Neutrinos and Matter-Antimatter Asymmetries in String Universes with Torsion

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SEVENTH AEGEAN SUMMER SCHOOL
BEYOND EINSTEIN'S THEORY OF GRAVITY

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OUTLINE

Motivation: There is dominance of matter over antimatter

Higgs scalar gives masses to Standard Model (SM) particles except neutrinos

Seesaw mechanism can account for smallness of mass of active neutrino species through right-handed neutrinos in extensions of SM

Role of (heavy) Majorana Right-handed Neutrinos
In Leptogenesis/Baryogenesis: extra CP Violation & Dark Matter

In the talk: discuss role of pseudoscalar fields as responsible for both right-handed Majorana neutrino masses beyond seesaw and matter-antimatter asymmetry in the Universe with no need to adjust extra amount of CP Violation

PLAN OF THE TALK

General Remarks on Matter-induced Torsion relevant to our discussion

Motivation for using bosonic Kalb-Ramond (KR) torsion from
string theory to generate matter/antimatter asymmetry
in a (toy) string Universe

A scenario: Background KR torsion yields matter/antimmater asymmetry
Quantum torsion may yield Majorana neutrino masses
relevant to dark matter….

Why torsion?

REMARKS ON TORSIONFUL GEOMETRIES

Einstein theories of gravity → postulate that torsion is absent (constraint)

Einstein-Cartan and Kibble (1961) Sciama (1964) theories , this constraint is not used and torsion is considered as a dynamical field: vielbeins independent from spin connection

Fermions in gravitational field, when formulated in the so called first-order formalism, where spin connection and vierbein are treated as independent the **torsion** does ***not*** vanish

Fermions and Torsion

Gravity with Torsion contains ***Antisymmetric*** parts in the connection:

$$\omega_\mu^{ab} = \overline{\omega}_\mu^{ab} + K_\mu^{ab}$$

$$\overline{\omega}_\mu{}^{ab} = e^a_\nu \partial_\mu e^{\nu b} + e^a_\nu e^{\sigma b} \Gamma^\nu_{\sigma\mu} = e^a_\nu e^{\nu b}{}_{;\mu}$$

$$K_\mu^{ab} = e^a_\nu e^b_\rho K^{\nu\rho}_{\ \ \mu} \ , \quad K^{\nu\rho}_{\ \ \mu} = -K^{\rho\nu}_{\ \ \mu}$$

Contorsion tensor

Torsion $T^\mu_{\nu\rho}$

$$K^\nu_{\rho\mu} = \frac{1}{2}\left(T^\nu_{\rho\mu} - T_\rho{}^\nu{}_\mu - T_\mu{}^\nu{}_\rho\right)$$

Curvature tensor in first order torsionful formalism

$$R^{ab}_{\ \ \mu\nu} = 2\partial_{[\mu,}\, \omega^{ab}_{\ \ \nu]} + 2\omega^a_{\ c[\mu}\, \omega^{cb}_{\ \ \nu]}$$

Covariant derivative (on fermions):

$$D_\mu \psi = \left(\partial_\mu - \frac{i}{4} \omega_\mu^{a\,b} \sigma_{ab} \right) \psi \,, \qquad \sigma^{ab} = \frac{i}{2} \left[\tilde{\gamma}^a, \tilde{\gamma}^b \right]$$

$$\{ \tilde{\gamma}^a, \tilde{\gamma}^b \} = 2\eta^{ab} \ .$$

The resulting Dirac spin-$\frac{1}{2}$ fermion Lagrangian reads:

$$\mathcal{L}_{QED-fermions} = \overline{\psi} \gamma^\mu \left(iD_\mu - eA_\mu \right) \psi \equiv \overline{\psi} \gamma^\mu \left(i\mathcal{D}_\mu \right) \psi$$

$$\{ \gamma^\mu, \gamma^\nu \} = 2g^{\mu\nu}$$

$$\boxed{\begin{aligned} S_G + S_{\mathrm{QED-fermions}} &= \frac{1}{2\kappa} \int d^4x \, e \, e_a^\mu e_b^\nu R^{ab}{}_{\mu\nu}(\omega) + \\ &\frac{i}{2} \int d^4x \, e \left(\overline{\psi} \gamma^\mu \mathcal{D}_\mu(\omega, A) \psi - \overline{\mathcal{D}_\mu(\omega, A) \psi} \, \gamma^\mu \psi \right) + \dots \end{aligned}}$$

In form notation:
Curvature 2-form

$$R^{ab} = d\omega^{ab} + \omega_c^a \wedge \omega^{cb}$$

$$\mathbf{A} = \mathbf{A}_{\mu\nu} \, dx^\mu \wedge dx^\nu$$

$$\omega^{ab} = \omega_\mu^{ab} \, dx^\mu$$

Torsion and Induced Four-Fermi Interactions - The Einstein Cartan Theory

To study the effects of torsion in 1st order formalism, it is convenient to Decompose the torsion tensor into irreducible components according to their Transformation under the local Lorentz group of transformations:

$$T_{\mu\nu\rho} = \frac{1}{3}(T_\nu g_{\mu\rho} - T_\rho g_{\mu\nu}) - \frac{1}{6}\epsilon_{\mu\nu\rho\sigma}S^\sigma + q_{\mu\nu\rho}$$

$T_\mu \equiv T^\nu_{\ \mu\nu}$ *torsion trace vector,*

$S_\mu \equiv \epsilon_{\mu\nu\rho\sigma}T^{\nu\rho\sigma}$ *pseudotrace axial vector*

$q^\nu_{\ \rho\nu} = 0 = \epsilon^{\sigma\mu\nu\rho}q_{\mu\nu\rho}$ antisymmetric tensor

$$S_G = \frac{1}{2\kappa}\int d^4 x e \left(e^{\mu}_{a} e^{\nu}_{b} R^{ab}_{\ \ \mu\nu}(\overline{\omega}) + \frac{1}{24} S_\mu S^\mu - \frac{2}{3} T_\mu T^\mu + \frac{1}{2} q_{\mu\nu\rho} q^{\mu\nu\rho} \right)$$

Using

$$\tilde{\gamma}^a \tilde{\gamma}^b \tilde{\gamma}^c = \eta^{ab}\tilde{\gamma}^c + \eta^{bc}\tilde{\gamma}^a - \eta^{ac}\tilde{\gamma}^b - i\epsilon^{abcd}\tilde{\gamma}_5\tilde{\gamma}_d \ , \qquad \tilde{\gamma}_5 = i\,\tilde{\gamma}^0\tilde{\gamma}^1\tilde{\gamma}^2\tilde{\gamma}^3$$

We obtain for the fermionic part of the action

$$S_{\text{QED-fermions}} = \frac{i}{2}\int d^4 x e \left(e^{\mu}_{a} \left(\overline{\psi}\tilde{\gamma}^a \mathcal{D}_\mu(\overline{\omega}, A)\psi - \overline{\mathcal{D}_\mu(\overline{\omega}, A)\psi}\,\tilde{\gamma}^a \psi \right) + \frac{i}{4} S_\mu J^{\mu}_{(A)} \right) \ ,$$

$$S_G = \frac{1}{2\kappa}\int d^4x e\left(e^\mu_a e^\nu_b R^{ab}_{\mu\nu}(\overline{\omega}) + \frac{1}{24}S_\mu S^\mu - \frac{2}{3}T_\mu T^\mu + \frac{1}{2}q_{\mu\nu\rho}q^{\mu\nu\rho}\right)$$

Using

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Variation of the comined actions S_G + $S_{FERMION}$ w.r.t. T, S and q components of torsion, viewed as ***independent*** fields, yields:

$$T^{\mu} = 0, \quad \frac{1}{24\kappa}S^{\mu} - \frac{1}{8}J^{\mu}_{(A)} = 0 \ , \quad q_{\mu\nu\rho} = 0$$

$J^{\mu}_{(A)} \equiv \overline{\psi}\gamma_5\gamma^{\mu}\psi$ is the spinor *axial* current.

These solutions allow for the spin connection with torsion to be written as:

$$\boxed{\omega_{\mu}^{ab} = \overline{\omega}_{\mu}^{ab} + \frac{\kappa}{4}\epsilon^{ab}{}_{cd}e^{c}_{\mu}J^{d}_{(A)}}$$

So that torsion is associated with the spinor axial current.

Total Action in Einstein-Cartan theory

$$S_{\rm total} = \frac{1}{2\kappa}\int d^4x e\, e^\mu_a e^\nu_b R^{ab}_{\ \ \mu\nu}(\overline{\omega}) +$$
$$\frac{i}{2}\int d^4x e\left(e^\mu_a(\overline{\psi}\tilde{\gamma}^a\mathcal{D}_\mu(\overline{\omega},A)\psi - \overline{\mathcal{D}_\mu(\overline{\omega},A)\psi}\,\tilde{\gamma}^a\psi)\right) - \int d^4x e\,\frac{3}{16}\kappa J^\mu_{(A)}J_{(A)\mu}\ .$$

$\kappa = 8\,\pi\, G_N$ is the gravitational constant

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$\kappa = 8\,\pi\, G_N$ is the gravitational constant

fermion self interactions induced by quantum torsion

Why torsion? SUMMARY ON TORSION CONNECTIONS

(i) Dirac spin ½ fermions in Einstein-Cartan

$$\boxed{\omega_\mu^{ab} = \overline{\omega}_\mu^{ab} + \frac{\kappa}{4}\epsilon^{ab}{}_{cd}e_\mu^c J^d_{(A)}}$$

$J^\mu_{(A)} \equiv \overline{\psi}\gamma_5\gamma^\mu\psi$ is the spinor *axial* current.

(ii) N=1 SUGRA

$$\omega_\mu{}^{ab} = \omega_\mu{}^{ab}(e,\psi) \equiv \omega_\mu{}^{ab}(e) + \kappa_\mu{}^{ab}(\psi)$$

$$J^\mu(\psi) = \tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}_\nu\gamma_\alpha\psi_\beta.$$

$$\boxed{\kappa_{\mu\alpha\beta}(\psi) = \tfrac{1}{4}[\bar{\psi}_\alpha\gamma_\mu\psi_\beta + \bar{\psi}_\mu\gamma_\alpha\psi_\beta - \bar{\psi}_\mu\gamma_\beta\psi_\alpha].}$$

(iii) N=2 SUGRA

$$\omega_\mu{}^{ab} = \omega_\mu{}^{ab}(e,\psi) \equiv \omega_\mu{}^{ab}(e) + \kappa_\mu{}^{ab}(\psi)$$

$$J^\mu(\psi) = \epsilon^{\mu\nu\alpha\beta}\bar{\psi}^I_\nu\gamma_\alpha\psi_{I\beta}$$

$$\boxed{\kappa_{\mu\alpha\beta}(\psi) = \tfrac{1}{4}[\bar{\psi}^I_\alpha\gamma_\mu\psi_{I\beta} + \bar{\psi}^I_\mu\gamma_\alpha\psi_{I\beta} - \bar{\psi}^I_\mu\gamma_\beta\psi_{I\alpha} + \text{c.c.}].}$$

(iv) N=4 SU(4) SUGRA

$$\omega_{\mu ab} = \omega_{\mu ab}(e,\psi,\Lambda) \equiv \omega_{\mu ab}(e) + \kappa_{\mu ab}$$

$$J^\mu(\psi) = \epsilon^{\mu\nu\alpha\beta}\bar{\psi}^I_\nu\gamma_\alpha\psi_{I\beta}$$

$$J^\mu(\Lambda) = e\bar{\Lambda}_I\gamma^\mu\Lambda^I.$$

$$\begin{aligned}
\kappa_{\mu\alpha\beta} &= \kappa_{\mu\alpha\beta}(\psi) + \kappa_{\mu\alpha\beta}(\Lambda),\\
\kappa_{\mu\alpha\beta}(\psi) &= \frac{1}{4}[\bar{\psi}^I_\alpha\gamma_\mu\psi_{I\beta} + \bar{\psi}^I_\mu\gamma_\alpha\psi_{I\beta} - \bar{\psi}^I_\mu\gamma_\beta\psi_{I\alpha} + \text{c.c.}],\\
\kappa_{\mu\alpha\beta}(\Lambda) &= -\frac{1}{4e}\epsilon_{\mu\alpha\beta\sigma}\bar{\Lambda}_I\gamma^\sigma\Lambda^I. \qquad (31)
\end{aligned}$$

Ambiguities in the Einstein-Cartan Theory-The Immirzi parameter

Actions can be modified by total derivatives, without effects to on-shell Physics (i.e. equations of motion). But such terms may play a role in quantum Physics….

In pure gravity without torsion one can add the following identically zero term Due to Bianchi identities/symmetry properties of the Riemann tensor

$$\epsilon^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}(\bar{\omega}) = 0$$

No longer true in the presence of torsion

$$S_{\rm Holst} = -\frac{\beta}{4\kappa}\int d^4x e\, e^{\mu}_{a} e^{\nu}_{b} \epsilon^{ab}_{\ \ cd} R^{cd}_{\ \ \mu\nu}$$

non trivial in torsionful geometries

$\tilde{R}^{ab}_{\mu\nu} \equiv \epsilon^{ab}_{\ \ cd} R^{cd}_{\ \ \mu\nu}$ Is the dual of the curvature tensor

BUT

in case of torsion ….equations of motion imply:

$$T^{\mu} = \frac{3\kappa}{4}\frac{\beta}{\beta^2+1}J^{\mu}_{(A)} \ , \quad S^{\mu} = \frac{3\kappa}{\beta^2+1}J^{\mu}_{(A)} \ , \quad q_{\mu\nu\rho} = 0 \ .$$

Equates a vector T with a PSEUDOVECTOR $J_{(A)}$

Hence adding just the Holst term into the action of first order Einstein-Cartan theory (gravity with fermions) is

MATHEMATICALLY INCONSISTENT

Holst term needs modification by the addition of fermions

TO BECOME TOTAL DERIVATIVE ***(S. Mercuri, PR D77 (2008)***)

Add to the Holst action *additional* fermion-piece of the form

$$S_{\text{Holst-fermi}} = +\frac{\alpha}{2}\int d^4x e \left(\overline{\psi}\gamma^\mu\gamma_5\mathcal{D}_\mu(\omega)\psi + \overline{\mathcal{D}_\mu(\omega)}\gamma^\mu\gamma_5\psi\right) \ , \quad \alpha = \text{const.}$$

Together with Dirac kinetic terms, the fermion action the reads:

$$S_{\text{Dirac-Holst-fermi}} = \frac{i}{2}\int d^4x e \left[\overline{\psi}\gamma^\mu\left(1 - i\alpha\gamma_5\right)\mathcal{D}_\mu\psi + \overline{\mathcal{D}_\mu\psi}\gamma^\mu\left(1 - i\alpha\gamma_5\right)\psi\right]$$

IF $\boxed{\alpha = \beta}$ Holst total gravity-fermion term becomes total derivative

$$\boxed{S_{\text{Holst-total}} = -i\frac{\beta}{2}\int d^4x\left[I_{\text{NY}} + \partial_\mu J^\mu_{(A)}\right]}$$

Theory equivalent
To Einstein-Cartan
Independent of Immirzi β

$$I_{\text{NY}} \equiv \epsilon^{\mu\nu\rho\sigma}\left(T_{\mu\nu}{}^{a}T_{\rho\sigma\,a} - \frac{1}{2}e^a_\mu e^b_\nu R_{\rho\sigma ab}(\omega)\right)$$

$$I_{\text{NY}} = \epsilon^{\mu\nu\rho\sigma}\partial_\mu T_{\nu\rho\sigma}$$

Nieh-Yan
Topological invariant
Density – unique Lorentz
Invariant torsion structure

SUMMARY ON (SUPER)HOLST MODIFICATIONS

(i) Dirac spin ½ fermions

$$\boxed{S_{\mathrm{Holst-total}} = \frac{i\beta}{4}\int d^4x \partial_\mu J^\mu_{(A)} = -\frac{i\beta}{6}\int d^4x \epsilon^{\mu\nu\rho\sigma}\partial_\mu T_{\nu\rho\sigma}(\psi)}$$

$J^\mu_{(A)} \equiv \overline{\psi}\gamma_5\gamma^\mu\psi$ is the spinor *axial* current.

Tsuda, Kaul,

(ii) N=1 SUGRA

$J^\mu(\psi) = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}_\nu\gamma_\alpha\psi_\beta.$

$$\boxed{\begin{aligned} S_{\mathrm{SHolst1}}[\omega(e,\psi)] &= -\frac{i\eta}{4}\int d^4x \partial_\mu J^\mu(\psi) \\ &= \frac{i\eta}{2}\int d^4x \epsilon^{\mu\nu\alpha\beta}\partial_\mu T_{\nu\alpha\beta}(\psi) \end{aligned}}$$

(iii) N=2 SUGRA

$J^\mu(\psi) = \epsilon^{\mu\nu\alpha\beta}\bar{\psi}^I_\nu\gamma_\alpha\psi_{I\beta}$

$$\boxed{S_{\mathrm{SHolst2}}[\omega(e,\psi)] = -\frac{i\eta}{4}\int d^4x \partial_\mu J^\mu(\psi)}$$

(iv) N=4 SU(4) SUGRA

$J^\mu(\psi) = \epsilon^{\mu\nu\alpha\beta}\bar{\psi}^I_\nu\gamma_\alpha\psi_{I\beta}$

$J^\mu(\Lambda) = e\bar{\Lambda}_I\gamma^\mu\Lambda^I.$

$$\boxed{S_{\mathrm{SHolst4}}[\omega(e,\psi,\Lambda)] = -\frac{i\eta}{4}\int d^4x \partial_\mu[J^\mu(\psi) - J^\mu(\Lambda)]}$$

DISCUSS A DIFFERENT KIND OF **BOSONIC** TORSION CORRESPONDING TO A PSEUDOSCALAR DEGREE OF FREEDOM (STRING INSPIRED, **KALB-RAMOND ``AXION”**)

SCALAR FIELDS IN EARLY UNIVERSE

Scalar Fields in Early Universe: **Plenty of them in extended particle physics models: inflaton(s), dilaton, axions, moduli ….*Various effects…***

OUTLINE

Scalar Fields in Early Universe: Plenty of them in extended particle physics models: inflaton(s), dilaton, axions, moduli ….*Various effects…*

Concentrate on:

String-Inspired Models and Torsionful Geometries:

Torsionful Geometries of early universe due to a Kalb-Ramond Pseudoscalar (KR ``axion”):

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String-Inspired Models and Torsionful Geometries:
Torsionful Geometries of early universe due to a Kalb-Ramond Pseudoscalar (KR ``axion”):

(i) Matter/antimatter asymmetry in the Universe (Leptogenesis/Baryogenesis) right-handed neutrinos no need for extra CP Violation

(ii) Majorana Right-handed Neutrino masses from KR ``axions” (quantum torsion) beyond seesaw, through anomalies…

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Anomalous Majorana Neutrino Mass Generation due to quantum KR torsion

N.E.M. and A. Pilaftsis,
arXiv: 1209.6387
Phys. Rev. D86 (2012), 124038

General Remarks on CP Violation

- Within Standard Model, CP Violation not enough to produce observed Baryon over antibaryon Asymmetry (assume CPT in early Universe & B-, C-, & CP – Violation: Sakharov Conditions)

- Several Ideas to go beyond the SM (e.g. GUT models, Supersymmetry, extra dimensional models etc.)

