Seventh Agean Summer School in Paros

Screening the fifth force in the Horndeski's most general scalar-tensor theories

R. Kase and S. Tsujikawa, JCAP 1308 (2013) 054

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1. Introduction

Discovery of late-time cosmic acceleration

In 1998, the discovery of late-time cosmic acceleration based on Type Ia supernovae is reported. The source for this acceleration is named dark energy.

Modified gravitational theories as candidates for dark energy

- There must be a stable accelerating solution which explains cosmic acceleration.
- They should be observationally distinguished from the Λ–CDM model.
- **These models need to recover General Relativistic (GR) behavior at short distances to satisfy solar system constraints.**

f(R) gravity,
Scalar–tensor theories.
→ with potential term

DGP braneworld,
Galileon gravity.
→ without potential term

There are two different mechanisms.

Recovery of GR behavior at short distances

1. Chameleon mechanism

f(R) gravity,
Brans–Dicke theory.

The potential term of a scalar field produces effective mass in the region of high density. The models recover GR if the effective mass is large enough.

→ **However a fine tuning of initial conditions is required to realize a viable cosmology.**

2. Vainshtein mechanism

DGP braneworld,
Galileon gravity.

In the DGP braneworld the non–linear effect of the field self–interaction term $\Box\phi(\partial_\mu\phi\partial^\mu\phi)$ allows the possibility to recover GR at short distances.

→ **However the DGP model suffers from a ghost, in addition to the difficulty of consistency with the combined data analysis.**

Horndeski's action

Horndeski's action:
The most general scalar-tensor theories in four dimensions.

$$S=\int d^4x\sqrt{-g}\left[\sum_{i=2}^{5}\mathcal{L}_i+\mathcal{L}_m\right]$$

The Lagrangian of the matter field

- **Brans-Dicke**
- **k-essence**
- **Galileon**
- **Gauss-Bonnet**

and so on

$$\mathcal{L}_2=K(\phi,\,X)\,,\mathcal{L}_3=-G_3(\phi,\,X)\,\Box\phi\,,$$
$$\mathcal{L}_4=G_4(\phi,\,X)\,R+G_{4,X}\times(\text{field derivatives})\,,$$
$$\mathcal{L}_5=G_5(\phi,\,X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\phi)-\frac{1}{6}G_{5,X}\times(\text{field derivatives})\,.$$

・A. De Felice, R. Kase and S. Tsujikawa, Phys. Rev. D 85, 044059

・R. Kimura, T. Kobayashi and K. Yamamoto, Phys. Rev. D 85, 024023

・K. Koyama, G. Niz and G. Tasinato, arXiv:1305.0279 [hep-th]

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- **Brans-Dicke**
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and so on

$$\mathcal{L}_2 = K(\phi,\ X)\,, \mathcal{L}_3 = -G_3(\phi,\ X)\,\Box\phi\,,$$

$$\mathcal{L}_4 = G_4(\phi,\ X)\,R + G_{4,X} \times (\text{field derivatives})\ ,$$

$$\mathcal{L}_5 = G_5(\phi,\ X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X} \times (\text{field derivatives})\,.$$

Spherically symmetric space-time

$$ds^2 = -e^{2\Psi(r)}dt^2 + e^{2\Phi(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

$X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$

$G_{i,X} \equiv \partial G_i/\partial X$

2. FIELD EQUATIONS OF MOTION

For example, the (0,0) component of the full equation of motion is the following

$$\boxed{\frac{2G_4}{r}\Phi' + \frac{G_4}{r^2}\left(e^{2\Phi}-1\right) - e^{2\Phi}\rho_m}$$

Dominant terms when the theory behaves as GR.

$$+\boxed{\left[\left(G_4' - 2\phi' X G_{3,X} + \ldots\right)\Phi' - G_4'' - \frac{2G_4'}{r} + 2\phi'' X G_{3,X} + \ldots\right]} = 0\,,$$

$$G_4' \equiv \partial G_4/\partial r$$

$$\varepsilon_{G4\phi} = \frac{r\phi' G_{4,\phi}}{G_4}\,,\ \ldots$$

These quantities should be suppressed inside the Vainshtein radius.

$$|\epsilon_i| \ll 1$$

We use the weak-gravity approximation under the above condition: $\{|\Phi|, |\epsilon_i|\} \ll 1$

Combining EOMs we obtain following two equations.

$$\Box\Psi = \mu_1 \rho_m + \boxed{\mu_2 \Box\phi} + \mu_3$$

$$\Box\phi = \mu_4\,\rho_m + \mu_5$$

$$\mu_i = \mu_i(K, G_3, G_4, G_5)$$

The scalar field gives rise to the modification to the gravitational potential, i.e. the fifth force.

Modified Poisson equation

$$\Box\Psi = 4\pi G_{\rm eff}\rho_m + \mu_2\mu_5 + \mu_3\,,$$

$$G_{\rm eff} \equiv \frac{1}{16\pi G_4}\left[1+\boxed{\frac{r}{G_4\beta}\alpha\left(\alpha-\frac{1}{2}\phi'\beta\right)}+3\Phi+\mathcal{O}(\varepsilon_j)\right]\,,$$

$$\boxed{\alpha \equiv G_{4,\phi}+2XG_{4,\phi X}-XG_{3,X}-\frac{2X(G_{5,X}+XG_{5,XX})}{r^2}\,.}$$

The modification of gravity manifests itself for the theories characterized by $\alpha \neq 0$.
The representative example having a non-zero value of α is the dilatonic coupling:

$$G_4(\phi) = \frac{M_{\rm pl}^2}{2}e^{-2Q\phi/M_{\rm pl}}$$

In this case,

$$G_{\rm eff} \simeq G\{1+2Q^2[1+\phi' r/(2QM_{\rm pl})]+3\Phi+\mathcal{O}(\varepsilon_j)\}$$

For $|Q| \sim \mathcal{O}(1)$, the gravitational theory is to be modified significantly. In the following we study how this modification is suppressed because of the term $G_i(\phi, X)$.

3. VAINSHTEIN MECHANISM

$$\Box\phi = \mu_4\,\rho_m + \mu_5$$

In the DGP model, a non-linear field self-interaction of the form $X\Box\phi$ can lead to the recovery of GR within the so-called Vainshtein radius r_V.

$G_3(\phi, X) = X$

$$\mu_4 \simeq -\frac{r}{4G_4\beta}\left[2\,G_{4,\phi} + 4\,XG_{4,\phi X} - 2\,XG_{3,X} - \phi'\beta - \frac{4\,X\,(G_{5,X} + XG_{5,XX})}{r^2}\right]$$

$$\mu_5 \simeq -\frac{1}{r\beta}\Big[(K_{,\phi} - 2\,XK_{,\phi X} + 2\,XG_{3,\phi\phi})\,r^2 - 4\,X\,(K_{,XX} - 2\,G_{3,\phi X} + 2\,G_{4,\phi\phi X})\,\phi' r$$
$$- 4X(3\,G_{3,X} + 4\,XG_{3,XX} - 9G_{4,\phi X} - 10\,XG_{4,\phi XX} + XG_{5,\phi\phi X})$$
$$+ \frac{8X\phi'\,(3\,G_{4,XX} + 2\,XG_{4,XXX} - 2\,G_{5,\phi X} - XG_{5,\phi XX})}{r}\Big],$$

$$\beta \equiv (K_{,X} + 2\,XK_{,XX} - 2\,G_{3,\phi} - 2\,XG_{3,\phi X})\,r - 4\,\phi'\,(G_{3,X} + XG_{3,XX} - 3\,G_{4,\phi X} - 2\,XG_{4,\phi XX})$$
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3. VAINSHTEIN MECHANISM

$$\Box\phi = \mu_4\,\rho_m + \mu_5$$

Non-linear terms dominant

$$\Box\phi = \mu_4\,\rho_m + \mu_5$$

Linear terms dominant

0 $\qquad r_V \qquad r$

$$r \gg r_V$$

$$\mu_4 \simeq -\frac{r}{4G_4\beta}\left[2\,G_{4,\phi} + 4\,XG_{4,\phi X} - 2\,XG_{3,X} - \phi'\beta - \frac{4\,X\,(G_{5,X} + XG_{5,XX})}{r^2}\right]$$

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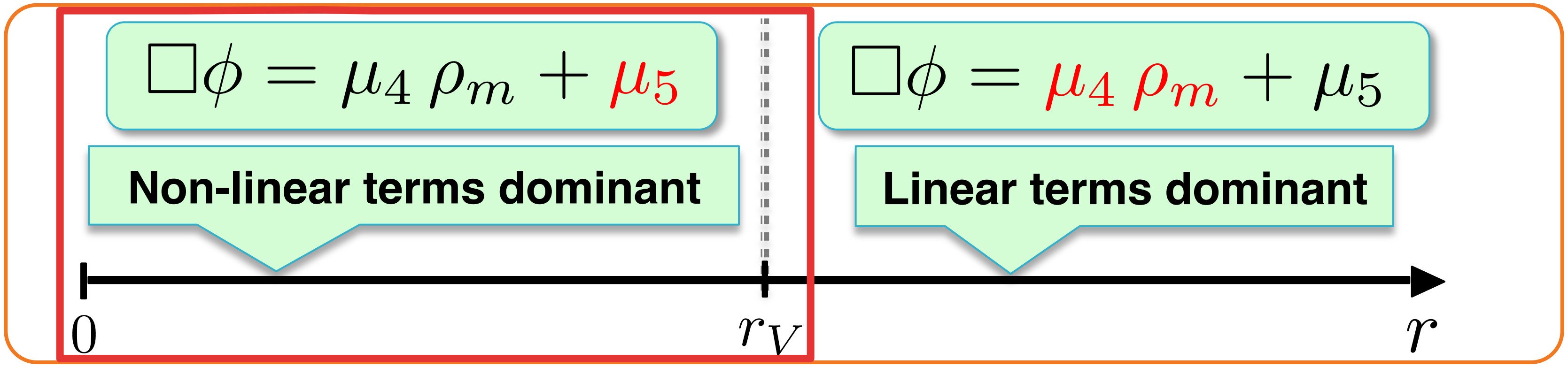
Linear terms

