Gradient expansion of superhorizon perturbations in G-Inflation

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Based on
Noemi Frusciante, Shuang-Yong Zhou, Thomas P. Sotiriou
JCAP 07, 20, (2013), arXiv:1303.6628 [astro-ph.CO]

7^{th} Aegean Summer School
"Beyond Einstein's Theory of Gravity",
Parikia, Paros
23-28 September, 2013

Motivation

- *Inflation paradigm*
  - *described by a scalar field/fields*
  - *to solve horizon and flatness problems*
  - *the origin of density fluctuations*
  - *Test of inflationary models: Non-Gaussianity*

- *Gradient expansion method*
  - *Fully non linear analysis on superhorizon scale*
  - *Quantities are expanded in terms of their inverse wavelengths compared to the Hubble length*
  - *Every spatial derivative adds one perturbative order*
  - *It is complementary to the usual second order cosmological perturbation theory inside the horizon*

D.S. Salopek and J.R. Bond, Phys. Rev. D **42**, *3936 (1990)*
Y. Tanaka and M. Sasaki, Prog. Theor. Phys., **117**, *633 (2007)*

Inflation with a shift-symmetric Galileon

- *We want to explore the phenomenology associated with the non linear derivative interactions of the scalar field in a galileon like action.*
- *We consider the action with shift symmetry* $\phi \to \phi + c$

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_{pl}^2}{2}R + K(X) - G(X)\Box\phi\right), \qquad (1)$$

where $X = -\partial^\mu\phi\partial_\mu\phi/2$.

C. Deffayet, O. Pujolas, I. Sawicki, A. Vikman, JCAP **1010***, 026 (2010).*

- *Truncated version of Generalized Galileon*
- *Second order field equations*
- *Self accelerating solutions: Inflation and the recent accelerated expansion*

Inflation with a shift-symmetric Galileon

The Einstein equations are given by

$$M_{pl}^2 G_{\mu\nu} = T^{\phi}_{\mu\nu}, \tag{2}$$

with

$$T^{\phi}_{\mu\nu} = (K_X - G_X \Box\phi)\partial_\mu\phi\partial_\nu\phi - 2\partial_{(\mu}G\partial_{\nu)}\phi + g_{\mu\nu}(K + \partial_\sigma G \partial^\sigma\phi), \tag{3}$$

the energy momentum tensor takes the form of an imperfect fluid.

O. Pujolas, I. Sawicki, A. Vikman, JHEP, **1111**, *156 (2011)*

The EoM for ϕ *can be given in terms of the current*

$$J^\mu = (K_X - G_X \Box\phi)\partial^\mu\phi - G_X \partial^\mu X, \tag{4}$$

as

$$\nabla_\mu J^\mu = 0. \tag{5}$$

3+1 Decomposition & Definitions

We decompose the metric according to the ADM-prescription

$$\mathrm{d}s^2 = -N^2 \mathrm{d}t^2 + \gamma_{ij}(\mathrm{d}x^i + N^i \mathrm{d}t)(\mathrm{d}x^j + N^j \mathrm{d}t).$$

We work in synchronous gauge:

$$\begin{aligned} N &= 1 \rightarrow \textit{proper time slicing}, \\ N^i &= 0 \rightarrow \textit{clocks are synchronized}, \end{aligned}$$

The extrinsic curvature

$$\mathcal{K}_{ij} = -\Gamma^t_{ij} = -\frac{1}{2}\dot{\gamma}_{ij},$$

and its trace is defined as $\mathcal{K} = \gamma^{ij}\mathcal{K}_{ij}$.
It is useful to decompose the spatial metric and the extrinsic curvature as

$$\gamma_{ij} = a^2(t)e^{2\zeta(t,\mathbf{x})}h_{ij}(t,\mathbf{x}), \tag{6}$$

$$\mathcal{K}^i_j = \frac{1}{3}\mathcal{K}(t,\mathbf{x})\delta^i_j + A^i_j(t,\mathbf{x}). \tag{7}$$

Gradient expansion method

It describes a NON-LINEAR cosmological perturbation theory on super-horizon scale.
The equations are expanded in terms of ϵ

$$\epsilon \sim \frac{H^{-1}}{L_{phys}} \ll 1$$

then

- *Super-horizon scale :* $L_{phys} >> H^{-1}$,
- *Assumption:* $L_{phys} \sim \mathcal{O}(1/\epsilon)$,
- $\partial_i \sim \mathcal{O}(\epsilon)$,
- $\partial_t \sim \mathcal{O}(\epsilon^0)$,
- *in the limit* $L_{phys} \to \infty$ $(\epsilon \to 0)$ *FLRW Universe*