- Massive ν are simplest extension of SM

- Right-handed massive ν may provide extensions of SM with:

  extra CP Violation and thus Origin of Universe's matter-antimatter asymmetry due to neutrino masses, Dark Matter

Within the Standard Model, lowest CP Violating structures

$$d_{CP} = \sin(\theta_{12})\sin(\theta_{23})\sin(\theta_{13})\sin\delta_{CP}$$
$$\cdot(m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)$$

Kobayashi-Maskawa CP Violating phase

Shaposhnikov

$$D = \mathrm{Im}\ \mathrm{Tr}\left[\mathcal{M}_u^2\mathcal{M}_d^2\mathcal{M}_u\mathcal{M}_d\right]$$

$$\boxed{\delta_{KM}^{CP} \sim \frac{D}{T^{12}} \sim 10^{-20}} \quad \text{<<} \quad \frac{n_B - \overline{n}_B}{n_B + \overline{n}_B} \sim \frac{n_B - \overline{n}_B}{s} = (8.4 - 8.9) \times 10^{-11}$$

$$T \simeq T_{\mathrm{sph}}$$

$$T_{\mathsf{sph}}(m_H) \in [130, 190]\mathsf{GeV}$$

This CP Violation Cannot be the Source of Baryon Asymmetry in The Universe

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SM Extension with N extra right-handed neutrinos

$\nu\mathrm{MSM}$

$$\boxed{L = L_{SM} + \bar{N}_I i\partial_\mu \gamma^\mu N_I - F_{\alpha I}\,\bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2}\,\bar{N}_I^c N_I + \mathrm{h.c.}}$$

Yukawa couplings Matrix

$$F = \widetilde{K}_L\, f_d\, \widetilde{K}_R{}^\dagger$$

νMSM

$$L = L_{SM} + \bar{N}_I i\partial_\mu \gamma^\mu N_I - F_{\alpha I}\, \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2}\, \bar{N}_I^c N_I + \text{h.c.}$$

Light Neutrino Masses through see saw

Minkowski,Yanagida, Mohapatra, Senjanovic Sechter, Valle …

$$m_\nu = -M^D \frac{1}{M_I}[M^D]^T .$$

$$M_D = F_{\alpha I} v$$

$$v = \langle \phi \rangle \sim 175 \text{ GeV}$$

$$M_D \ll M_I$$

SM Extension with N extra right-handed neutrinos

$$\nu\mathrm{MSM}$$

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From Constraints (compiled ν oscillation data) on (light) sterile neutrinos:
Giunti, Hernandez ...
N=1 *excluded by data*

Yukawa couplings Matrix (N=2 or 3) $F = \widetilde{K}_L\, f_d\, \widetilde{K}_R^\dagger$

Model with 2 or 3 singlet fermions works well in reproducing Baryon Asymmetry and is consistent with Experimental Data on neutrino oscillations

Model with N=3 also works fine, and in fact it allows ***one*** of the Majorana fermions to almost ***decouple*** from the rest of the SM fields, thus providing candidates for ***light*** (keV region of mass) sterile neutrino ***Dark Matter.***

SM Extension with N extra right-handed neutrinos

νMSM

PERTURBATIVELY CONSISTENT (STABLE)
STANDARD MODEL HIGGS SECTOR
WHEN RIGHT-HANDED NEUTRINOS PRESENT SEEMS TO REQUIRE :

THREE RIGHT-HANDED NEUTRINOS
TWO DEGENERATE IN MASS (HEAVIER THAN GeV)
ONE LIGHT (in keV region) → dark matter candidate
CPT IS ASSUMED AN EXACT SYMMETRY OF THE EARLY UNIVERSE
CP Violation enhanced due to degeneracy

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SM Extension with N extra right-handed neutrinos

Non SUSY νMSM

Boyarski, Ruchayskiy, Shaposhnikov

$$L = L_{SM} + \bar{N}_I i\partial_\mu \gamma^\mu N_I - F_{\alpha I}\,\bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2}\,\bar{N}_I^c N_I + \text{h.c.}$$

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It can also provide consistent model for Leptogenesis and, through, sphaleron processes, Baryogenesis.

Important:

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STANDARD MODEL HIGGS SECTOR
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CPT IS ASSUMED AN EXACT SYMMETRY OF THE EARLY UNIVERSE
CP Violation enhanced due to degeneracy but
may **not be sufficient** to reproduce Baryon Asymmetry:

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CPT IS ASSUMED AN EXACT SYMMETRY OF THE EARLY UNIVERSE
CP Violation enhanced due to degeneracy but
may **not be sufficient** to reproduce Baryon Asymmetry:
Mechanism for baryon asymmetry through coherent oscillations of degenerate in mass neutrinos

...rather delicate !

Thermal Properties

Ashaka, Shaposhnikov…

If M_I < M_W (electroweak scale), *e.g.* $M_I = O(1)$ GeV

Keep light neutrino masses in right order , Yukawa couplings must be:

$$F_{\alpha I} \sim \frac{\sqrt{m_{\rm atm} M_I}}{v} \sim 4 \times 10^{-8}$$

Baryogenesis through coherent oscillations right-handed singlet fermions

Akhmedov, Rubakov, Smirnov

Higgs mass stability vs higher loops → Mass degeneracy $N_{2,3}$

Assume ***Mass degeneracy*** $N_{2,3}$, hence **enhanced CP violation**
Coherent Oscillations between these singlet fermions

$$\omega \sim \frac{|M_2^2 - M_3^2|}{E_I} \sim \frac{M_2 \Delta M(T)}{T}$$

$$E_I \sim T$$
$$\Delta M(T) \ll M_2 \approx M_3$$

FOR CP VIOLATION TO OCCUR MUST HAVE: Oscillation rate > Hubble rate H(T)

Baryogenesis occurs @: $T_B \sim \left(M_I \Delta M(T) M_0\right)^{1/3}$ eg O(100) GeV

$$\frac{n_B}{s} \simeq 1.7 \cdot 10^{-10}\, \delta_{CP} \left(\frac{10^{-5}}{\Delta M(T)/M_2}\right)^{\frac{2}{3}} \left(\frac{M_2}{10\ \ \mathrm{GeV}}\right)^{\frac{5}{3}}$$

Mass N_2 (N_3) / (Mass N_1) = O(10^5)

N_1 Lightest Sterile nutrino is a natural DARK MATTER candidate

BAU ESTIMATES

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Baryogenesis occurs @: $T_B \sim \left(M_I \Delta M(T) M_0\right)^{1/3}$ eg O(100) GeV

Quite effective Mechanism: Maximal Baryon asymmetry $\Delta \equiv \frac{n_B - n_{\bar{B}}}{n_B + n_{\bar{B}}} \sim 1$

for $T_B = T_{sph} = T_{eq}$

Assumption: Interactions with plasma of SM particles
do not destroy quantum mechanical coherence of oscillations

BAU ESTIMATES

Assume ***Mass degeneracy*** $N_{2,3}$, hence enhanced CP violation
Coherent Oscillations between these singlet fermions

$$\boxed{\omega \sim \frac{|M_2^2 - M_3^2|}{E_I} \sim \frac{M_2 \Delta M(T)}{T}}$$

$$E_I \sim T$$

$$\Delta M(T) \ll M_2 \approx M_3$$

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for $T_B = T_{sph} = T_{eq}$

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do not destroy quantum mechanical coherence of oscillations

CONCLUSIONS FO FAR:

PERTURBATIVELY CONSISTENT (STABLE)
STANDARD MODEL HIGGS SECTOR
WHEN RIGHT-HANDED NEUTRINOS PRESENT
SEEMS TO REQUIRE :

THREE RIGHT-HANDED NEUTRINOS
TWO DEGENERATE IN MASS (HEAVIER THAN GeV)
ONE LIGHT (in keV region) → **dark matter candidate**

CPT IS ASSUMED AN EXACT SYMMETRY OF THE EARLY UNIVERSE
CP Violation enhanced due to degeneracy but
may **not be sufficient** to reproduce Baryon Asymmetry

Can we have alternatives to this CP Violation but with the important role of Right-handed Neutrinos maintained?

νMSM

This talk: fluctuations of pseudoscalar-induced torsion → dynamical mass M_I generation + background torsion → matter/antimatter asymmetry

$$\boxed{L = L_{SM} + \bar{N}_I i\partial_\mu \gamma^\mu N_I - F_{\alpha I}\, \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2}\, \bar{N}_I^c N_I + \text{h.c.}}$$

Minkowski, Yanagida, Mohapatra, Senjanovic Sechter, Valle …

Light Neutrino Masses through see saw

$$\boxed{m_\nu = -M^D \frac{1}{M_I} [M^D]^T\ .}$$

$$M_D = F_{\alpha I} v$$

$$v = \langle \phi \rangle \sim 175 \text{ GeV}$$

$$M_D \ll M_I$$

PART II
CPT VIOLATION IN (TORSIONFUL) GEOMETRIES OF THE EARLY UNIVERSE DUE TO KR ``AXIONS”

CPT VIOLATION IN THE EARLY UNIVERSE

GENERATE Baryon and/or Lepton ASYMMETRY without Heavy Sterile Neutrinos?

CPT Invariance Theorem :

(i) Flat space-times
(ii) Lorentz invariance
(iii) Locality
(iv) Unitarity

Schwinger, Pauli, Luders, Jost, Bell revisited by:
Greenberg, Chaichian, Dolgov, Novikov…

(ii)-(iv) Independent reasons for violation

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Background induced

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GRAVITATIONAL BACKGROUNDS GENERATING **CPT VIOLATING EFFECTS** IN THE EARLY UNIVERSE:

PARTICLE-ANTIPARTICLE DIFFERENCES IN DISPERSION RELATIONS →

Differences in populations

→ freeze out → Baryogenesis or

→ Leptogenesis → Baryogenesis

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REVIEW VARIOUS SCENARIOS

GRAVITATIONAL BACKGROUNDS GENERATING **CPT VIOLATING EFFECTS** IN THE EARLY UNIVERSE:

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B-L conserving GUT
or
Sphaleron

REVIEW VARIOUS SCENARIOS

CPTV Effects of different Space-Time-Curvature/Spin couplings between neutrinos/ antineutrinos

B. Mukhopadhyay, U. Debnath, N. Dadhich, M. Sinha Lambiase, Mohanty

Curvature Coupling to **fermion spin** may lead to different dispersion relations between neutrinos and antineutrinos (assumed ***dominant*** in the Early eras) in **non-spherically symmetric** geometries in the Early Universe.

Dirac Lagrangian

$$\mathcal{L} = \sqrt{-g}\left(i\,\bar{\psi}\,\gamma^a D_a\psi - m\,\bar{\psi}\psi\right)$$

$$D_a = \left(\partial_a - \frac{i}{4}\omega_{bca}\sigma^{bc}\right),$$

Gravitational covariant derivative including spin connection

$$\sigma^{ab} = \tfrac{i}{2}\left[\gamma^a, \gamma^b\right]$$

$$\omega_{bca} = e_{b\lambda}\left(\partial_a e_c^\lambda + \Gamma^\lambda_{\gamma\mu} e_c^\gamma e_a^\mu\right).$$

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_I = \sqrt{-g}\bar{\psi}\left[(i\gamma^a\partial_a - m) + \gamma^a\gamma^5 B_a\right]\psi,$$

$$B^d = \epsilon^{abcd} e_{b\lambda}\left(\partial_a e_c^\lambda + \Gamma^\lambda_{\alpha\mu} e_c^\alpha e_a^\mu\right)$$

Dirac Lagrangian (for concreteness, it can be extended to Majorana neutrinos)

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for the Majorana neutrinos, above $\mathcal{L}_I$ turns out explicitly as

$$\mathcal{L}_I = \psi_L^\dagger \gamma^a \psi_L B_a, \qquad \mathcal{L}_I = -\psi_L^{c\,\dagger} \gamma^a \psi_L^c B_a$$

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Standard Model Extension type Lorentz-violating coupling (Kostelecky *et al.*)

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For homogeneous and isotropic **Friedman-Robertson-Walker** geometries the resulting B^μ ***vanish***

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Can be constant in a given local frame in Early Universe **axisymmetric (Bianchi) cosmologies** or **near rotating Black holes**

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Need Non-diagonal metric components

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Can be constant in a given local frame in Early Universe
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BIANCHI COSMOLOGIES

Debnath et al. hep-ph/0510351

$$ds^2 = -dt^2 + S(t)^2\,dx^2 + R(t)^2\,[dy^2 + f(y)^2\,dz^2] - S(t)^2\,h(y)\,[2dx - h(y)\,dz]\,dz$$

$$f(y) = \{y, \sinh y, \sin y\}, \qquad h(y) = \{-y^2/2, -\cosh y, \cos y\}$$

we have $$B^0 = \frac{S[-f^2R^2(hf'R + Sh') + h^2S^2(hf'R + Sh') + 2fhRS(Rf' - hh'S)]}{f^4R^4 + f^2h^2R^2S^2}$$

$$B^2 = \frac{h[-f^2R^2 + 2fRS + h^2S^2][RS' - R'S]}{f^3R^4 + fh^2R^2S^2}.$$

and thus for $\quad$ Bianchi II $$B^0 = \frac{4R^3S + 3y^2RS^3 - 2y\,S^4}{8R^4 + 2y^2R^2S^2}$$

$$B^2 = \frac{(4y\,R^2 - 8R\,S - y^3S^2)(R\,S' - R'S)}{8R^4 + 2y^2R^2S^2}$$

Bianchi VIII:

$$B^0 = \frac{S[2\cosh^2 y(\cosh 2y - 3)R\,S^2 - 4\cosh^2 y\,\sinh y\,S^3 + 4R^3\,\cosh^2 y\,\sinh^2 y - R^2 S(5\,\sinh y + \sinh 3y)}{4(\cosh^2 y\,\sinh^2 y\,R^2S^2 + R^4\sinh^4 y)}$$

$$B^2 = \frac{\cosh y(S^2\cosh^2 y + 2R\,S\,\sinh y - R^2\,\sinh^2 y)(RS' - R'S)}{\cosh^2 y\,\sinh y\,R^2S^2 + R^4\sinh^3 y},$$

and for Bianchi IX:

$$B^0 = \frac{S[2\cos^2 y(3 - \cos 2y)RS^2 - 4\cos^2 y\,\sin y\,S^3 - 4R^3\cos^2 y\,\sin^2 y + R^2 S(5\sin y + \sin 3y)]}{4(\cos^2 y\,\sin^2 y\,R^2S^2 + R^4\sin^4 y)}$$

$$B^2 = \frac{\cos y(S^2\cos^2 y + 2RS\,\sin y - R^2\sin^2 y)(RS' - R'S)}{\cos^2 y\,\sin y\,R^2S^2 + R^4\sin^3 y}.$$

Neutrino/antineutrino asymmetry around black holes

B. Mukhopadhyay, astro-ph/0505460

Consider the metric of a Kerr **(rotating)** black hole

$$ds^2 = \eta_{ij}\, dx^i\, dx^j - \left[\frac{2\alpha}{\rho} s_i\, v_j + \alpha^2\, v_i\, v_j\right] dx^i\, dx^j$$

$$\alpha = \frac{\sqrt{2Mr}}{\rho}, \qquad \rho^2 = r^2 + \frac{a^2 z^2}{r^2} \qquad v_i = \left(1,\ \frac{ay}{a^2 + r^2},\ \frac{-ax}{a^2 + r^2},\ 0\right)$$

$$s_i = \left(0,\ \frac{rx}{\sqrt{r^2 + a^2}},\ \frac{ry}{\sqrt{r^2 + a^2}},\ \frac{z\sqrt{r^2 + a^2}}{r}\right)$$

and we have $$r^4 - r^2\left(x^2 + y^2 + z^2 - a^2\right) - a^2 z^2 = 0$$

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Mass

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angular momentum

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and we have $$r^4 - r^2\left(x^2+y^2+z^2-a^2\right) - a^2 z^2 = 0$$

Calculate the vector B^μ can show that it is non vanishing, hence there is CPT Violation induced in this case

Neutrino/antineutrino asymmetry around black holes

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$$B^0 = e_{1\lambda}\left(\partial_3 e_2^\lambda - \partial_2 e_3^\lambda\right) + e_{2\lambda}\left(\partial_1 e_3^\lambda - \partial_3 e_1^\lambda\right) + e_{3\lambda}\left(\partial_2 e_1^\lambda - \partial_1 e_2^\lambda\right) = -\frac{4a\sqrt{M}z}{\bar{\rho}^2\sqrt{2r^3}},$$

where $\bar{\rho}^2 = 2r^2 + a^2 - x^2 - y^2 - z^2$.

Consider the neutrino-antineutrino population difference

$$\Delta n = \frac{g}{(2\pi)^3}\int_{R_i}^{R_f} dV \int d^3|\vec{p}|\left[\frac{1}{1+exp(E_\nu/T)} - \frac{1}{1+exp(E_{\bar{\nu}}/T)}\right]$$

Consider Kerr black holes for which $\vec{B}.\vec{p} \ll B_0\, p^0$, can show that

$$\Delta n = \frac{g}{(2\pi)^2} T^3 \int_{R_i}^{R_f}\int_0^\infty\int_0^\pi \left[\frac{1}{1+e^u\, e^{B_0/T}} - \frac{1}{1+e^u\, e^{-B_0/T}}\right] u^2\, d\theta\, du\, dV$$

$$u = |\vec{p}|/T$$

Then, if $B^0 \ll T$

$$\Delta n \sim g\, T^3 \left(\frac{\overline{B_0}}{T}\right)$$

averaged over the spatial volume V

DISPERSION RELATIONS OF NEUTRINOS ARE *DIFFERENT* FROM THOSE OF ANTINEUTRINOS IN *SUCH* GEOMETRIES IF *B^a* = CONSTANT IN A LOCAL FRAME

$(p_a \pm B_a)^2 = m^2,$ ± refers to chiral fields (here neutrino/antineutrino)

CPTV Dispersion relations

$$E = \sqrt{(\vec{p} - \vec{B})^2 + m^2} + B_0 \;, \quad \bar{E} = \sqrt{(\vec{p} + \vec{B})^2 + m^2} - B_0$$

but (bare) masses are equal between particle/anti-particle sectors

Abundances of neutrinos in Early Universe, then, ***different*** from those of antineutrinos if **B_0** is ***non-trivial***, **ALREADY IN THERMAL EQUILIBRIUM**

Abundances of neutrinos in Early Universe **different** from those of antineutrinos if $B^0 \neq 0$ asnd constant in a local frame

$$\Delta n = \frac{g}{(2\pi)^3} \int d^3\mathrm{p} \left[\frac{1}{1+\exp(E_\nu/T)} - \frac{1}{1+\exp(E_{\bar{\nu}}/T)} \right]$$

$$\Delta n = \frac{g}{(2\pi)^2} T^3 \int_0^\infty \int_0^\pi \left[\frac{1}{1+e^u\, e^{B_0/T}} - \frac{1}{1+e^u\, e^{-B_0/T}} \right] u^2\, d\theta\, du$$

$$u = |\vec{p}|/T$$

$$\boxed{\Delta n_\nu \equiv n_\nu - n_{\overline{\nu}} \sim g^\star T^3 \left(\frac{B_0}{T} \right)}$$

with $g^\star$ the number of degrees of freedom for the (relativistic) neutrino.

DISPERSION RELATIONS OF NEUTRINOS ARE *DIFFERENT* FROM THOSE OF ANTINEUTRINOS IN *SUCH* GEOMETRIES

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Lepton Asymmetry, *e.g*. for neutrinos

$$\boxed{\Delta L(T < T_d) = \frac{\Delta n_\nu}{s} \sim \frac{B_0}{T_d}}$$

CPTV BARYOGENESIS through B-L conserving sphalerons? NO NEED FOR ENHANCED CP VIOLATION IN EARLY UNVIERSE?

$$\Delta n \sim g T^3 \left(\frac{\overline{B_0}}{T} \right)$$

REMARKS

Asymmetry depends on the sign of $\boldsymbol{B^0}$

PRIMORDIAL BLACK HOLES WITH MASSES $M_{BH} < 10^{15}$ gm have evaporated today, only BH with masses $M_{BH} > 10^{15}$ gm may survive today

Hawking temperature $T = \dfrac{\hbar}{8\pi k_B M} \sim 10^{-7} K \left(\dfrac{M_\odot}{M} \right)$

$T \sim 10^{11}$ K $\sim 1.6 \times 10^{-5}$ erg, $\overline{B_0} \sim 1.6 \times 10^{-6}$ erg, then $\Delta n \sim 10^{-16}$.

