On Lorentz violation in superluminal theories

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Outline

- Motivation: modified gravity theories
- A toy model with superluminality
- Instabilities and Lorentz violation
- Pathologies in stress-energy tensor
- Conclusions

dRGT Massive Gravity

Features

- Free of Boulware-Deser ghosts
- Allows self-accelerated cosmological solution

Modified gravity theories: examples

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- Free of Boulware-Deser ghosts
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Galileon theory

Features

- Ghost-free
- Self-accelerated cosmological solution
- Galilean Genesis as alternative to inflation

Presence of superluminal modes on non-trivial backgrounds.

Key moment: number of derivatives in self-coupling should be greater than 2; the sign of the coupling should be minus.

Adams et al., 2006

$$\Downarrow$$

Example: Galileon theory

$$\mathcal{L} = \partial_\mu \pi \, \partial^\mu \pi + \frac{c}{\Lambda^4} \partial_\mu^2 \pi (\partial_\nu \pi)^2 + \ldots ; \quad c < 0$$

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- Possible problems with causality (a principal possibility to make a closed time-like curves)
- Problems with UV-completion (scattering amplitudes do not satisfy S-matrix analyticity axioms)

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Superluminal effective theories could not be UV-completed to local QFT or unitary string theory!

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Toy model with superluminality in D=2

$$S = \int d^2x \left[\frac{1}{2}(\partial_\mu\phi\,\partial^\mu\phi) - \frac{m^2}{2}\phi^2 - \frac{1}{4\Lambda^2}(\partial_\mu\phi\,\partial^\mu\phi)^2 + M\delta(x)\phi \right]$$

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Consider classical perturbations on a background of a static solution. It is enough to consider EOM without non-linear ϕ term.

$$\phi'' - m^2\phi = -M\delta(x) \;\Rightarrow\; \tilde{\phi} = \frac{M}{2m}\,\mathrm{e}^{-m|x|}$$

Superluminality

Consider quadratic action for classical perturbations $\phi = \tilde{\phi} + \xi$.
Collect terms up to the order $\frac{1}{\Lambda^2}$

$$S_\xi = \int d^2x \left[\frac{1}{2} Z^{\mu\nu} \partial_\mu \xi \, \partial_\nu \xi - \frac{m^2}{2} \xi^2 \right] ;$$

$$Z^{\mu\nu} = \begin{pmatrix} 1 + \frac{M^2}{4\Lambda^2} \, \mathrm{e}^{-2m|x|} & 0 \\ 0 & -1 - \frac{3M^2}{4\Lambda^2} \, \mathrm{e}^{-2m|x|} \end{pmatrix}$$

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Dispersion relation

$$\omega_\pm = \pm \sqrt{-\frac{Z^{11}}{Z^{00}}} \, k \quad \Rightarrow \quad \left| \frac{d\omega}{dk} \right| > 1$$

Group velocity of the excitations exceeds the speed of light.

Looking for a classical instability

In a static frame

$\det Z^{\mu\nu}_{static} < 0 \quad \Rightarrow$ the solution $\tilde{\phi}$ is classically stable

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In a boosted frame $(x \to \gamma|x + \beta t|)$

$$Z^{\mu\nu}_{boost} = \begin{pmatrix} 1 - \frac{M^2}{4\Lambda^2}\gamma^2(3\beta^2 - 1)\,\mathrm{e}^{-2m\gamma|x+\beta t|} & \frac{M^2}{2\Lambda^2}\gamma^2\beta\,\mathrm{e}^{-2m\gamma|x+\beta t|} \\ \frac{M^2}{2\Lambda^2}\gamma^2\beta\,\mathrm{e}^{-2m\gamma|x+\beta t|} & -1 - \frac{M^2}{4\Lambda^2}\gamma^2(3 - \beta^2)\,\mathrm{e}^{-2m\gamma|x+\beta t|} \end{pmatrix}$$

However, $\det Z^{\mu\nu}_{boost} = \det \Lambda^{\mu}_{\rho}\Lambda^{\nu}_{\sigma} Z^{\rho\sigma}_{static} = \det Z^{\mu\nu}_{static}$

$$\Downarrow$$

The solution $\tilde{\phi}$ is classically stable in any frame.

Looking for a quantum instability

As Z^{00} change its sign in a boosted frame $\Rightarrow$ Ghosts appear!

$$Z^{00} = 1 - \frac{M^2}{4\Lambda^2}\gamma^2(3\beta^2 - 1)\,\mathrm{e}^{-2m\gamma|x+\beta t|} < 0$$

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Critical boost factor $\gamma > \frac{\Lambda\sqrt{2}}{M}\,\mathrm{e}^{m\gamma|x+\beta t|}$

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Looking for UV dynamics; we can take $m = 0$ and regard boosted background as a constant

$$S_\xi = \int d^2x \left[\frac{1}{2} Z^{\mu\nu}\partial_\mu\xi\,\partial_\nu\xi\right]; \; Z^{\mu\nu} = \begin{pmatrix} Z^{00} & Z^{01} \\ Z^{01} & Z^{11} \end{pmatrix} = \text{const}$$

Looking for evolution of negative energy modes.

