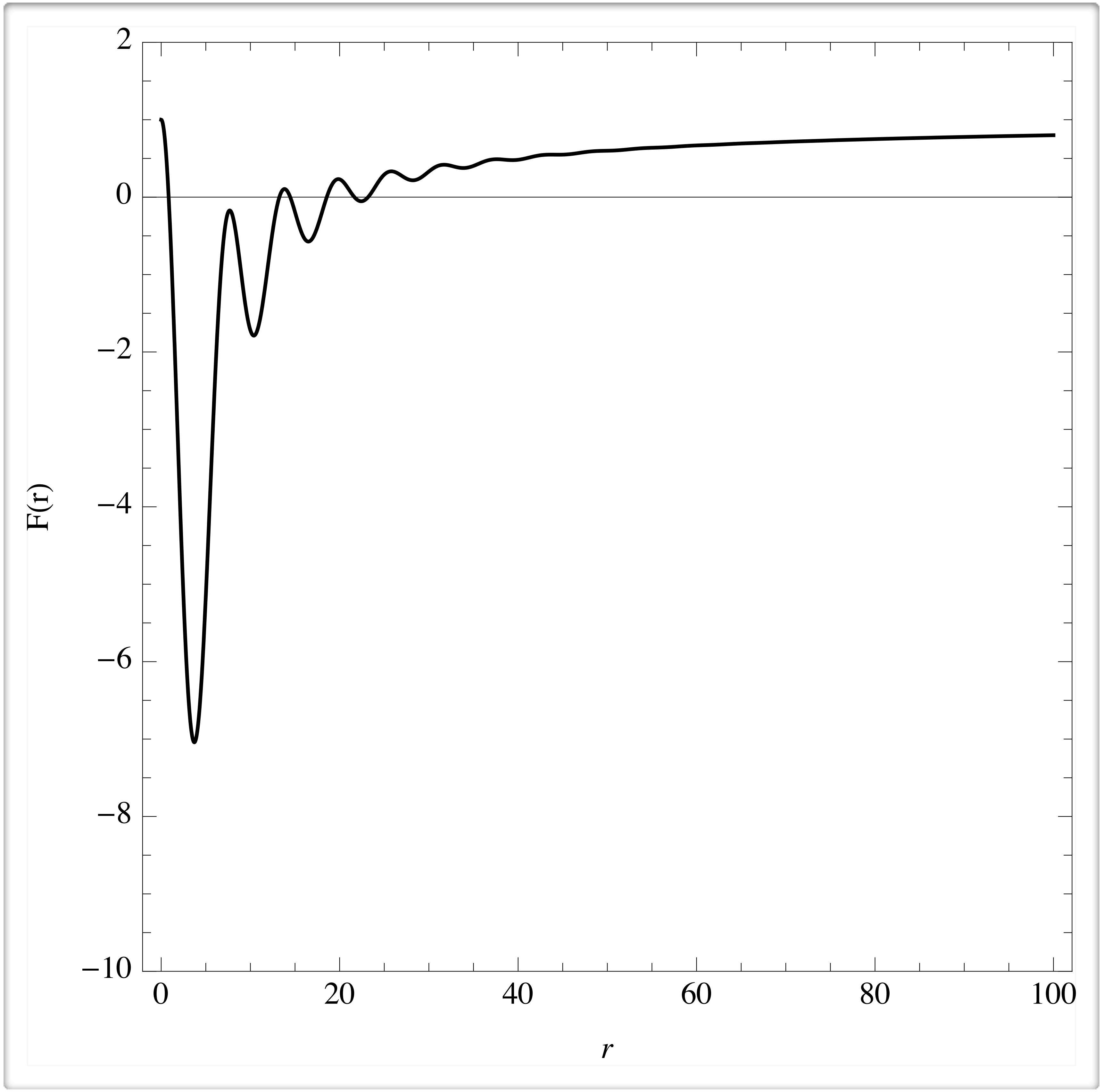


$\Lambda = M_P \ , \quad m = M_P \, .$

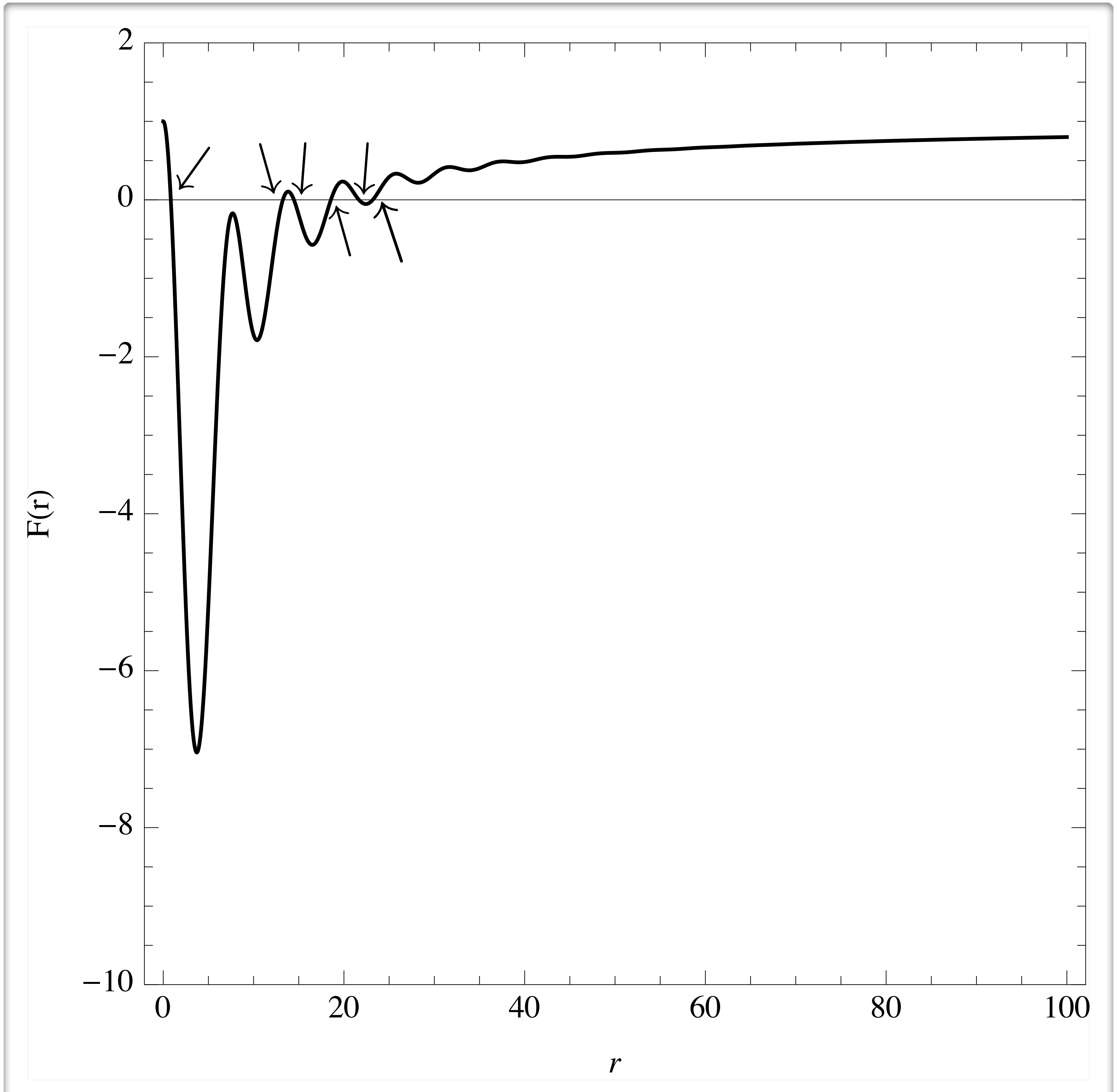


$\Lambda = M_P \ , \quad m = 10\, M_P \, .$

Multi-horizons Black Holes



Multi-horizons Black Holes



Singularity-free Black Holes in String Field Theory

P. Nicolini, L.M.

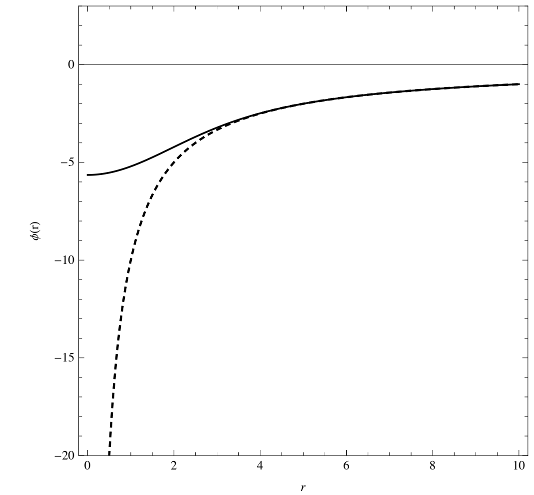
Theory II

Spacetime Structure

Gravitational Potential

$$h_{00} = -8\pi G_N \int d^4y\, G(x,y) T_{00}\,,$$

$$\phi(r) = -\frac{GM}{r}\,\mathrm{Erf}\left(\frac{\Lambda r}{2}\right).$$

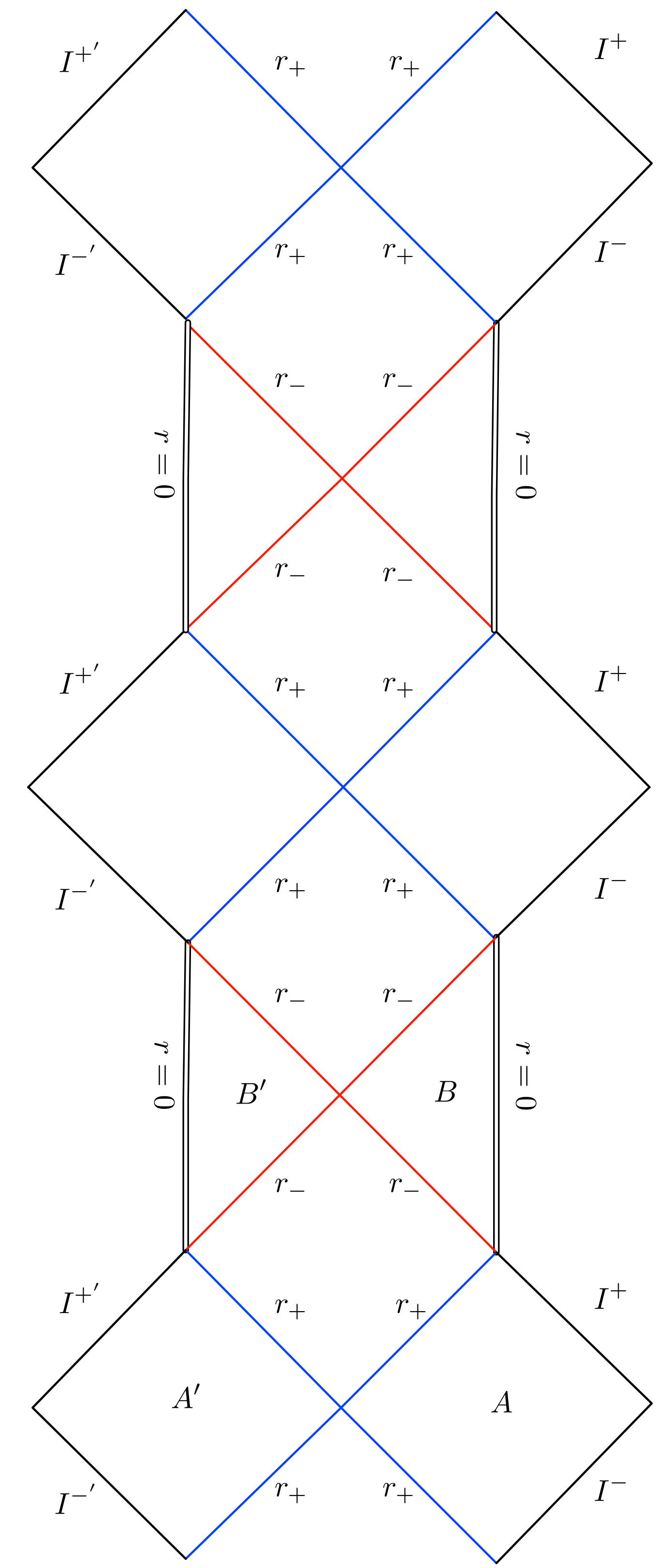


Black Holes

L.M. , J. Moffat, P. Nicolini.

$$ds^2 = -\left(1-\frac{2m(r)}{r}\right)dt^2 + \frac{dr^2}{\left(1-\frac{2m(r)}{r}\right)} + r^2 d\Omega^{(2)}\,,$$

$$m(r) = M\left[1 - \frac{\Gamma\left(3/2; r^2\Lambda^2/4\right)}{\Gamma(3/2)}\right].$$

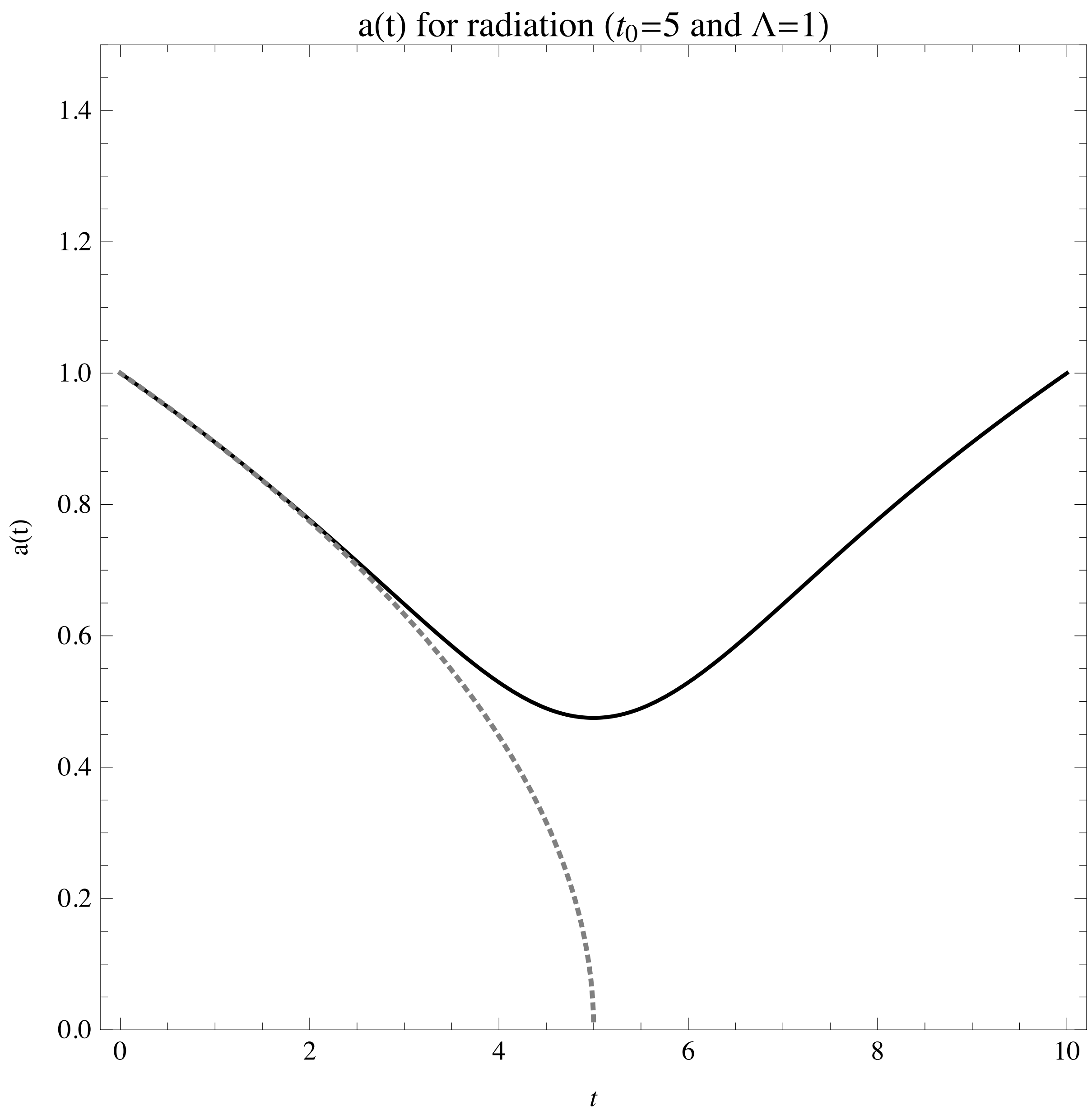


Terminating Black Holes

C. Bambi, D. Malafarina, L.M.

From the propagator & asymptotic freedom : $D(k) \sim \dfrac{V(k)}{k^2}\,.$

Radiation : $$a^2(t) = \frac{2e^{-\frac{1}{4}\Lambda^2(t-t_0)^2}}{\Lambda\sqrt{\pi}\,t_0} + \frac{(-t+t_0)\,\mathrm{erf}\left(\frac{\Lambda(-t+t_0)}{2}\right)}{t_0}.$$