To reproduce observed Baryon asymmetry $\Delta n = O(10^{-10})$ we need 10^6 BH with the same sign of B^0 → ***fine tuning …..***

Fermions in Gravity with TORSION

NEM & Sarben Sarkar, arXiv:1211.0968

Ellis, NEM, Sarkar 1304.5433

A SPECIFIC KIND OF TORSION (KALB-RAMOND FIELD STRENGTH)
INSPIRED FROM STRING THEORY (UV COMPLETE)
CAN DO THE JOB OF PROVIDING A CONSTANT B^0
AXIAL BACKGROUND IN A LOCAL FRAME OF FRW COSMOLOGY

Fermions in Gravity with TORSION

Dirac Lagrangian (for concreteness, it can be extended to Majorana neutrinos)

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$$e^\mu{}_a e^\nu{}_b \eta^{a\,b} = g^{\mu\nu}$$

If torsion then $\boldsymbol{\Gamma_{\mu\nu} \neq \Gamma_{\nu\mu}}$
antisymmetric part is the contorsion tensor, contributes to

$$B^d = \epsilon^{abcd} e_{b\lambda}\left(\partial_a e^\lambda_c + \Gamma^\lambda_{\alpha\mu} e^\alpha_c e^\mu_a\right)$$

vielbeins (tetrads) independent from spin connection $\omega_\mu{}^{ab}$ now ....

of interest to us here...cf below

- Field Theories with (Kalb-Ramond) torsion & axion fields : *String inspired models, loop quantum gravity effective field theories …*

UV complete models

- Majorana Neutrino Masses from (three-loop) ***anomalous*** terms with axion-neutrino couplings

Microscopic UV complete underlying theory of quantum gravity :

STRINGS

Massless Gravitational multiplet of (closed) strings: spin 0 scalar (dilaton)
spin 2 traceless symmetric rank 2 tensor (graviton)
spin 1 antisymmetric rank 2 tensor

KALB-RAMOND FIELD $B_{\mu\nu} = -B_{\nu\mu}$

Effective field theories (low energy scale E << M_s) `` gauge" invariant

$$B_{\mu\nu} \to B_{\mu\nu} + \partial_{[\mu}\theta(x)_{\nu]}$$

Depend only on field strength : $H_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]}$

Bianchi identity : $\boxed{\partial_{[\sigma}H_{\mu\nu\rho]} = 0 \to d \star \mathbf{H} = 0}$

Anomaly (gravitational vs gauge) **cancellation** in strings require redefinition of H so that Bianchi identity now is extended to :

$$\mathbf{H} = \mathbf{d}\ \mathbf{B} + \frac{\alpha'}{8\kappa}\Big(\Omega_L - \Omega_V\Big)$$

$$\kappa^2 = 8\pi G_N = \frac{8\pi}{M_P^2}$$

Lorentz (L) & Gauge (V) Chern-Simons three forms

EXTENDED BIANCHI IDENTITY

$$\mathbf{d}\ \mathbf{H} = \frac{\alpha'}{8\kappa}\mathrm{Tr}\Big(\mathbf{R}\wedge\mathbf{R} - \mathbf{F}\wedge\mathbf{F}\Big)$$

ROLE OF Kalb-Ramond H-FIELD AS TORSION

EFFECTIVE GRAVITATIONAL ACTION IN STRING LOW-ENERGY LIMIT CAN BE EXPRESSED IN TERMS OF ***A GENERALIZED CURVATURE*** RIEMANN TENSOR WHERE THE CHRISTOFFEL CONNECTION INCLUDES **H-FIELD *TORSION***

4-DIM PART

$$S^{(4)} = \int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}R - \frac{1}{6}H_{\mu\nu\rho}\,H^{\mu\nu\rho}\Big)$$
$$= \int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}\overline{R} - \frac{1}{3}\kappa^2 H_{\mu\nu\rho}\,H^{\mu\nu\rho}\Big)$$

$$\overline{\Gamma}^{\mu}_{\nu\rho} = \Gamma^{\mu}_{\nu\rho} + \frac{\kappa}{\sqrt{3}}H^{\mu}_{\nu\rho} \neq \overline{\Gamma}^{\mu}_{\rho\nu}$$

IN 4-DIM DEFINE DUAL OF H AS :

$$\boxed{-3\sqrt{2}\partial_\sigma b = \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}}$$

***b(x)* = Pseudoscalar (Kalb-Ramond (KR) axion)**

FERMIONS COUPLE TO H –TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

$$S_\psi = \frac{i}{2}\int d^4x\sqrt{-g}\Big(\overline{\psi}\gamma^\mu \overline{\mathcal{D}}_\mu \psi - (\overline{\mathcal{D}}_\mu \overline{\psi})\gamma^\mu \psi\Big)$$

TORSIONFUL CONNECTION, FIRST-ORDER FORMALISM

$$\overline{\mathcal{D}}_\mu = \overline{\nabla}_\mu - ieA_\mu$$

$$\overline{\omega}_{ab\mu} = \omega_{ab\mu} + K_{ab\mu}$$

gauge field

contorsion

$$K_{abc} = \frac{1}{2}\Big(T_{cab} - T_{abc} - T_{bca}\Big)$$

$$H_{cab}$$

Non-trivial contributions to $\boldsymbol{B^\mu}$

$$B^d = \epsilon^{abcd} e_{b\lambda}\left(\partial_a e_c^\lambda + \Gamma^\lambda_{\alpha\mu} e_c^\alpha e_a^\mu\right)$$

$$\overline{\Gamma}^\mu_{\nu\rho} = \Gamma^\mu_{\nu\rho} + \frac{\kappa}{\sqrt{3}} H^\mu_{\nu\rho} \neq \overline{\Gamma}^\mu_{\rho\nu}$$

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Constant *H* ?

$$\overline{\Gamma}^\mu_{\nu\rho} = \Gamma^\mu_{\nu\rho} + \frac{\kappa}{\sqrt{3}} H^\mu_{\nu\rho} \neq \overline{\Gamma}^\mu_{\rho\nu}$$

Exact (conformal Field Theories on World-sheet) Solutions from String theory

Antoniadis, Bachas, Ellis, Nanopoulos

Cosmological Solutions, non-trivial time-dependent dilatons, axions

In Einstein frame *E* (Scalar curvature term in gravitational effective action has canonical normalisation):

$$ds^2 = g^E_{\mu\nu}(x)dx^\mu dx^\nu = dt^2 - a(t)^2 \delta_{ij} dx^i dx^j$$

$$a(t) = t$$

$$\Phi = -\ln a(t) + \phi_0$$

$$H_{\mu\nu\rho} = e^{2\Phi}\epsilon_{\mu\nu\rho\sigma}\partial^\sigma b(x) \qquad b(x) = \sqrt{2}e^{-\phi_0}\sqrt{Q^2}\,\frac{M_s}{\sqrt{n}}t$$

Central charge of uderlying world-sheet conformal field theory $n \in Z^+$

$$c = 4 - 12Q^2 - \frac{6}{n+2} + c_I$$

``internal'' dims central charge

Kac-Moody algebra level

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Conformal Invariance (world-sheet) Conditions for the string to O(α')
(lowest non-trivial order in derivatives in the effective target-space action):

$$\beta^{g(1)}_{\mu\nu} = R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \phi - \frac{1}{4} H_{\mu\rho\sigma} H_\nu{}^{\rho\sigma} \quad = 0$$

$$\beta^{B(1)}_{\mu\nu} = -\frac{1}{2} \nabla^\rho H_{\rho\mu\nu} + H_{\rho\mu\nu} \partial^\rho \phi \quad = 0$$

$$\beta^{\phi(1)} = \frac{D-26}{6\alpha'} - \frac{1}{2}\nabla^2 \phi + \partial^\rho \phi \partial_\rho \phi - \frac{1}{24} H_{\mu\nu\rho} H^{\mu\nu\rho} \quad = 0$$

Linear dilaton solution in string frame (or logarithmic in FRW time in Einstein-frame with conformally flat Einstein-frame target space time → exact (to all orders in α')

Exact (conformal Field Theories on World-sheet) Solutions from String theory

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H-torsion & CPTV

NEM & Sarben Sarkar, arXiv:1211.0968

Covariant Torsion tensor

$$\overline{\Gamma}^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\mu\nu} + e^{-2\Phi} H^{\lambda}_{\mu\nu} \equiv \Gamma^{\lambda}{}_{\mu\nu} + T^{\lambda}{}_{\mu\nu}$$

$$\boxed{T_{ijk} \sim \epsilon_{ijk} \dot{b} = \epsilon_{ijk} \sqrt{2Q^2} e^{-\phi_0} \frac{M_s}{\sqrt{n}}}$$

Constant

constant B^0

$$\boxed{B^0 \sim \sqrt{2Q^2} e^{-\phi_0} \frac{M_s}{\sqrt{n}} \text{ GeV} > 0.}$$

Lepton Asymmetry as in previous cases , *e.g.* for neutrinos

$$\Delta L(T < T_d) = \frac{\Delta n_\nu}{s} \sim \frac{B_0}{T_d}$$

DISPERSION RELATIONS OF NEUTRINOS ARE *DIFFERENT* FROM THOSE OF ANTINEUTRINOS IN *SUCH* GEOMETRIES

$(p_a \pm B_a)^2 = m^2,$ ± refers to chiral fields (here neutrino/antineutrino)

CPTV Dispersion relations

$$E = \sqrt{\vec{p}^2 + m^2} + B_0, \qquad E = \sqrt{\vec{p}^2 + m^2} - B_0$$

but (bare) masses are equal between particle/anti-particle sectors

Abundances of neutrinos in Early Universe, then, ***different*** from those of antineutrinos if **B_0** is ***non-trivial***, **ALREADY IN THERMAL EQUILIBRIUM**

Lepton Asymmetry, *e.g*. for neutrinos

$$\boxed{\Delta L(T < T_d) = \frac{\Delta n_\nu}{s} \sim \frac{B_0}{T_d}}$$

CPTV BARYOGENESIS through B-L conserving sphalerons? NO NEED FOR ENHANCED CP VIOLATION IN EARLY UNVIERSE?

CPTV → neutrino/antineutrino mixing & oscillations

M Sinha & B. Mukhopadhyay
arXiv: 0704.2593

$$\mathcal{L} = det(e)\,\bar{\Psi}\left(\frac{i}{2}\gamma^a \overleftrightarrow{\partial}_a - m + \gamma^a\gamma^5 B_a\right)\Psi$$

$$B^d = \epsilon^{abcd}\omega_{bca}$$

$$E_\nu = \sqrt{(\vec{p}-\vec{B})^2 + m^2} + B_0,$$

$$E_{\nu^c} = \sqrt{(\vec{p}+\vec{B})^2 + m^2} - B_0$$

Assume Majorana neutrino in Weyl rep (Lepton number violation unavoidable)

$$\Psi = \begin{pmatrix} \psi_L^c \\ \psi_L \end{pmatrix} \qquad \mathcal{D}_\mu \equiv (\partial_0, \partial_i + \gamma^5 B_i).$$

Majorana mass term violates L number

$$\boxed{(-g)^{-1/2}\mathcal{L} = \begin{pmatrix} \psi^{c\dagger} & \psi^\dagger \end{pmatrix} \frac{i}{2}\gamma^0\gamma^\mu \overleftrightarrow{\mathcal{D}}_\mu \begin{pmatrix} \psi^c \\ \psi \end{pmatrix} - \begin{pmatrix} \psi^{c\dagger} & \psi^\dagger \end{pmatrix} \begin{pmatrix} -B_0 & -m \\ -m & B_0 \end{pmatrix} \begin{pmatrix} \psi^c \\ \psi \end{pmatrix}}$$

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$$\Psi = \begin{pmatrix} \psi_L^c \\ \psi_L \end{pmatrix} \qquad \mathcal{D}_\mu \equiv (\partial_0, \partial_i + \gamma^5 B_i).$$

Lead to CP Violating neutrino/antineutrino mixing & oscillations

$$\boxed{(-g)^{-1/2} \mathcal{L} = \left(\psi^{c\dagger} \;\; \psi^\dagger \right) \frac{i}{2} \gamma^0 \gamma^\mu \overleftrightarrow{\mathcal{D}}_\mu \begin{pmatrix} \psi^c \\ \psi \end{pmatrix} - \left(\psi^{c\dagger} \;\; \psi^\dagger \right) \begin{pmatrix} -B_0 & -m \\ -m & B_0 \end{pmatrix} \begin{pmatrix} \psi^c \\ \psi \end{pmatrix}}$$

mass eigenstates ν_1 and ν_2 as

$$|\nu_1\rangle = \frac{1}{N}\left\{\left(B_0 + \sqrt{B_0^2 + m^2}\right)|\psi^c\rangle + m\,|\psi\rangle\right\}$$
$$|\nu_2\rangle = \frac{1}{N}\left\{-m\,|\psi^c\rangle + \left(B_0 + \sqrt{B_0^2 + m^2}\right)|\psi\rangle\right\}.$$

$$m_{1,2} = \mp\sqrt{B_0^2 + m^2}.$$

$$|\nu_1\rangle = \cos\theta\,|\psi^c\rangle + \sin\theta\,|\psi\rangle$$
$$|\nu_2\rangle = -\sin\theta\,|\psi^c\rangle + \cos\theta\,|\psi\rangle$$

NB: neutrino CPTV mass shifts

neutrino/antineutrino mixing

$$\tan\theta = \frac{m}{B_0 + \sqrt{B_0^2 + m^2}}.$$

$$|\psi^c\rangle = \cos\theta\,|\nu_1\rangle - \sin\theta\,|\nu_2\rangle$$
$$|\psi\rangle = \sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle.$$

oscillations

$$\mathcal{P}(t) = \sin^2 2\theta\,\sin^2\delta(t)$$
$$= \frac{m^2}{B_0^2 + m^2}\,\sin^2\{(B_0 - |\vec{B}|)t\}$$

$$\delta(t) = \frac{|E_\nu - E_{\nu^c}|t}{2},$$

mass eigenstates ν_1 and ν_2 as

$$|\nu_1\rangle \;=\; \frac{1}{N}\left\{\left(B_0+\sqrt{B_0^2+m^2}\right)\,|\psi^c\rangle \;+\; m\,|\psi\rangle\right\}$$

$$|\nu_2\rangle \;=\; \frac{1}{N}\left\{-m\,|\psi^c\rangle \;+\; \left(B_0+\sqrt{B_0^2+m^2}\right)\,|\psi\rangle\right\}.$$

$$m_{1,2} = \mp\sqrt{B_0^2+m^2}.$$

$$|\nu_1\rangle \;=\; \cos\theta\,|\psi^c\rangle \;+\; \sin\theta\,|\psi\rangle$$
$$|\nu_2\rangle \;=\; -\sin\theta\,|\psi^c\rangle \;+\; \cos\theta\,|\psi\rangle$$

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$$\tan\theta \;=\; \frac{m}{B_0+\sqrt{B_0^2+m^2}}.$$

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oscillations

$$\mathcal{P}(t) \;=\; \sin^2 2\theta\,\sin^2\delta(t)$$
$$=\; \frac{m^2}{B_0^2+m^2}\,\sin^2\{(B_0-|\vec{B}|)t\}$$

oscilaltion lengt $\lambda = \dfrac{\pi}{B_0-|\vec{B}|}$.

$$\delta(t) = \frac{|E_\nu - E_{\nu^c}|t}{2},$$

Ellis, NEM, Sarkar 1304.5433

SCENARIO: Phase transitions in the early string Universe may induce

B^0 = 0 @ $T = T_d$

T > T_d : $B^0 \neq 0$ ➔ **Majorana Neutrino-Antineutrino Oscillations** (Pontecorvo-type)

Extra CP Violation but no need to tune any decay width

4-component chiral Majorana spinor

$$\Psi = \begin{pmatrix} \psi_L^c \\ \psi_L \end{pmatrix}$$

$$\mathcal{L}_\nu = \sqrt{-g}\Big[\left(\psi^{c\dagger} \quad \psi^\dagger\right)\frac{i}{2}\gamma^0\gamma^\mu D_\mu \begin{pmatrix} \psi^c \\ \psi \end{pmatrix} - \left(\psi^{c\dagger} \quad \psi^\dagger\right)\begin{pmatrix} -B_0 & -m \\ -m & B_0 \end{pmatrix}\begin{pmatrix} \psi^c \\ \psi \end{pmatrix}$$

Mass eigenstates

$$|\chi_1\rangle = \mathcal{N}^{-1}\left\{\left(B_0 + \sqrt{B_0^2 + m^2}\right)|\psi^c\rangle + m\,|\psi\rangle\right\} ,$$

$$|\chi_2\rangle = \mathcal{N}^{-1}\left\{-m\,|\psi^c\rangle + \left(B_0 + \sqrt{B_0^2 + m^2}\right)|\psi\rangle\right\}$$

with eigenvalues

$$m_{1,2} = \mp\sqrt{B_0^2 + m^2}$$

$$\mathcal{N} \equiv \left[2\left(B_0^2 + m^2 + B_0\sqrt{B_0^2 + m^2}\right)\right]^{1/2}$$

Neutrino/antineutrino Mixing & Oscillations

$$\nu \equiv \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \psi^c \\ \psi \end{pmatrix} \qquad \tan\theta \equiv \frac{m}{B_0 + \sqrt{B_0^2 + m^2}}$$

Number operators

$$N_{\chi_1} = <: \chi_1^\dagger \chi_1 :>= \cos^2\theta \;\; <: \psi^{c\dagger} \psi^c :> +\sin^2\theta \;\; <: \psi^\dagger \psi :>$$

$$N_{\chi_2} = <: \chi_2^\dagger \chi_2 :>= \sin^2\theta \;\; <: \psi^{c\dagger} \psi^c :> +\cos^2\theta \;\; <: \psi^\dagger \psi :>$$

$$N_{\chi_1} - N_{\chi_2} = \cos 2\theta \Big(< n_{\psi^c} > - < n_\psi > \Big)$$ $\neq 0$ for $\theta \neq \pi/4, \; i.e., \; B_0 \neq 0$

Probability of Oscillation

$$\boxed{\mathcal{P}(t) = |\langle \nu_1(t) | \nu_2(0) \rangle|^2 = \sin^2\theta \sin^2\left(\frac{E_\nu - E_{\nu^c}}{2} \, t \right) = \frac{m^2}{B_0^2 + m^2} \sin^2(B_0 \, t)}$$

Oscillation length $$L = \frac{\pi \hbar c}{|B_0|} = \frac{6.3 \times 10^{-14} \, \mathrm{GeV}}{B_0} \, \mathrm{cm}$$

Neutrino/Antineutrino oscillations are the local processes in the early Universe responsible for the CPT (& also CP) Violation

They occur provided the oscillation length is smaller than the Hubble horizon

For **T_d = O(10^9 GeV)** the Hubble horizon size is 10^{-12} cm & for **B_0 = O(0.1) GeV** the oscillation length is 10^{-13} cm, i.e. smaller than Hubble Horizon, so oscilaltions can occur → provide chemical equilibrium for $T > T_d$

NB: If neutrinos have also Dirac mass, i.e. mass matrix in lagrangian of the form

$$\mathcal{M} = \begin{pmatrix} m_D - B_0 & -m \\ -m & m_D + B_0 \end{pmatrix}$$

$$\mathcal{P}^s(t) \simeq \frac{m^2}{B_0^2 + m^2} \sin^2\left(\frac{m_D\sqrt{B_0^2+m^2}}{E}\, t\right), \qquad p \gg m$$

with oscillation length depending on the neutrino energy *E*

$$L^s \simeq \frac{\pi E}{m_D\sqrt{B_0^2 + m^2}}$$

Neutrino/natineutrino oscillation processes are ***UNIQUE*** to Majorana neutrinos

Charged leptons and quarks of the Standard Model, which also couple to H-torsion, cannot exhibit such oscillation due to electric charge conservation

Hence: above scenario for Leptogenesis @ T = 10^9 GeV and then Baryogenesis at T = O(100 GeV) through standard-model B-L conserving sphaleron processes appears **unique to Majorana Neutrinos**

Consistent with absence of observed CPTV today in neutrino sector

Torsion $B^0 = 0$ (or very small) today

Standard Thermal Leptogenesis

Independent of Initial Conditions @ $T \gg T_{eq}$

Heavy Right-handed Majorana neutrinos enter ***equilibrium at $T = T_{eq} > T_{decay}$***

Lepton number Violation

$$N_I \to H\nu,\ \bar{H}\bar{\nu}$$

enhanced CP V

Out of Equilibrium Decays

$$T \simeq T_{\rm decay} > T_{\rm sph}$$

Produce Lepton asymmetry

$$L = \frac{2}{M} l_L l_L \phi\phi + \text{H.c.}$$

where

$$l_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

Equilibrated electroweak B+L violating sphaleron interactions

Independent of Initial Conditions

Observed Baryon Asymmetry In the Universe (BAU)

Estimate BAU by solving Boltzmann equations for Heavy Neutrino Abundances

Standard Thermal Leptogenesis

Independent of Initial Conditions @ T >>T$_{eq}$

Heavy Right-handed Majorana neutrinos enter ***equilibrium at T = T$_{eq}$ > T$_{decay}$***

~~**Lepton number Violation**~~

~~$N_I \to H\nu,\ \bar{H}\bar{\nu}$~~

~~**enhanced CPV**~~

Out of Equilibrium Decays

$$T \simeq T_{\rm decay} > T_{\rm sph}$$

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Estimate BAU by solving Boltzmann equations for Heavy Neutrino Abundances

CPT Violating Thermal Leptogenesis

Ellis, NEM, Sarkar
1304.5433

Early Universe
$T > T_d = 10^9$ GeV

CPT Violation

No need for **tuned enhanced CPV.** Heavy Right-handed Majorana **neutrino/antineutrino CPV & CPTV oscillations**

particle/antiparticle populations difference already in thermal equilibrium

KR axion →
torsion B^0 vector

Produce Lepton asymmetry

T = O(100) GeV

$$L = \frac{2}{M} l_L l_L \phi\phi + \mathrm{H.c.}$$

where

$$l_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

Equilibrated electroweak B+L violating sphaleron interactions

Independent of Initial Conditions

Observed Baryon Asymmetry In the Universe (BAU)

Estimate BAU by fixing CPTV background parameters
In some models this may imply fine tuning ….