Add interaction with matter scalar field χ.

$$S_\chi = \int d^2x \left[\frac{1}{2}(\partial_\mu \chi \, \partial^\mu \chi) - \frac{m_\chi^2}{2}\chi^2 + \frac{g}{2}\xi\chi\chi \right]$$

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Triples $\xi\chi\chi$ created from vacuum as $Z^{00} < 0$

Required hierarchy of parameters:

$$m_\chi < m\gamma \ll k \ll \Lambda \; ; \quad m \ll M \; ; \quad g\frac{M}{m} \ll m_\chi^2$$

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Estimations for background decay rate and energy loss rate

Integral saturated in IR

$$\frac{dE}{dt} \sim \Gamma \sim \frac{g^2}{2\pi m^2 \gamma^2} \lesssim \frac{m_\chi^4}{\Lambda^2}$$

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Small, but non-zero quantity $\Rightarrow$ Lorentz invariance is violated!

Naive estimation for $4D$

$$S_\chi = \int d^2x \left[\frac{1}{2} (\partial_\mu \chi \, \partial^\mu \chi) - \frac{m_\chi^2}{2} \chi^2 + \frac{g'}{2} \xi (\partial_\mu \chi \partial^\mu \chi) \right]$$

This simulates coupling $\frac{g'}{2} \xi^2 \chi^2$ in $4D$ as it have the same dimensionality.

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Estimation for background decay rate

Integral saturated in UV

$$\Gamma \sim \frac{g'^2}{8\pi}\Lambda^2$$

Estimation for energy loss rate

$$\frac{dE}{dt} \sim \frac{g'^2}{8\pi}\frac{\Lambda^3}{m\gamma} \, .$$

This can be very high and probably could destroy the classical solution.

Expect some features of $T^{\tilde{\phi}}_{\mu\nu}(\xi)$ as $\gamma \to \gamma_{critical}$

Below $\gamma_{critical}$, $T^{\tilde{\phi}\, boosted}_{\mu\nu}(\xi) = T^{\tilde{\phi}\, static}_{\mu\nu}(\xi)$

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Looking for accelerated solution $\tilde{\phi}$ with velocity $\beta = t/\tau \simeq 1$

$$Z^{\mu\nu} \simeq \begin{pmatrix} 1 - t^2/\alpha^2 & -t^2/\alpha^2 \\ -t^2/\alpha^2 & -1 - t^2/\alpha^2 \end{pmatrix} \; ; \quad \alpha = \text{const}$$

Stress-energy tensor pathology

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Solution of EOM $\partial_\mu[Z^{\mu\nu}\partial_\nu\xi] = 0$ with ansatz $\xi = \xi(t)\,\mathrm{e}^{ikx}$

$$\xi(t,x) = c_1\,\mathrm{e}^{-ik(t-x)} + c_2\,\mathrm{e}^{-ik(t-x-\alpha\log\frac{\alpha+t}{\alpha-t})}$$

We quantize this excitations considering $Z^{\mu\nu}$ as "curved" background.
Then we calculate renormalized $T_{\mu\nu}$

$$\langle T^{ren.}_{\mu\nu} \rangle = \lim_{x \to x'} \mathcal{D}_{\mu\nu} [G^{(1)}_{ex.}(x, x') - G^{(1)}_{pert.}(x, x')] \; ; \quad G^{(1)}_{ex.}(x, x') = \langle [\xi(x), \xi(x')] \rangle$$

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Using Schwinger–DeWitt technique we get

$$G^{(1)}_{ren.} = \lim_{x\to x'}[G^{(1)}_{ex.}(x,x') - G^{(1)}_{pert.}(x,x')] = \log\left[\frac{(\alpha^2 - {t'}^2)^2}{(\alpha^2 + {t'}^2)\alpha^2}\right] + \text{regular terms}$$

Indication for pathology as $t' \to \alpha$, although renormalization of $T_{\mu\nu}$ is needed

Work in progress...

Conclusions & Discussion

- Physical solution in the simple superluminal quantum field theory shows different behaviour in static and boosted frames. I.e. Lorentz invariance is broken.
    - Theory is pathological and we don't need to consider it.
    - As the theory is non-Lorentzian we need to complete a Lagrangian with other Lorentz-braking terms.
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    - Theory is pathological and we don't need to consider it.
    - As the theory is non-Lorentzian we need to complete a Lagrangian with other Lorentz-braking terms.
    - It is a spontaneous breaking of Lorentz invariance, so we need to consider a phase with a broken symmetry.
- This analysis seem to be suitable for other theories that have superluminal propagation on classical backgrounds.

Thank you!