The Calculation

G. Calcagni, L. M., P. Nicolini.

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}\,,$$
$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a(t)^2\, \delta_{ij} dx^i dx^j\,,$$
$$a^2(t) = 1 - \kappa h(t)\,, \qquad h(t = t_{\rm i}) = 0\,, \qquad g_{\mu\nu}(t = t_{\rm i}) = \eta_{\mu\nu}\,,$$
$$h_{\mu\nu}(t) = h(t)\,\mathrm{diag}(0, \delta_{ij}) =: h(t)\,\mathcal{I}_{\mu\nu}\,;$$

$$\bar{h}_{\mu\nu} := h'_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\, h'^{\lambda}_{\lambda} = h(t)\,\mathrm{diag}(0, -2\delta_{ij}) = -2h(t)\mathcal{I}_{\mu\nu}\,, \qquad \partial^\mu \bar{h}_{\mu\nu} = 0\,,$$

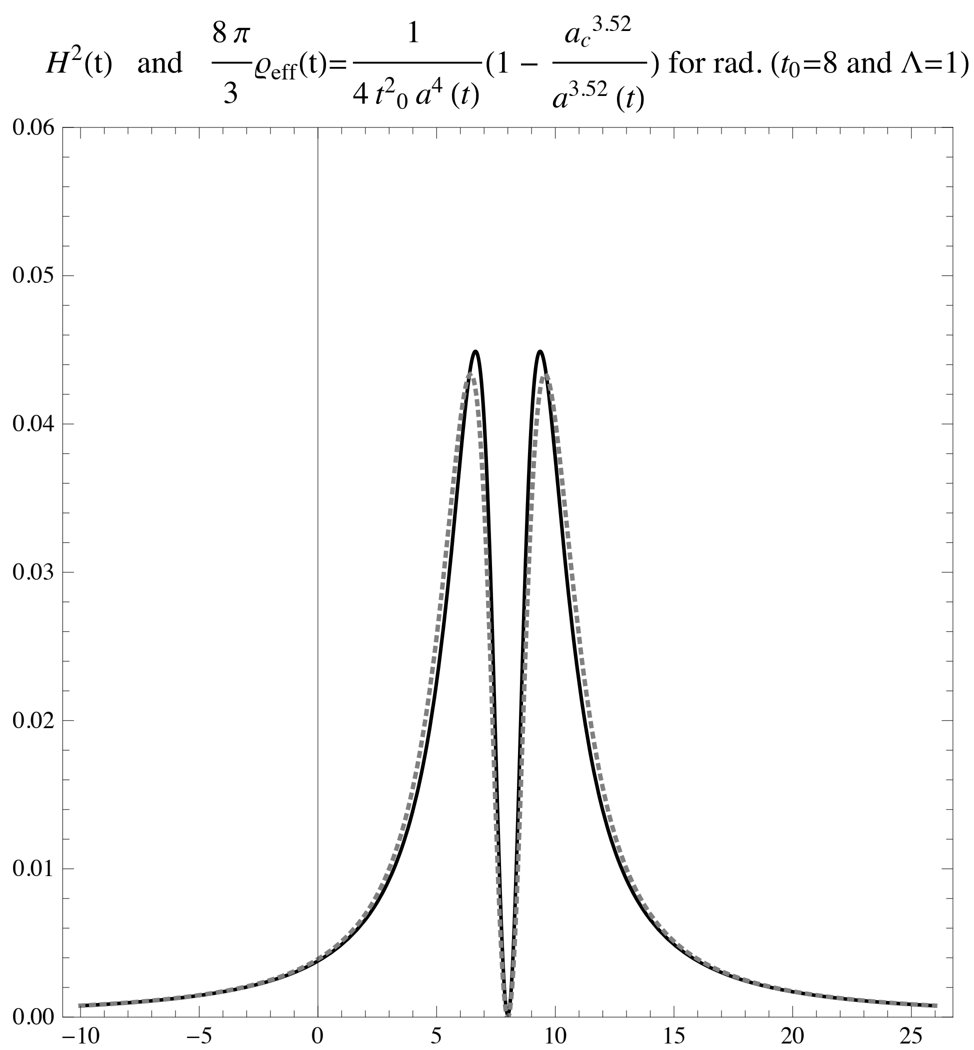
$$\tilde{\bar{h}}_{\mu\nu}(E, \vec{k}) = -2\tilde{h}(E)(2\pi)^3 \delta(\vec{k})\,\mathcal{I}_{\mu\nu}\,.$$

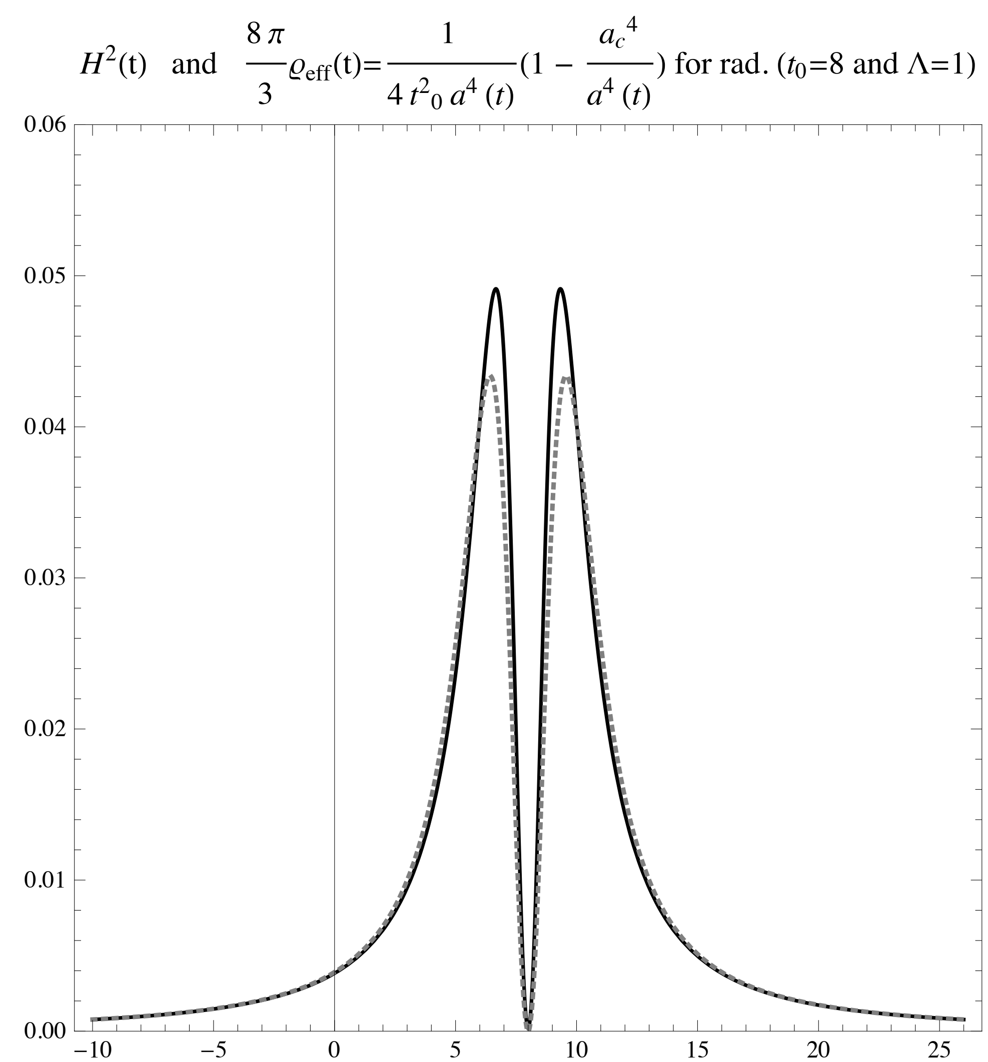
$$O^{-1}(k) = \frac{V(k^2)}{k^2}\left(P^{(2)} - \frac{P^{(0)}}{2}\right) \quad \Longrightarrow \quad \bar{h}_{\mu\nu}(x) = \kappa \int \frac{d^4k}{(2\pi)^4} O^{-1}_{\mu\nu,\rho\sigma}(k)\tilde{T}^{\rho\sigma}(k)\, e^{-ik\cdot x}\,,$$

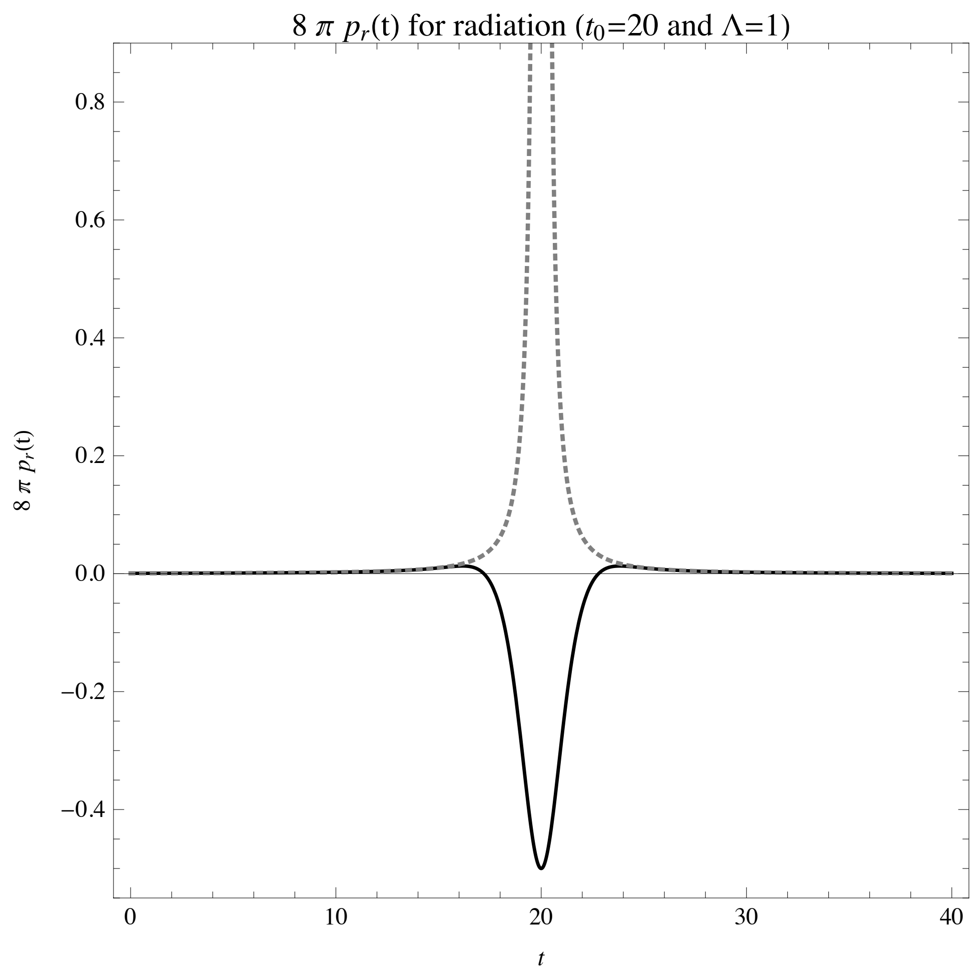
$$h(t) = -\frac{\kappa}{2}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 V^{-1}(k^2)}\,\tilde{\rho}(E, \vec{k})\, e^{-ik\cdot x} = \int \frac{dE}{2\pi}\,\tilde{h}(E)\, e^{iEt}\,.$$

Effective Theory

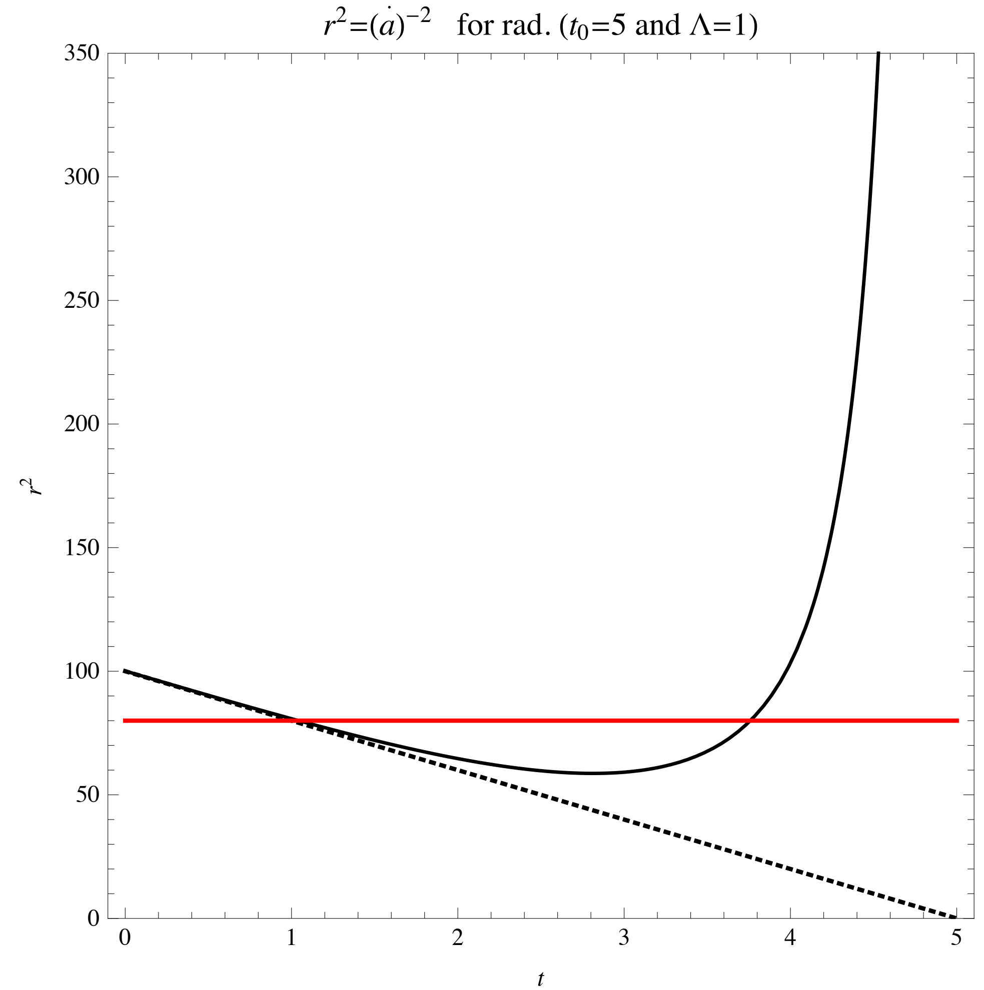
$$H^2 := \frac{8\pi G}{3}\rho_{\rm eff} = \frac{8\pi G}{3}\rho\left(1 - \left(\frac{\rho}{\rho_{\rm cr}}\right)^{\alpha}\right) \quad \text{where} \quad \rho = \frac{\rho_0}{a^4}\,.$$



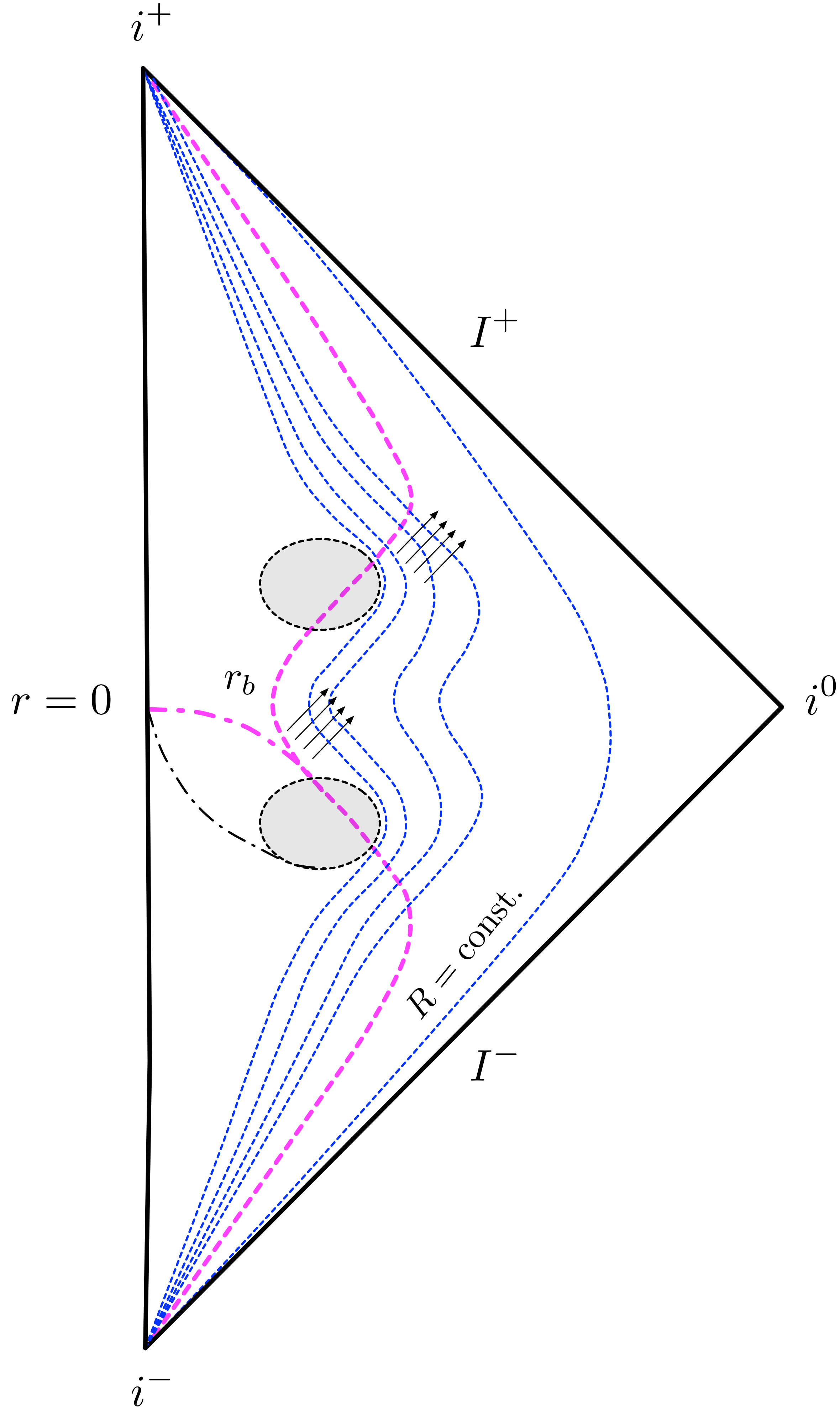




8 π p_r(t) for radiation (t_0=20 and Λ=1)
8 π p_r(t)
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
0
10
20
30
40
t