B^0 : (string) theory underwent a **phase transition** @ $T = T_d = 10^9$ GeV, **to** :

(i) **either $B^0 = 0$**

(ii) **or B^0 small today but non zero**

If a small B^a is present today

Standard Model Extension type coupling b_μ

Kostelecky, Mewes, Russell, Lehnert …

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_I = \sqrt{-g}\bar{\psi}\left[(i\gamma^a\partial_a - m) + \gamma^a\gamma^5 B_a\right]\psi,$$

If a small B^a is present today

Standard Model Extension type coupling b_μ

Kostelecky, Mewes, Russell, Lehnert …

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_I = \sqrt{-g}\bar{\psi}\left[(i\gamma^a\partial_a - m) + \gamma^a\gamma^5 B_a\right]\psi,$$

If due to H-torsion, it should couple universally (gravity) to all particle species of the standard model (electrons etc)

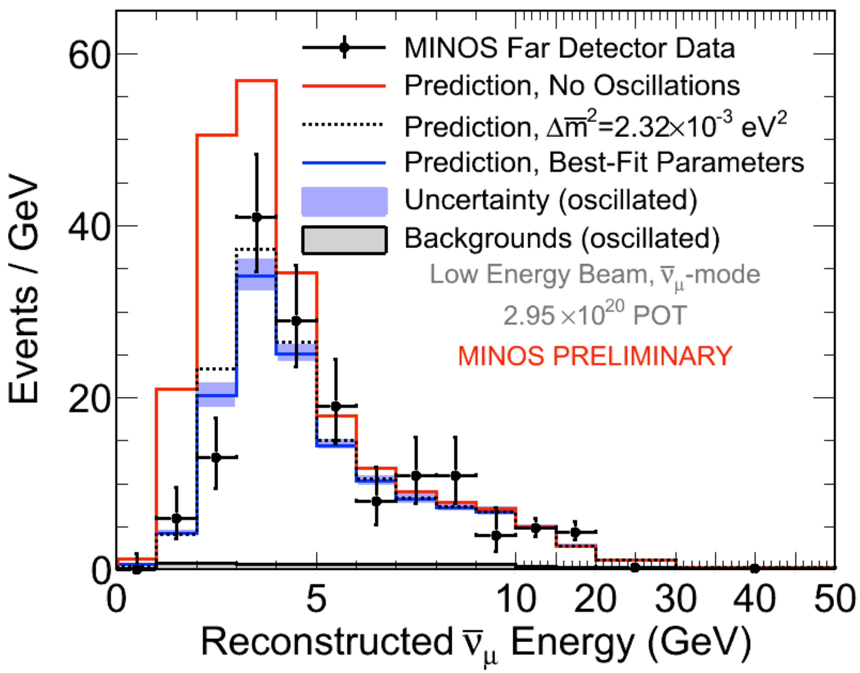
Very Stringent constraints from astrophysics on spatial ONLY components (e.g. Masers)

$$B_i \equiv b_i < 10^{-31}\,\mathrm{GeV} \qquad |B^0| < 10^{-2}\,\mathrm{eV}$$

+ STRINGENT CONSTRAINTS TODAY DUE TO NON-OBSERVATION OF NEUTRINO/ANTINEUTRINO OSCILLATIONS?

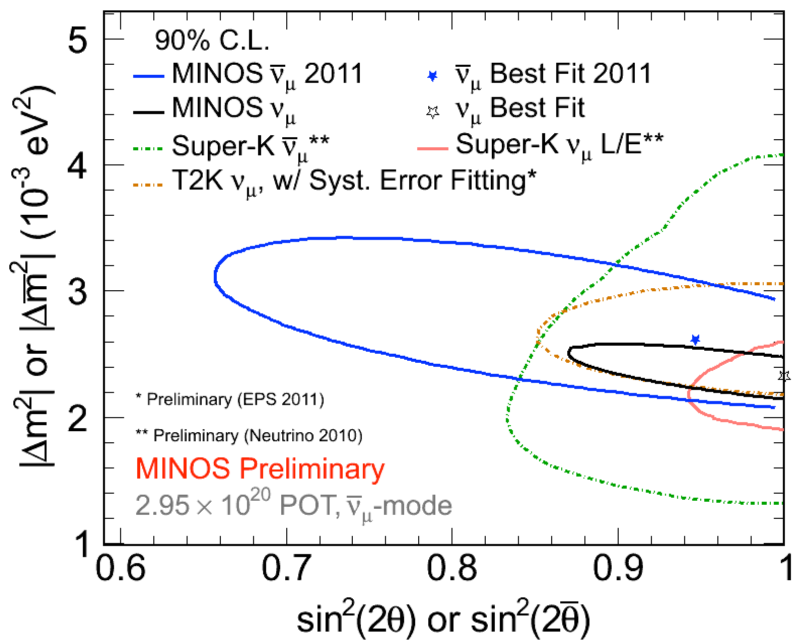
MINOS Exp. RESULTS ON Potential Neutrino-Antineutrino OSCILLATION PARAMETER DIFFERENCES

http://www-numi.fnal.gov



$\bar{\nu}_\mu$ disapearance-Energy spectrum

[arXiv:1108.1509]



$\bar{\nu}_\mu$ vs ν_μ Oscillation parameters

[arXiv:1104.0344] [arXiv1103.0340]

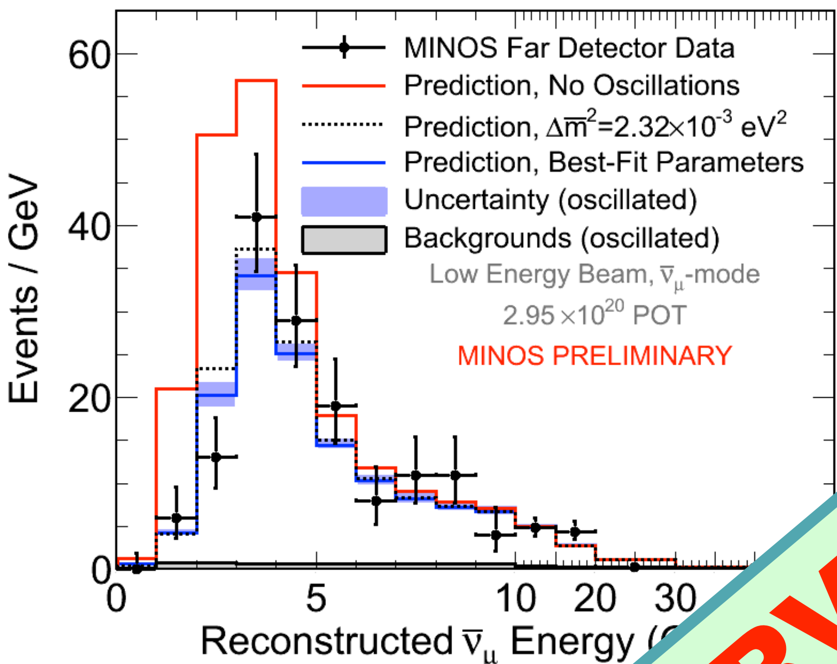
$\bar{\nu}_\mu$ disappearance $\Delta\bar{m}^2=(2.62+0.31-0.28$ (stat.) ±0.09 (syst.) $)\times10^{-3}$ eV^2, $\sin^2(2\Theta)=0.95$ $+0.10-0.11$ (stat.) ±0.01 (syst.).

ν_μ disappearance: $\Delta m^2=(2.32+0.12-0.08)\times10^{-3}$ eV^2 , $\sin^2(2\Theta)=1.00$ ($\sin^2(2\Theta)>0.90$ @ 90% CL

Consistent with equality of mass differences between particle/antiparticles

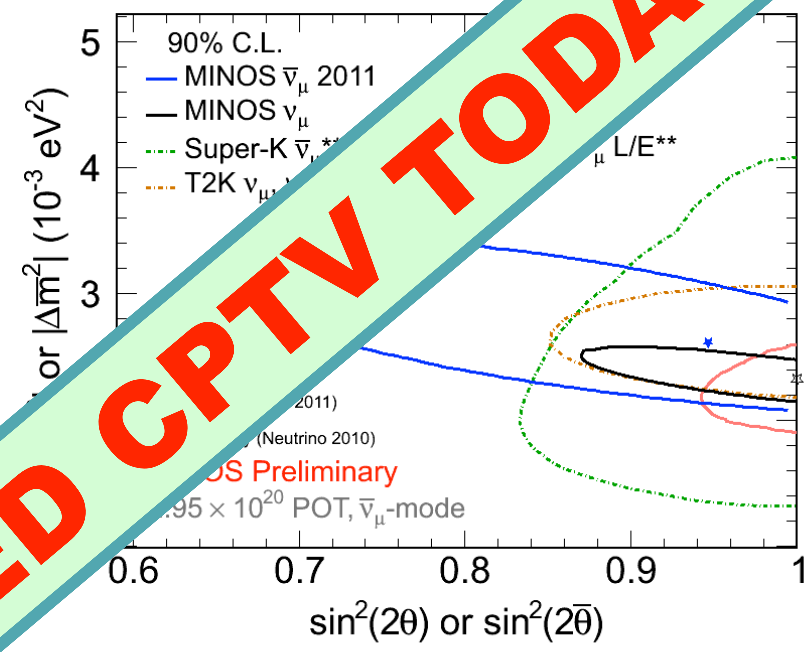
MINOS Exp. RESULTS ON Potential Neutrino-Antineutrino OSCILLATION PARAMETER DIFFERENCES

http://ww... l.gov



$\bar{\nu}_\mu$ disapearance-Energ...

...9]



$\bar{\nu}_\mu$ vs ν_μ Oscillation parameters

[arXiv:1104.0344] [arXiv1103.0340]

NO OBSERVED CPTV TODAY

$\bar{\nu}_\mu$ disappearanc... +0.31-0.28 (stat.) ±0.09 (syst.))x10^{-3} eV^2,
...)=0.95 +0.10-0.11 (stat.) ±0.01 (syst.).

ν_μ disappe... m^2=(2.32+0.12-0.08)x10^{-3} eV^2 , $\sin^2(2\Theta)$ =1.00 ($\sin^2(2\Theta)$ > 0.90 @ 90% CL

Consisten... with equality of mass differences between particle/antiparticles

PART III:
QUANTUM H-TORSION-INDUCED MASS HIERARCHY FOR RIGHT-HANDED NEUTRINO

ABOVE: We have seen how a background of Kalb-Ramond H-Torsion generates Matter-Antimatter Asymmetry via (Right-handed) neutrino/antineutrino oscillations in the Early Universe

ABOVE: We have seen how a background of
Kalb-Ramond H-Torsion generates
Matter-Antimatter Asymmetry via
(Right-handed) neutrino/antineutrino oscillations
in the Early Universe

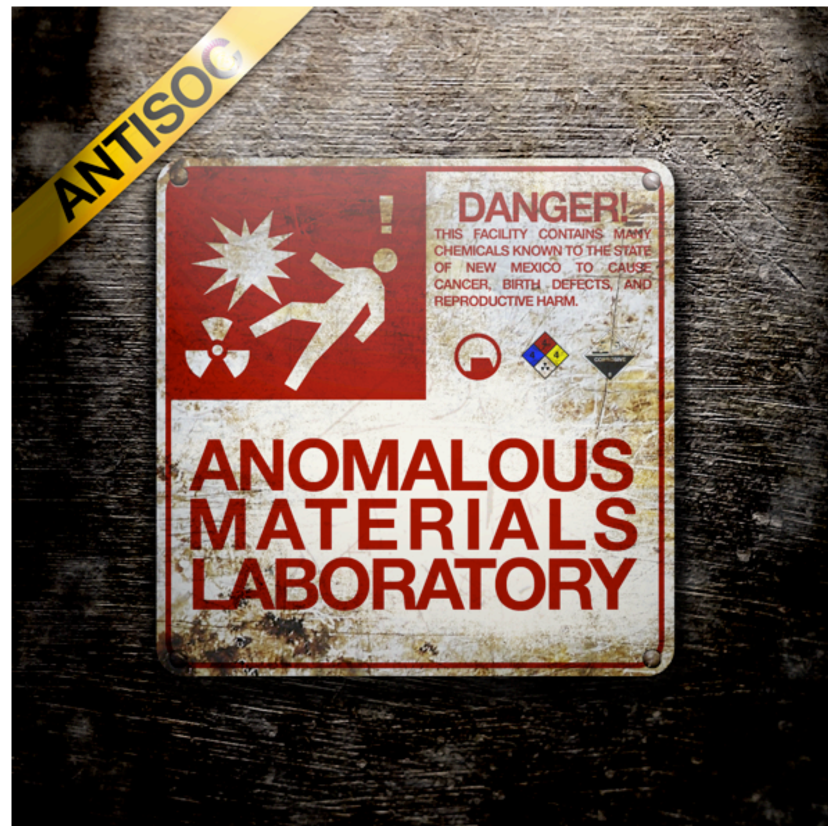
What About the Quantum Fluctuations of the H-torsion ?

Physical Effect in Generating Majorana masses for neutrinos
via coupling to ordinary axion fields

ANOMALOUS GENERATION OF RIGHT-HANDED MAJORANA NEUTRINO MASSES THROUGH TORSIONFUL QUANTUM GRAVITY

UV complete string models ?

NEM & Pilaftsis 2012
PRD 86, 124038
arXiv:1209.6387



Fermionic Field Theories with H-Torsion
EFFECTIVE ACTION AFTER INTEGRATING OUT QUANTUM TORSION FLUCTUATIONS

Fermions: $$S_\psi \ni -\frac{3}{4}\int d^4\sqrt{-g}\, S_\mu \bar{\psi}\gamma^\mu\gamma^5\psi = -\frac{3}{4}\int S \wedge {}^\star J^5$$

+ standard Dirac terms without torsion

$$\mathbf{S} = {}^\star\mathbf{T}$$

$$S_d = \frac{1}{3!}\epsilon^{abc}{}_d T_{abc}$$

$$T_{abc} \to H_{cab} = \epsilon_{cabd}\,\partial^d b$$

conserved ``torsion '' charge

Bianchi identity $$\mathbf{d}\,{}^\star\mathbf{S} = 0$$

classical

$$Q = \int {}^\star\mathbf{S}$$

Postulate conservation at quantum level by adding counterterms

Implement $\mathbf{d}\,{}^\star\mathbf{S} = 0$ via $\delta(d\,{}^\star\mathbf{S})$ constraint
→ lagrange multiplier in Path integral → b-field

Fermionic Field Theories with H-Torsion
EFFECTIVE ACTION AFTER INTEGRATING OUT QUANTUM TORSION FLUCTUATIONS

Fermions: $S_\psi \ni -\frac{3}{4}\int d^4\sqrt{-g}\, S_\mu \bar{\psi}\gamma^\mu\gamma^5\psi = -\frac{3}{4}\int S \wedge {}^\star J^5$

+ standard Dirac terms without torsion

$$\mathbf{S} = {}^\star\mathbf{T}$$

$$S_d = \frac{1}{3!}\epsilon^{abc}{}_d T_{abc}$$

$$T_{abc} \to H_{cab} = \epsilon_{cabd}\,\partial^d b$$

Bianchi identity

$$\mathrm{d}\,{}^\star\mathrm{S} = 0$$

classical

conserved ``torsion '' charge

$$Q = \int {}^\star\mathrm{S}$$

Postulate conservation at quantum level by adding counterterms

Implement $\mathrm{d}\,{}^\star\mathrm{S} = 0$ via $\delta(d\,{}^\star\mathrm{S})$ constraint
→ lagrange multiplier in Path integral → b-field

$$\int D\mathbf{S}\, Db \exp\Big[i\int \frac{3}{4\kappa^2}\mathbf{S}\wedge{}^{\star}\mathbf{S} - \frac{3}{4}\mathbf{S}\wedge{}^{\star}\mathbf{J}^5 + \Big(\frac{3}{2\kappa^2}\Big)^{1/2} b\, d{}^{\star}\mathbf{S}\Big]$$

$$= \int Db \exp\Big[-i\int \frac{1}{2}\mathrm{d}b\wedge{}^{\star}\mathrm{d}b + \frac{1}{f_b}\mathrm{d}b\wedge{}^{\star}\mathbf{J}^5 + \frac{1}{2f_b^2}\mathbf{J}^5\wedge\mathbf{J}^5\Big]$$

multiplier field $\Phi(x) \equiv (3/\kappa^2)^{1/2} b(x)$.

$$f_b = (3\kappa^2/8)^{-1/2} = \frac{M_P}{\sqrt{3\pi}}$$

$$\int D\mathbf{S}\, Db \exp\Big[i \int \frac{3}{4\kappa^2}\mathbf{S}\wedge {}^\star\mathbf{S} - \frac{3}{4}\mathbf{S}\wedge {}^\star\mathbf{J}^5 + \Big(\frac{3}{2\kappa^2}\Big)^{1/2} b\, d^\star\mathbf{S}\Big]$$
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partial integrate

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partial integrate

Use chiral anomaly equation (one-loop) in curved space-time:

$$\begin{aligned}\nabla_\mu J^{5\mu} &= \frac{e^2}{8\pi^2}F^{\mu\nu}\widetilde{F}_{\mu\nu} - \frac{1}{192\pi^2}R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma} \\ &\equiv G(\mathbf{A},\omega)\,.\end{aligned}$$

Hence, effective action of torsion-full QED

$$\int Db\, \exp\left[-i\int \frac{1}{2}\mathrm{d}b\wedge{}^\star\mathrm{d}b - \frac{1}{f_b}bG(\mathbf{A},\omega) + \frac{1}{2f_b^2}\mathbf{J}^5\wedge\mathbf{J}^5\right].$$