Considering that the linear term in $K(\phi, X)$ gives the dominant contribution, we have $K(\phi, X) = e^{-\lambda_2\phi/M_{\rm pl}}X$. Picking up the dominant contributions to the EOM…

$$\phi'(r) = \frac{QM_{\rm pl}r_g}{r^2}$$

$$r_g \equiv \frac{1}{M_{\rm pl}^2}\int_0^r \rho_m \tilde{r}^2 d\tilde{r}$$

3. VAINSHTEIN MECHANISM



$r \ll r_V$

$$\mu_5 \simeq -\frac{1}{r\beta}\Big[(K_{,\phi} - 2\,XK_{,\phi X} + 2\,XG_{3,\phi\phi})\,r^2 - 4\,X\,(K_{,XX} - 2\,G_{3,\phi X} + 2\,G_{4,\phi\phi X})\,\phi' r$$
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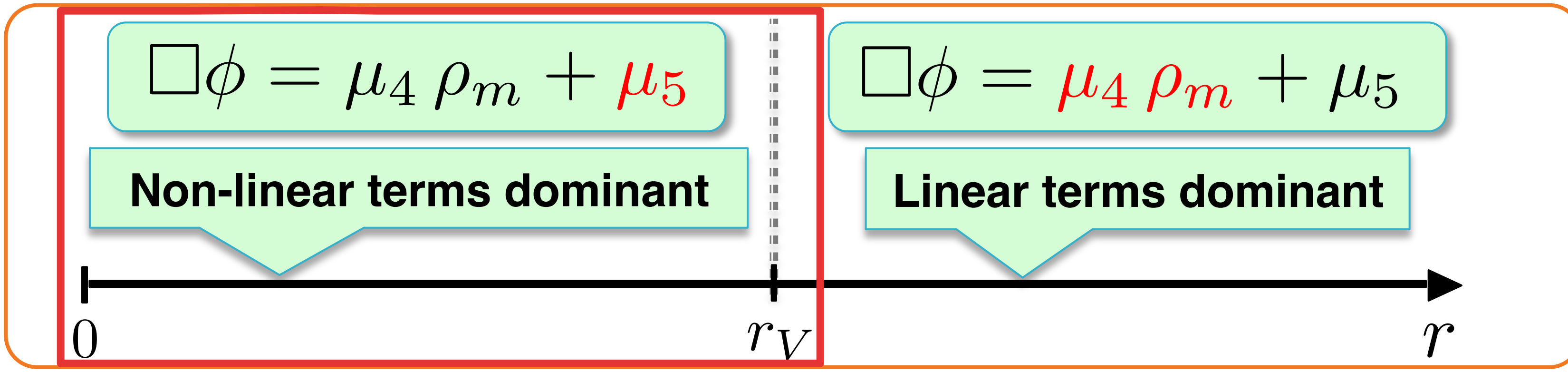

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For example in the case of $G_3 = 0 = G_5$,

$$\frac{d}{dr}(r^2\phi') \simeq 2r\phi' \quad \Rightarrow \quad \phi'(r) = \text{Constant}.$$

3. VAINSHTEIN MECHANISM

$$\Box\phi = \mu_4\,\rho_m + \mu_5 \qquad \Box\phi = \mu_4\,\rho_m + \mu_5$$

Non-linear terms dominant | **Linear terms dominant**

$0 \qquad r_V \qquad r$

$$r \ll r_V$$

$$\mu_5 \simeq -\frac{1}{r\beta}\Big[(K_{,\phi} - 2\,XK_{,\phi X} + 2\,XG_{3,\phi\phi})\,r^2 - 4\,X\,(K_{,XX} - 2\,G_{3,\phi X} + 2\,G_{4,\phi\phi X})\,\phi' r$$

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$$+\boxed{\frac{8X\phi'\,(3\,G_{4,XX} + 2\,XG_{4,XXX} - \cdots}{r}}$$

$$\beta \equiv (K_{,X} + 2\,XK_{,XX} - 2\,G_{3,\phi} - 2\,XG_{3,\phi X})\,r - 4\,\phi'\,(G_{3,X} + \cdots$$

$$\boxed{-\frac{4\,X\,(3\,G_{4,XX} + 2 \cdots}{\cdots}}$$

$$\Box\Psi = \mu_1\rho_m + \mu_2\Box\phi + \mu_3$$

The fifth force propagate via the second derivative of ϕ. Thus constant solution may suppress the fifth force.

For example in the case of $G_3 = 0 = G_5$,

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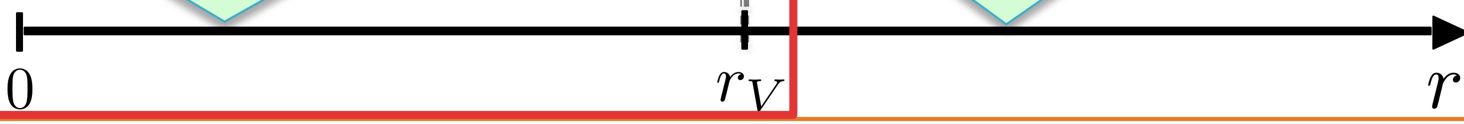

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$$-\frac{4\,X\,(3\,G_{4,XX} + 2 \ldots}{\ldots}$$

When $G_4(X) = X^p$,

$p \geq 2$ is required.

$\Box\Psi = \mu_1\rho_m + \mu_2\Box\phi + \mu_3$

The fifth force propagate via the second derivative of ϕ. Thus constant solution may suppress the fifth force.

For example in the case of $G_3 = 0 = G_5$,

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4. COVARIANT GALILEON

$$G_3(X) = \frac{c_3}{M^3} X\,, \quad G_4(\phi, X) = \frac{M_{\rm pl}^2}{2} e^{-2Q\phi/M_{\rm pl}} + \frac{c_4}{M^6} X^2\,, \quad G_5(X) = \frac{c_5}{M^9} X^2\,,$$

A. Nicolis, R. Rattazzi, E. Tricherini (2009)
C. Deffayet, G. Esposito-Farese, A. Vikman (2009)

- Vainshtein radius

$$\phi' \simeq Q M_{\rm pl} r_g / r^2 \quad (r \gg r_V)$$

$$\frac{M^3 r_V^3}{Q M_{\rm pl} r_g} = c_V$$

For DE, $M^3 \simeq M_{\rm pl} H_0^2$

$$r_V \simeq 10^{20}\ {\rm cm}$$

$$c_V = 2c_3 \pm \sqrt{4c_3^2 - 12c_4}$$

or

$$c_V = -2c_3 \pm \sqrt{4c_3^2 - 12c_4}$$

- The solution in the regime $r \ll r_V$

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$$\mu_5 \simeq -\frac{1}{r\beta}\Bigg[(K_{,\phi} - 2XK_{,\phi X} + 2XG_{3,\phi\phi})\,r^2 - 4X\,(K_{,XX} - 2G_{3,\phi X} + 2G_{4,\phi\phi X})\,\phi' r$$
$$-4X(3G_{3,X} + 4XG_{3,XX} - 9G_{4,\phi X} - 10XG_{4,\phi XX} + XG_{5,\phi\phi X})$$
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- The solution in the regime $r \ll r_V$

$$\Box\phi = \mu_4\, \rho_m + \mu_5$$

$$r\phi'^2(r) - \frac{2c_4}{c_3 M^3}\phi'^3(r) = C$$

4. COVARIANT GALILEON

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G_3

G_4

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4. COVARIANT GALILEON

(i) G_3 dominant $(|c_3| \sim 1$ and $|c_4| \ll 1)$

G_3 **dominant**

$$r\phi'^2(r) - \frac{2c_4}{c_3 M^3}\phi'^3(r) = C$$

$$\phi'(r) \simeq \frac{Q M_{\rm pl} r_g}{r_V^{3/2} r^{1/2}} \propto r^{-1/2}$$

$$\Phi \simeq \frac{r_g}{2r}\left[1 - 2Q^2\left(\frac{r}{r_V}\right)^{3/2} + \frac{4c_5 Q^2}{5c_V^3}\frac{r_g r_V^{3/2}}{r^{5/2}}\right],$$

$$\Psi \simeq -\frac{r_g}{2r}\left[1 - 4Q^2\left(\frac{r}{r_V}\right)^{3/2} + \frac{8c_5 Q^2}{35c_V^3}\frac{r_g r_V^{3/2}}{r^{5/2}}\right]$$

G_5

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G_3 dominant

$$r\phi'^2(r) - \frac{2c_4}{c_3 M^3}\phi'^3(r) = C$$

$(c_5^2 r_g^2 r_V^3)^{1/5}$
$\simeq 10^{14}$ cm.
(for the Sun)

In this region this model cannot satisfy solar system constraints.

$$\phi'(r) \simeq \frac{Q M_{\rm pl} r_g}{r_V^{3/2} r^{1/2}} \propto r^{-1/2}$$

$$\Phi \simeq \frac{r_g}{2r}\left[1 - 2Q^2\left(\frac{r}{r_V}\right)^{3/2} + \frac{4c_5 Q^2}{5c_V^3}\frac{r_g r_V^{3/2}}{r^{5/2}}\right],$$

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G_5

4. COVARIANT GALILEON

(ii) G_4 dominant ($|c_3| \ll 1$ and $|c_4| \sim 1$)

$$\Box\phi = \mu_4\,\rho_m + \mu_5$$

G_4 dominant

$$r\phi'^2(r) - \frac{2c_4}{c_3 M^3}\phi'^3(r) = C$$

$$\phi'_{\rm out}(r) = QM_{\rm pl}r_g/r_V^2$$

$$\Phi \simeq \frac{r_g}{2r}\left[1 - 2Q^2\left(\frac{r}{r_V}\right)^2 - \frac{3c_5Q^2r_g^2}{c_V^3 r_V r}\right],$$

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$$|\gamma - 1| \simeq 0$$

$$\Phi \simeq \frac{r_g}{2r}\left[1 - 2Q^2\left(\frac{r}{r_V}\right)^2 - \frac{3c_5Q^2r_g^2}{c_V^3 r_V r}\right],$$

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The observational constraint on post−Newtonian parameter $\gamma \equiv -\Phi/\Psi$

$$|\gamma - 1| < 2.3 \times 10^{-5}$$

4. COVARIANT GALILEON

(ii) G_4 dominant ($|c_3| \ll 1$ and $|c_4| \sim 1$)

$$\Box\phi = \mu_4 \rho_m + \mu_5$$

Inside the surface of the star $r \lesssim r_s$, ρ_m tends to be large.