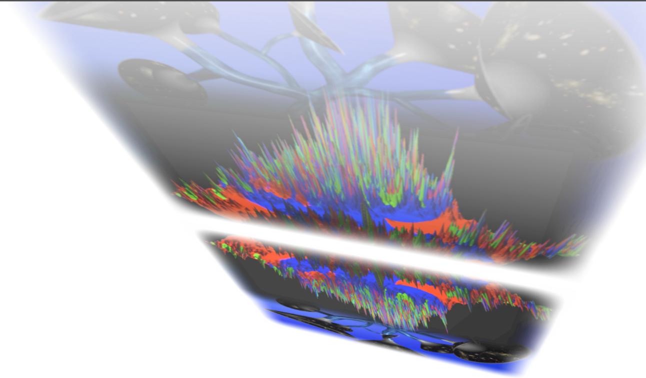


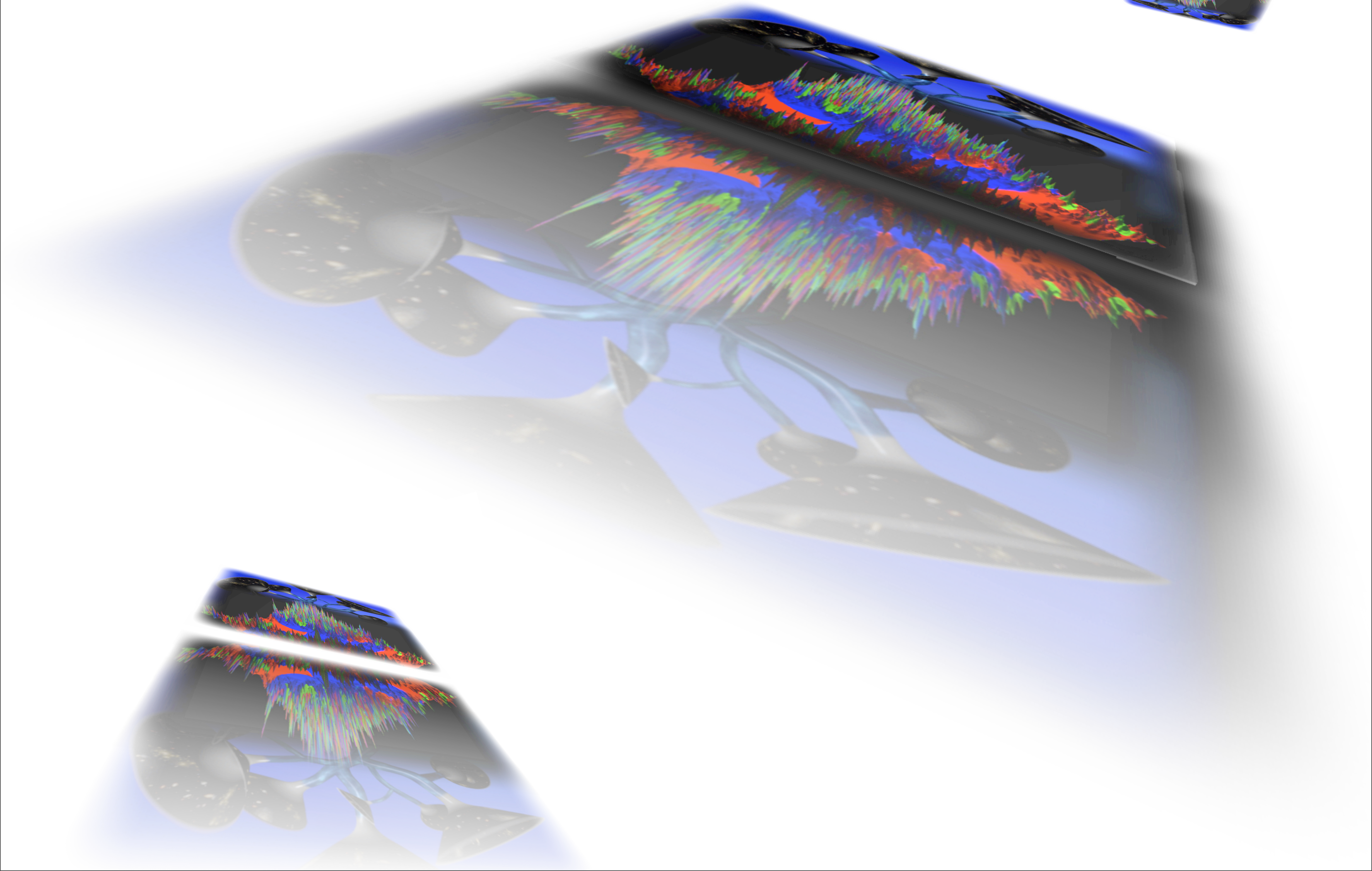
$r^2=(\dot{a})^{-2}$ for rad. (t_0=5 and Λ=1)
r^2
350
300
250
200
150
100
50
0
0
1
2
3
4
5
t



i^+
I^+
$r = 0$
r_b
i^0
$R = \text{const.}$
I^-
i^-

Cosmology





Super-accelerating bouncing cosmology in asymptotically-free non-local gravity

G. Calcagni, L. M., P. Nicolini.

The Calculation

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}\,,$$
$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a(t)^2\,\delta_{ij}\,dx^i dx^j\,,$$
$$a^2(t) = 1 - \kappa h(t)\,, \qquad h(t=t_{\rm i}) = 0\,, \qquad g_{\mu\nu}(t=t_{\rm i}) = \eta_{\mu\nu}\,,$$
$$h_{\mu\nu}(t) = h(t)\,\mathrm{diag}(0,\delta_{ij}) =: h(t)\,\mathcal{I}_{\mu\nu}\,;$$

$$\bar{h}_{\mu\nu} := h'_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\,h'^{\lambda}_{\lambda} = h(t)\,\mathrm{diag}(0,-2\delta_{ij}) = -2h(t)\mathcal{I}_{\mu\nu}\,, \qquad \partial^\mu \bar{h}_{\mu\nu} = 0\,,$$

$$\tilde{\bar{h}}_{\mu\nu}(E,\vec{k}) = -2\tilde{h}(E)(2\pi)^3\delta(\vec{k})\,\mathcal{I}_{\mu\nu}\,.$$

$$O^{-1}(k) = \frac{V(k^2)}{k^2}\left(P^{(2)} - \frac{P^{(0)}}{2}\right) \quad \Longrightarrow \quad \bar{h}_{\mu\nu}(x) = \kappa\int\frac{d^4k}{(2\pi)^4}O^{-1}_{\mu\nu,\rho\sigma}(k)\tilde{T}^{\rho\sigma}(k)\,e^{-ik\cdot x}\,,$$

$$h(t) = -\frac{\kappa}{2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2V^{-1}(k^2)}\,\tilde{\rho}(E,\vec{k})\,e^{-ik\cdot x} = \int\frac{dE}{2\pi}\,\tilde{h}(E)\,e^{iEt}\,.$$

The Solution

$$a_{\rm cl}(t) = \left|\frac{t}{t_{\rm i}}\right|^{p} \quad \text{and} \quad h_{\rm cl}(t) = \frac{1}{\kappa}\left[1 - \left|\frac{t}{t_{\rm i}}\right|^{2p}\right],$$

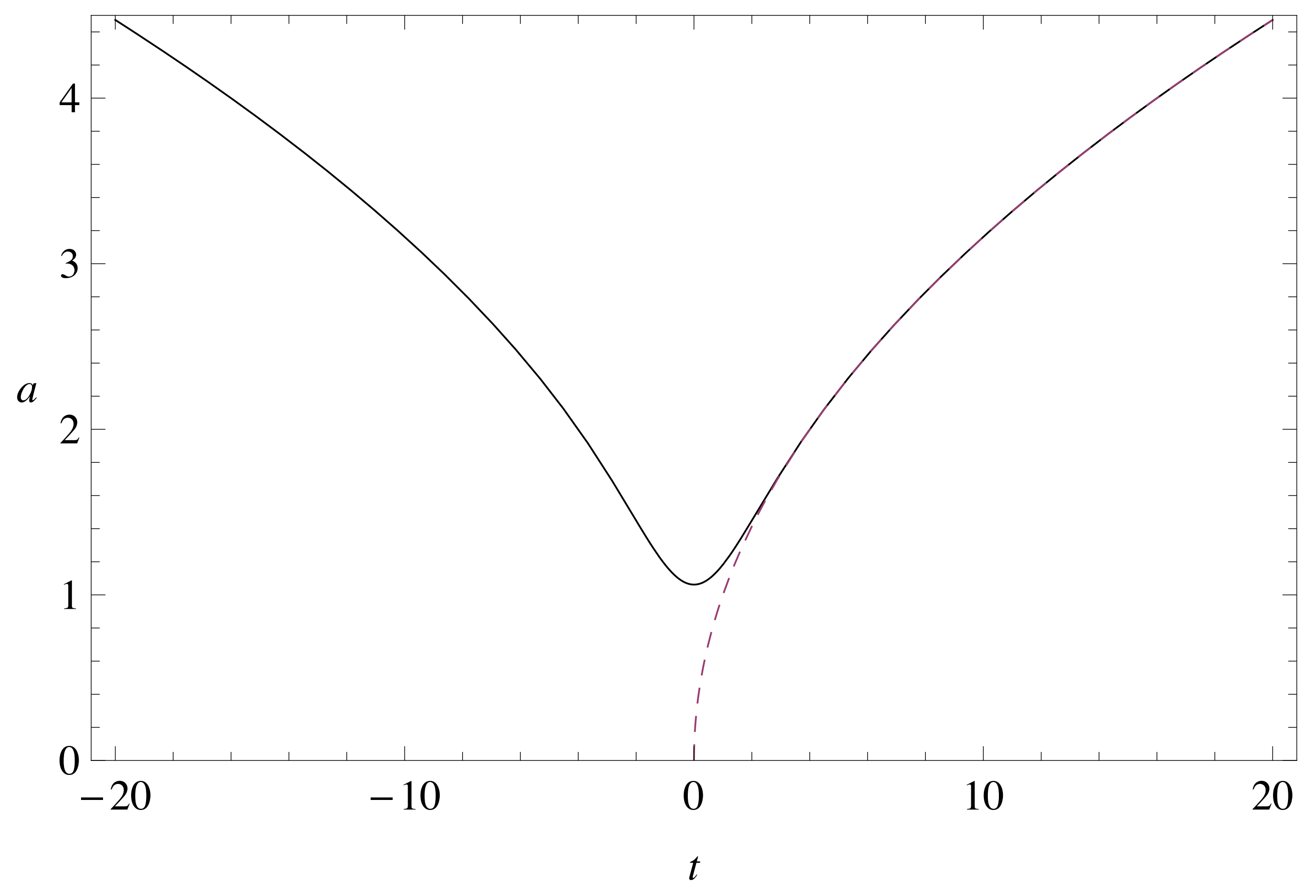
$$\kappa\, h(t) = 1 - \left(\frac{2}{\Lambda t_{\rm i}}\right)^{2p} \frac{\Gamma\left(\frac{1}{2}+p\right)}{\sqrt{\pi}}\, {}_1F_1\left(-p; \frac{1}{2}; -\frac{1}{4}t^2\Lambda^2\right),$$

$$a(t) = \left(\frac{2}{\Lambda t_i}\right)^{p} \sqrt{\frac{\Gamma\left(\frac{1}{2}+p\right)}{\sqrt{\pi}}\, {}_1F_1\left(-p; \frac{1}{2}; -\frac{1}{4}t^2\Lambda^2\right)}.$$

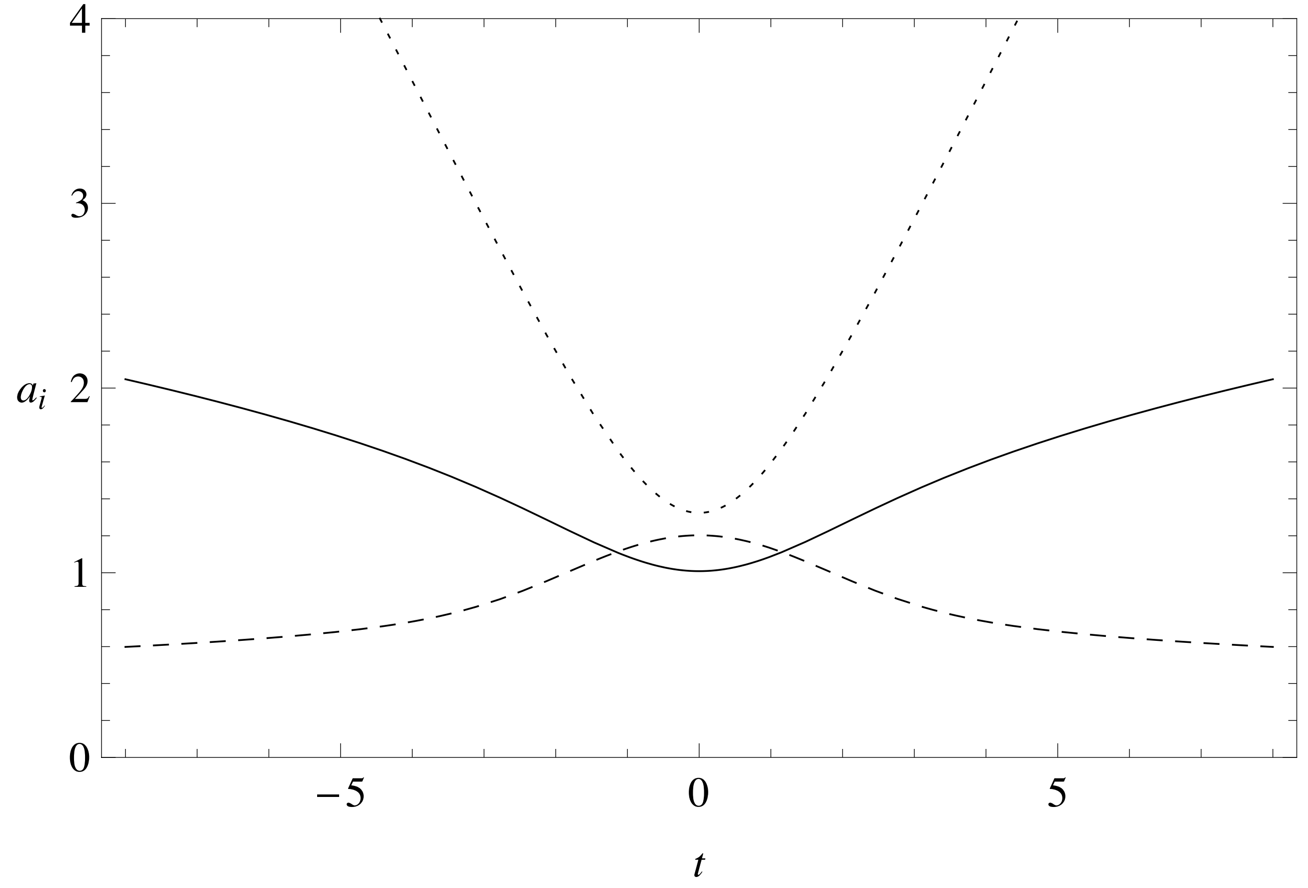
Radiation $p = 1/2$: $$a(t) = \sqrt{\frac{2e^{-\frac{1}{4}\Lambda^2 t^2}}{\sqrt{\pi}\,\Lambda t_{\rm i}} + \frac{t}{t_{\rm i}}\,\mathrm{erf}\left(\frac{\Lambda t}{2}\right)}.$$