SUMMARY ON (SUPER)HOLST MODIFICATIONS

(i) Dirac spin ½ fermions

$$S_{\rm Holst-total} = \frac{i\beta}{4}\int d^4x\, \partial_\mu J^\mu_{(A)} = -\frac{i\beta}{6}\int d^4x\, \epsilon^{\mu\nu\rho\sigma}\partial_\mu T_{\nu\rho\sigma}(\psi)$$

promotion here of the constant coefficient
of the super Holst action
to a dynamical pseudoscalar field (KR axion)

$$\int D\mathbf{S}\, Db\, \exp\left[i\int \frac{3}{4\kappa^2}\mathbf{S}\wedge{}^\star\mathbf{S} - \frac{3}{4}\mathbf{S}\wedge{}^\star\mathbf{J}^5 + \left(\frac{3}{2\kappa^2}\right)^{1/2} b\, d{}^\star\mathbf{S}\right]$$

$$= \int Db\, \exp\left[-i\int \frac{1}{2}\mathbf{d}b\wedge{}^\star\mathbf{d}b + \frac{1}{f_b}\mathbf{d}b\wedge{}^\star\mathbf{J}^5 + \frac{1}{2f_b^2}\mathbf{J}^5\wedge\mathbf{J}^5\right]$$

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$$\boxed{bR\tilde{R} - bF\tilde{F}}$$

coupling

Hence, effective action of torsion-full QED

$$\int Db\, \exp\left[-i\int \frac{1}{2}\mathbf{d}b\wedge{}^\star\mathbf{d}b - \frac{1}{f_b}bG(\mathbf{A},\omega) + \frac{1}{2f_b^2}\mathbf{J}^5\wedge\mathbf{J}^5\right].$$

Fermionic Field Theories with H-Torsion
EFFECTIVE ACTION AFTER INTEGRATING OUT QUANTUM TORSION FLUCTUATIONS

$$\mathcal{S} = \int d^4x\sqrt{-g}\,\Big[\frac{1}{2}(\partial_\mu b)^2 + \frac{b(x)}{192\pi^2 f_b} R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma} + \frac{1}{2f_b^2} J^5_\mu J^{5\mu} \; +$$

+ Standard Model terms for fermions

SHIFT SYMMETRY b(x) → b(x) + c

$c\,R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma}$ and $c\,F^{\mu\nu}\widetilde{F}_{\mu\nu}$ total derivatives

Mavromatos, Pilaftsis arXiv: 1209.6387

ANOMALOUS MAJORANA NEUTRINO MASS TERMS from QUANTUM TORSION

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$c\, R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma}$ and $c\, F^{\mu\nu}\widetilde{F}_{\mu\nu}$. total derivatives

OUR SCENARIO ***Break*** such ***shift symmetry*** by coupling first b(x) to another pseudoscalar field such as QCD axion a(x) (or e.g. other string axions)

$$\mathcal{S} = \int d^4x\sqrt{-g}\Big[\frac{1}{2}(\partial_\mu b)^2 + \frac{b(x)}{192\pi^2 f_b}R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma} + \frac{1}{2f_b^2}J^5_\mu J^{5\mu} + \gamma(\partial_\mu b)\,(\partial^\mu a) + \frac{1}{2}(\partial_\mu a)^2 - y_a i a\left(\overline{\psi}_R^{\ C}\psi_R - \overline{\psi}_R\psi_R^{\ C}\right)\Big]\,,$$

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ANOMALOUS MAJORANA NEUTRINO MASS TERMS from QUANTUM TORSION

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Yukawa

neutrino fields

Field redefinition

$$b(x) \to b'(x) \equiv b(x) + \gamma a(x)$$

so, effective action becomes

$$\mathcal{S} = \int d^4x \sqrt{-g}\, \Big[\frac{1}{2}(\partial_\mu b')^2 + \frac{1}{2}\left(1-\gamma^2\right)(\partial_\mu a)^2 + \frac{1}{2f_b^2} J^5_\mu J^{5\mu} + \frac{b'(x) - \gamma a(x)}{192\pi^2 f_b} R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma} - y_a i a\left(\overline{\psi}_R^C \psi_R - \overline{\psi}_R \psi_R^C\right)\Big] .$$

must have $\boxed{|\gamma| < 1}$ ***otherwise axion field a(x) appears as a ghost → canonically normalised kinetic terms***

$$\mathcal{S}_a = \int d^4x \sqrt{-g}\, \Big[\frac{1}{2}(\partial_\mu a)^2 - \frac{\gamma a(x)}{192\pi^2 f_b \sqrt{1-\gamma^2}} R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma} - \frac{i y_a}{\sqrt{1-\gamma^2}}\, a\left(\overline{\psi}_R^C \psi_R - \overline{\psi}_R \psi_R^C\right) + \frac{1}{2f_b^2} J^5_\mu J^{5\mu}\Big] .$$

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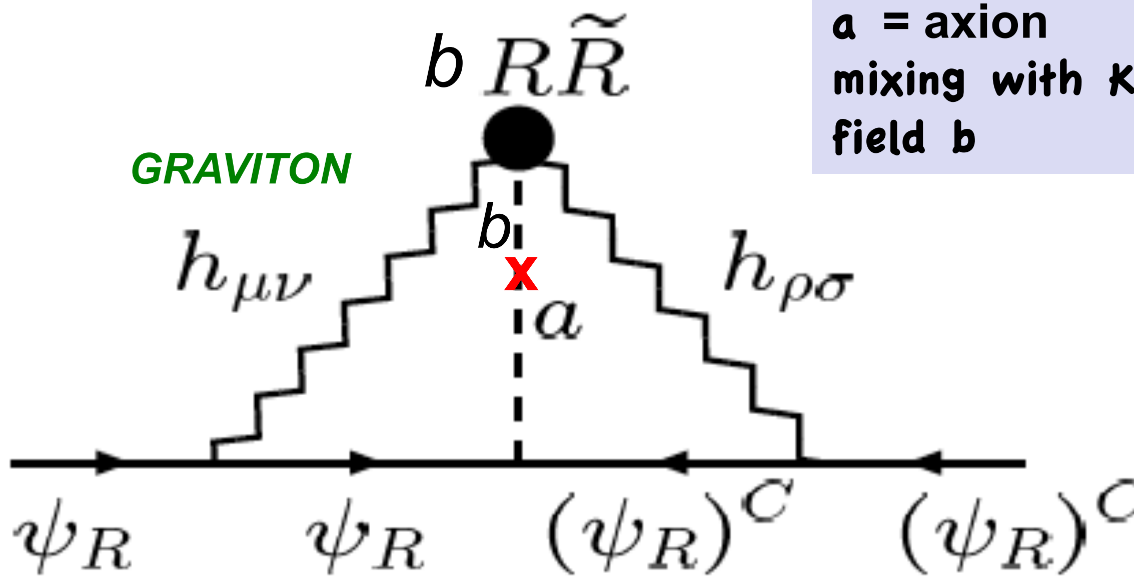
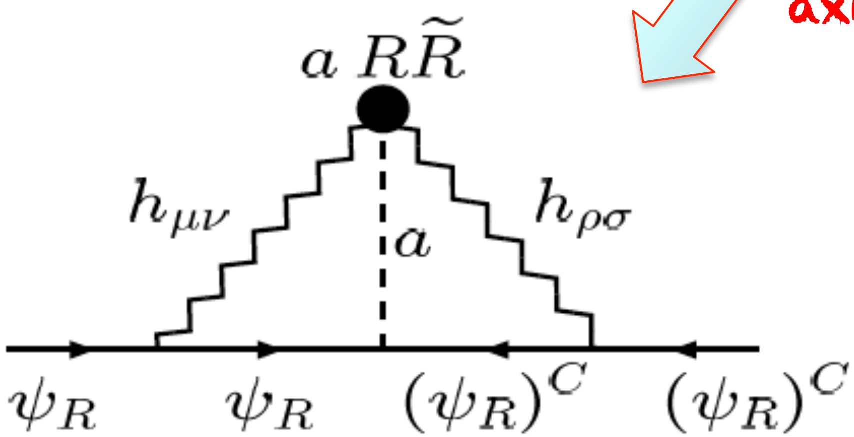
CHIRALITY CHANGE

NEM, Pilaftsis arXiv: 1209.6387

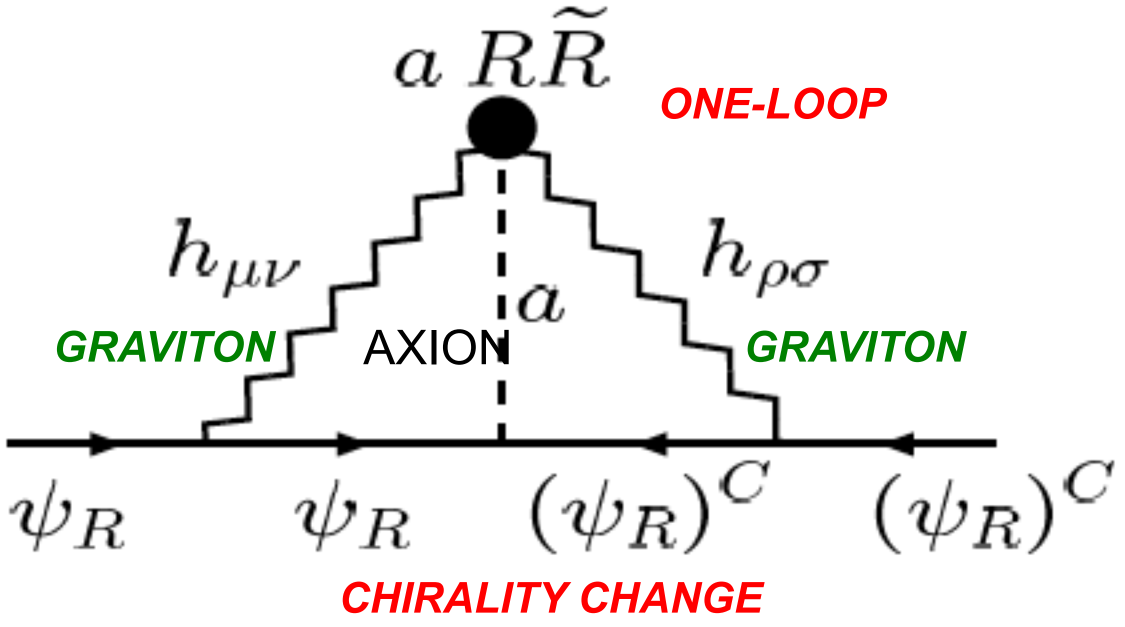
a = axion
mixing with KR
field b



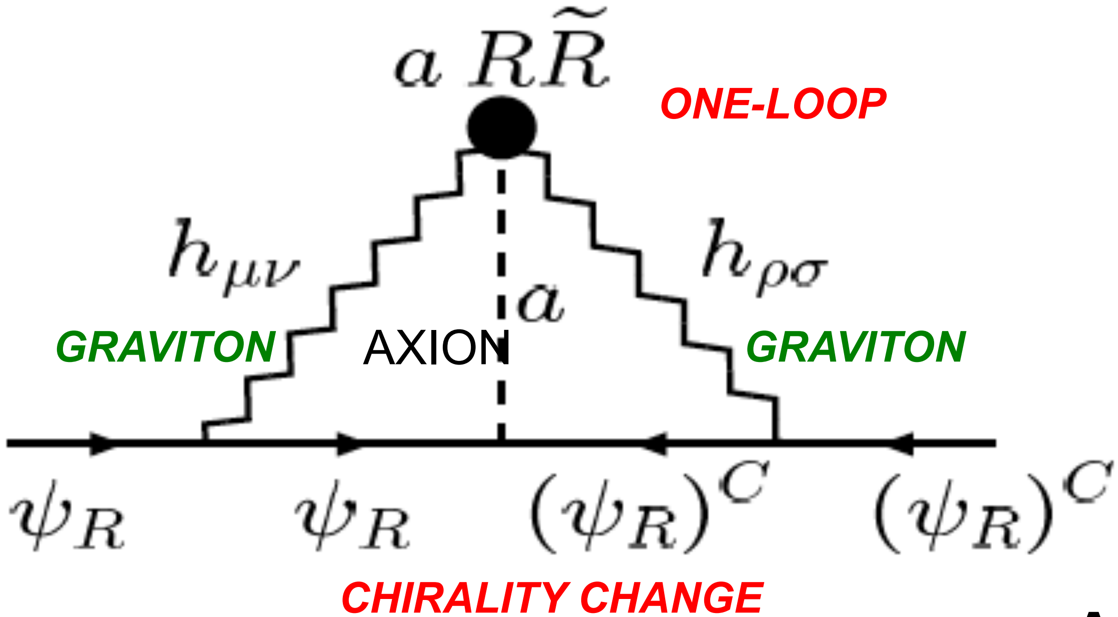
axion field redefinition



THREE-LOOP ANOMALOUS FERMION MASS TERMS



THREE-LOOP ANOMALOUS FERMION MASS TERMS



Λ = UV cutoff

$$\boxed{M_R \sim \frac{1}{(16\pi^2)^2}\,\frac{y_a\,\gamma\,\kappa^4\Lambda^6}{192\pi^2 f_b(1-\gamma^2)} = \frac{\sqrt{3}\,y_a\,\gamma\,\kappa^5\Lambda^6}{49152\sqrt{8}\,\pi^4(1-\gamma^2)}}$$

SOME NUMBERS

$\Lambda = 10^{17}$ GeV

$\gamma = 0.1$

M_R is at the TeV for $y_a = 10^{-3}$

$\Lambda = 10^{16}$ GeV

$M_R \sim 16$ keV,

$y_a = \gamma = 10^{-3}$

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$M_R \sim 16$ keV,

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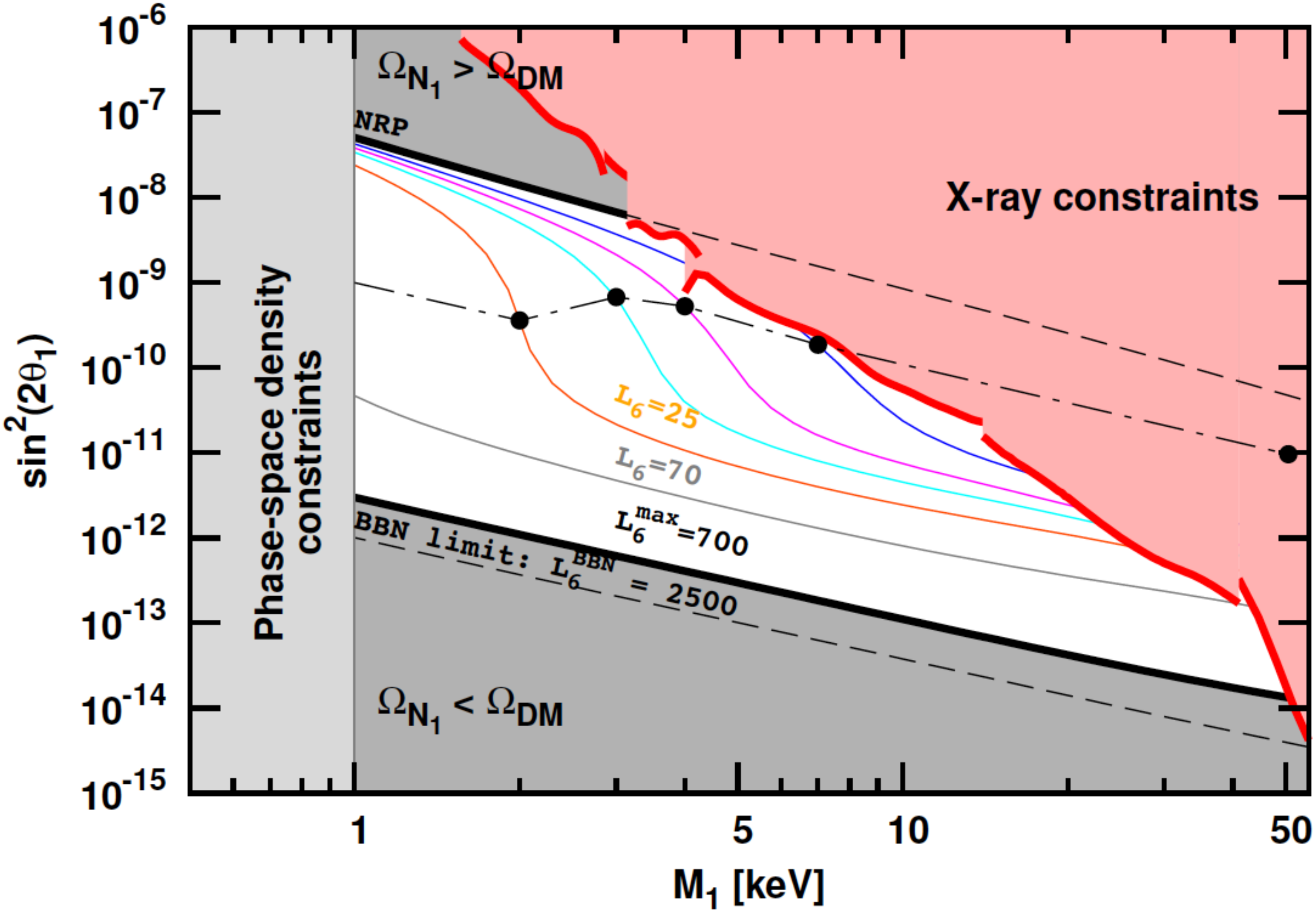
INTERESTING WARM DARK MATTER REGIME

Appropriate Hierarchy for the other two massive Right-handed neutrinos for Leptogenesis-Baryogenesis & Dark matter cosntraints can be arranged by choosing Yukawa couplings

νMSM

Boyarski, Ruchayskiy, Shaposhnikov…

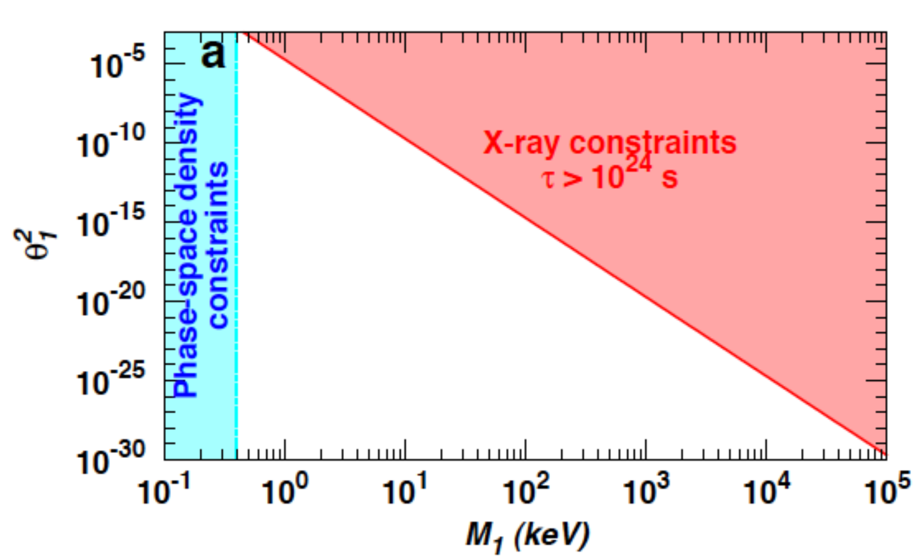
MODEL CONSISTENT WITH BBN, STRUCTURE FORMATION DATA IN THE UNIVERSE & ALL OTHER ASTROPHYSICAL CONSTRAINTS