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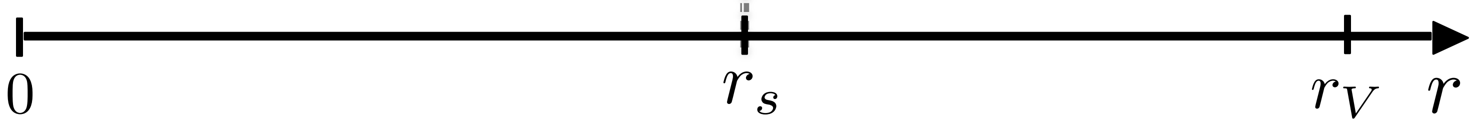

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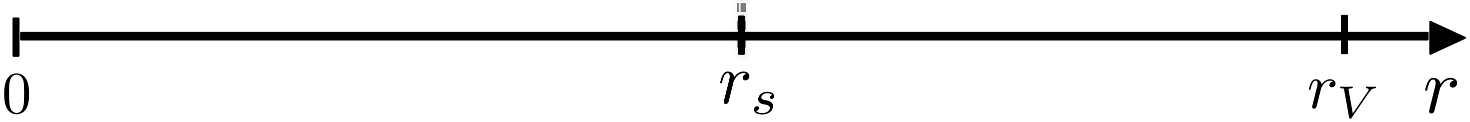
G_4 **dominant**



(a) $\rho_m = \rho_c$: constant matter density

regularity condition: $\phi'(0) = 0$

$$\phi'_{\rm in}(r) \simeq \left(\frac{Q\rho_c}{12c_4 M_{\rm pl}}\right)^{1/3} M^2 r$$

$$r\phi'^2(r) - \frac{2c_4}{c_3M^3}\phi'^3(r) = C$$

$$\phi'_{\rm out}(r) = QM_{\rm pl}r_g/r_V^2$$

$$\Phi \simeq \frac{r_g}{2r}\left[1 - 2Q^2\left(\frac{r}{r_V}\right)^2 - \frac{3c_5Q^2r_g^2}{c_V^3 r_V r}\right],$$

$$\Psi \simeq -\frac{r_g}{2r}\left[1 - 2Q^2\left(\frac{r}{r_V}\right)^2 - \frac{3c_5Q^2r_g^2}{c_V^3 r_V r}\right],$$

Comparing two solutions at the surface of the star, we found...

$$|\phi'_{\rm in}(r_s)|/|\phi'_{\rm out}(r_s)| \simeq 1$$

4. COVARIANT GALILEON

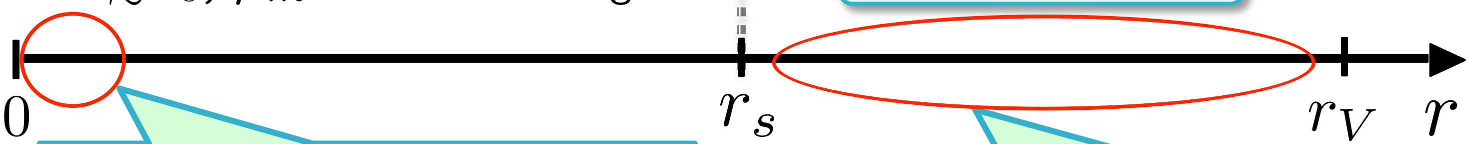
(ii) G_4 dominant $(|c_3| \ll 1$ and $|c_4| \sim 1)$

$$\Box\phi = \mu_4\rho_m + \mu_5$$

$$\Box\phi = \mu_4\,\rho_m + \mu_5$$

Inside the surface of the star $r \lesssim r_s$, ρ_m tends to be large.

G_4 dominant



$$\phi'_{\rm in}(r) \simeq \left(\frac{Q\rho_c}{12c_4 M_{\rm pl}}\right)^{1/3} M^2 r$$

$$\phi'_{\rm out}(r) = Q M_{\rm pl} r_g / r_V^2$$

$$\mu_4 \simeq -\frac{r}{4G_4\beta}\left[2\,G_{4,\phi} + 4\,XG_{4,\phi X} - 2\,XG_{3,X} - \phi'\beta - \frac{4\,X\,(G_{5,X} + XG_{5,XX})}{r^2}\right]$$

The term $4X(G_{5,X} + G_{5,XX})/r^2$ dose not dominate over the other term $2G_{4,\phi}$ inside the radius of the star r_s for...

$$|2G_{4,\phi}| \gg |4X(G_{5,X} + XG_{5,XX})/r^2|$$

$$|c_5| < 10 M M_{\rm pl}^{7/3}/(r_s^2 \rho_c^{4/3})$$

4. COVARIANT GALILEON

(b) $\rho_m(r)$: varying matter density

$$\rho_m = \rho_c \exp(-r^2/r_t^2)\,,$$

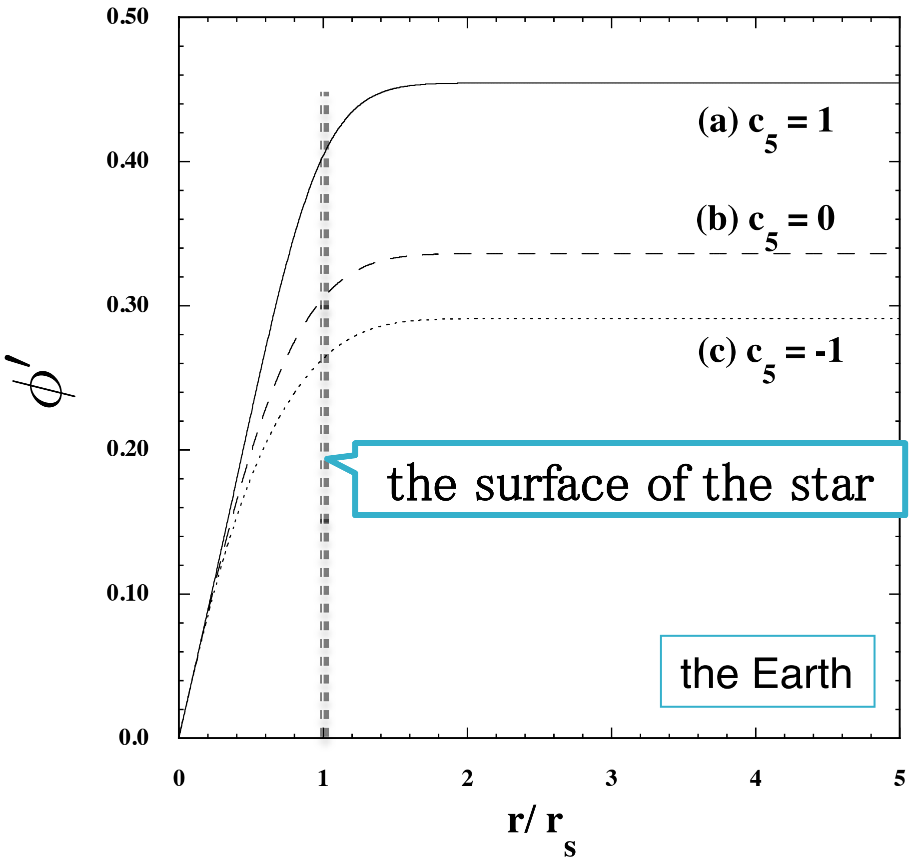
r_t : the distance where the density starts to decrease

ρ_c : the density around the origin

The effect of the term G_5 dose not manifest itself inside the star for...

$$|c_5| < 10 M M_{\rm pl}^{7/3}/(r_s^2 \rho_c^{4/3})$$

- the Earth $|c_5| \lesssim 1$
- the Sun $|c_5| \lesssim 10^{-7}$



4. COVARIANT GALILEON

(b) $\rho_m(r)$: varying matter density $\rho_m = \rho_c \exp(-r^2/r_t^2)\,,$

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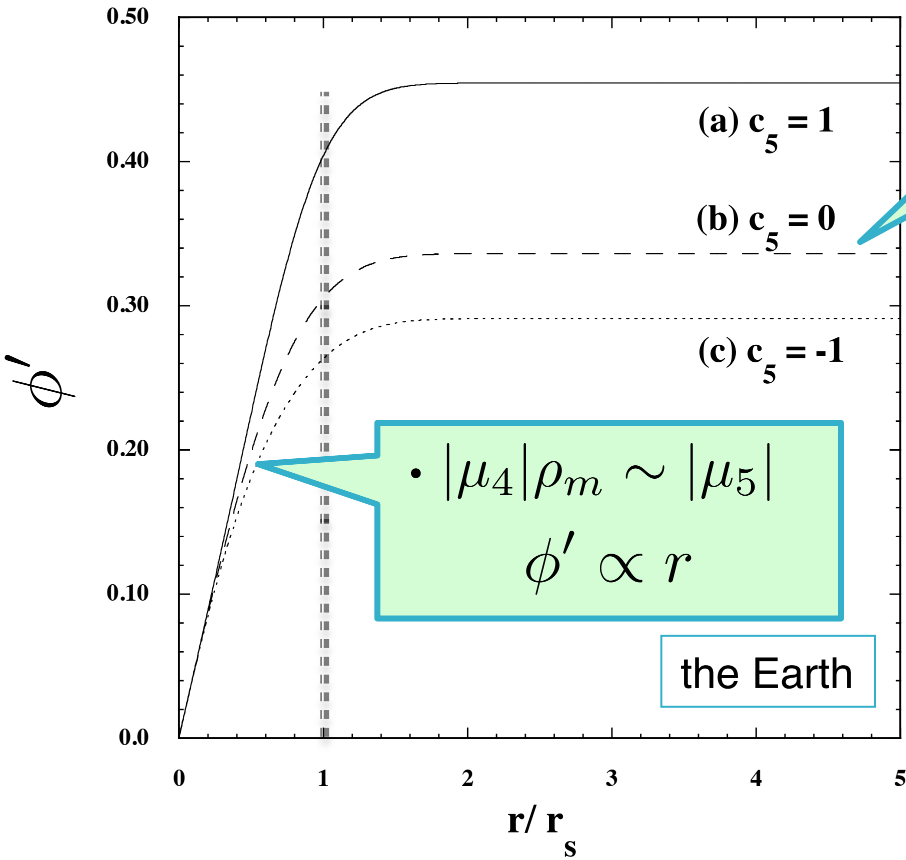

- $|\mu_4|\rho_m \sim |\mu_5|$

  $\phi' \propto r$

- μ_5 dominant

  $\phi' \simeq$ Constant.