Bouncing

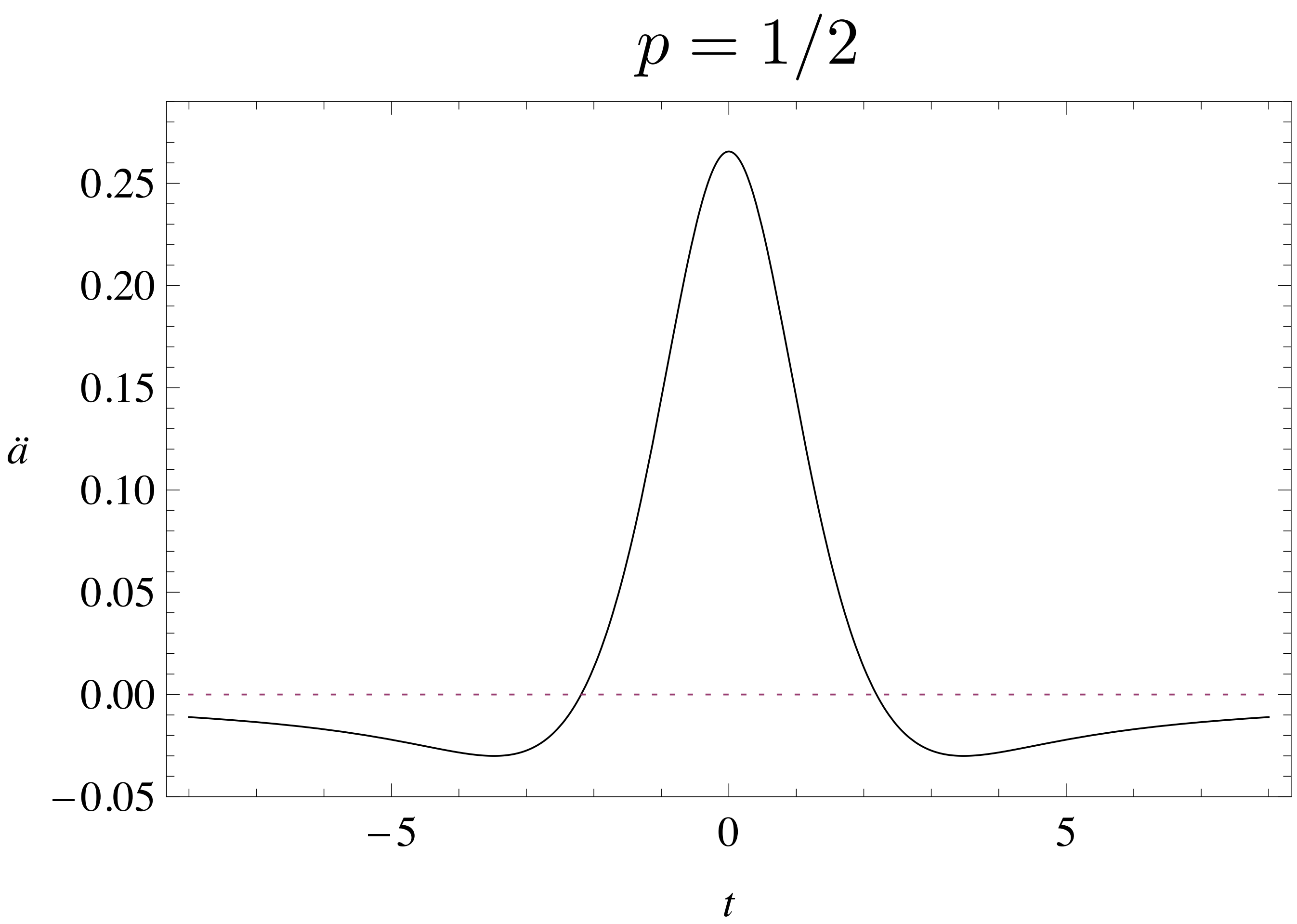
$p = 1/2$:

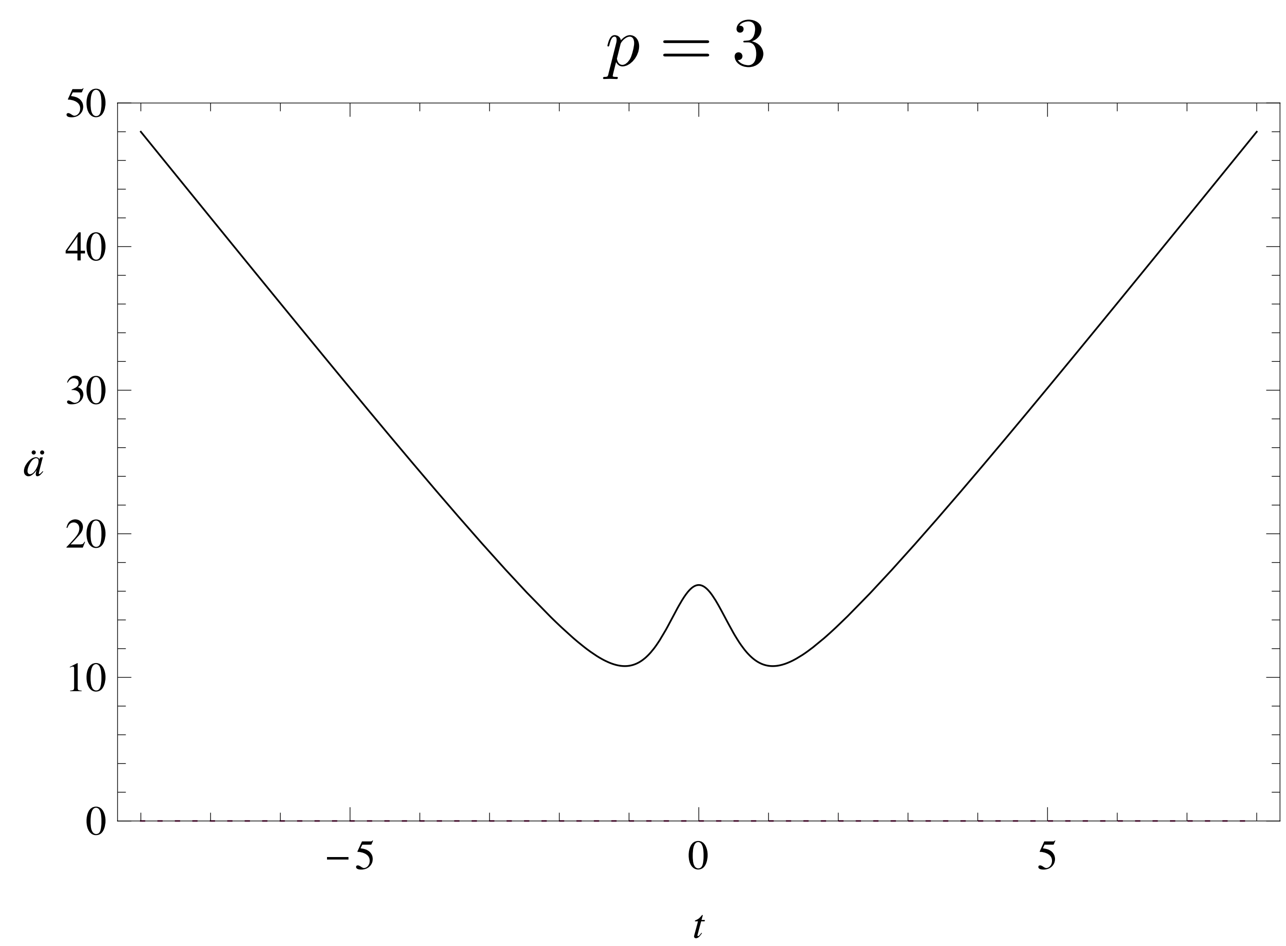


$p_1 = -1/4$ (dashed curve),
$p_2 = (5 - \sqrt{5})/8 \approx 0.35$ (solid curve),
$p_3 = (5 + \sqrt{5})/8 \approx 0.90$:



Super-acceleration





$$H(t) = \frac{\dot{a}}{a} = \frac{p\Lambda^2 t}{2} \frac{{}_1F_1\left(1-p; \frac{3}{2}; -\frac{1}{4}t^2\Lambda^2\right)}{{}_1F_1\left(-p; \frac{1}{2}; -\frac{1}{4}t^2\Lambda^2\right)} \,.$$

For $t \to 0 \quad H(t) \sim \dfrac{p\Lambda^2}{2} t$,

$$a(t) \sim e^{\frac{p}{4}\Lambda^2 t^2} \,.$$

Effective Theory

$$H^2 := \frac{8\pi G}{3}\rho_{\rm eff} = \frac{8\pi G}{3}\rho\left(1 - \left(\frac{\rho}{\rho_{\rm cr}}\right)^{\alpha}\right) \quad \text{where} \quad \rho = \frac{\rho_0}{a^4} \,.$$

Quantum Supergravity

Supersymmetric multiplet consists of :

- the spin-2 graviton, $h_{\mu\nu}$,
- the spin-3/2 gravitino ψ_μ ,
- three auxiliary fields S, P and A_μ.

$$\mathcal{L} = -\frac{\gamma}{2}\underbrace{(\kappa^{-2} R + \text{more})}_{\mathcal{L}_R} + \underbrace{(\kappa^{-2} R\,\alpha(\Box)\,R + \text{more})}_{\mathcal{L}_{R^2}}$$
$$\underbrace{+\,\kappa^{-2}\left(R_{\mu\nu}\,\beta(\Box)R^{\mu\nu} - \frac{1}{3}R\,\beta(\Box)\,R\right) + \text{more}}_{\mathcal{L}_{C^2}}\,.$$

$$\mathcal{L}_R = \kappa^{-2}R + e^{-1}\bar{\psi}_\mu\,\epsilon^{\mu\nu\rho\sigma}\,\gamma_5\gamma_\nu\mathcal{D}_\rho\psi_\sigma + \frac{2}{3}\left(S^2 + P^2 - A_\mu^2\right)\;,$$

$$\mathcal{L}_{R^2} = \frac{1}{\kappa^2}R\alpha(\Box)R + \bar{R}\cdot\gamma\;\partial\!\!\!/\,\gamma\,\alpha(\Box)\cdot R - 4\,(\partial_\mu S)\alpha(\Box)(\partial^\mu S) - 4\,(\partial_\mu P)\alpha(\Box)(\partial^\mu P) + 4\,(\partial_\mu A^\mu)\alpha(\Box)(\partial^\nu A^\nu),$$

$$\mathcal{L}_{C^2} = \kappa^{-2}\left(R_{\mu\nu}\,\beta(\Box)R^{\mu\nu} - \frac{1}{3}R\,\beta(\Box)\,R\right) - \bar{\psi}_\mu(\Box\delta_{\mu\nu} - \partial_\mu\partial_\nu)\beta(\Box)\left(R^\nu - \frac{1}{3}\gamma^\nu\gamma\cdot R\right) - \frac{1}{3}F_{\mu\nu}\beta(\Box)F^{\mu\nu}\,.$$

Propagators

$$\langle h\,h\rangle = \frac{P^2}{\frac{\gamma\Box}{8}+\frac{\beta(\Box)\Box^2}{4}} - \frac{P^{0,s}}{\frac{\gamma\Box}{4}-3\alpha(\Box)\,\Box^2}\,,$$

$$\langle \psi\,\psi\rangle = -\frac{P^{3/2}}{\not\partial\left(\beta(\Box)\,\Box+\frac{\gamma}{2}\right)} + \frac{P^{0,s}}{\not\partial\left(-12\,\alpha(\Box)\Box+\gamma\right)}\,,$$

$$\langle A\,A\rangle = \frac{P^1}{\frac{\gamma}{3}+\frac{2\beta(\Box)\Box}{3}} + \frac{P^0}{\frac{\gamma}{3}-4\alpha(\Box)\,\Box}\,,$$

$$\langle S\,S\rangle = \langle P\,P\rangle = \frac{1}{4\alpha(\Box)\Box-\frac{\gamma}{3}}\,.$$

$$\alpha(\Box) := \alpha_0 + h_0(\Box)\,,\quad \beta(\Box) := \beta_2 + h_2(\Box)\,,$$

$$h_2(z) = \frac{\tilde\gamma(e^{H(z)}-1)-2\,\tilde\beta_2\,z}{2\,z},$$

$$h_0(z) = -\frac{\tilde\gamma(e^{H(z)}-1)+12\,\tilde\alpha_0\,z}{12\,z}\,.$$

→

$$\langle h\,h\rangle = 8\frac{e^{-H(\Box)}}{\gamma\,\Box}\left[P^2-\frac{P^{0,s}}{2}\right],$$

$$\langle \psi\,\psi\rangle = -2\frac{e^{-H(\Box)}}{\gamma\,\not\partial}\left[P^{3/2}-2P^{0,s}\right],$$

$$\langle A\,A\rangle = 3e^{-H(\Box)}\left[P^1+P^0\right]\gamma^{-1}\,,$$

$$\langle S\,S\rangle = \langle P\,P\rangle = -3e^{-H(\Box)}\,\gamma^{-1}.$$

Finiteness

$$\Delta\mathcal{L} = \frac{1}{\epsilon}\left[x R_{\mu\nu}^2 + y R^2 + z R_{\mu\nu\rho\sigma} M^{\mu\nu\rho\sigma}(\Phi) + w N(\Phi)\right],$$

Einstein gravity :

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = -\kappa^2 T_{\mu\nu} \quad \longrightarrow \quad \Delta\mathcal{L} = \frac{1}{\epsilon}\left[4x\kappa^4 T_{\mu\nu}^2 + 4y\kappa^4 T^2 + z R \cdot M(\Phi) + w N(\Phi)\right] .$$

$$\langle h_{\mu\nu} \dots | \Delta\mathcal{L} | \psi_\mu \dots \rangle \propto \langle h_{\mu\nu} \dots | \Delta\mathcal{L} | h_{\rho\sigma} \dots \rangle = 0 \quad \text{on shell} .$$

Super-renormalizable gravity :

$$E_{\mu\nu} = -\kappa^2 e^{-H(\Box)} T_{\mu\nu} \, ,$$

$$E_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + O(R^2) + \dots + O(\Box^{a-1} R^2) \, ,$$

$$\langle h_{\mu\nu} \dots | \Delta\mathcal{L} | h_{\rho\sigma} \dots \rangle \propto \langle h_{\mu\nu} \dots | O(R^2) + \dots | h_{\rho\sigma} \dots \rangle \neq 0 \quad \text{on shell} .$$

Extended supergravity

Beta function spin $-\, s$ particle : $\beta_s = -(-1)^{2s}\left[(2s)^2 - \frac{1}{3}\right]$,

$N = 4$ supergravity :

$$\beta = \sum_{s=2,\frac{3}{2},1,\frac{1}{2},0} \beta_s = -\frac{47}{3}\cdot 1 + \frac{26}{3}\cdot 4 - \frac{11}{3}\cdot 6 + \frac{2}{3}\cdot 4 + \frac{1}{3}\cdot 1 = 0\,.$$

$N = 4$ (or $N = 8$) extended supergravity is off shell finite.

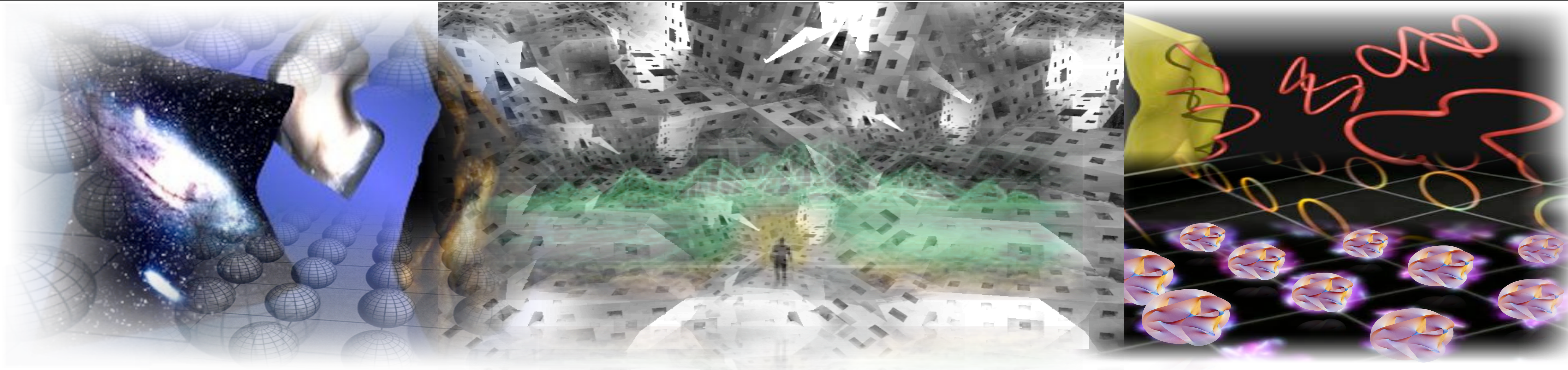
Summary and Conclusions

Gravity Theory

$$S = \int d^{D_{\rm odd}}x \, 2\kappa^{-2} \sqrt{|g|} \left[R - G_{MN} \left(\frac{e^{H(-\Box_\Lambda)} - 1}{\Box} \right) R^{MN} \right]$$

$$e^{H(z)} = |p_{\gamma+{\rm N}+1}(z)| \, e^{\frac{1}{2}\left[\Gamma\left(0, p^2_{\gamma+{\rm N}+1}(z)\right) + \gamma_E\right]}, \quad p_{\gamma+{\rm N}+1}(z): \text{ real polynomial of degree } \gamma + {\rm N} + 1 \, .$$