E.M. Lifshitz and I.M. Khalatnikov, Adv. Phys. **12**, *185 (1963)*
K. Tomita, Prog. Theor. Phys. **54**, *730 (1975)*

Gradient expansion: order analysis

- *Starting orders for various quantities of interest:*

$$\text{FLRW background} \rightarrow \; \dot{h}_{ij} = \mathcal{O}(\epsilon),$$
$$\text{from the scalar eq.} \rightarrow \; \partial_i \phi = \mathcal{O}(\epsilon^2).$$

Different assumption: $\dot{h}_{ij} = \mathcal{O}(\epsilon^2)$

e.g. Y.-i. Takamizu and T. Kobayashi, Prog. Theor. Exp. Phys., 063E03 (2013)

- *Expand all quantities and hence the equations in* ϵ*,*
- *Solve the EoM order by order (up to* $\mathcal{O}(\epsilon^2)$*),*
  - *FLRW background EoM,*
  - *In our case the ti Einstein equations at* $\mathcal{O}(n+1)$ *are constraint equations for* $\mathcal{O}(n)$*-quantities.*

Summary of the expanded quantities

In summary, the various quantities of interest, to be determined, are expanded as follows:

$$\zeta = \zeta^{(0)}(\mathbf{x}) + \zeta^{(1)}(t,\mathbf{x})\epsilon + \zeta^{(2)}(t,\mathbf{x})\epsilon^2 + \mathcal{O}(\epsilon^3),$$
$$\phi = \phi^{(0)}(t) + \phi^{(1)}(t,\mathbf{x})\epsilon + \phi^{(2)}(t,\mathbf{x})\epsilon^2 + \mathcal{O}(\epsilon^3)\,,$$
$$A^i_j = A^{(1)}{}^i_j(t,\mathbf{x})\epsilon + A^{(2)}{}^i_j(t,\mathbf{x})\epsilon^2 + \mathcal{O}(\epsilon^3),$$
$$h_{ij} = h^{(0)}_{ij}(\mathbf{x}) + h^{(1)}_{ij}(t,\mathbf{x})\epsilon + h^{(2)}_{ij}(t,\mathbf{x})\epsilon^2 + \mathcal{O}(\epsilon^3)\,,$$
$$\mathcal{K}^i_j = -H(t)\delta^i_j + \mathcal{K}^{(1)}{}^i_j(t,\mathbf{x})\epsilon + \mathcal{K}^{(2)}{}^i_j(t,\mathbf{x})\epsilon^2 + \mathcal{O}(\epsilon^3),$$
$$\mathcal{K} = -3H(t) + \dot{\zeta}^{(1)}(t,\mathbf{x})\epsilon + \dot{\zeta}^{(2)}(t,\mathbf{x})\epsilon^2 + \mathcal{O}(\epsilon^3).$$

NOTE: We solve the equations order by order in full generality for any background !

N. Frusciante, S.-Y. Zhou and T.P. Sotiriou, arXiv:1303.6628 (2013)

Example:Late time G-Inflation

Assumption: Quasi-de Sitter Expansion

Background equations can be simplified to

$$K_X\dot{\phi} + 6HG_X X \propto a(t)^{-3}, \;\; J^t \to 0,$$
$$K = -3M_{pl}^2 H^2,$$
$$K_X = -3G_X H\dot{\phi},$$

and $\dot{\phi} = const,\ H = const \quad \to \quad$ *de Sitter Solution*

Kobayashi et al. Prog. Theor. Phys. 126,511 (2011).

The background quantities now all become constant:

$$\mathcal{A}^0,\;\mathcal{C}^0,\;\mathcal{C}_1^0,\;\mathcal{C}_2^0,\;\mathcal{C}_3^0,\;\mathcal{D}^0,\;\mathcal{E}_1^0,\;\mathcal{E}_2^0,\;\mathcal{E}_3^0.$$