$$\Box\phi = \mu_4\,\rho_m + \mu_5\,,$$

$$\phi'_{\rm in}(r) \simeq \left(\frac{Q\rho_c}{12 c_4 M_{\rm pl}}\right)^{1/3} M^2 r$$

$$\phi'_{\rm out}(r) = Q M_{\rm pl} r_g / r_V^2 \sim \mathcal{O}(0.1)$$

4. COVARIANT GALILEON

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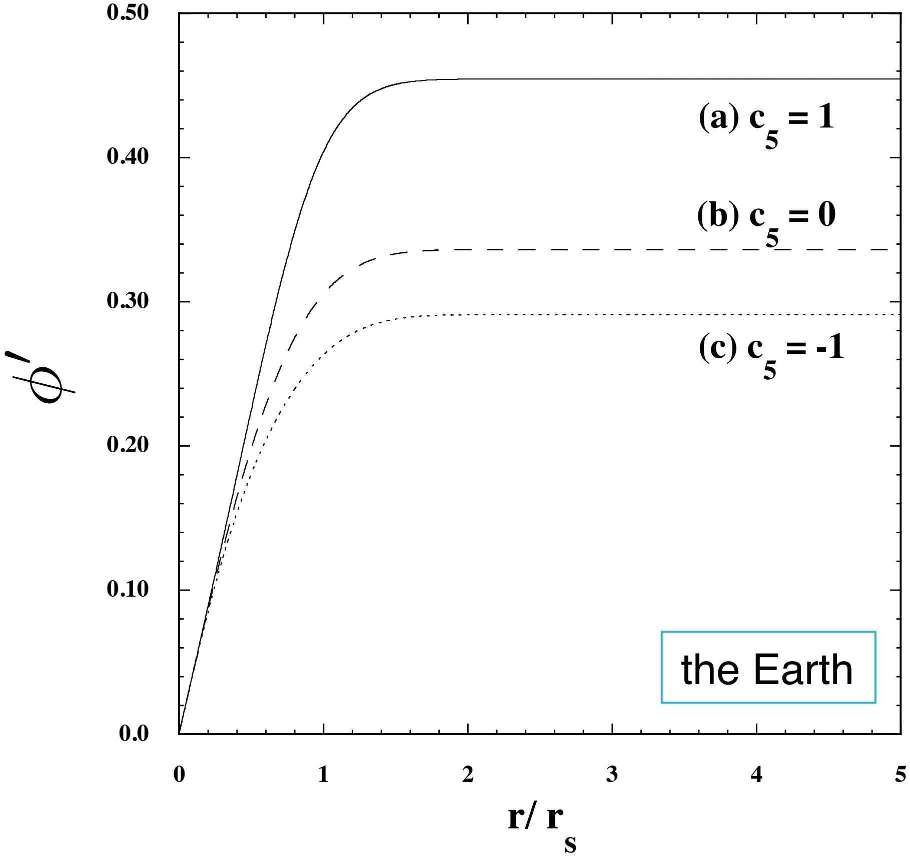

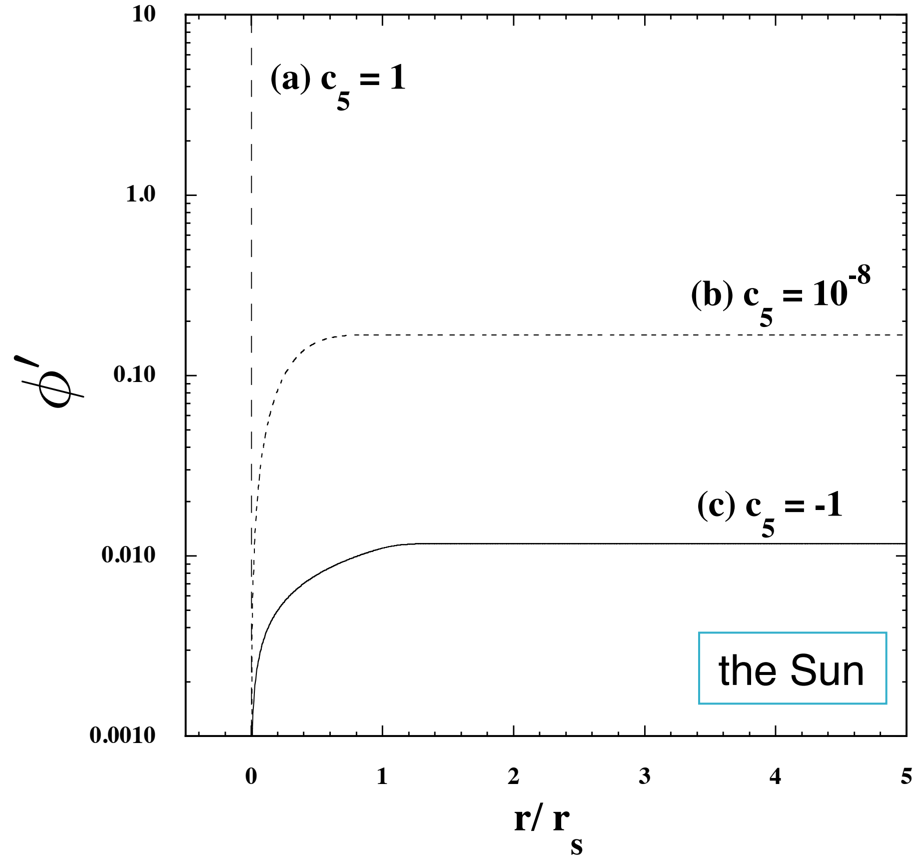

5. APPLICATION TO OTHER MODELS

- **DBI Galileons with Gauss-Bonnet and other terms**

C. de Rham and A. J. Tolley (2010)

$$S=\int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}e^{-2Q\phi/M_{\rm pl}}R-f_1(\phi)^2\mu^4\left(\sqrt{1-\frac{2f_1(\phi)^{-1}X}{\mu^4}}-1\right)+f_2(\phi)\frac{X^2}{\mu^4}\right.$$
$$\left.+f_3(\phi)\frac{X}{M^3}\Box\phi+f_4(\phi)c_{\rm GB}R_{\rm GB}^2+f_5(\phi)\frac{1}{m^2}G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right],$$

- the Vainshtein radius $\quad r_V\simeq(4|Q|M_{\rm pl}r_g)^{1/3}/M$
- the solution in the regime $r\ll r_V$ $\quad \phi'\simeq QM_{\rm pl}r_g/(r_V^{3/2}r^{1/2})$

This solution is valid in the regime $r_s\ll r\ll r_V$ under following conditions

$$|c_{\rm GB}|\ll\frac{r_V^3r_sM_{\rm pl}^2}{Q^2r_g^2},\qquad \mu\gg\left(\frac{Q^2M_{\rm pl}^2r_g^2}{r_V^3r_s}\right)^{1/4},\qquad m\gg\left(\frac{r_g^2}{r_V^3r_s^3}\right)^{1/4}.$$

In the case of the Sun and the Vainshtein radius $r_V=10^{20}\,{\rm cm}$, these conditions are translated to

$$|c_{\rm GB}|\ll10^{124},\qquad \mu\gg10^{-13}\,{\rm GeV},\qquad m\gg10^{-34}\,{\rm GeV}.$$

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$$\left.+f_3(\phi)\frac{X}{M^3}\Box\phi+f_4(\phi)c_{\rm GB}R_{\rm GB}^2+f_5(\phi)\frac{1}{m^2}G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right],$$

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This solution is valid in the regime $r_s\ll r\ll r_V$ under following conditions

$$|c_{\rm GB}|\ll\frac{r_V^3r_sM_{\rm pl}^2}{Q^2r_g^2},\qquad \mu\gg\left(\frac{Q^2M_{\rm pl}^2r_g^2}{r_V^3r_s}\right)^{1/4},\qquad m\gg\left(\frac{r_g^2}{r_V^3r_s^3}\right)^{1/4}.$$

Moreover if we demand that $|\gamma-1|<2.3\times10^{-5}$ is satisfied up to $r=10\,{\rm AU}=10^{14}\,{\rm cm}$, then we need $r_V>10^{17}\,{\rm cm}$. This condition corresponds to...

$$\boxed{M\lesssim10^{-18}\,{\rm GeV}}$$

5. CONCLUSIONS

- In most general second-order scalar-tensor theories we have studied how the Vainshtein mechanism works in a spherically symmetric background with a matter source.
- We derived the full equation of motion, general formula for the Vainshtein radius, and several conditions under which the Vainshtein mechanism can be at work sufficiently.
- As it was discussed in several articles, the term G_5 can violate the Vainshtein mechanism. We derived the analytic conditions for the success of the Vainshtein mechanism in the context of covariant Galileons by studying the connection between interior and exterior solutions. And we also confirmed that numerically.
- We applied our results to several models such as conformal/ extended Galileon models, DBI Galileons with Gauss-Bonnet and other terms.