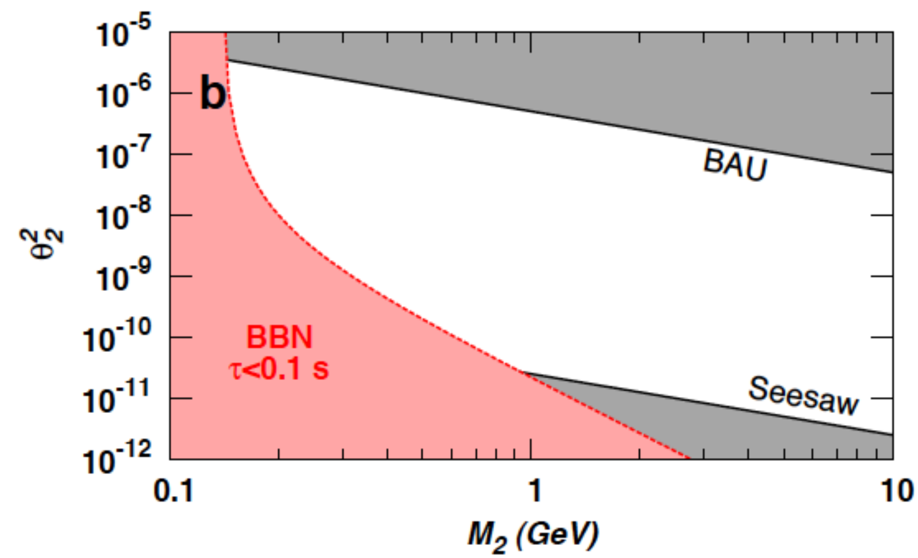


More than one sterile neutrino needed to reproduce Observed oscillations

νMSM

Boyarski, Ruchayskiy, Shaposhnikov…





Constraints on two heavy degenerate singlet neutrinos

N_1 DM production estimation in Early Universe must take into account its interactions with $N_{2,3}$ heavy neutrinos

FINITENESS OF THE MASS

MULTI-AXION SCENARIOS (e.g. string axiverse)

$$\mathcal{S}_a^{\rm kin} = \int d^4x\,\sqrt{-g}\,\Big[\frac{1}{2}\sum_{i=1}^{n}\Big((\partial_\mu a_i)^2 - M_i^2\Big) + \gamma(\partial_\mu b)(\partial^\mu a_1) - \frac{1}{2}\sum_{i=1}^{n-1}\delta M_{i,i+1}^2\, a_i a_{i+1}\Big];$$

$$\delta M^2_{i,i+1} \;<\; M_i M_{i+1}$$

positive mass spectrum
for all axions

simplifying all mixing equals

$$M_R \sim \frac{\sqrt{3}\,y_a\,\gamma\,\kappa^5\,\Lambda^{6-2n}(\delta M_a^2)^n}{49152\sqrt{8}\,\pi^4(1-\gamma^2)} \qquad n \leq 3$$

$$M_R \sim \frac{\sqrt{3}\,y_a\,\gamma\,\kappa^5(\delta M_a^2)^3}{49152\sqrt{8}\,\pi^4(1-\gamma^2)}\,\frac{(\delta M_a^2)^{n-3}}{(M_a^2)^{n-3}} \quad n > 3$$

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$$\delta M_{i,i+1}^2 \;<\; M_i M_{i+1}$$

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M_R : UV finite for n=3 @ 2-loop independent of axion mass

CONCLUSIONS

- ***nuMSM*: Three Sterile neutrinos necessary for Higgs sector stability ?**

- ***These Sterile Neutrinos may explain matter-antimatter origin in the Universe***

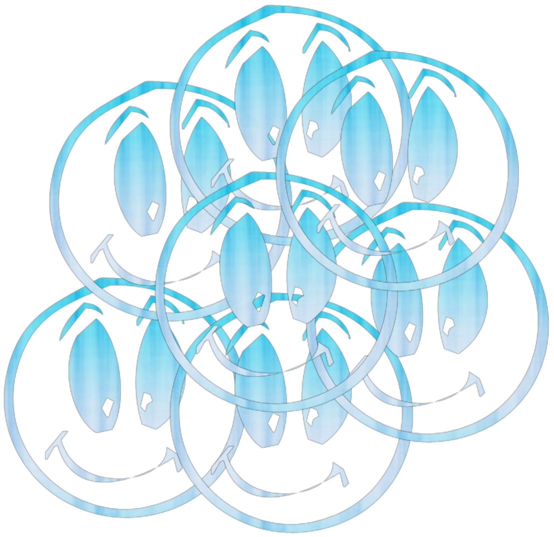
- **Lightest of them provide interesting Dark matter Candidates**

- ***Gravitationally-induced anomalous Right-handed Majorana neutrino masses possible, without the need for see-saw…***

- Interesting Physics for the Early Universe to be investigated, e.g. H-torsion-induced CPTV Leptogenesis/ Baryogenesis

OUTLOOK

ASTROPHYSICS



- keV dark matter (generic) may play important role in galactic structures (core & haloes)

- Right-handed neutrino matter with scalar and/or vector interactions may provide consistent galactic profiles for haloes and core structure → core contains significant amounts of right-handed neutrinos (rhn)

role of rhn condensates?

(Ruffini, Arguelles, Rueda,...NEM)

IS THIS CPTV ROUTE WORTH FOLLOWING? ….

CPT Violation

**Construct Microscopic Early Universe or Quantum Gravity models with strong CPT Violation in Early Universe eras, but maybe weak today… Fit with all available data…
Estimate in this way matter-antimatter asymmetry in Universe.**

BEFORE THE END ….

…. BEYOND NEUTRINOS

BARYON-ASYMMETRY FROM TORSIONFUL GEOMETRIES?

N. Poplawski, Physical Review D
Vol. 83, No. 8 (2011) 084033

Einstein theories of gravity → postulate that torsion is absent (constraint)

Einstein-Cartan and Kibble (1961) Sciama (1964) theories , this constraint is not used and torsion is considered as a dynamical field.

Fermions in gravitational field, when formulated in the so called first-order formalism, where spin connection and vierbein are treated as independent the **torsion** does ***not*** vanish

Quantum (totally antisymmetric) torsion S^μ

$$\int DS\, Db \exp\left[i \int \frac{3}{4\kappa^2} S \wedge {}^\star S - \frac{3}{4} S \wedge {}^\star J^5 + \left(\frac{3}{2\kappa^2}\right)^{1/2} b\, d^\star S\right]$$
$$= \int Db \exp\left[-i \int \frac{1}{2} db \wedge {}^\star db + \frac{1}{f_b} db \wedge {}^\star J^5 + \frac{1}{2 f_b^2} J^5 \wedge J^5\right],$$

$$\boxed{J^{5\,\mu} = \overline{\psi}\, \gamma^5\, \gamma^\mu\, \psi}$$

quantum-torsion-induced
four-fermi interactions

$$\text{multiplier field } \Phi(x) \equiv (3/\kappa^2)^{1/2} b(x).$$

$$f_b = (3\kappa^2/8)^{-1/2} = \frac{M_P}{\sqrt{3\pi}}$$

even in absence of any background field b ...

$$\mathfrak{L} = \frac{i}{2}\mathfrak{e} e_a^\mu(\bar{\psi}\gamma^a\psi_{;\mu} - \bar{\psi}_{;\mu}\gamma^a\psi) - m\mathfrak{e}\bar{\psi}\psi.$$

$$\mathfrak{e} = \det(e_\mu^a)$$

Variation with respect to torsion yields connection of torsion to axial currents then substitution back to fermion equations yields the higher order equation

$$\boxed{i e_a^\mu \gamma^a \psi_{;\mu} = m\psi - \frac{3\kappa}{8}(\bar{\psi}\gamma^5\gamma_a\psi)\gamma^5\gamma^a\psi,}$$

Hehl-Datta equation

$$\mathfrak{L} = \frac{i}{2}\mathfrak{e}e^{\mu}_{a}(\bar{\psi}\gamma^{a}\psi_{;\mu} - \bar{\psi}_{;\mu}\gamma^{a}\psi) - m\mathfrak{e}\bar{\psi}\psi,$$

$$\mathfrak{e} = \det(e^{a}_{\mu})$$

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$$\boxed{ie^{\mu}_{a}\gamma^{a}\psi_{:\mu} = m\psi - \frac{3\kappa}{8}(\bar{\psi}\gamma^{5}\gamma_{a}\psi)\gamma^{5}\gamma^{a}\psi,}$$

Hehl-Datta equation

obtained from effective lagrangian

$$\mathfrak{L}_{e} = \frac{i}{2}\mathfrak{e}e^{\mu}_{a}(\bar{\psi}\gamma^{a}\psi_{:\mu} - \bar{\psi}_{:\mu}\gamma^{a}\psi) - m\mathfrak{e}\bar{\psi}\psi + \frac{3\kappa\mathfrak{e}}{16}(\bar{\psi}\gamma^{5}\gamma_{a}\psi)(\bar{\psi}\gamma^{5}\gamma^{a}\psi)$$

repulsive four fermion interaction

Complex conjugate equation

$$\psi^c = -i\gamma^2\psi^*, \;\; \psi^* = -i\gamma^2\psi^c$$

$$\gamma^{5*} = \gamma^5$$

$$\boxed{ie^\mu_a\gamma^a\psi^c_{:\mu} \;\; = m\psi^c + \frac{3\kappa}{8}(\overline{\psi^c}\gamma^5\gamma_a\psi^c)\gamma^5\gamma^a\psi^c}$$

Complex conjugate equation $\psi^c = -i\gamma^2\psi^*,\ \ \psi^* = -i\gamma^2\psi^c$

$$\gamma^{5*} = \gamma^5$$

$$ie^\mu_a\gamma^a\psi^c_{:\mu} = m\psi^c + \frac{3\kappa}{8}(\overline{\psi^c}\gamma^5\gamma_a\psi^c)\gamma^5\gamma^a\psi^c$$

Go compare

$$ie^\mu_a\gamma^a\psi_{:\mu} = m\psi - \frac{3\kappa}{8}(\bar{\psi}\gamma^5\gamma_a\psi)\gamma^5\gamma^a\psi,$$

opposite signs

Using plane wave approximation we obtain ***different dispersion relations*** between particles/antiparticles ***AT FINITE DENSITIES ASSUMING CONSTANT BACKGROUND TORSION***

fermion

$$\omega = m + \alpha\kappa N$$

antifermion

$$\omega = m - \alpha\kappa N.$$

Using plane wave approximation we obtain ***different dispersion relations*** between particles/antiparticles ***AT FINITE DENSITIES ASSUMING CONSTANT BACKGROUND TORSION***

fermion

$$\omega = m + \alpha\kappa N$$

antifermion

$$\omega = m - \alpha\kappa N.$$

NB:

Lagrangian is C (harge conjugation) symmetric but Hehl-Datta equation is ***NOT***

$$\overline{\psi^c}\,\psi^c = -\overline{\psi}\,\psi$$

$$(\overline{\psi^c}\gamma^\mu\gamma^5\psi^c)(\overline{\psi^c}\gamma_\mu\gamma^5\psi^c) = (\overline{\psi}\gamma^\mu\gamma^5\psi)(\overline{\psi}\gamma_\mu\gamma^5\psi)$$

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Only chiral matter
(e.g. right-handed neutrinos)
can form condensates

$$\langle \overline{\psi}\gamma^5\,\gamma^0 \rangle \equiv N \neq 0$$

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$$\omega = m - \alpha\kappa N.$$

Only chiral matter
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$$\langle \overline{\psi}\gamma^5\,\gamma^0 \rangle \equiv N \neq 0$$

inverse normalization
of Dirac spinor wavefunction
of order of (energy scale)^3

$$\boxed{n_X - n_{\bar{X}} = \frac{g}{(2\pi)^3}\int d^3\mathbf{p}\left(\frac{1}{1+e^{E_X/T}} - \frac{1}{1+e^{E_{\bar{X}}/T}}\right)}$$

$$N \sim T^3.$$

for the Universe
at temperature T

particle/antiparticle asymmetry can
thus be generated from torsion ?

SPARES

FURTHER OUTLOOK

Scalar Fields in Early Universe: Plenty of them in extended particle physics models: inflaton(s), dilaton, axions, moduli ….*Various effects…*

In stringy model our model:
Linear in cosmic time background KR pseudoscalar : $b \sim t$

b-induced torsion that produces matter/antimatter asymmetry

Logarithmic dilaton $\phi \sim \mathrm{Ln}\, a(t) = \mathrm{Ln}\, (t)$ →
→ linearly expanding Universe

FURTHER OUTLOOK

Scalar Fields in Early Universe: **Plenty of them in extended particle physics models: inflaton(s), dilaton, axions, moduli ….*Various effects…***

PERSONAL PERSPECTIVE

Running dilaton

$$\frac{d\mathcal{N}}{dt} + 3H\mathcal{N} = \Gamma(t)\mathcal{N} + C[f],$$

Dilution of dark matter relic density due to coupling with running dilatons

$$\Gamma(t) = \dot{\phi}, \qquad \phi = -|\phi_0| \ln a(t)$$

$$\lambda \equiv \sigma_0\, m_X^3 / H_m \qquad H = H_m\, x^{-2} \qquad x = m_X/T$$

$$\langle \sigma v \rangle = \sigma_0\, x^{-n}$$

Lahanas, NEM, Nanopoulos, Bender, Sarkar

$$\boxed{Y \equiv \frac{\mathcal{N}}{s} \sim \frac{(n+1+|\phi_0|)}{\lambda(n+1+|\phi_0|+x_f)}\, x^{-|\phi_0|}\ .}$$

FURTHER OUTLOOK

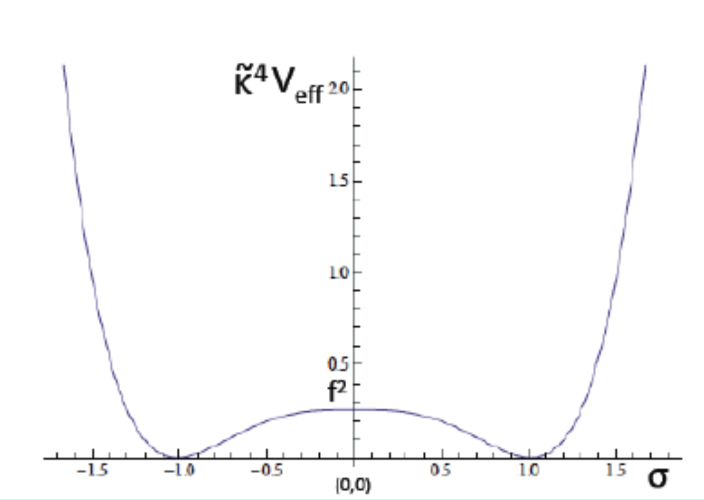
Scalar Fields in Early Universe: **Plenty of them in extended particle physics models: inflaton(s), dilaton, axions, moduli ….*Various effects…***

PERSONAL PERSPECTIVE

Inflation in the model

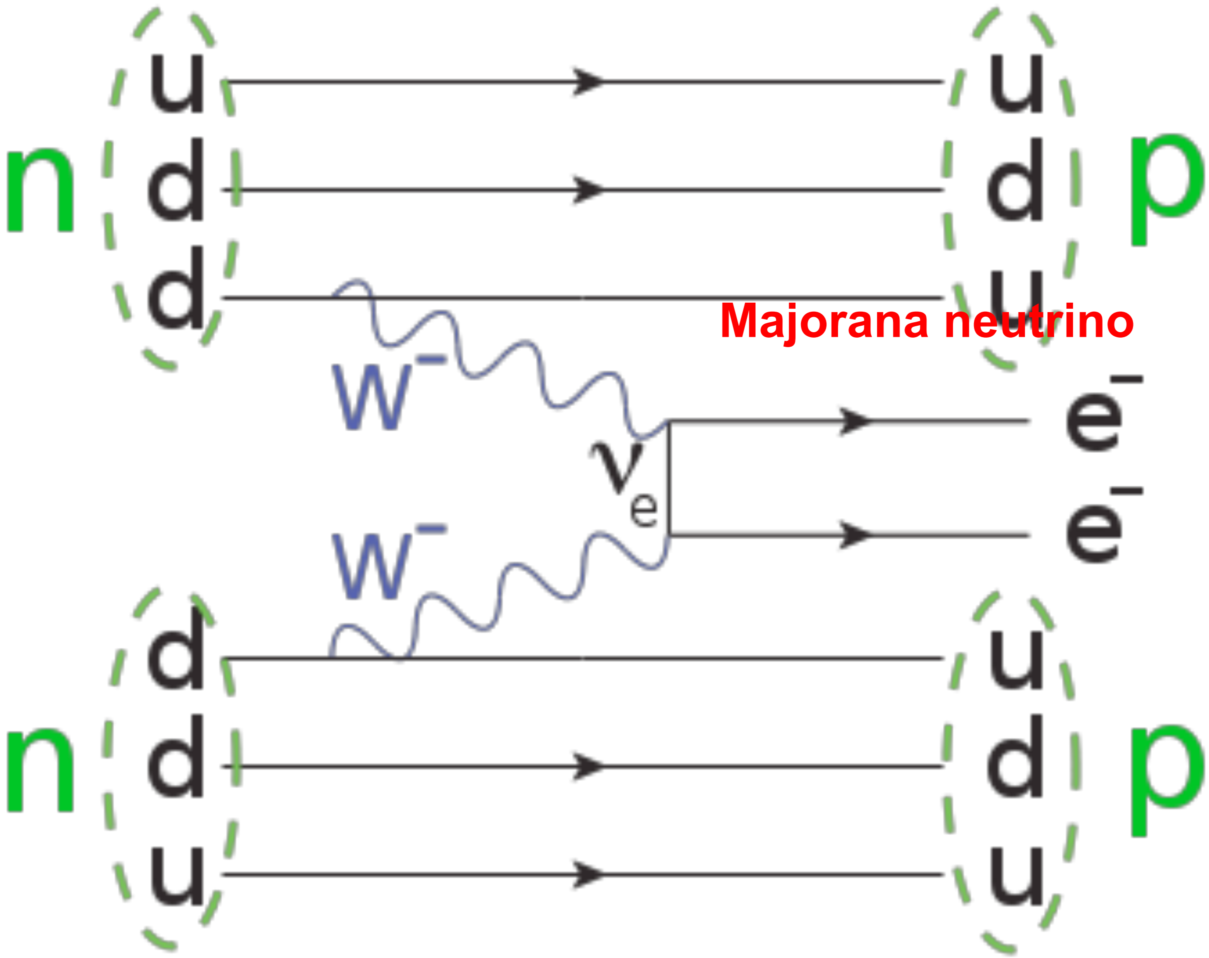
Condensation of gravitino fields → dynamical SUGRA BREAKING
→ inflation : small-field inflation due to flat one-loop effective potential of gravitino condensate (σ = inflaton)

Ellis, NEM,
Alexandre, Houston



Modifications in Neutrinoless 2Beta decay rate in 2 flavour mixing
(due to CPTV modified effective mass)

M Sinha & B. Mukhopadhyay
arXiv: 0704.2593



If a small B^0 is present today

ignore neutrino/ antineutrino mixing here:

Amplitude $A \propto \sqrt{B_0^2 + m_e^2}.$

If a small B^a is present today

Sinha, Mukhopadhyay, 0704.2593

...will also affect flavour oscillations

2-flavour toy example

$$\Psi = \begin{pmatrix} \psi_L^c \\ \psi_L \end{pmatrix} \qquad \boxed{\nu_{e,\mu} = \mathcal{U}^\dagger_{e,\mu} \Psi_{e,\mu}} \qquad \mathcal{U}_{e,\mu} = \begin{pmatrix} \cos\theta_{e,\mu} & -\sin\theta_{e,\mu} \\ \sin\theta_{e,\mu} & \cos\theta_{e,\mu} \end{pmatrix}.$$

neutrino-antineutrino mixing on top of flavour mixing

FLAVOUR OSCILALTIONS IN THE PRESENCE OF GRAVITY

2 SETS OF FLAVOUR EIGENSTATES

$$
\begin{aligned}
|\nu_{e1}\rangle &= \cos\phi_1|\chi_1\rangle - \sin\phi_1|\chi_2\rangle \\
|\nu_{\mu 1}\rangle &= \sin\phi_1|\chi_1\rangle + \cos\phi_1|\chi_2\rangle
\end{aligned}
\qquad
\begin{aligned}
|\nu_{e2}\rangle &= \cos\phi_2|\chi_3\rangle - \sin\phi_2|\chi_4\rangle \\
|\nu_{\mu 2}\rangle &= \sin\phi_2|\chi_3\rangle + \cos\phi_2|\chi_4\rangle.
\end{aligned}
$$

$$\boxed{\begin{aligned}\mathcal{P}_{fg1} &= \sin^2 2\phi_1 \sin^2 \delta_{fg1}(t), \\ \delta_{fg1}(t) &= \frac{|M_1^2 - M_2^2|}{4E}\, t\end{aligned}}
\qquad
\boxed{\begin{aligned}\mathcal{P}_{fg2} &= \sin^2 2\phi_2 \sin^2 \delta_{fg2}(t), \\ \delta_{fg2}(t) &= \frac{|M_3^2 - M_4^2|}{4E}\, t\end{aligned}}$$

OSCILLATION LENGTH

$$\lambda_{fg1} = \frac{4\pi E}{|M_1^2 - M_2^2|} \qquad \lambda_{fg2} = \frac{4\pi E}{|M_3^2 - M_4^2|}$$

$$\Delta M^2 = |M_1^2 - M_2^2| = |M_3^2 - M_4^2| = \left(\sqrt{B_0^2 + m_\mu^2} + \sqrt{B_0^2 + m_e^2}\right)$$
$$\times\sqrt{\left\{\left(\sqrt{B_0^2 + m_\mu^2} - \sqrt{B_0^2 + m_e^2}\right)^2 + 4m_{e\mu}^2\right\}}.$$

FLAVOUR OSCILALTIONS IN THE PRESENCE OF GRAVITY

2 SETS OF FLAVOUR EIGENSTATES

$$|\nu_{e1}\rangle = \cos\phi_1|\chi_1\rangle - \sin\phi_1|\chi_2\rangle$$
$$|\nu_{\mu1}\rangle = \sin\phi_1|\chi_1\rangle + \cos\phi_1|\chi_2\rangle$$

$$|\nu_{e2}\rangle \ldots \sin\phi_2|\chi_4\rangle$$
$$|\nu \ldots \cos\phi_2|\chi_4\rangle.$$

$$\mathcal{P}_{fg1} = \sin^2 2\phi_1 \sin^2\delta_{fg1}(t),$$
$$\delta_{fg1}(t) = \frac{|M_1^2 - M_2^2 \ldots}{4E \ldots}$$

$$\ldots \sin^2 2\phi_2 \sin^2\delta_{fg2}(t),$$
$$\delta_{fg2}(t) = \frac{|M_3^2 - M_4^2|}{4E}\, t$$

OSCILLATION LENGTH

$$\ldots M_2^2|$$

$$\lambda_{fg2} = \frac{4\pi E}{|M_3^2 - M_4^2|}$$

$$\Delta M \ldots |M_3^2 - M_4^2| = \left(\sqrt{B_0^2 + m_\mu^2} + \sqrt{B_0^2 + m_e^2}\right) \times \sqrt{\left\{\left(\sqrt{B_0^2 + m_\mu^2} - \sqrt{B_0^2 + m_e^2}\right)^2 + 4m_{e\mu}^2\right\}}.$$

STRINGENT CONSTRAINTS TODAY DUE TO NON-OBSERVATION OF NEUTRINO/ANTINEUTRINO OSCILLATIONS?