Solution for the late time of Inflation

$$\zeta(t,\mathbf{x}) = \zeta^{(0)}(\mathbf{x}) - \frac{C_\zeta^{(1)}(\mathbf{x})}{3Ha^3} + \frac{\mathcal{E}_1^0\left(C_\zeta^{(1)}(\mathbf{x})\right)^2}{18H^2a^6} + \frac{\mathcal{E}_2^0 C_A^{(1)}{}^k{}_l(\mathbf{x})C_A^{(1)}{}^l{}_k(\mathbf{x})}{18H^2a^6} - \frac{\mathcal{E}_3^0\,{}^{[3]}R^{(2)}(\mathbf{x})}{2H^2a^2} + \mathcal{O}(\epsilon^3),$$

$$\phi(t,\mathbf{x}) = \phi^{(0)}(t) + C_\phi^{(1)}(\mathbf{x}) - \frac{\mathcal{A}^0 C_\zeta^{(1)}(\mathbf{x})}{3Ha^3} + \mathcal{A}^0\zeta^{(2)}(t,\mathbf{x}) + \frac{\mathcal{C}^0\left(C_\zeta^{(1)}(\mathbf{x})\right)^2}{6Ha^6} - \frac{\mathcal{C}_3^0\,{}^{[3]}R^{(2)}(\mathbf{x})}{4Ha^2} + \frac{\mathcal{C}_3^0 C_A^{(1)}{}^k{}_j(\mathbf{x})C_A^{(1)}{}^j{}_k(\mathbf{x})}{12Ha^6} + \mathcal{O}(\epsilon^3),$$

$$A^i_j(t,\mathbf{x}) = \frac{C_A^{(1)}{}^i{}_j(\mathbf{x})}{a^3} + \frac{C_\zeta^{(1)}(\mathbf{x})C_A^{(1)}{}^i{}_j(\mathbf{x})}{Ha^6} + \frac{{}^{[3]}R^{(2)}{}^i{}_j(\mathbf{x}) - \frac{1}{3}\delta^i_j\,{}^{[3]}R^{(2)}(\mathbf{x})}{Ha^2} + \mathcal{O}(\epsilon^3),$$

$$h_{ij}(t,\mathbf{x}) = h_{ij}^{(0)}(\mathbf{x}) + \frac{2h_{ik}^{(0)}(\mathbf{x})C_A^{(1)}{}^k{}_j(\mathbf{x})}{3Ha^3} + \frac{2h_{ik}^{(0)}(\mathbf{x})\left({}^{[3]}R^{(2)}{}^k{}_j(\mathbf{x}) - \frac{1}{3}\delta^k_j\,{}^{[3]}R^{(2)}(\mathbf{x})\right)}{3H^2a^2} + \frac{3h_{ik}^{(0)}(\mathbf{x})C_\zeta^{(1)}(\mathbf{x})C_A^{(1)}{}^k{}_j(\mathbf{x}) + 2h_{il}^{(0)}(\mathbf{x})C_A^{(1)}{}^l{}_k(\mathbf{x})C_A^{(1)}{}^k{}_j(\mathbf{x})}{9H^2a^6} + \mathcal{O}(\epsilon^3).$$

DoF Counting

- *Synchronous gauge is not uniquely defined $\rightarrow$ residual DoF*
- *Re-slicing and requiring that the synchronous gauge conditions are respected*
  - *1 re-slicing initial surface*
  - *3 spatial diffeomorphism trasformation*

Therefore

$$\begin{aligned}
\zeta^{(0)}(\mathbf{x}) \quad & \text{1 scalar growing mode} \; = \; \text{1 component,}\\
h^{(0)}_{ij}(\mathbf{x}) \quad & \text{2 tensor growing modes} \; = \; \text{5 components} - \text{3 gauge DoFs,}\\
C^{(1)}_{\zeta}(\mathbf{x}) \quad & \text{1 scalar decaying mode} \; = \; \text{1 component,}\\
C^{(1)}_{A}{}^{i}{}_{j}(\mathbf{x}) \quad & \text{2 tensor decaying modes} \; = \; \text{5 components} - \text{3 constraints.}
\end{aligned}$$

Conclusion

- *We developed the gradient expansion formalism for shift-symmetric Galileon type action.*
- *We solved the EoM of G-Inflation up to second order in ϵ.*
- *We obtained a general solution without imposing extra conditions on first order quantities.*
- *We have identified the physical DoF in the solution.*
- *We defined a curvature perturbation $\mathcal{R}^{(1)}$ conserved up to first order.*
- *One can evaluate non-Gaussianities in G-Inflation at superhorizon scale and, combined with the non-linear perturbation analysis, one can then use the existing data to constrain the model parameters.*