constant EOS

- Planck+WP+SNLS

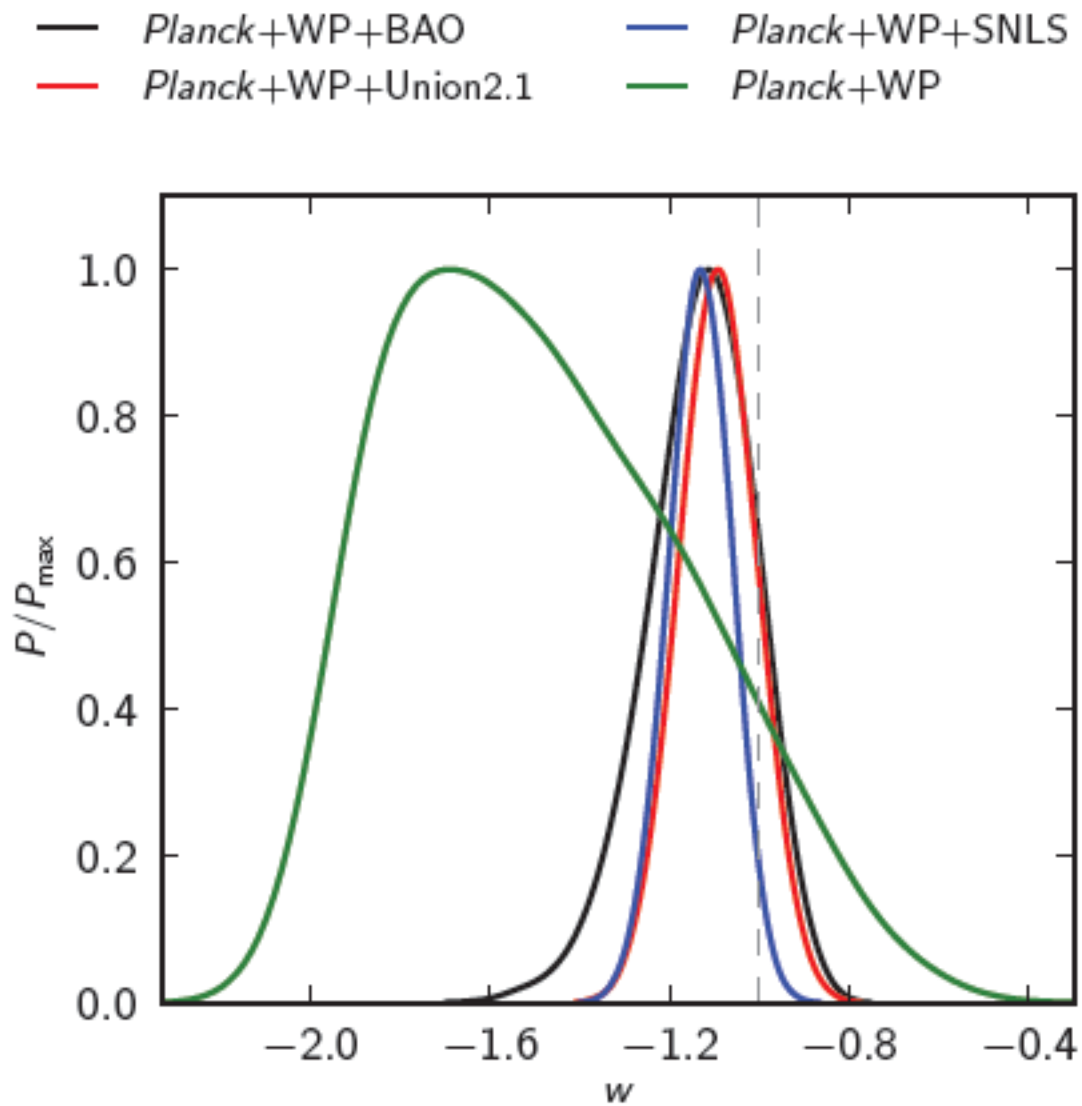
$$w = -1.13^{+0.13}_{-0.14}$$

- Planck+WP+BAO

$$w = -1.13^{+0.24}_{-0.25}$$

- Planck+WP+Union2.1

$$w = -1.09 \pm 0.17$$



Planck collaboration arXiv:1303.5076 [astro-ph.CO]

1. Motivation

- **Discovery of late-time cosmic acceleration**

In 1998, the discovery of late-time cosmic acceleration based on Type Ia supernovae is reported. The source for this acceleration is named dark energy.

The equation of state defined below characterizes dark energy.

$$w \equiv P/\rho$$

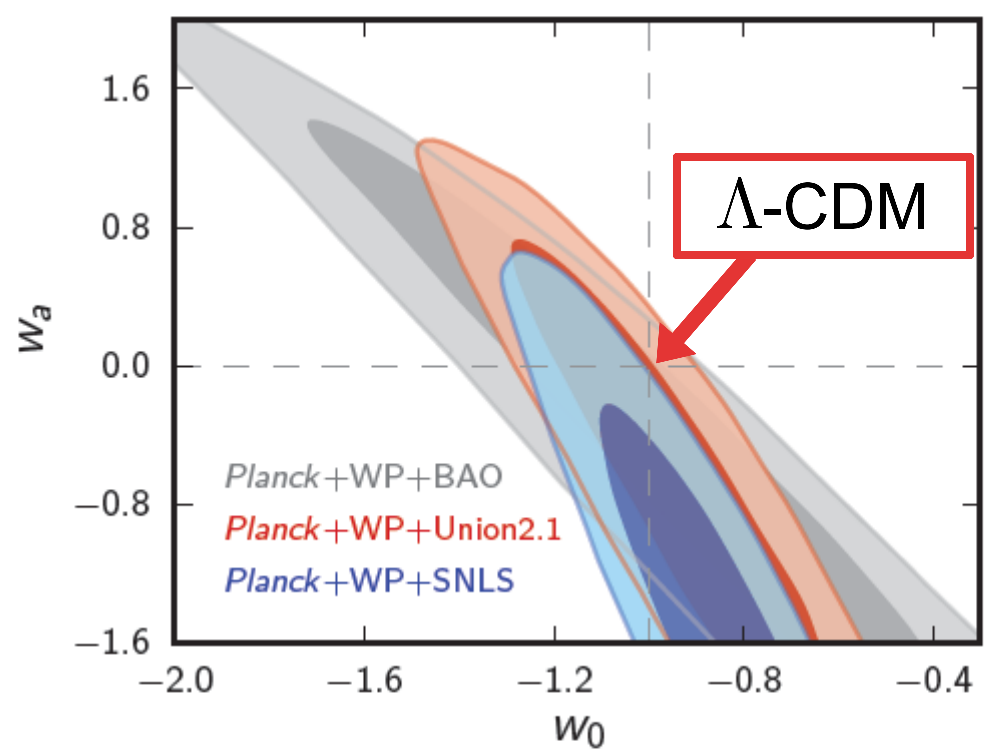
Condition for acceleration :

$$w < -1/3$$

Constraint of the equation of state from observations

- Chevallier-Polarski-Linder (CPL) Parameterization

$$w_{\rm DE}(a) = w_0 + w_a(1-a)$$



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Candidates for dark energy

Cosmological constant (Λ)

$$w = -1$$

It corresponds to vacuum energy in particle physics. However, observational value is far smaller than theoretically predicted value!!

Dark energy problem may imply some modification of gravity on large scales.

Modified gravitational theories

$$w = w(t)$$

- There must be a stable accelerating solution which explains cosmic acceleration.
- They should be observationally distinguished from the Λ–CDM model.
- **These models need to recover General Relativistic (GR) behavior at short distances to satisfy solar system constraints.**

f(R) gravity,
Scalar–tensor theories.

with potential term

DGP braneworld,
Galileon gravity.

without potential term

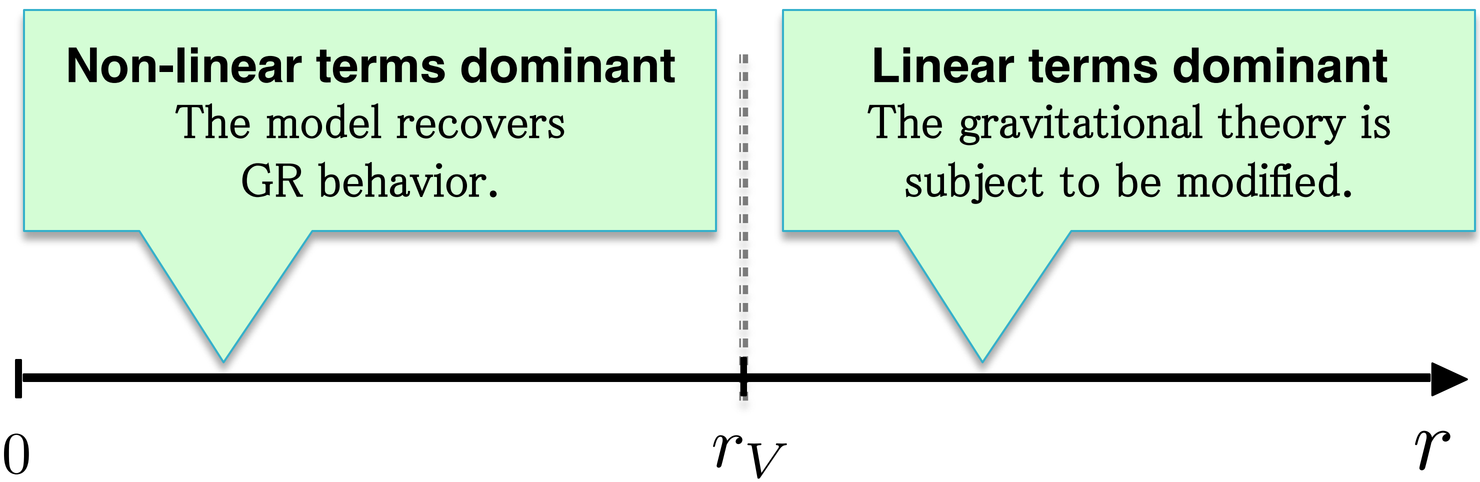
There are two different mechanisms.

- Recovery of GR behavior at short distances

2. Vainshtein mechanism

DGP braneworld,
Galileon gravity.

In the DGP braneworld the non-linear effect of the field self-interaction term $\Box\phi(\partial_\mu\phi\partial^\mu\phi)$ allows the possibility to recover GR at short distances.



The schematic view of the Vainshtein mechanism.

However the DGP model suffers from a ghost, in addition to the difficulty of consistency with the combined data analysis.

Recovery of GR behavior at short distances

1. Chameleon mechanism

f(R) gravity,
Brans–Dicke theory.

The potential term of a scalar field produces effective mass in the region of high density. The models recover GR if the effective mass is large enough.

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}F(\phi)R + \omega(\phi)X - V(\phi)\right] + \int d^4x\,\mathcal{L}_m(g_{\mu\nu},\Psi_m)$$

conformal transformation:

$$\hat{g}_{\mu\nu} = F(\phi)g_{\mu\nu},\ \chi \equiv \int d\phi\sqrt{\frac{3}{2}\left(\frac{M_{\rm pl}F_{,\phi}}{F}\right)^2 + \frac{\omega}{F}},\ \hat{V}(\chi) \equiv \frac{V}{F^2},\ A^2(\chi) \equiv F^{-1}(\phi)$$

$$\hat{\mathcal{S}} = \int d^4x\sqrt{-\hat{g}}\left[\frac{M_{\rm pl}^2}{2}\hat{R} - \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \hat{V}(\chi)\right] + \int d^4x\,\mathcal{L}_m(A^2(\chi)\hat{g}_{\mu\nu},\Psi_m)$$

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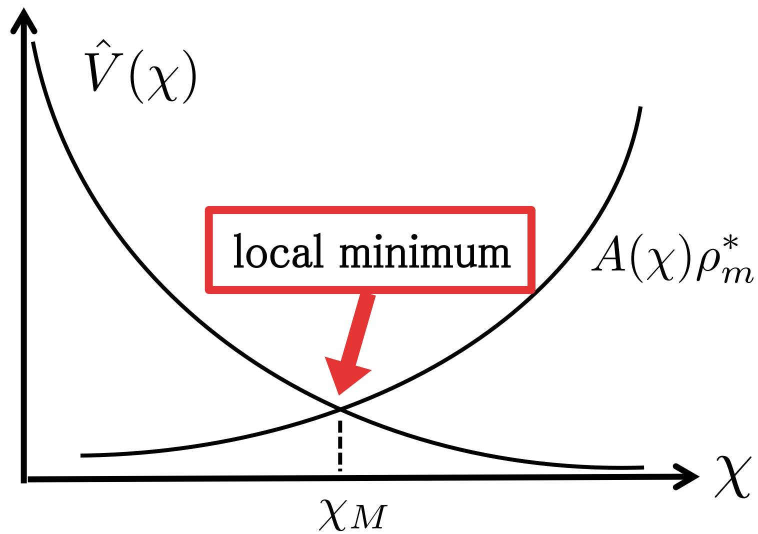
$$\hat{\mathcal{S}} = \int d^4x \sqrt{-\hat{g}} \left[\frac{M_{\rm pl}^2}{2} \hat{R} - \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \hat{V}(\chi) \right] + \int d^4x \, \mathcal{L}_m(A^2(\chi) \hat{g}_{\mu\nu}, \Psi_m)$$

・Equation of motion

$$\hat{\Box}\chi = \hat{V}(\chi) + A(\chi)\rho_m^*$$

$$\rho_m^* = A^3 \rho_m$$



However a fine tuning of initial conditions is required to realize a viable cosmology.