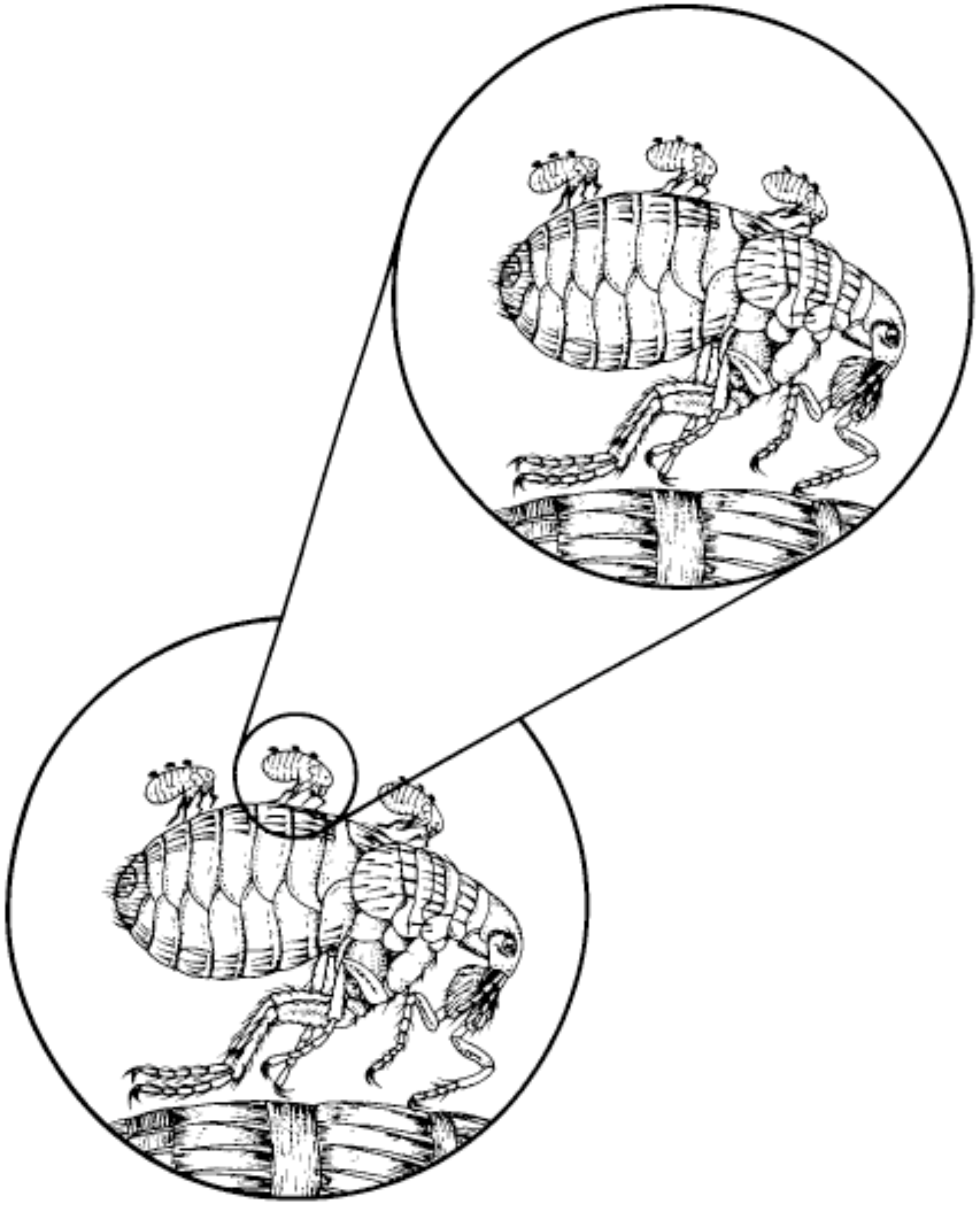
- Non-polynomial Higher Derivative Gravity,
- Super-renormalizable & Unitary (no ghosts) Quantum Gravity.
- Finite Quantum Gravity.
- Quantum Supergravity & M-Theory.
- Super-renormalizable completion of the Starobinsky theory.
- New Non-local Massive Gravity.



- Regular (multi-horizon-)black holes,
- Gravitational collapse and terminating black holes,
- Bouncing cosmology,
- Starobinsky inflation,
- Fractal properties of the spacetime,
- Infrared non-polynomial modifications.


MANY WORLDS

Other Applications

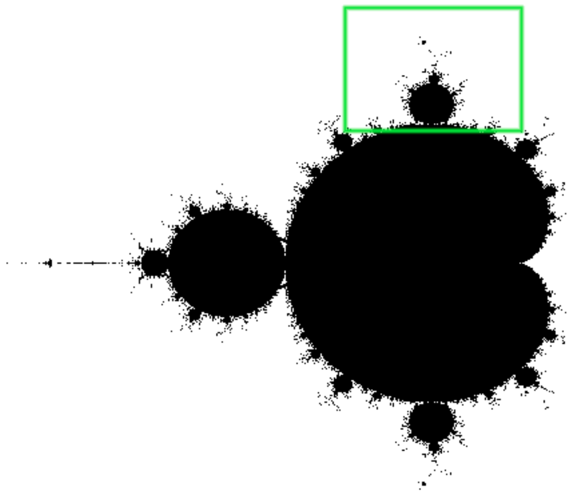


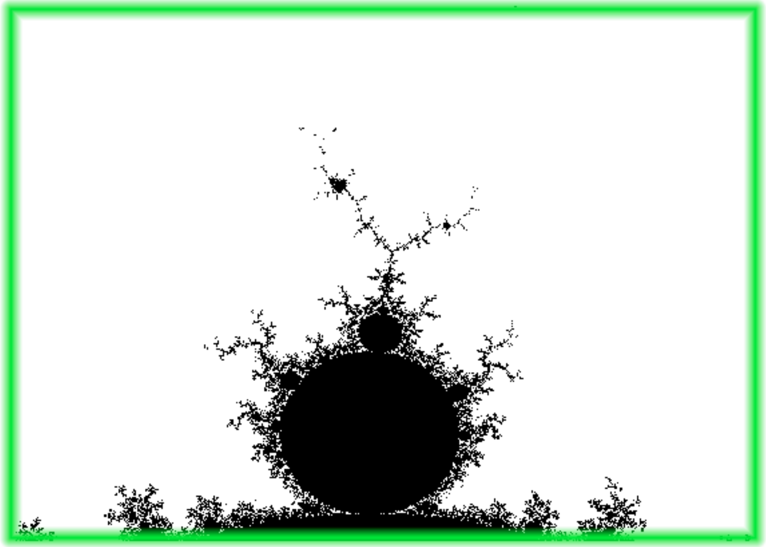
Why are we interested in the “spectral dimension of spacetime”?

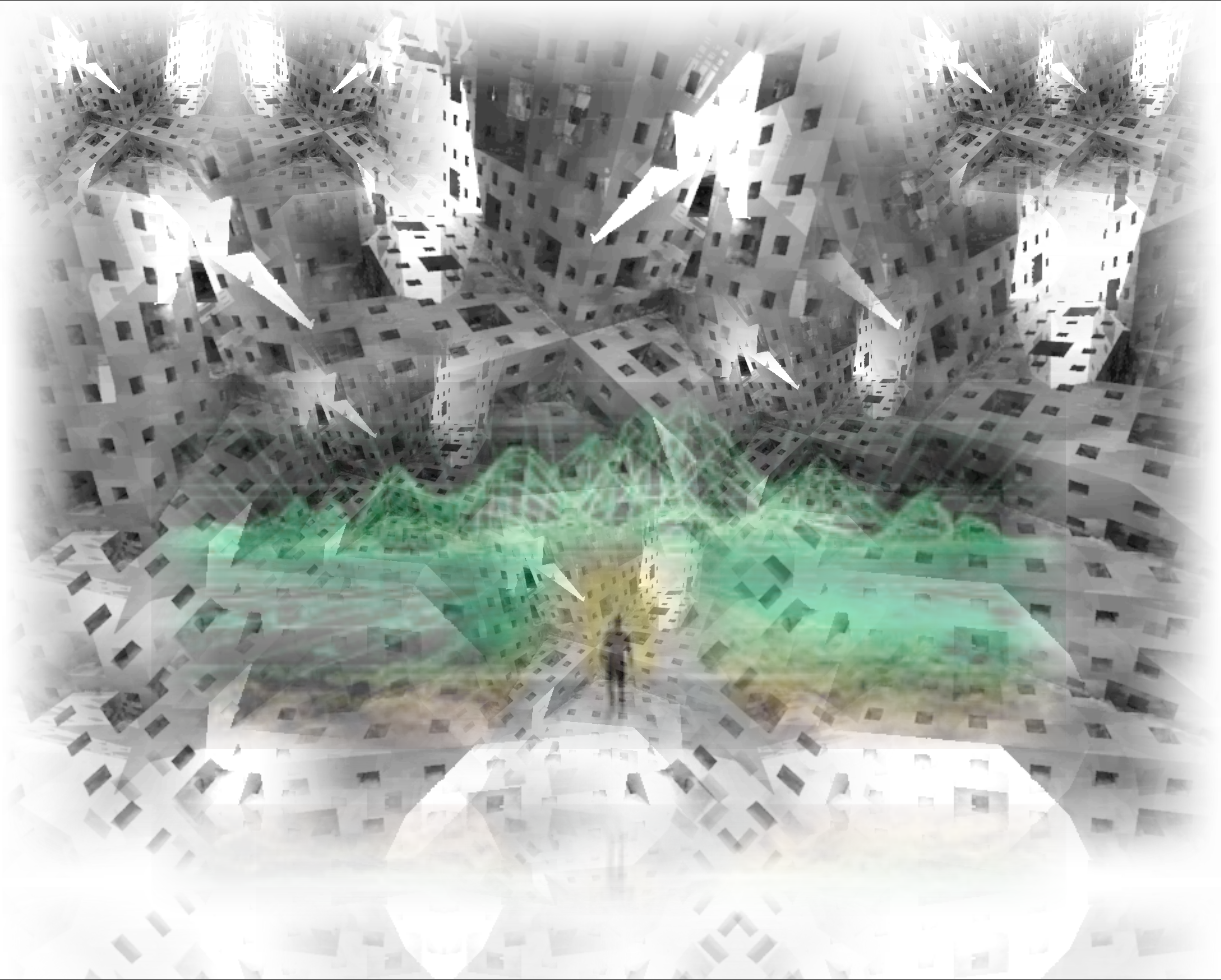
Motivations :

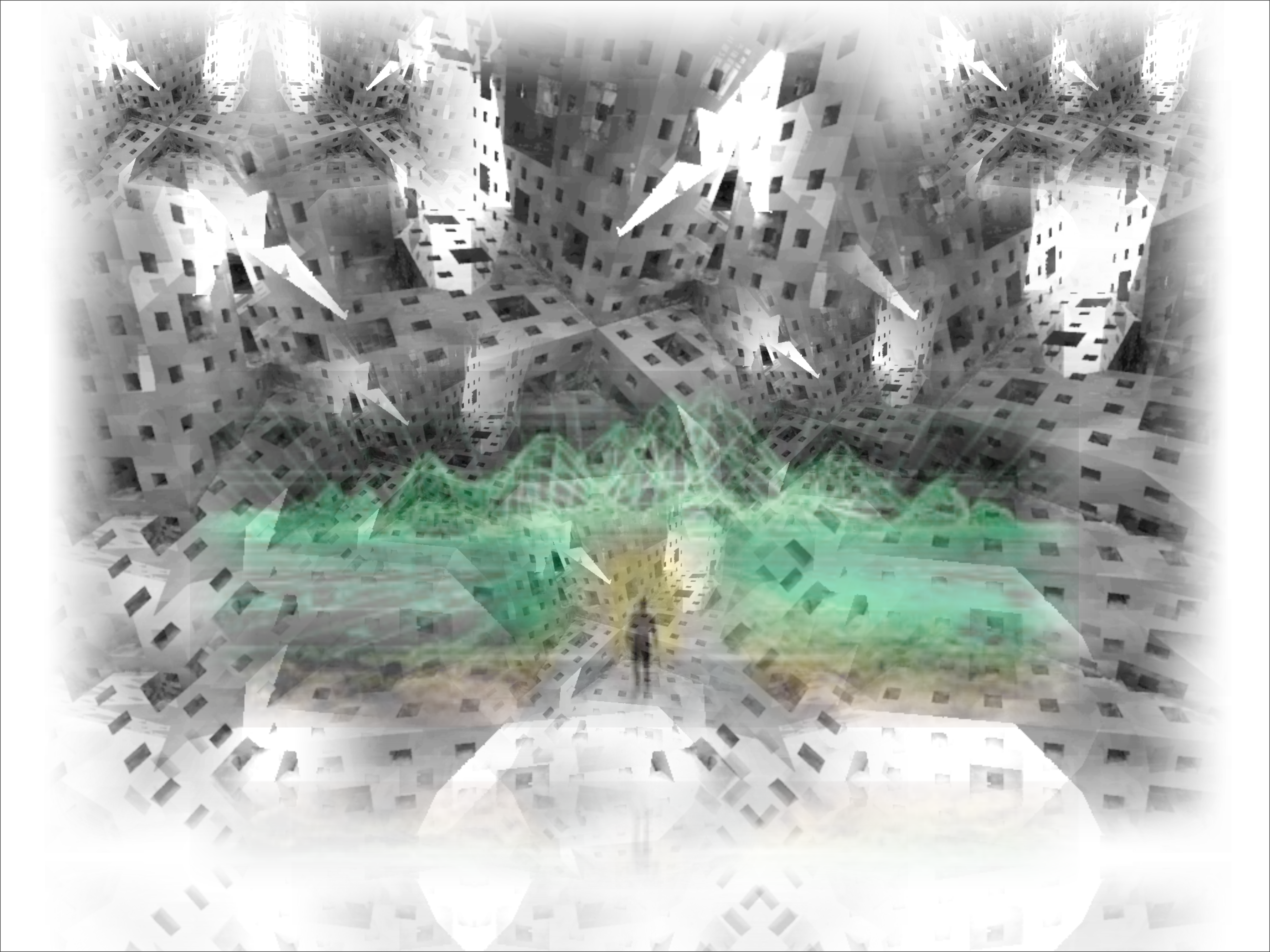
1. $d_s \iff$ Gravitational potential,

2. comparing different approaches to quantum gravity (CDT and ASQG, M. Reuter et. al.).









Spectral Dimension

Diffusion of a probe particle on a d-dimensional manifold :

$$K_g(x,x';T) = \langle x'|\exp(T\,\Delta_g^{\mathrm{eff}})|x\rangle.$$

(probability to diffuse from x$'$ to x in a time T),

$$\partial_T K_g(x,x';T) = \Delta_g^{\mathrm{eff}} K_g(x,x';T).$$

Average return probability :

$$P_g(T) \equiv V^{-1}\int \mathrm{d}^d x\,\sqrt{g(x)}\,K_g(x,x;T) \equiv V^{-1}\,\mathrm{Tr}\,\exp(T\,\Delta_g^{\mathrm{eff}}) \quad\rightarrow\quad d_s \equiv -2\frac{\mathrm{d}\ln P_g(T)}{\mathrm{d}\ln T}\,.$$

Propagator ⟺ Heat-kernel

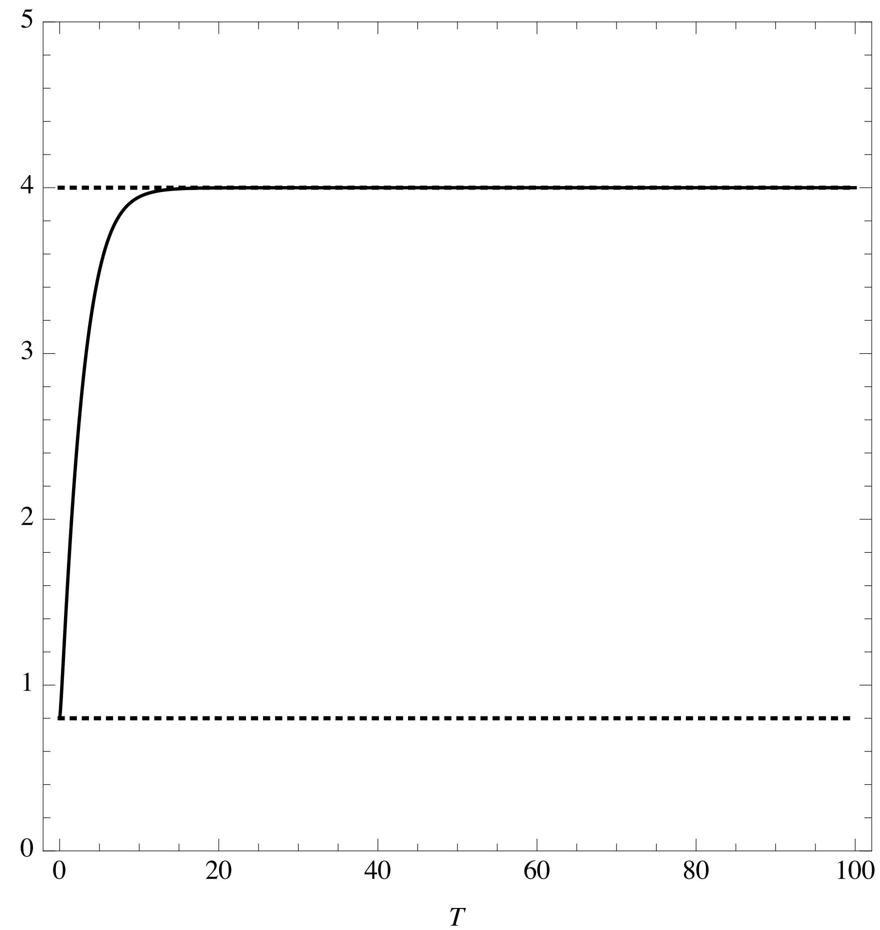
$$G(x,x') = \int_0^{+\infty}\mathrm{d}T\,K_g(x,x';T) \propto \int \mathrm{d}^4k\,e^{ik(x-x')}\int_0^{+\infty}\mathrm{d}T\,K_g(k;T)\,.$$

Spectral Dimension Flux

$$D(k) \propto \frac{1}{k^2\,\bar{h}(k^2/\Lambda^2)} \underset{\text{UV}}{\rightarrow} \frac{1}{k^{2\gamma+4}} \quad \Rightarrow \quad K_g(k;T) \propto e^{-k^2\,\bar{h}(k^2/\Lambda^2)\,T}$$