NB:Some Remarks on Right-Handed Neutrinos & CMB

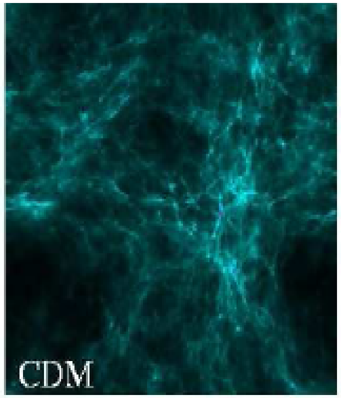
PHYSICS: WMAP and Dark Matter

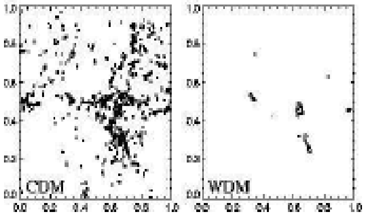
WMAP results so far:

- Disfavor strongly hot dark matter (neutrinos), $\Omega_\nu h^2 < 0.0076$ ($< m_\nu >_e < 0.23$ eV).
- Warm Dark Matter (gravitino) disfavoured by evidence for re-ionization at redshift $z \sim 20$.
- Cold Dark Matter (CDM) remains: axions, supersymmetric dark matter (lightest SUSY particle (LSP)), superheavy (masses $\sim 10^{14\pm5}$ GeV)

  WMAP results: $\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$ (matter), $\Omega_b h^2 = 0.0224 \pm 0.0009$ (baryons), hence, assuming CDM is the difference, $\Omega_{CDM} h^2 = 0.1126^{+0.0161}_{-0.0181}$, ($2\sigma$ level).

WMAP excludes WARM Dark Matter





Numerical simulations for structure formation in Cold Dark Matter (CDM) (top) and Warm Dark Matter (WDM) (middle) with mass $m_X = 10$ KeV at $z = 20$. Bottom: Dark halos with mass $> 10^5 M_\odot$ for CDM (left) and for WDM (right).

IMPORTANT COMMENTS:

Such structure formation arguments can only place a lower bound on mass of the WDM candidate: $m_X > 10$ KeV.

Above results exclude Light Gravitino Models ($m_X < 0.5 KeV$) of Particle Physics as DM candidates.

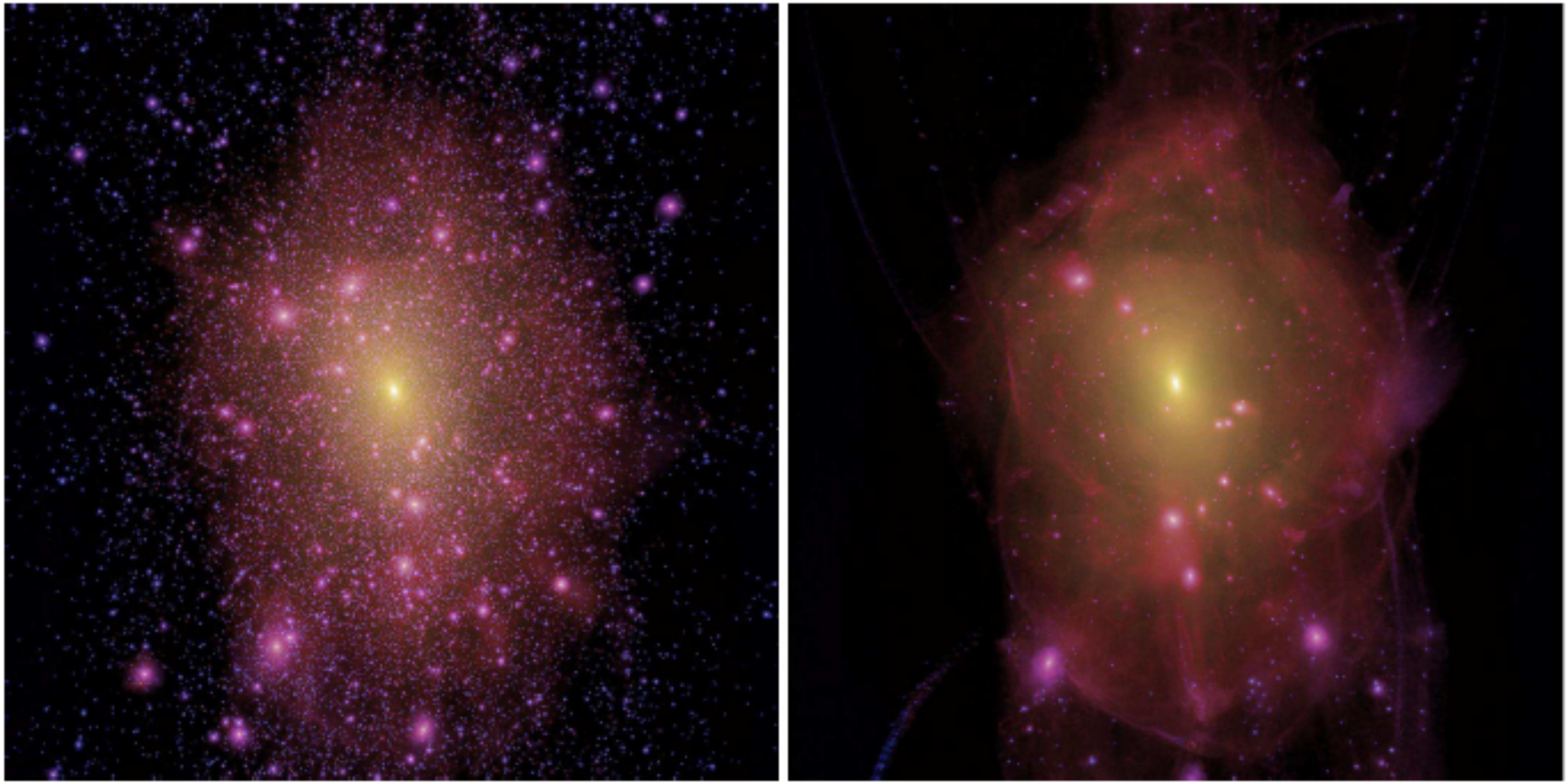
NB! WDM with $m_X \geq 100$ KeV becomes indistinguishable from Cold Dark Matter, as far as structure formation is concerned.

Further Evidence in fvour of Warm Dark Matter & Structure Formation

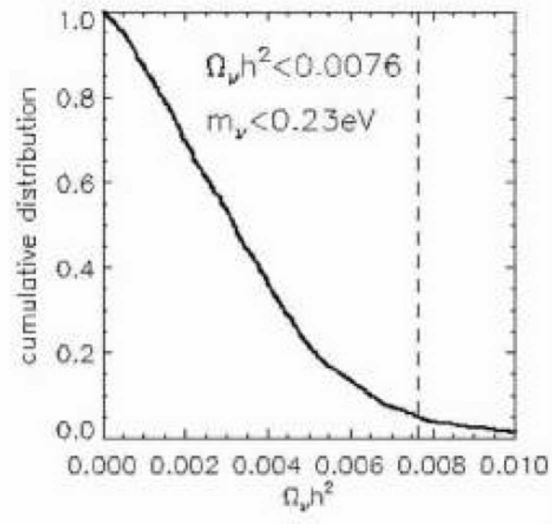
Warm Dark Matter (WDM) Universe may solve puzzle of Satelite Galaxies to Milky Way whose properties are incompatible with Cold Dark Matter (CDM) Model : Satelite internal dynamics and structure formation ***inexplicable*** by CDM

Lovell, Eke, Frenk *et al.*, arXiv: 1104.2929

Satellite Galaxies in WDM 5



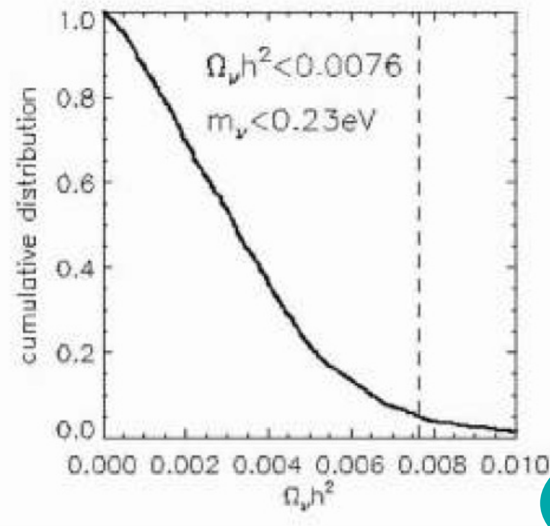
WMAP excludes HOT Dark Matter



Contribution of neutrinos to energy density of Universe: $\Omega_\nu h^2 = \frac{\sum_i m_i}{94.0 \text{ eV}}$
(sum over light neutrino species (decouple while still relativistic)).
WMAP and other experiments (the Lyman α data etc) $\Omega_\nu h^2 < 0.0076 \Rightarrow < m_\nu >_e < 0.23$ eV :
Excludes HOT DM.

NB: WMAP still consistent with Majorana neutrinos, and also marginally with $\beta\beta$-decay (Heidelberg-Moscow Coll.).

WMAP excludes HOT Dark Matter



PLANCK

$$\sum_\nu m_\nu \leq 0.023 \text{ eV}$$

PLANCK + BAO

$$\sum_\nu m_\nu \leq 0.002 \text{ eV}$$

Contribution of neutrinos to energy density of Universe: $\Omega_\nu h^2 = \frac{\sum_i m_i}{94.0 \text{ eV}}$
(sum over light neutrino species (decouple while still relativistic)).
WMAP and other experiments (the Lyman α data etc) $\Omega_\nu h^2 < 0.0076 \Rightarrow < m_\nu >_e < 0.23$ eV :
Excludes HOT DM.

NB: WMAP still consistent with Majorana neutrinos, and also marginally with $\beta\beta$-decay (Heidelberg-Moscow Coll.).

PLANCK SATELLITE RESULTS ON STERILE NEUTRINOS

Effective number of excited neutrino species contributing to radiation content of the Universe as measured by CMB data within ΛCDM

$$N_{\mathrm{eff\ WMAP}} = 4.34^{+0.86}_{-0.88}$$

PLANCK + WMAP+ BAO + high-multiple CMB data

$$N_{\mathrm{eff\ Planck}} = 3.33 \pm 0.27$$

@ 68 % C.L.

still leaves room for almost an extra light neutrino species?

May be, with active-sterile mass squared splitting in the range **Δm^2_{i4} = (10^{-5} – 10^2) eV^2** and active/sterile mixing **$\sin^2\theta_{i4} < 10^{-2.5}$**

Mirizzi *et al.* 1303.5368

de Vega, Sanchez, 1304.0759

BUT…Subtle connection of N_{eff} to real number of active plus **keV** sterile neutrinos

One warm (**keV**) dark matter sterile neutrino contributes to N_{eff} at recombination (rc)

$$\Delta N^{WDM} = \left(\frac{T_d}{T_{rc}}\right)^4 = \left[\frac{g_{rc}}{g(T_d)}\right]^{4/3}$$

(entropy conservation requirements)

@ z=1094 (redshift of rc), one can estimate for, e.g. standard model

$$\Delta N^{WDM}_{SM} = 0.02771\ldots$$

too small to be detected by CMB

Since (see previous slide) one sterile (Majorana) neutrino up to a few eV mass range is compatible with Planck → important, since, in view of the above, this may imply also the existence of keV sterile neutrinos playing the role of dark matter → as in the νMSM extension of Standard Model

SUMMARY ON TORSION CONNECTIONS

(i) Dirac spin ½ fermions

$$\boxed{\omega_\mu^{ab} = \overline{\omega}_\mu^{ab} + \frac{\kappa}{4}\epsilon^{ab}{}_{cd}e^c_\mu J^d_{(A)}}$$

$J^\mu_{(A)} \equiv \overline{\psi}\gamma_5\gamma^\mu\psi$ is the spinor *axial* current.

(ii) N=1 SUGRA

$$\omega_\mu{}^{ab} = \omega_\mu{}^{ab}(e,\psi) \equiv \omega_\mu{}^{ab}(e) + \kappa_\mu{}^{ab}(\psi)$$

$$J^\mu(\psi) = \tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}_\nu\gamma_\alpha\psi_\beta.$$

$$\boxed{\kappa_{\mu\alpha\beta}(\psi) = \tfrac{1}{4}[\bar{\psi}_\alpha\gamma_\mu\psi_\beta + \bar{\psi}_\mu\gamma_\alpha\psi_\beta - \bar{\psi}_\mu\gamma_\beta\psi_\alpha].}$$

(iii) N=2 SUGRA

$$\omega_\mu{}^{ab} = \omega_\mu{}^{ab}(e,\psi) \equiv \omega_\mu{}^{ab}(e) + \kappa_\mu{}^{ab}(\psi)$$

$$J^\mu(\psi) = \epsilon^{\mu\nu\alpha\beta}\bar{\psi}^I_\nu\gamma_\alpha\psi_{I\beta}$$

$$\boxed{\kappa_{\mu\alpha\beta}(\psi) = \tfrac{1}{4}[\bar{\psi}^I_\alpha\gamma_\mu\psi_{I\beta} + \bar{\psi}^I_\mu\gamma_\alpha\psi_{I\beta} - \bar{\psi}^I_\mu\gamma_\beta\psi_{I\alpha} + \text{c.c.}].}$$

(iv) N=4 SU(4) SUGRA

$$\omega_{\mu ab} = \omega_{\mu ab}(e,\psi,\Lambda) \equiv \omega_{\mu ab}(e) + \kappa_{\mu ab}$$

$$J^\mu(\psi) = \epsilon^{\mu\nu\alpha\beta}\bar{\psi}^I_\nu\gamma_\alpha\psi_{I\beta}$$

$$J^\mu(\Lambda) = e\bar{\Lambda}_I\gamma^\mu\Lambda^I.$$

$$\boxed{\begin{aligned}
\kappa_{\mu\alpha\beta} &= \kappa_{\mu\alpha\beta}(\psi) + \kappa_{\mu\alpha\beta}(\Lambda),\\
\kappa_{\mu\alpha\beta}(\psi) &= \frac{1}{4}[\bar{\psi}^I_\alpha\gamma_\mu\psi_{I\beta} + \bar{\psi}^I_\mu\gamma_\alpha\psi_{I\beta} - \bar{\psi}^I_\mu\gamma_\beta\psi_{I\alpha} + \text{c.c.}],\\
\kappa_{\mu\alpha\beta}(\Lambda) &= -\frac{1}{4e}\epsilon_{\mu\alpha\beta\sigma}\bar{\Lambda}_I\gamma^\sigma\Lambda^I. \qquad (31)
\end{aligned}}$$

STANDARD MODEL EXTENSION WITH MASSIVE RIGHT-HANDED MAJORANA NEUTRINOS

SM Extension with N extra right-handed neutrinos

Non SUSY $\nu\mathrm{MSM}$

Boyarski, Ruchayskiy, Shaposhnikov

$$\boxed{L = L_{SM} + \bar{N}_I i\partial_\mu \gamma^\mu N_I - F_{\alpha I}\, \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2}\, \bar{N}_I^c N_I + \mathrm{h.c.}}$$

SM Extension with N extra right-handed neutrinos

$$\boxed{L = L_{SM} + \bar{N}_I i\partial_\mu \gamma^\mu N_I - F_{\alpha I}\,\bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2}\,\bar{N}_I^c N_I + \text{h.c.}}$$

Right-handed
Massive **Majorana**
neutrinos *I=1,…N*

Leptons

SM Extension with N extra right-handed neutrinos

$$L = L_{SM} + \bar{N}_I i\partial_\mu \gamma^\mu N_I - F_{\alpha I}\,\bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2}\,\bar{N}_I^c N_I + \text{h.c.}$$

Higgs scalar SU(2)
Dual: $\tilde{\phi}_i = \epsilon_{ij}\phi_j^*.$

SM Extension with N extra right-handed neutrinos

$$L = L_{SM} + \bar{N}_I i\partial_\mu \gamma^\mu N_I - F_{\alpha I}\,\bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2}\,\bar{N}_I^c N_I + \mathrm{h.c.}$$

Yukawa couplings Matrix

$$F = \widetilde{K}_L\, f_d\, \widetilde{K}_R{}^\dagger$$

SM Extension with N extra right-handed neutrinos

νMSM

Boyarski, Ruchayskiy, Shaposhnikov

$$\boxed{L = L_{SM} + \bar{N}_I i\partial_\mu \gamma^\mu N_I - F_{\alpha I}\, \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2}\, \bar{N}_I^c N_I + \text{h.c.}}$$

Yukawa couplings Matrix (N=3)

$$F = \widetilde{K}_L\, f_d\, \widetilde{K}_R{}^\dagger$$

$$f_d = \mathrm{diag}(f_1, f_2, f_3)\,, \quad \widetilde{K}_L = K_L P_\alpha\,, \quad \widetilde{K}_R{}^\dagger = K_R{}^\dagger P_\beta$$

$$P_\alpha = \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, 1)\,, \quad P_\beta = \mathrm{diag}(e^{i\beta_1}, e^{i\beta_2}, 1)$$

Majorana phases

Mixing

$$K_L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{L23} & s_{L23} \\ 0 & -s_{L23} & c_{L23} \end{pmatrix} \begin{pmatrix} c_{L13} & 0 & s_{L13}e^{-i\delta_L} \\ 0 & 1 & 0 \\ -s_{L13}e^{i\delta_L} & 0 & c_{L13} \end{pmatrix} \begin{pmatrix} c_{L12} & s_{L12} & 0 \\ -s_{L12} & c_{L12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$c_{Lij} = \cos(\theta_{Lij})$ and $s_{Lij} = \sin(\theta_{Lij})$.