Modification of gravity

Non-minimal couplings generally appear in higher-dimensional theories.
For example...

Gauss-Bonnet gravity

$$\mathcal{S} = \int \mathrm{d}^{D+N}\mathrm{x}\sqrt{-g^{(D+N)}}\left(R + \alpha R_{\mathrm{GB}}\right)$$

dimensional reduction

$$\mathcal{S} = \int \mathrm{d}^{D}\mathrm{x}\sqrt{-g^{(D)}}\,\boxed{e^{N\pi/2}}\left[R^{(D)} + c_1(\partial\pi)^2 + \alpha\left(R_{\mathrm{GB}}^{(D)} + c_2 G_{\mu\nu}\partial^\mu\pi\partial^\nu\pi + c_3(\partial\pi)^4 + c_4(\partial\pi)^2\Box\pi\right)\right]$$

dilatonic coupling

The action of this family is covered by the so-called Horndeski's action.

3. VAINSHTEIN MECHANISM

$$\boxed{\Box\phi = \mu_4\,\rho_m + \mu_5\,,}$$

In the DGP model, a non-linear field self-interaction of the form $X\Box\phi$ can lead to the recovery of GR within the so-called Vainshtein radius r_V.

$$\boxed{r \ll r_V}$$

$$\boxed{|\mu_4|\rho_m \ll |\mu_5|}$$

$$\begin{aligned}
&|(\lambda_2/M_{\rm pl} + 2G_{3,\phi\phi})r^2 + 4(2G_{3,\phi X} - 2G_{4,\phi\phi X})\phi' r| \\
&\quad \ll |4(3G_{3,X} + 4XG_{3,XX} - 9G_{4,\phi X} - 10XG_{4,\phi XX} + XG_{5,\phi\phi X}) \\
&\qquad - 8\phi'(3G_{4,XX} + 2XG_{4,XXX} - 2G_{5,\phi X} - XG_{5,\phi XX})/r|\,.
\end{aligned}$$

Considering that the linear term in $K(\phi, X)$ gives the dominant contribution, we have $K(\phi, X) = e^{-\lambda_2\phi/M_{\rm pl}}X$.

$$\begin{aligned}
r \ll &|2(G_{3,\phi} + XG_{3,\phi X})r + 4(G_{3,X} + XG_{3,XX} - 3G_{4,\phi X} - 2XG_{4,\phi XX})\phi'(r) \\
&- 2(3G_{4,XX} + 2XG_{4,XXX} - 2G_{5,\phi X} - XG_{5,\phi XX})\phi'^2(r)/r|.
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&\qquad - 8\phi'(3G_{4,XX} + 2XG_{4,XXX} - 2G_{5,\phi X} - XG_{5,\phi XX})/r|\,.
\end{aligned}$$

Using this condition, the Vainshtein radius can be represented as following.

$$\begin{aligned}
r_V =& |2(G_{3,\phi} + XG_{3,\phi X})r_V + 4(G_{3,X} + XG_{3,XX} - 3G_{4,\phi X} - 2XG_{4,\phi XX})\phi'(r_V) \\
&- 2(3G_{4,XX} + 2XG_{4,XXX} - 2G_{5,\phi X} - XG_{5,\phi XX})\phi'^2(r_V)/r_V|.
\end{aligned}$$

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$$r \gg r_V$$

$$|\mu_4|\rho_m \gg |\mu_5|$$

$$|\phi/M_{\rm pl}| \ll 1$$

$$|2G_{4,\phi}| \gg |4XG_{4,\phi X} - 2XG_{3,X} - \phi'\beta - 4X(G_{5,X} + XG_{5,XX})/r^2|\,.$$

$$M_{\rm pl}^2 e^{-2Q\phi/M_{\rm pl}}/2 \gg e^{-\lambda_4\phi/M_{\rm pl}} g_4(X)\,.$$

$$r \gg |2(G_{3,\phi} + XG_{3,\phi X})r + 4(G_{3,X} + XG_{3,XX} - 3G_{4,\phi X} - 2XG_{4,\phi XX})\phi'(r) - 2(3G_{4,XX} + 2XG_{4,XXX} - 2G_{5,\phi X} - XG_{5,\phi XX})\phi'^2(r)/r|.$$

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$G_3(\phi, X) = X$

$r \gg r_V$

$$\beta \equiv (K_{,X} + 2\,XK_{,XX} - 2\,G_{3,\phi} - 2\,XG_{3,\phi X})\,r - 4\,\phi'\,(G_{3,X} + XG_{3,XX} - 3\,G_{4,\phi X} - 2\,XG_{4,\phi XX}) - \frac{4\,X\,(3\,G_{4,XX} + 2\,XG_{4,XXX} - 2\,G_{5,\phi X} - XG_{5,\phi XX})}{r}\,.$$

$$\Rightarrow\quad r_V = |2(G_{3,\phi} + XG_{3,\phi X})r_V + 4(G_{3,X} + XG_{3,XX} - 3G_{4,\phi X} - 2XG_{4,\phi XX})\phi'(r_V) - 2(3G_{4,XX} + 2XG_{4,XXX} - 2G_{5,\phi X} - XG_{5,\phi XX})\phi'^2(r_V)/r_V|.$$

Considering that the linear term in $K(\phi, X)$ gives the dominant contribution, we have $K(\phi, X) = e^{-\lambda_2\phi/M_{\rm pl}}X$.

$$\frac{d}{dr}(r^2\phi') \simeq QM_{\rm pl}\frac{dr_g}{dr}$$

$$r_g \equiv \frac{1}{M_{\rm pl}^2}\int_0^r \rho_m \tilde{r}^2 d\tilde{r}$$

$$\Rightarrow\quad \phi'(r) = \frac{QM_{\rm pl}r_g}{r^2}$$

4. COVARIANT GALILEON

(i) initially G_3 dominant ($|c_3| \sim 1$ and $|c_4| \ll 1$)

$$r_{43} \simeq \left(\frac{2|c_4|}{|c_3 c_V|}\right)^{2/3} r_V$$

$$c_5 = 0$$

G_4 dominant

$$r\phi'^2(r) - \frac{2c_4}{c_3 M^3}\phi'^3(r) = C$$

G_3 dominant

$$r\phi'^2(r) - \frac{2c_4}{c_3 M^3}\phi'^3(r) = C$$

$0 \quad r_{43} \quad r_V \quad r$

$$\phi'(r) \simeq \frac{M(QM_{\rm pl}r_g)^{2/3}}{(-2c_4/c_3)^{1/3} r_V} = \frac{QM_{\rm pl}r_g}{r_V^{3/2} r_{43}^{1/2}}$$

$$\phi'(r) \simeq \frac{QM_{\rm pl}r_g}{r_V^{3/2} r^{1/2}} \propto r^{-1/2}$$

$$\Phi \simeq \frac{r_g}{2r}\left(1 - 2Q^2 \frac{r^2}{r_V^{3/2} r_{43}^{1/2}}\right)$$

$$\Psi \simeq -\frac{r_g}{2r}\left(1 - 2Q^2 \frac{r^2}{r_V^{3/2} r_{43}^{1/2}}\right)$$

$$\Phi \simeq \frac{r_g}{2r}\left[1 - 2Q^2 \left(\frac{r}{r_V}\right)^{3/2}\right]$$

$$\Psi \simeq -\frac{r_g}{2r}\left[1 - 4Q^2 \left(\frac{r}{r_V}\right)^{3/2}\right]$$

$$|\gamma - 1| \simeq 0$$

$$|\gamma - 1| \simeq 2Q^2 (r/r_V)^{3/2}$$

The observational bound can be satisfied for $r \ll r_V$

4. COVARIANT GALILEON

$c_5 = 0$

- The solution in the regime $r \ll r_V$

$$\underbrace{r\phi'^2(r)}_{G_3} - \underbrace{\frac{2c_4}{c_3 M^3}\phi'^3(r)}_{G_4} = C$$

(i) initially G_4 dominant ($|c_3| \ll 1$ and $|c_4| \sim 1$)

$$r\phi'^2(r) - \frac{2c_4}{c_3 M^3}\phi'^3(r) = C$$

$$\phi'(r) \simeq \frac{Q M_{\rm pl} r_g}{r_V^2} = \text{Constant.}$$

The solution in $r \gg r_V$

$$\phi'(r) = \frac{Q M_{\rm pl} r_g}{r^2}$$

- Gravitational potentials in the regime $r \ll r_V$

correction terms

$$\Phi \simeq \frac{r_g}{2r}\left[1 - \boxed{2Q^2\left(\frac{r}{r_V}\right)^2}\right], \quad \Psi \simeq -\frac{r_g}{2r}\left[1 - \boxed{2Q^2\left(\frac{r}{r_V}\right)^2}\right].$$

We introduce the post−Newtonian parameter $\gamma \equiv -\Phi/\Psi$ whose experimental bound is $|\gamma - 1| < 2.3 \times 10^{-5}$. Since we obtain $|\gamma - 1| \simeq 0$ in this case, the above results satisfies the observational bound in the regime $r \ll r_V$.

4. COVARIANT GALILEON

$c_5 = 0$ ▸ The solution around the origin $(r \sim 0)$

(i) initially G_4 dominant $(|c_3| \ll 1$ and $|c_4| \sim 1)$

(b) $\rho_m(r)$: varying matter density

However if the matter density varies in the star, it is not obvious whether the interior and exterior solutions connect each other successfully.