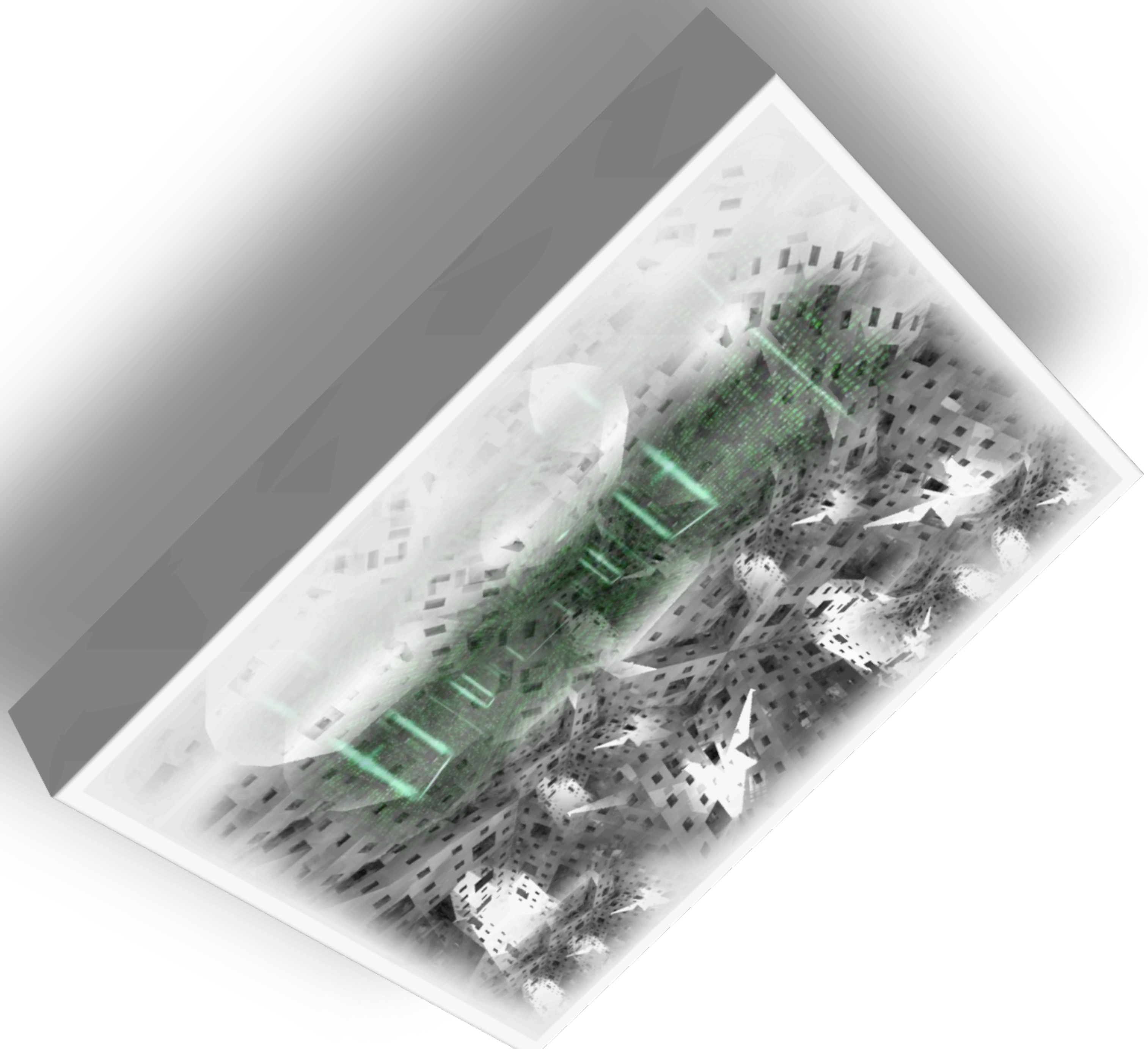
$$P_g(T) \propto \int \mathrm{d}^4k\, e^{-k^2\,\bar{h}(k^2/\Lambda^2)\,T} \quad \Rightarrow \quad \text{In the UV} \quad d_s = \frac{4}{\gamma+2}\,.$$

$d_s(T)$
for $\gamma = 3$.



Theory 2

Stringy Renormalizable Gravity



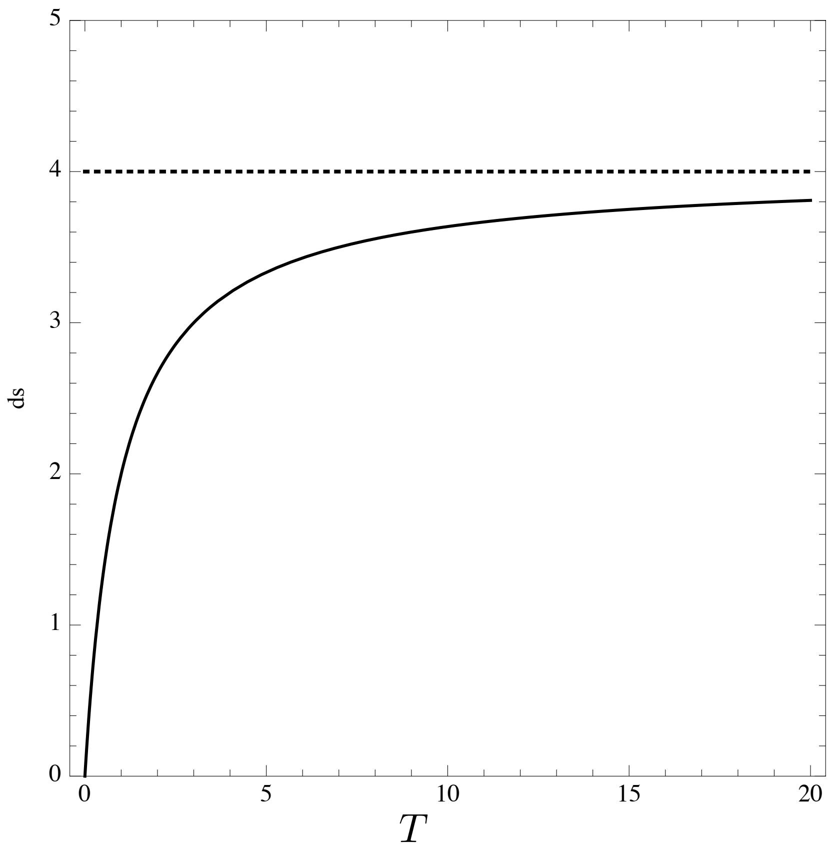
Fractal Properties

An Analytically Solvable Example

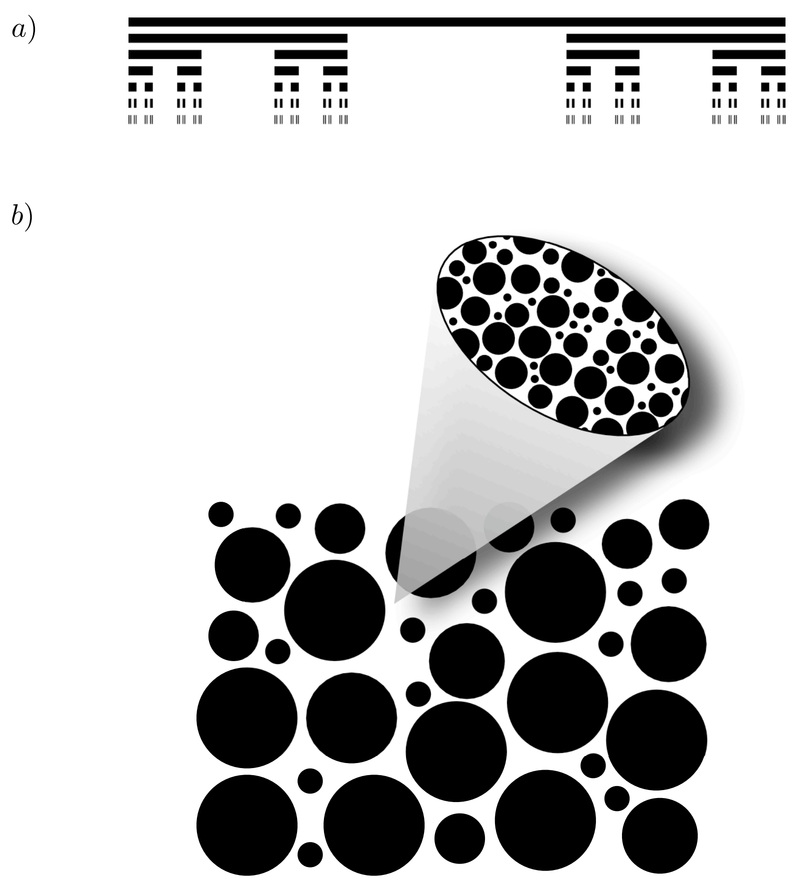
P. Nicolini, L.M.

$$\bar{h}_2(z) = \bar{h}_0(z) \propto e^z \quad \Rightarrow \quad D(k) \propto \frac{e^{-k^2/\Lambda^2}}{k^2} \quad \Rightarrow \quad K(x, x'; T) = \frac{e^{-\frac{(x-x')^2}{4(T+1/\Lambda^2)}}}{\left[4\pi\left(T + 1/\Lambda^2\right)\right]^2}\,.$$

$$d_s(T) = \frac{4\,T}{T + 1/\Lambda^2}.$$

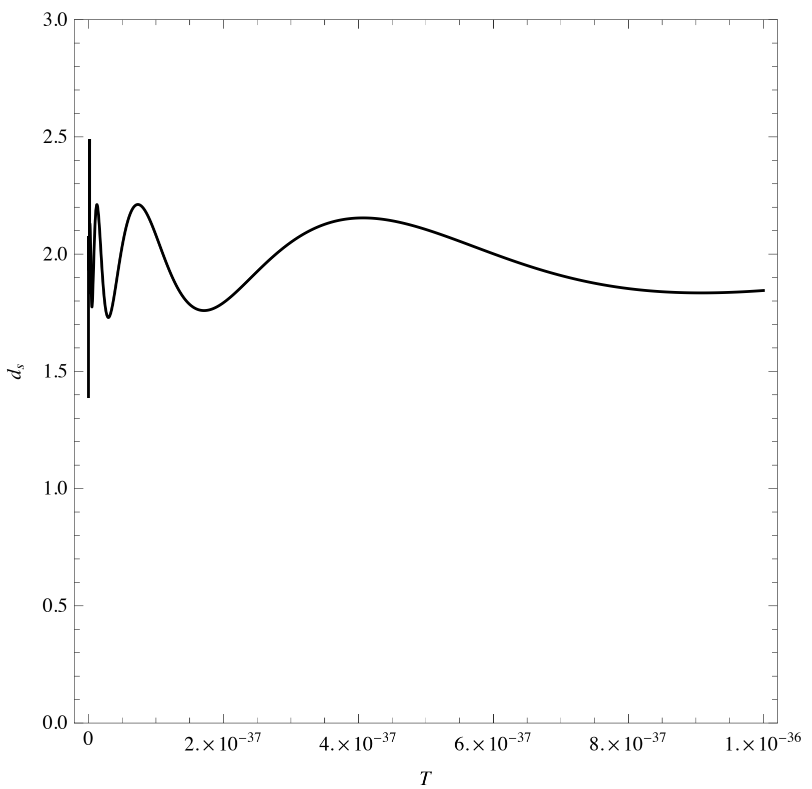


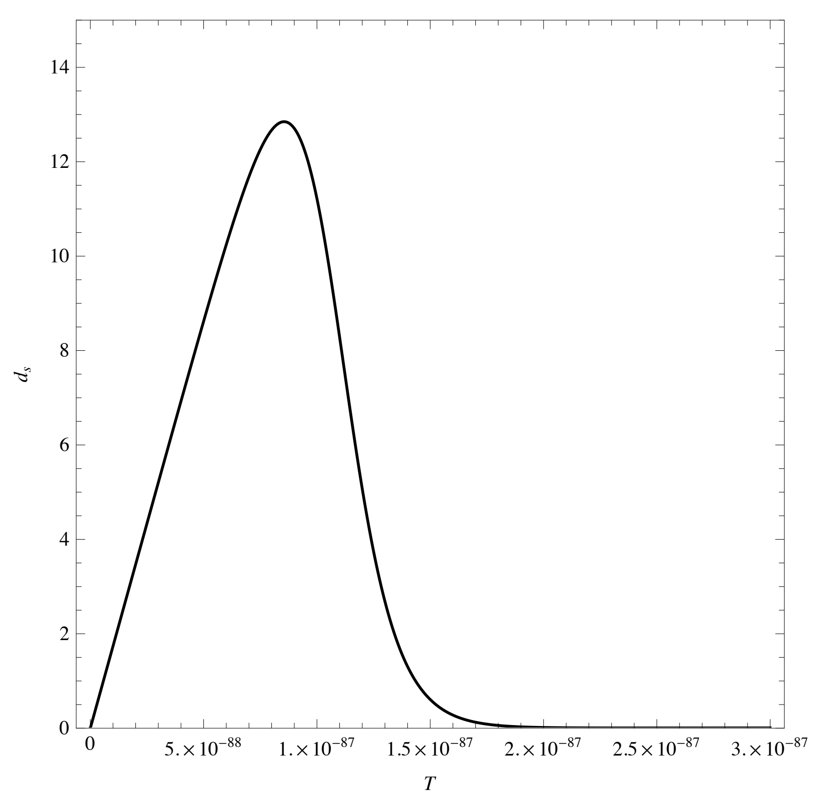
Spacetime as a Cantor Set

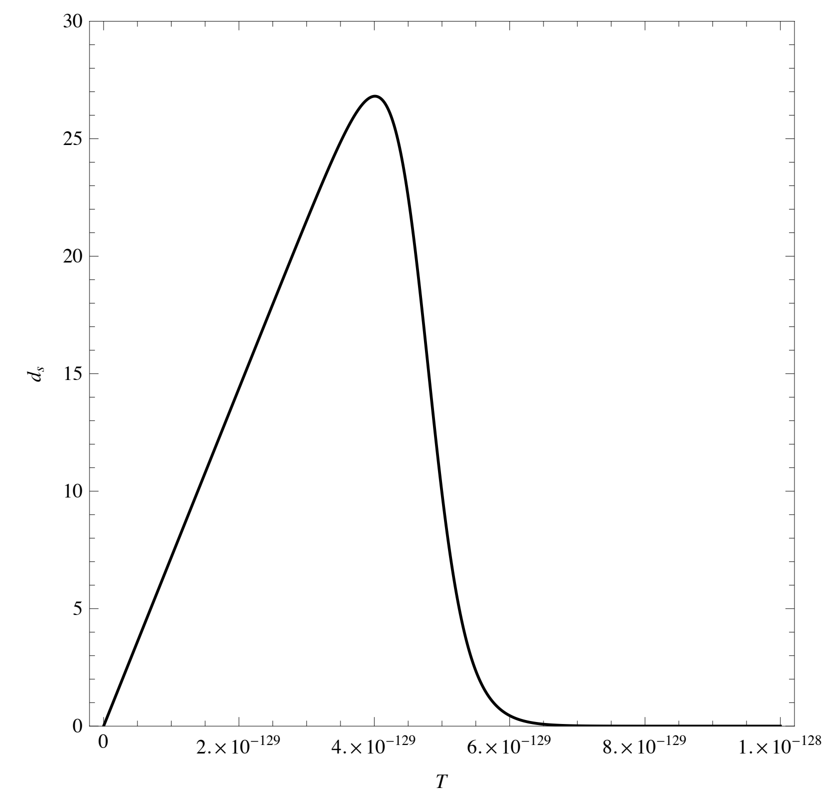


Fractal Quantum Spacetime

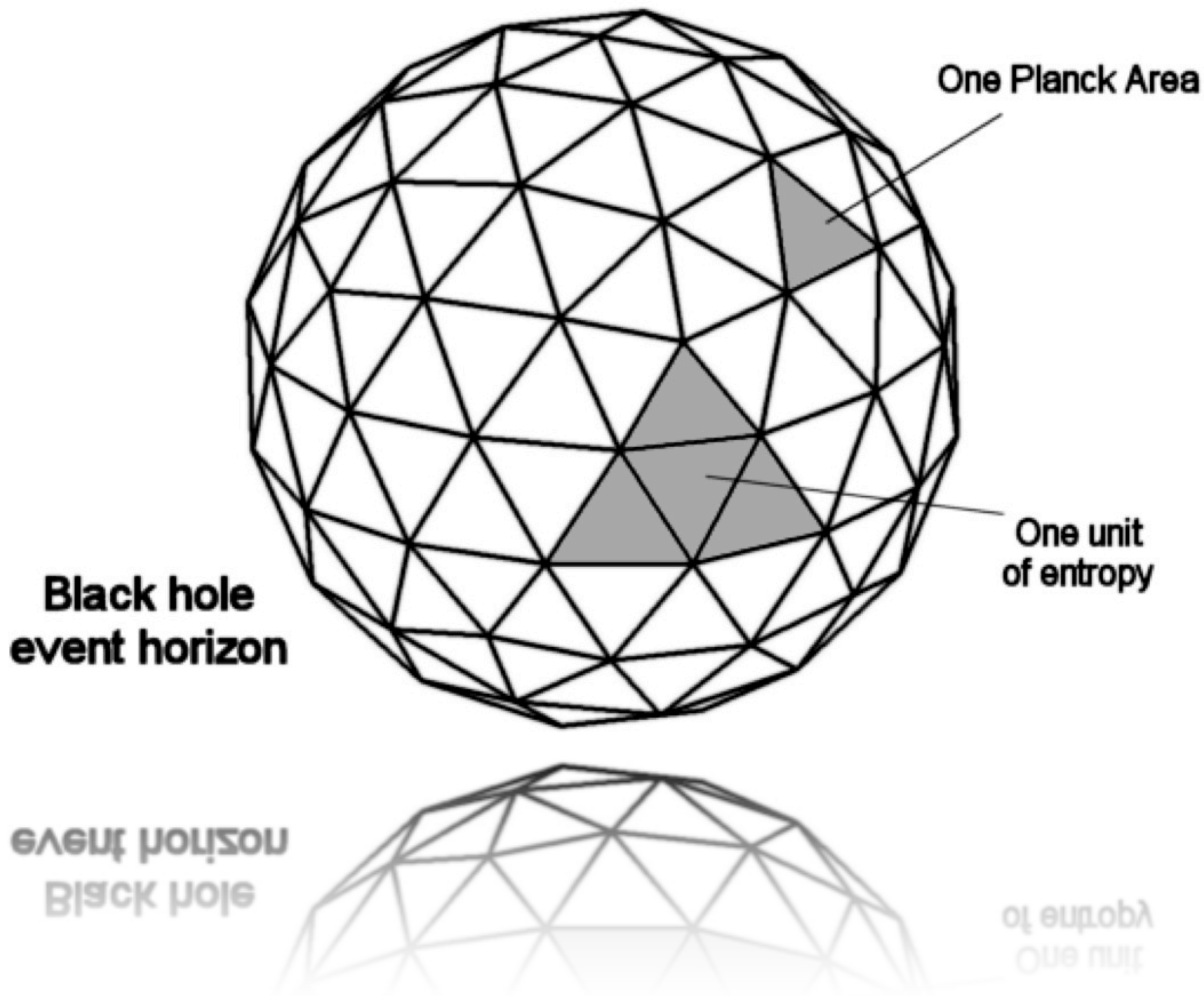
$$\text{Spin } 2: \quad D(p) \propto \frac{1}{p^2\left[1 + p^2\left(\text{const.} \times \log\frac{p^2}{\mu^2}\right)\right]}.$$







Entanglement Entropy



Entanglement Entropy

$$D(p)^{-1} \sim p^{2\gamma+4} \quad \text{and} \quad \gamma + 2 = k\,,$$

$$S = \frac{A}{4l_P^2} \times \underbrace{\frac{1}{\epsilon^{\frac{2}{k}}}}_{\approx 1 \;\; \text{for} \;\; k \gg 1\,.} .$$

What is the scale Λ ?