SM Extension with N extra right-handed neutrinos

νMSM

$$L = L_{SM} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I}\, \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2}\, \bar{N}_I^c N_I + \text{h.c.}$$

Majorana masses to (2 or 3) active neutrinos via ***seesaw***

Yukawa couplings Matrix (N=2 or 3)

$$F = \tilde{K}_L\, f_d\, \tilde{K}_R^\dagger$$

NB: Upon Symmetry Breaking
<Φ> = v ≠ 0 → Dirac mass term

νMSM

$$\boxed{L = L_{SM} + \bar{N}_I i\partial_\mu \gamma^\mu N_I - F_{\alpha I}\, \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2}\, \bar{N}_I^c N_I + \text{h.c.}}$$

Light Neutrino Masses through see saw

Minkowski,Yanagida,
Mohapatra, Senjanovic
Sechter, Valle …

$$\boxed{m_\nu = -M^D \frac{1}{M_I} [M^D]^T}$$

$$M_D = F_{\alpha I} v$$

$$v = \langle \phi \rangle \sim 175 \text{ GeV}$$

$$M_D \ll M_I$$

Canetti, Frossart, Shaposhnikov, arXiv: 1208.4607

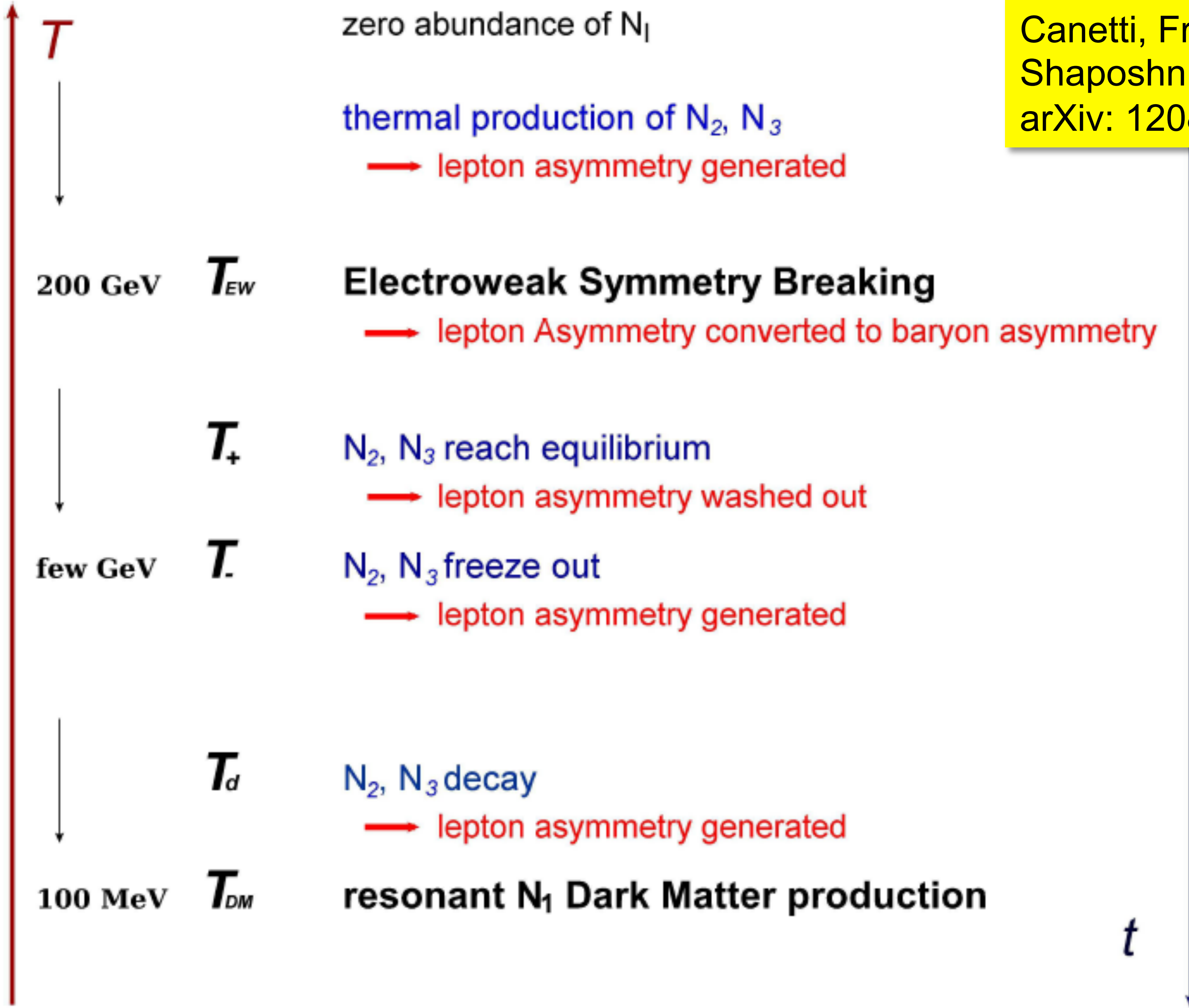


Figure 1: *The thermal history of the universe in the νMSM.*

Canetti, Frossart, Shaposhnikov, arXiv: 1208.4607

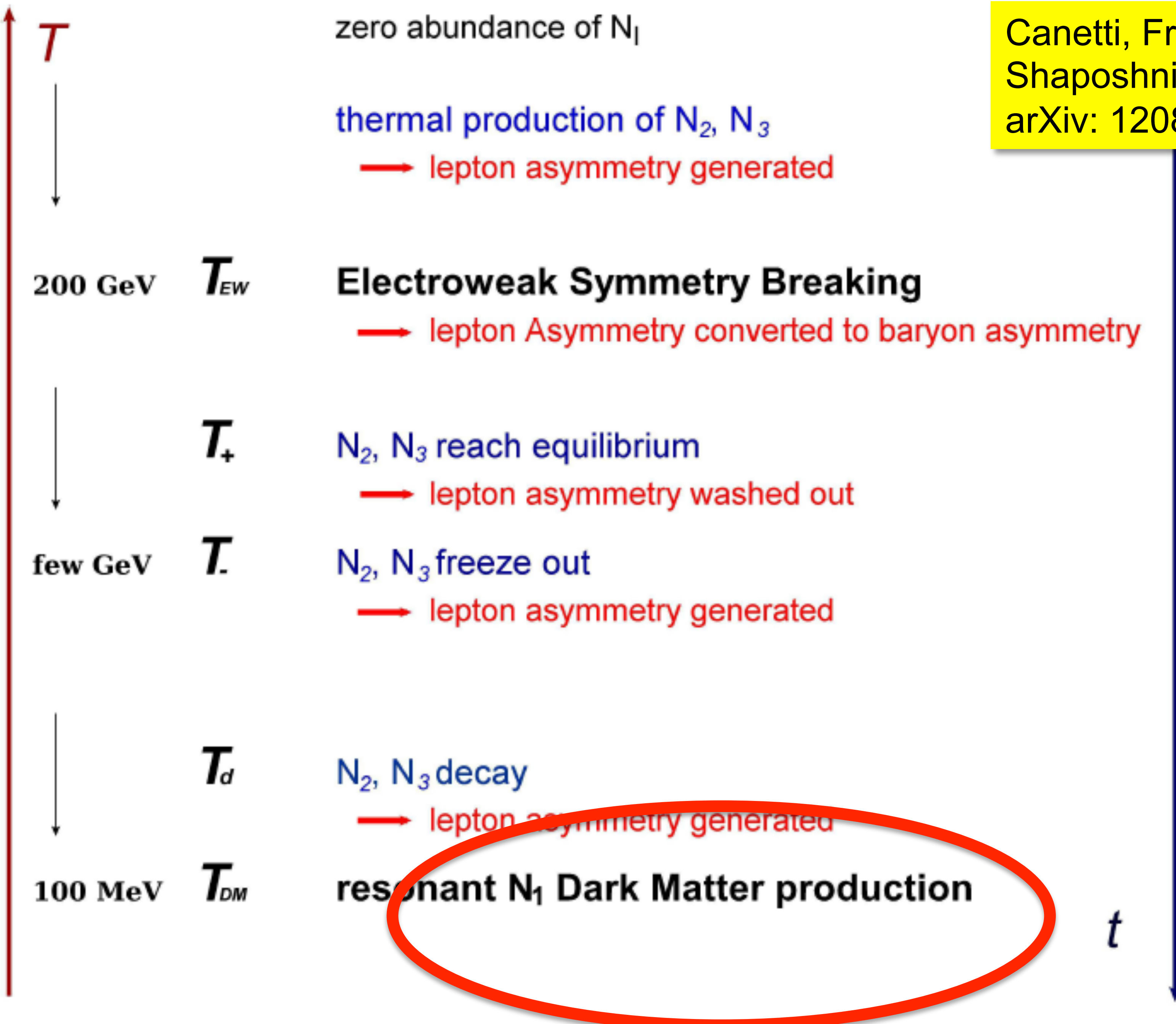
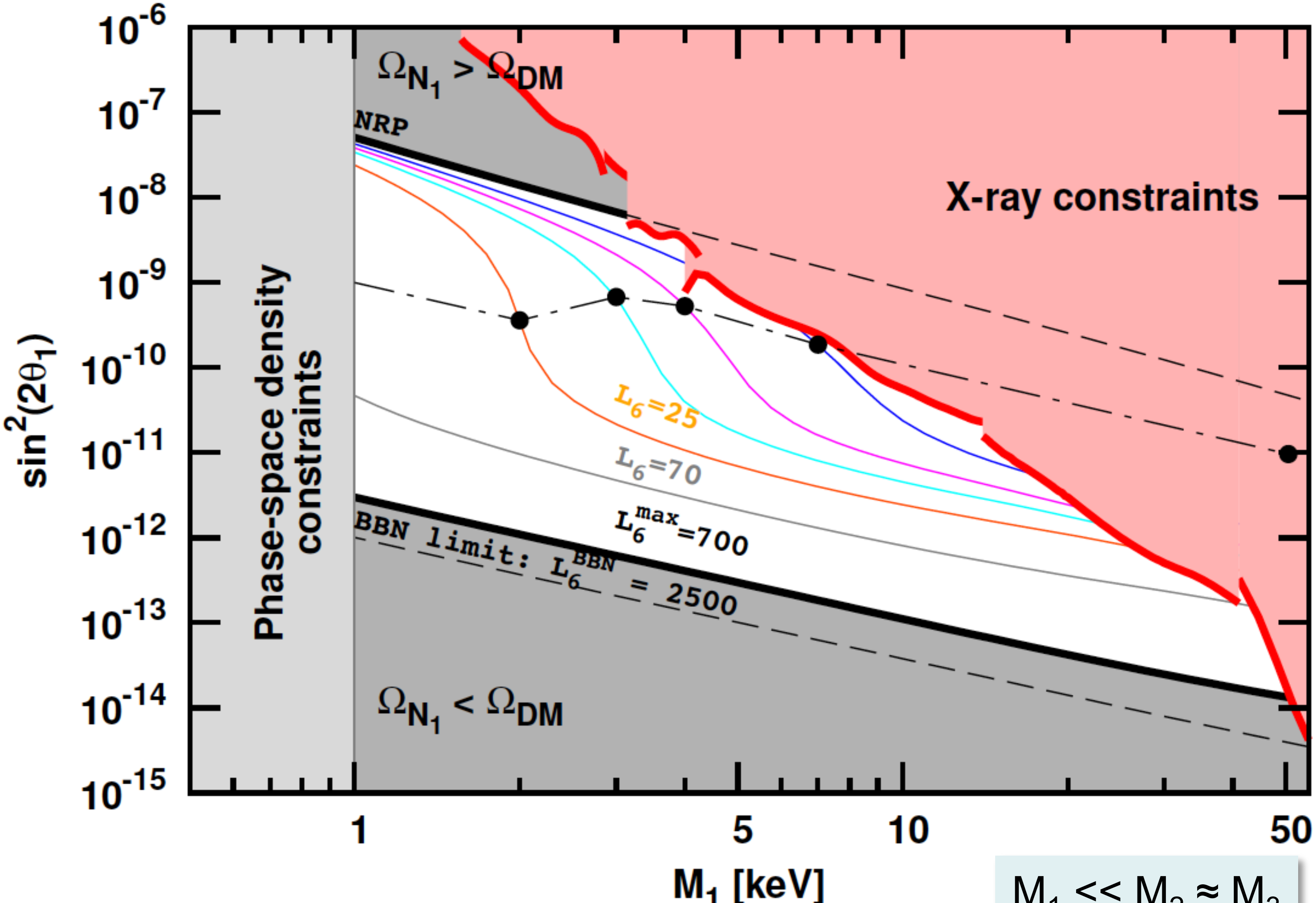


Figure 1: *The thermal history of the universe in the νMSM.*

νMSM

Boyarski, Ruchayskiy, Shaposhnikov…

MODEL CONSISTENT WITH BBN, STRUCTURE FORMATION DATA IN THE UNIVERSE & ALL OTHER ASTROPHYSICAL CONSTRAINTS



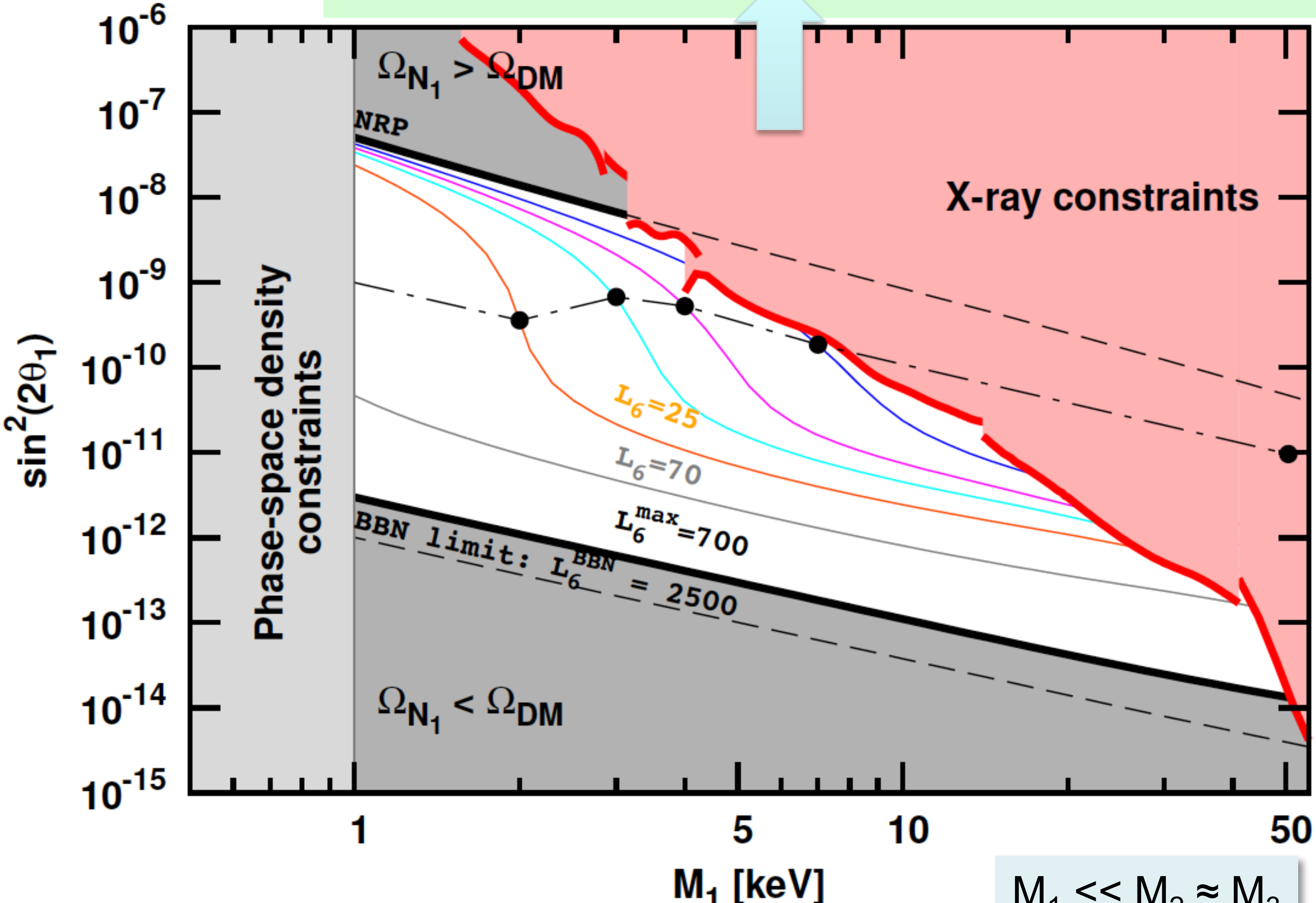
$M_1 << M_2 \approx M_3$

νMSM

Boyarski, Ruchayskiy, Shaposhnikov…

Decaying N_1 produces narrow spectral line in spectra of DM dominated astrophysical objects

MODEL CONS
UNIVERSE & A

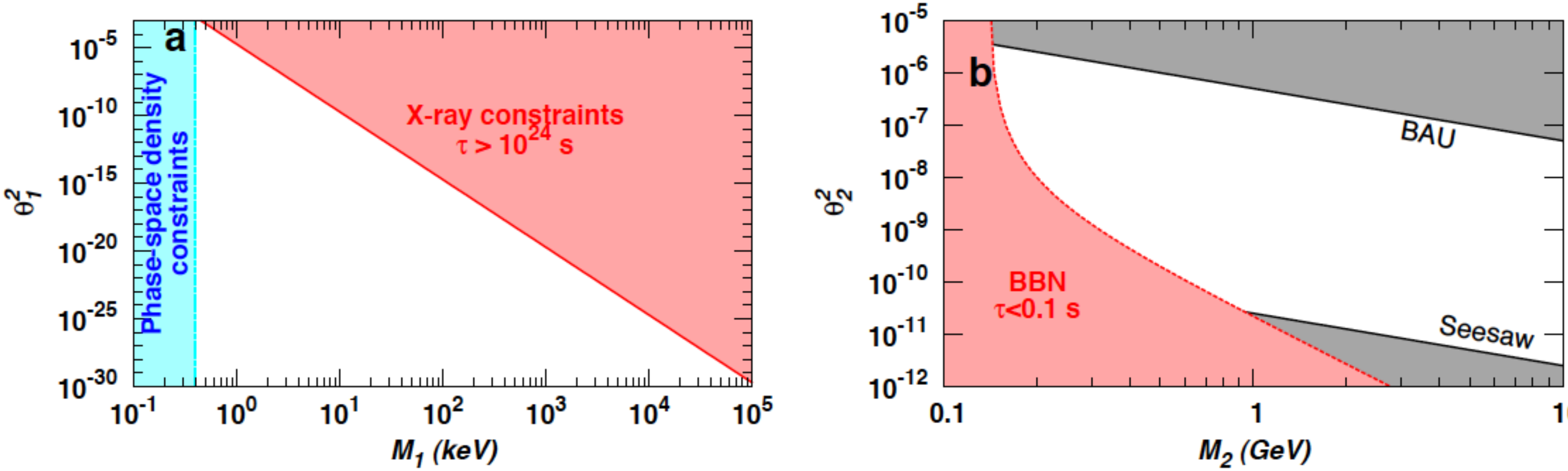


$M_1 << M_2 \approx M_3$

More than one sterile neutrino needed to reproduce Observed oscillations

νMSM

Boyarski, Ruchayskiy, Shaposhnikov…



Constraints on two heavy degenerate singlet neutrinos

N_1 DM production estimation in Early Universe must take into account its interactions with $N_{2,3}$ heavy neutrinos

$M_1 << M_2 \approx M_3$