In order to study the matching of the solutions properly, we employ the profile

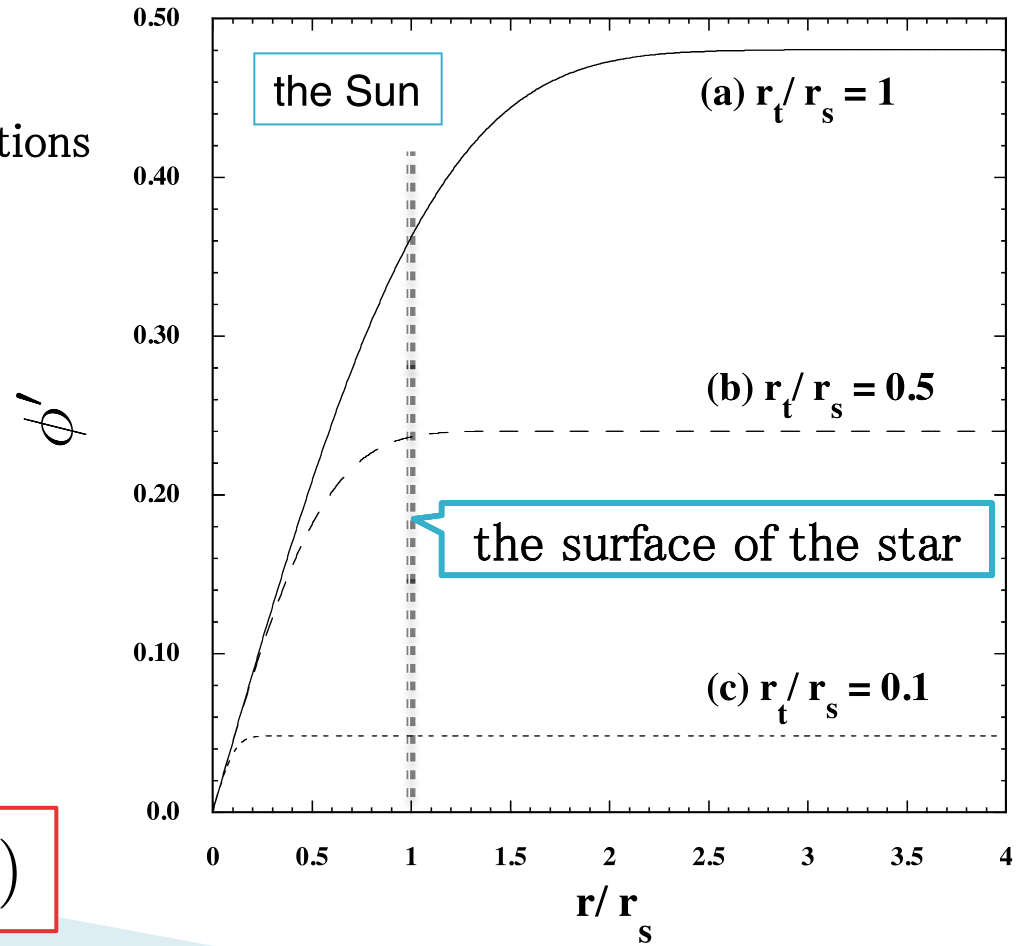
$$\rho_m = \rho_c \exp(-r^2/r_t^2)\,,$$

and solve the full EOM numerically.

ρ_c : the density around the origin

r_s : the radius of the star

r_t : the distance where the density starts to decrease



$\phi'_{\rm out}(r) = QM_{\rm pl}r_g/r_V^2$ ➡ $\mathcal{O}(0.1)$

4. COVARIANT GALILEON

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$\boxed{c_5 = 0}$

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and solve the full EOM numerically.

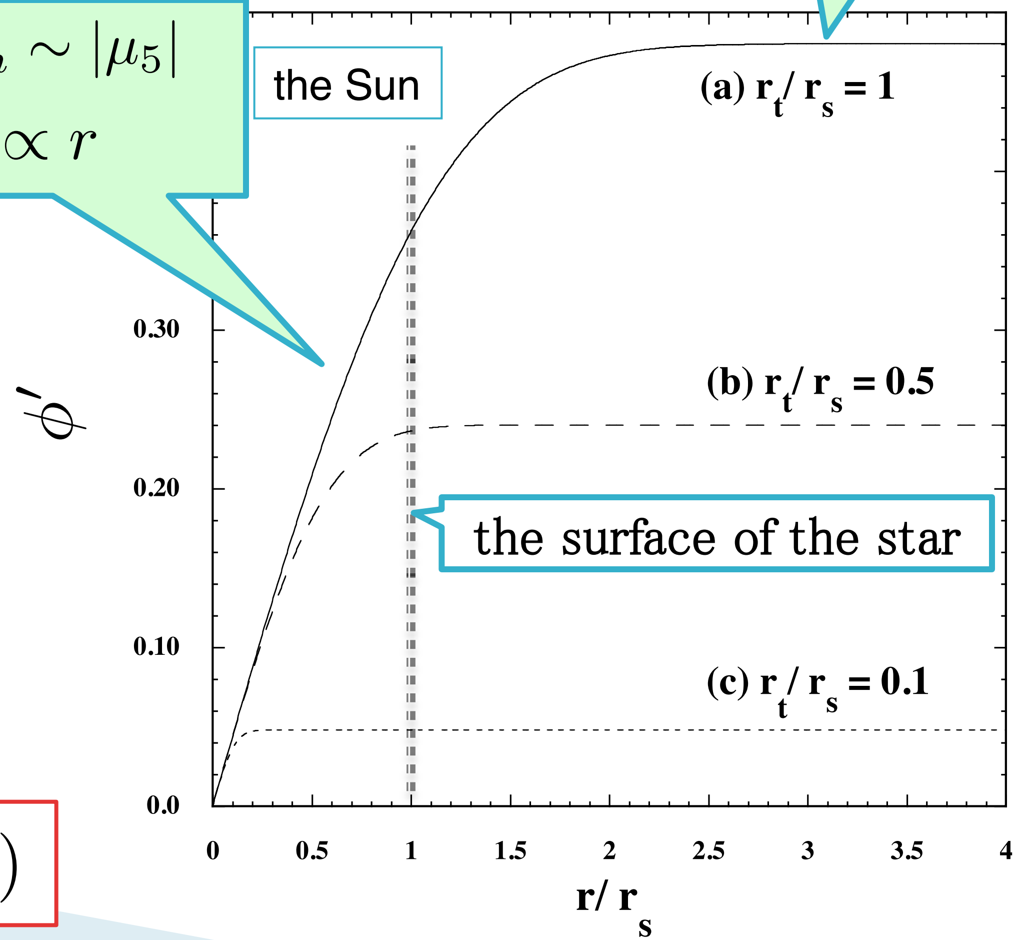
ρ_c : the density around the origin

r_s : the radius of the star

r_t : the distance where the density starts to decrease

- μ_5 dominant
  $\phi' \simeq$ Constant.

- $|\mu_4|\rho_m \sim |\mu_5|$
  $\phi' \propto r$



$$\phi'_{\rm out}(r) = QM_{\rm pl} r_g / r_V^2 \;\Rightarrow\; \mathcal{O}(0.1)$$

5. APPLICATION TO OTHER MODELS

- **Extended Galileon** $(G_5 = 0)$

$$G_3(X) = c_3 M^{1-4p_3} X^{p_3}\,, \quad G_4(\phi, X) = \frac{M_{\rm pl}^2}{2} e^{-2Q\phi/M_{\rm pl}} + c_4 M^{2-4p_4} X^{p_4}\,,$$

(i) G$_4$ dominant $(|c_3| \ll 1$ and $|c_4| \sim 1)$

$p_3 \geq 1$ and $p_4 \geq 2$

$$r_V \simeq (|Q| M_{\rm pl} r_g)^{\frac{2p_3-1}{4p_3-1}}/M\,, \;\; \phi'(r) \simeq Q M_{\rm pl} r_g / r_V^2 \quad (r \ll r_V)\,.$$

$$\Phi \simeq \frac{r_g}{2r}\left[1 - 2Q^2\left(\frac{r}{r_V}\right)^2\right], \; \Psi \simeq -\frac{r_g}{2r}\left[1 - 2Q^2\left(\frac{r}{r_V}\right)^2\right].$$

(ii) G$_3$ dominant $(|c_3| \sim 1$ and $|c_4| \ll 1)$

$$r_V \simeq (|Q| M_{\rm pl} r_g)^{\frac{p_4-1}{2p_4-1}}/M\,,$$

$$\phi'(r) \simeq Q M_{\rm pl} r_g r_V^{\frac{1-4p_3}{2p_3}} r^{-\frac{1}{2p_3}} \quad (r_{43} \ll r < r_V)\,,$$

$$\phi'(r) \simeq Q M_{\rm pl} r_g r_V^{\frac{1-4p_3}{2p_3}} r_{43}^{-\frac{1}{2p_3}} \quad (r \ll r_{43})\,.$$

$$\Phi \simeq \frac{r_g}{2r}\left[1 - 2Q^2\left(\frac{r}{r_V}\right)^{\frac{4p_3-1}{2p_3}}\right], \quad \Psi \simeq -\frac{r_g}{2r}\left[1 - \frac{4p_3}{2p_3-1}Q^2\left(\frac{r}{r_V}\right)^{\frac{4p_3-1}{2p_3}}\right] \quad (r_{43} \ll r < r_V),$$

$$\Phi \simeq \frac{r_g}{2r}\left(1 - 2Q^2 \frac{r^2}{r_V^{\frac{4p_3-1}{2p_3}} r_{43}^{\frac{1}{2p_3}}}\right), \quad \Psi \simeq -\frac{r_g}{2r}\left(1 - 2Q^2 \frac{r^2}{r_V^{\frac{4p_3-1}{2p_3}} r_{43}^{\frac{1}{2p_3}}}\right) \quad (r \ll r_{43}).$$

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$p_3 \geq 1$ and $p_4 \geq 2$

(i) G$_4$ dominant $(|c_3| \ll 1$ and $|c_4| \sim 1)$

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For $p_3 \gg 1$,
this solution reduces
to constant solution.