- SRG is an effective theory.
- SRG is asymptotically free.
- Λ is the Graviball mass.
- SRQG is self-complete.

MANY WORLDS

What is the scale Λ ?

- SRG is an effective theory.
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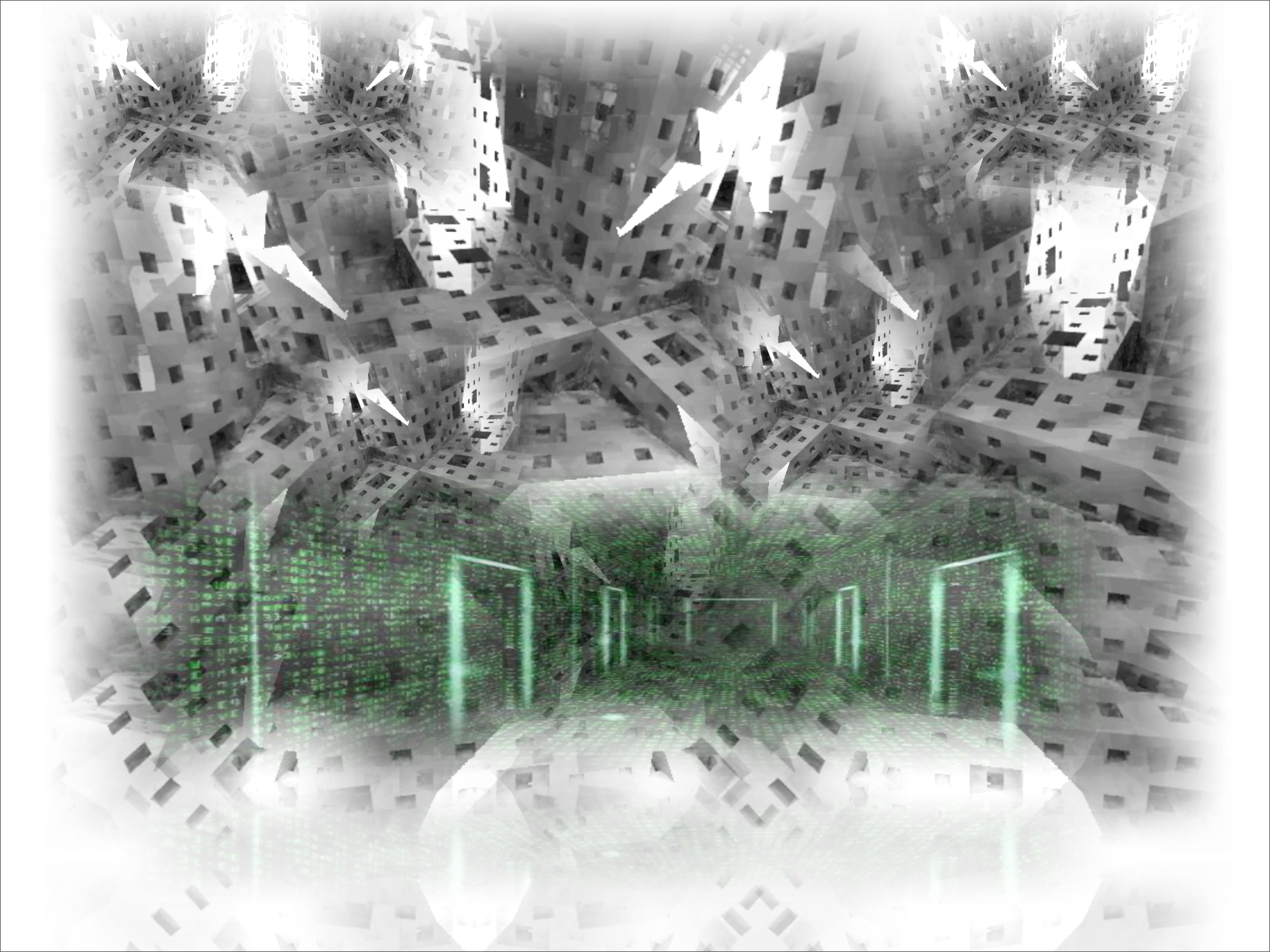
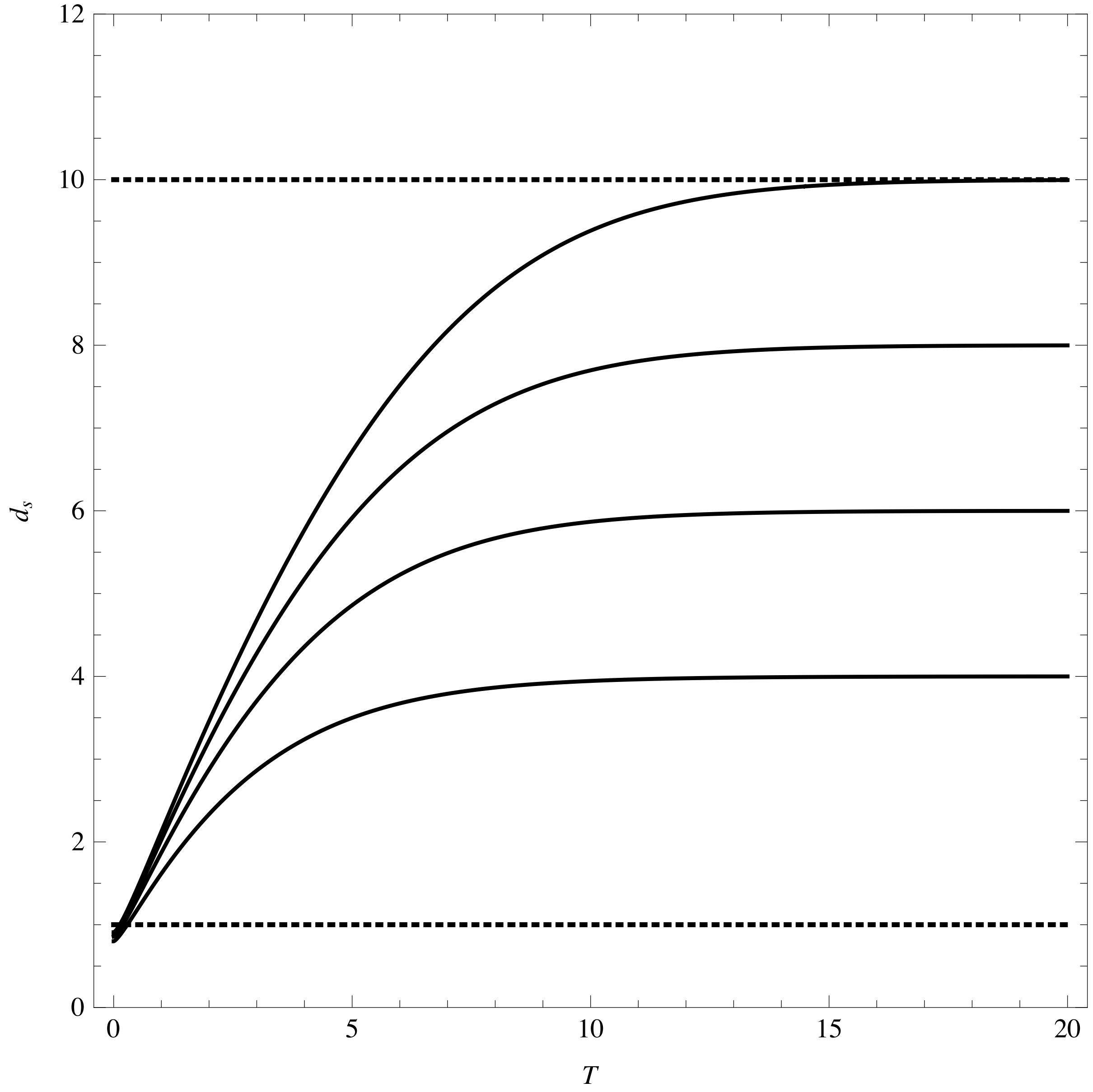
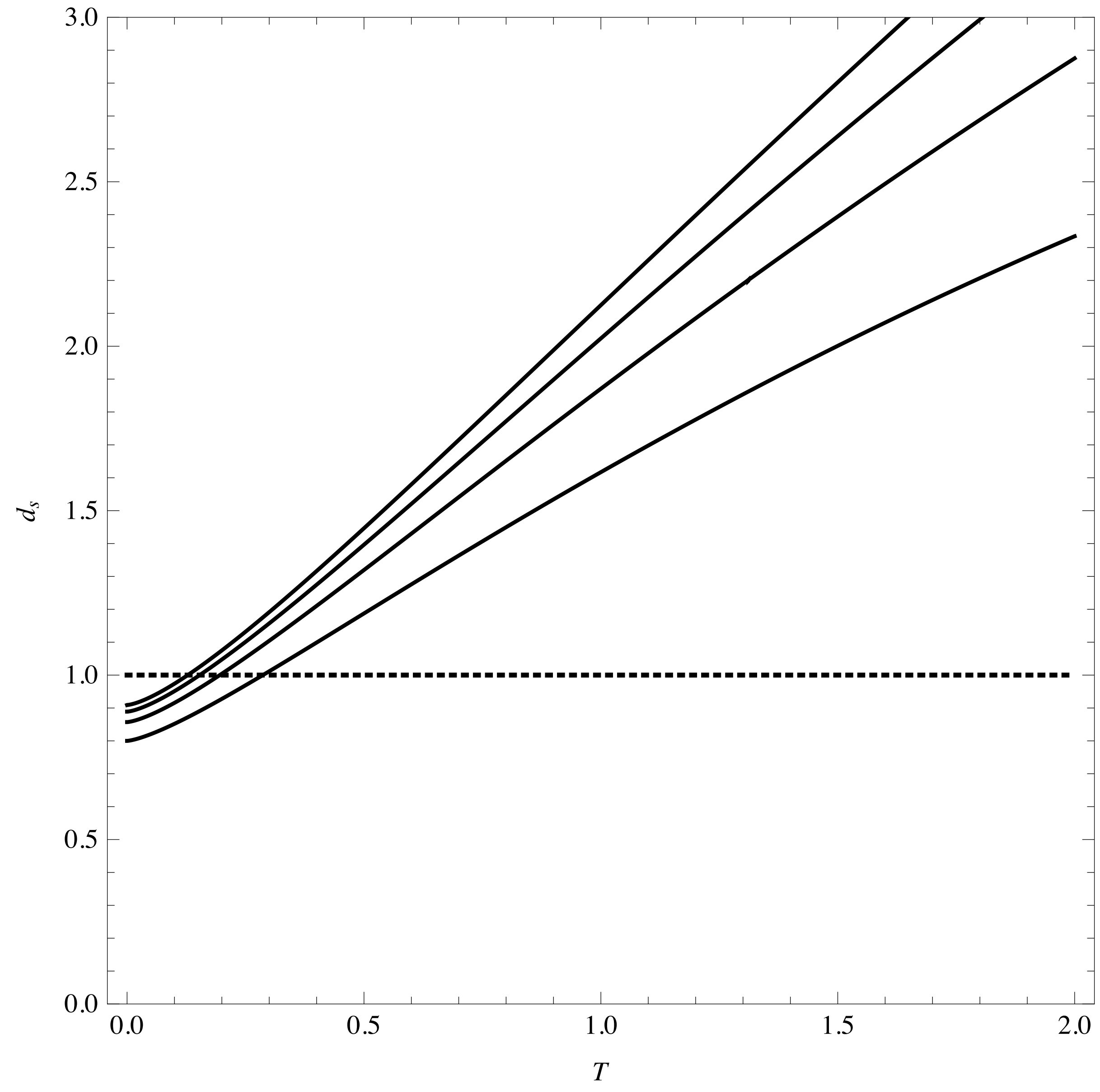


Fractal Dimension

Theory I

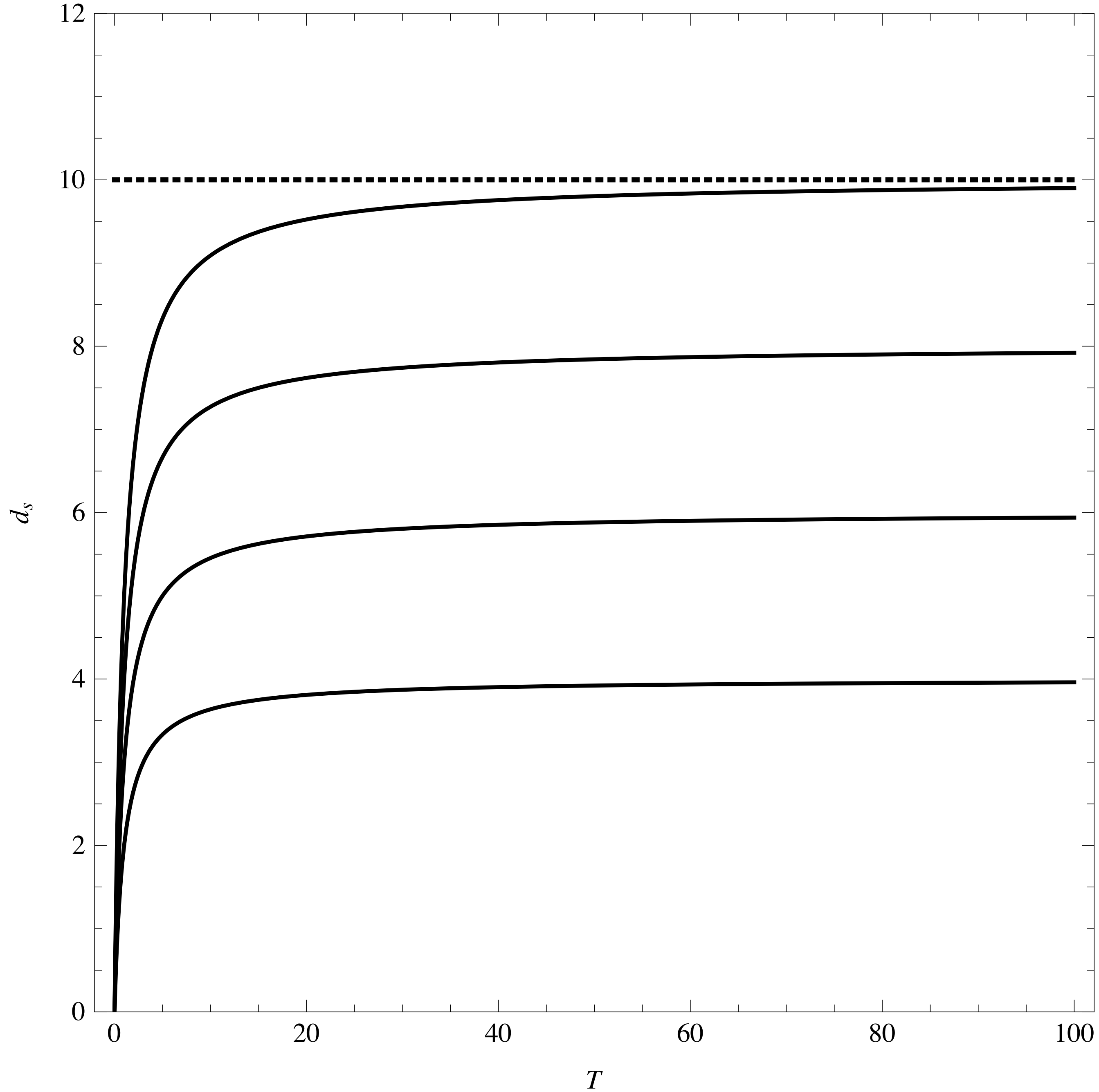
$$H_\gamma(z) = \frac{1}{2}\left[\gamma_E + \Gamma\left(0, z^{2\gamma+2N+2}\right)\right] + \log\left|z^{\gamma+N+1}\right|.$$