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5. APPLICATION TO OTHER MODELS

EXTENDED GALILEON

(ii) G_3 dominant ($|c_3| \sim 1$ and $|c_4| \ll 1$)

$$\Box\phi = \mu_4\, \rho_m + \mu_5$$

$$\phi'(r) = \left[\frac{(-2)^{p_3} Q M^{4p_3-1} \rho_c}{12 c_3 p_3 M_{\rm pl}}\, r^2\right]^{1/(2p_3)}$$

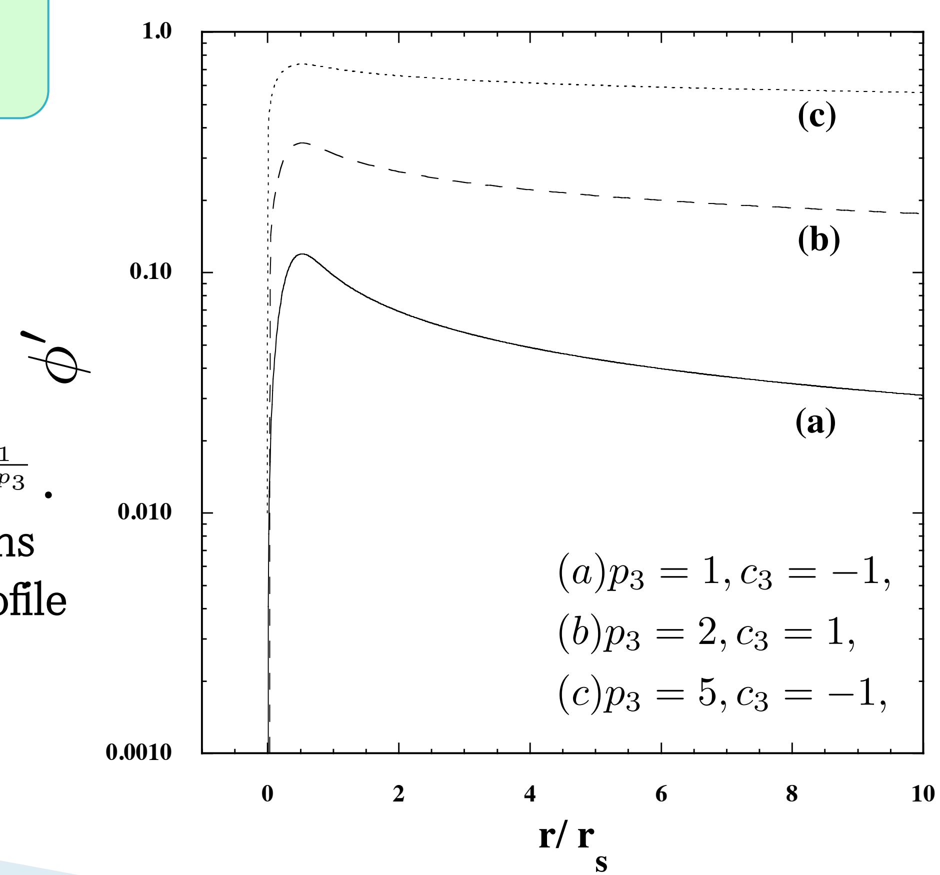
We require

$$c_3 Q < 0 \quad (p_3 : odd)\,,$$
$$c_3 Q > 0 \quad (p_3 : even)\,,$$

so that this solution connects with the exterior solution $\phi'_{\rm out} = Q M_{\rm pl} r_g r_V^{\frac{1-4p_3}{2p_3}}\, r^{-\frac{1}{2p_3}}$.
In order to study the matching of the solutions properly, we employ the following density profile

$$\rho_m = \rho_c \exp(-r^2/r_t^2)\,,$$

- The solution around the origin



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In order to study the matching of the solutions properly, we employ the following density profile

$$\rho_m = \rho_c \exp(-r^2/r_t^2) \,,$$

- The solution around the origin

• $|\mu_4|\rho_m \sim |\mu_5|$

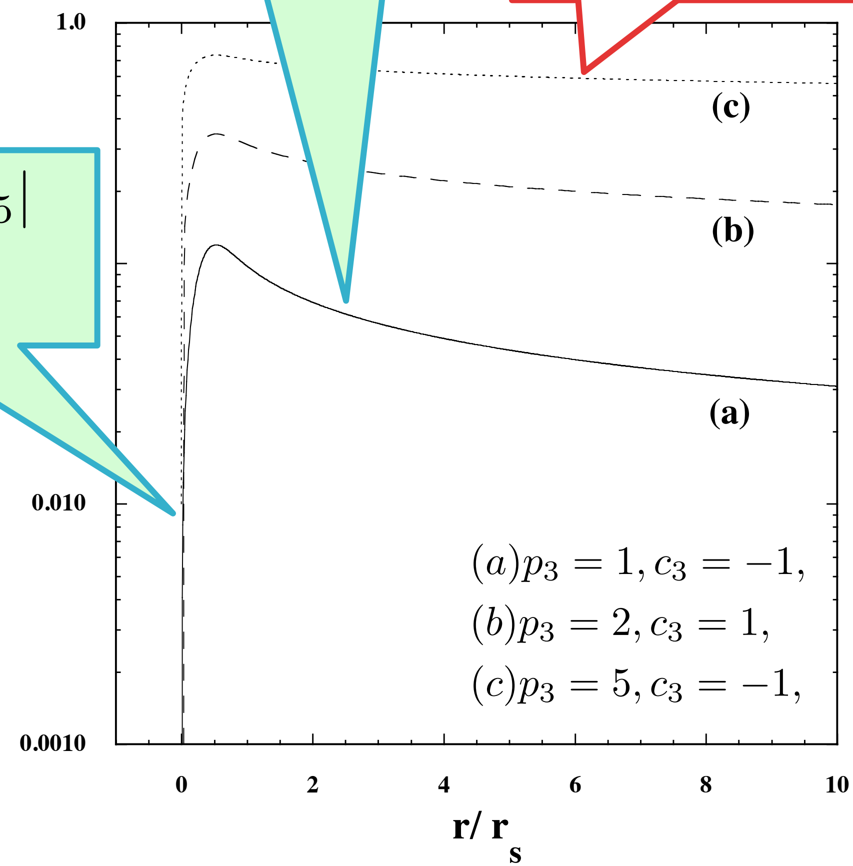
$\phi' \propto r^{1/p_3}$

• μ_5 dominant

$\phi' \propto r^{-1/(2p_3)}$

For $p_3 \gg 1$,

$\phi' \to$ Constant



5. APPLICATION TO OTHER MODELS

Galileons with dilatonic couplings

$$G_3(\phi,X)=\frac{c_3}{M^3}e^{-\lambda_3\phi/M_{\rm pl}}X\,,\;G_4(\phi,X)=\frac{M_{\rm pl}^2}{2}e^{-2Q\phi/M_{\rm pl}}+\frac{c_4}{M^6}e^{-\lambda_4\phi/M_{\rm pl}}X^2\,,\;G_5(\phi,X)=\frac{c_5}{M^9}e^{-\lambda_5\phi/M_{\rm pl}}X^2$$

$\boxed{c_3=0=c_4,\;c_5\neq 0}$

- The solution in the regime $r \ll r_V$

Since the terms such as $G_{5,\phi X}$, $G_{5,\phi XX}$ does not vanish, there appears the following solution

$$r_V=(6|c_5\lambda_5|Q^4M_{\rm pl}^3r_g^4/M^9)^{1/10}$$

$$\phi'_{\rm out}(r)=QM_{\rm pl}r_g/r_V^2 \quad (r\ll r_V)$$

However the term $|4X(G_{5,X}+XG_{5,XX})/r^2|$ gets larger in smaller distances

$$\mu_4\simeq-\frac{r}{4G_4\beta}\left[2\,G_{4,\phi}+4\,XG_{4,\phi X}-2\,XG_{3,X}-\phi'\beta\boxed{-\frac{4\,X\,(G_{5,X}+XG_{5,XX})}{r^2}}\right]$$

and the condition $|\mu_4|\rho_m\ll|\mu_5|$ is to be violated for $\rho_m(r)r>M_{\rm pl}^2r_g/r_V^2$.

- the Earth

$$\rho_m(r_s)>10^{-16}{\rm g/cm}^3$$

- the Sun

$$\rho_m(r_s)>10^{-17}{\rm g/cm}^3$$

5. APPLICATION TO OTHER MODELS

Galileons with dilatonic couplings

$$G_3(\phi,X)=\frac{c_3}{M^3}e^{-\lambda_3\phi/M_{\rm pl}}X\,,G_4(\phi,X)=\frac{M_{\rm pl}^2}{2}e^{-2Q\phi/M_{\rm pl}}+\frac{c_4}{M^6}e^{-\lambda_4\phi/M_{\rm pl}}X^2\,,G_5(\phi,X)=\frac{c_5}{M^9}e^{-\lambda_5\phi/M_{\rm pl}}X^2$$

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Since the terms such as $G_{5,\phi X}$, $G_{5,\phi XX}$ does not vanish, there appears the following solution

$$r_V=(6|c_5\lambda_5|Q^4M_{\rm pl}^3r_g^4/M^9)^{1/10}$$
$$\phi'_{\rm out}(r)=QM_{\rm pl}r_g/r_V^2 \quad (r\ll r_V)$$

- The solution around the origin $(r\sim 0)$

$$\phi'_{\rm in}(r)\simeq -\frac{\rho_c}{3\lambda_5M_{\rm pl}}r$$

We require $\lambda_5Q<0$ so that this solution connects with the exterior solution.

$$\left|\frac{\phi'_{\rm in}(r_s)}{\phi'_{\rm out}(r_s)}\right|\approx\frac{1}{|\lambda_5Q|}\left(\frac{r_V}{r_s}\right)^2\approx\left(\frac{r_V}{r_s}\right)^2$$

The two solutions connect each other when $r_V\sim r_s$. However in this case the model cannot satisfy solar system constraints.

5. APPLICATION TO OTHER MODELS

- **DBI Galileons with Gauss-Bonnet and other terms**

$$S = \int d^4x\sqrt{-g}\Big[\frac{M_{\rm pl}^2}{2}e^{-2Q\phi/M_{\rm pl}}R - f_1(\phi)^2\mu^4\left(\sqrt{1-\frac{2f_1(\phi)^{-1}X}{\mu^4}}-1\right) + f_2(\phi)\frac{X^2}{\mu^4}$$
$$+ f_3(\phi)\frac{X}{M^3}\Box\phi + f_4(\phi)c_{\rm GB}R_{\rm GB}^2 + f_5(\phi)\frac{1}{m^2}G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\Big]\,,$$

$$K(\phi,X) = -f(\phi)^2\mu^4\left(\sqrt{1-\frac{2f(\phi)^{-1}X}{\mu^4}}-1\right) + f(\phi)\frac{X^2}{\mu^4} + 8c_{\rm GB}f^{(4)}(\phi)X^2\left[3-\ln\left(\frac{X}{\mu^4}\right)\right]\,,$$
$$G_3(\phi,X) = -f(\phi)\frac{X}{M^3} + 4c_{\rm GB}f^{(3)}(\phi)X\left[7-3\ln\left(\frac{X}{\mu^4}\right)\right]\,,$$
$$G_4(\phi,X) = \frac{M_{\rm pl}^2}{2}e^{-2Q\phi/M_{\rm pl}} + 4c_{\rm GB}f^{(2)}(\phi)X\left[2-\ln\left(\frac{X}{\mu^4}\right)\right]\,,$$
$$G_5(\phi,X) = \frac{M_{\rm pl}}{\lambda m^2}f(\phi) - 4c_{\rm GB}f^{(1)}(\phi)\ln\left(\frac{X}{\mu^4}\right)\,.$$