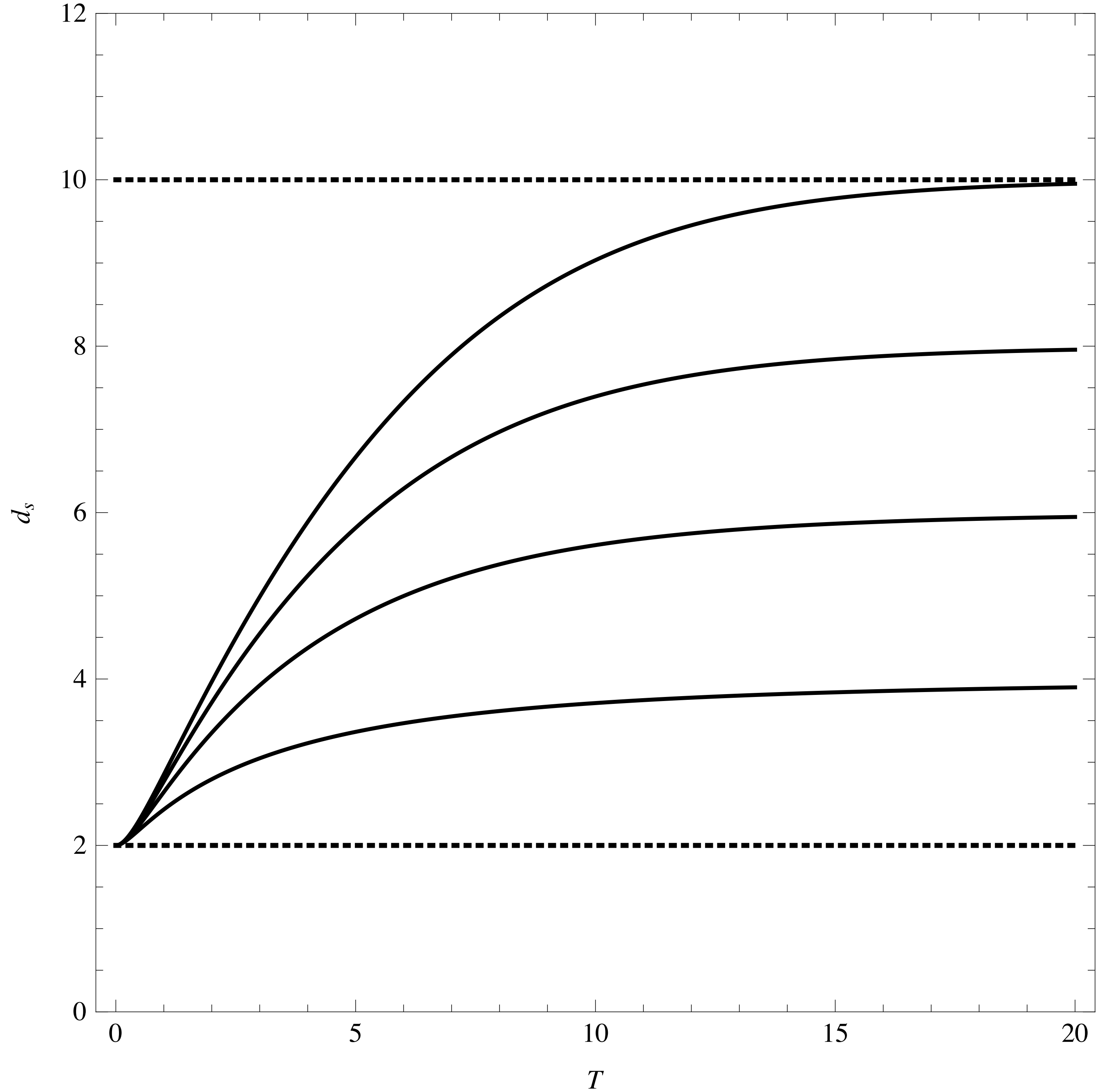
Theory II

$$H(-\Box_\Lambda) = -\Box_\Lambda$$

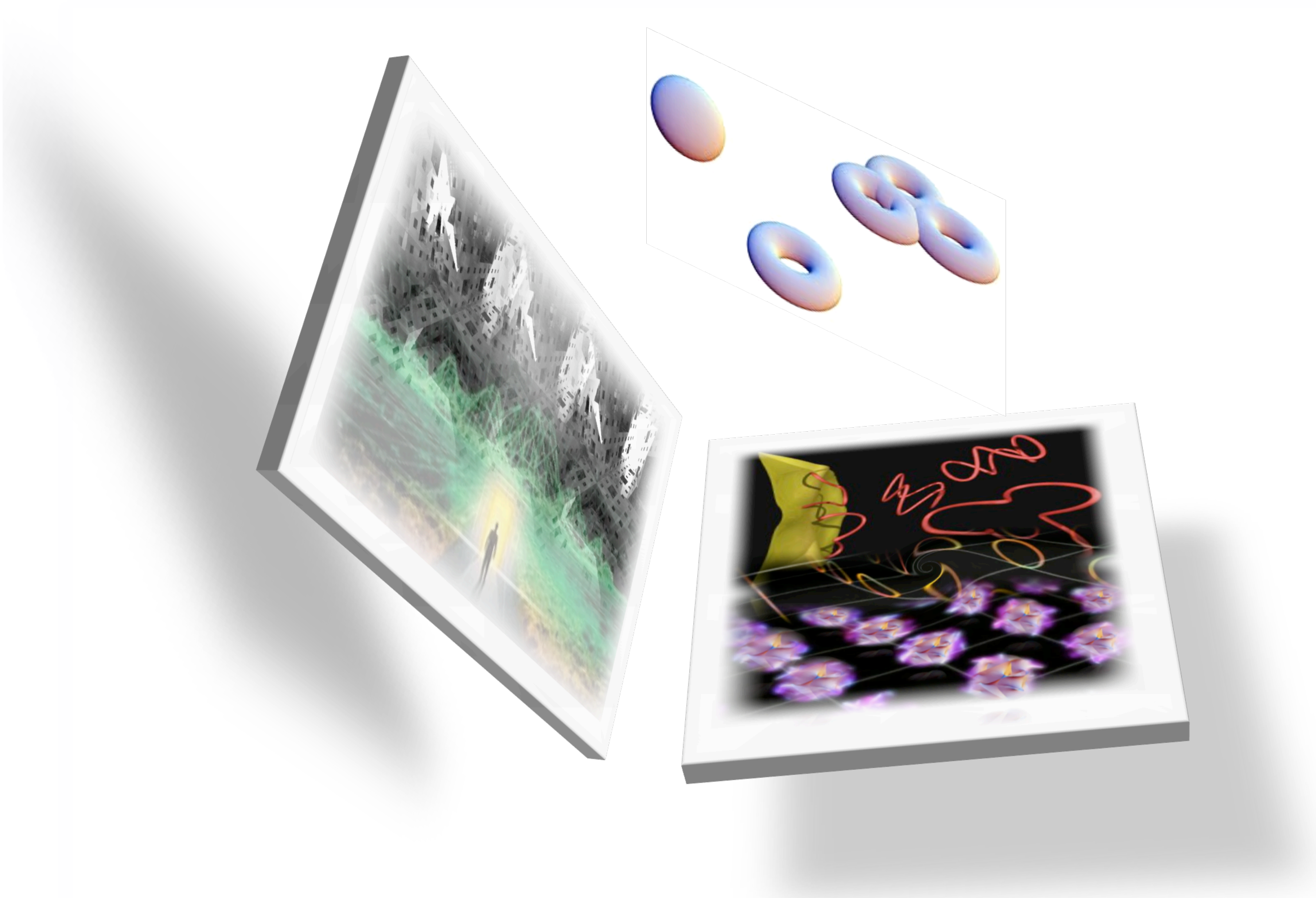


Theory III

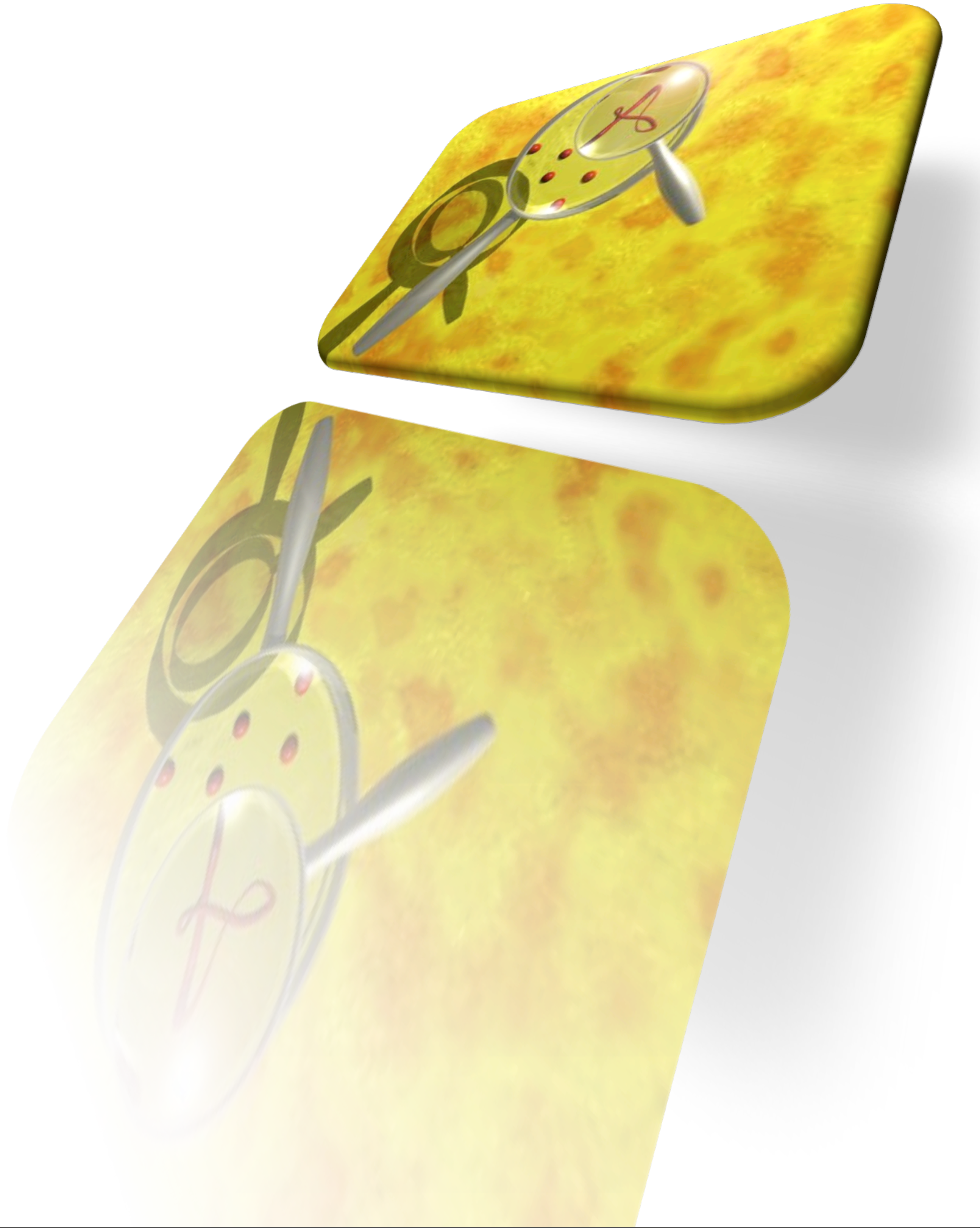
$$\gamma = 0.$$



Supergravity



Why non-locality?



Non-locality and Non-commutativity

(P. Nicolini, E. Spallucci, S. Alexander, A. Marciano', L.M.)

$$[x^\mu, x^\nu] = \theta^{\mu\nu} = \mathrm{diag}(\theta_1\, \epsilon^{ab} \theta_2 \epsilon^{ab}) \quad \rightarrow \quad \theta_1 = \theta_2 := \theta := 4/\Lambda^2\,.$$

For $a = 1, 2$:

$$\mathbf{z} = \frac{1}{\sqrt{2}}(\mathbf{x}_1 + i\mathbf{x}_2)\,, \;\; \mathbf{z}^\dagger = \frac{1}{\sqrt{2}}(\mathbf{x}_1 - i\mathbf{x}_2)\,,$$

$$|\alpha\rangle \equiv \exp\left[\frac{\Lambda^2}{4}\left(\alpha^*\mathbf{z} - \alpha\mathbf{z}^\dagger\right)\right]|0\rangle\,,$$

$$\mathbf{z}|\alpha\rangle = \alpha|\alpha\rangle\,, \;\; \langle\alpha|\mathbf{z}^\dagger = \langle\alpha|\alpha^*\,, \;\; \langle\alpha|\mathbf{x}_1|\alpha\rangle = \sqrt{2}\,\mathrm{Re}\,\alpha\,, \;\; \langle\alpha|\mathbf{x}_2|\alpha\rangle = \sqrt{2}\,\mathrm{Im}\,\alpha\,,$$

$$h_{\mu\nu}(x) = \int \frac{dp}{2p_0}\left[\mathbf{a}(p)^\dagger_{\mu\nu}\langle\alpha|e^{i\sum_1^2 \vec{p}_i\cdot\tilde{\mathbf{x}}_i}|\alpha\rangle + \mathrm{h.c.}\right].$$

$$\langle\alpha|e^{i\sum_1^2 \vec{p}_i\cdot\tilde{\mathbf{x}}_i}|\alpha\rangle = e^{-\sum_1^2 \frac{\vec{p}_i^2}{\Lambda^2} + i\sum_1^2 \vec{p}_i\cdot\vec{x}_i}\,,$$

$$\langle 0|T(h_{\mu\nu}(x)h_{\rho\sigma}(x')|0\rangle \equiv \langle 0|T(\langle\alpha|h_{\mu\nu}(\mathbf{x})|\alpha\rangle\,\langle\alpha'|h_{\rho\sigma}(\mathbf{x})|\alpha'\rangle)|0\rangle \propto \underbrace{\int d^2p\, \frac{e^{-\sum_1^2 \frac{p_i^2}{\Lambda^2}}\, e^{i\sum_1^2 (x_i - x_i')p_i}}{p^2} \times \mathrm{TS}}_{\text{SRQG-Propagator}}\,.$$

Stochastic Spacetime

Stochastic fluctuations of the light-cone

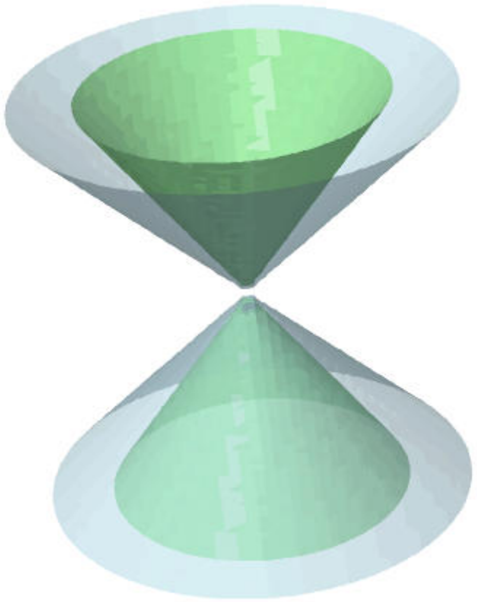
$$\hat{x} = (x_0 + ib_4, \mathbf{x} + \mathbf{b})\,, \;\; x_\mu = (x_0, \mathbf{x}) \;\; b_E = (b_4, \mathbf{b})\,,$$

$$s_R^2 := \langle s^2 \rangle = \int d^4 b_E \, w(b_E^2/l^2)[(x_0 + ib_4)^2 - (\mathbf{x}+\mathbf{b})^2] = x_0^2 - \mathbf{x}^2 - l^2\,,$$

$$\text{with measure} : w(b_E^2/l^2)\,, \;\; \int dw(b_E^2/l^2) = 1\,, \;\; dw(b_E^2/l^2) \geqslant 0\,.$$

$$\text{Field} : \;\; \phi_R(x) = \langle \phi(\hat{x}) \rangle = \int d^4 b_E \, w(b_E^2/l^2)\, \phi(x_0 + ib_4, \mathbf{x}+\mathbf{b})\,.$$

$$D(x_1 - x_2) = \langle 0|T[\phi(\hat{x}_1)\phi(\hat{x}_2)]|0\rangle = \int\int d^4 b_{1E}\, d^4 b_{2E}\, w(b_{1E}^2/l^2)\, w(b_{2E}^2/l^2) \times$$
$$\times \langle 0|T[\phi(x_{10} + ib_{14}, \mathbf{x_1}+\mathbf{b_1})\phi(x_{20}+ib_{24}, \mathbf{x_2}+\mathbf{b_2})]|0\rangle$$



$$\propto \int d^4 p \frac{e^{-p^2}}{p^2} \;\; \text{for} \;\; w(y^2) \propto e^{-2y^2}\,.$$