Imperfect fluids, dynamic Black Holes and generalized McVittie

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Outline

- The McVittie solution (recap)
- Generalized McVittie
  - Construction with an imperfect fluid
  - Properties
  - Apparent horizons
- Work in progress
- Remarks

McVittie solution

$$ds^2 = -\frac{\left(1 - \frac{m}{2\,a(t)\,r}\right)^2}{\left(1 + \frac{m}{2\,a(t)\,r}\right)^2} dt^2 + a(t)^2 \left(1 + \frac{m}{2\,a(t)\,r}\right)^4 \left(dr^2 + r^2\, d\,\Omega^2\right)$$

McVittie solution

$$ds^2 = -\frac{\left(1-\frac{m}{2a(t)r}\right)^2}{\left(1+\frac{m}{2a(t)r}\right)^2} dt^2 + a(t)^2 \left(1+\frac{m}{2a(t)r}\right)^4 \left(dr^2 + r^2 d\Omega^2\right)$$

- Spherically symmetric
- Shear free
- Perfect fluid
- Asymptotically FLRW
- Singularity

McVittie solution

$$ds^2 = -\frac{\left(1-\frac{m}{2a(t)r}\right)^2}{\left(1+\frac{m}{2a(t)r}\right)^2} dt^2 + a(t)^2 \left(1+\frac{m}{2a(t)r}\right)^4 \left(dr^2 + r^2 d\Omega^2\right)$$

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$a(t) \rightarrow 1 \Rightarrow$

McVittie solution

$$ds^2 = -\frac{\left(1-\frac{m}{2r}\right)^2}{\left(1+\frac{m}{2r}\right)^2} dt^2 + \left(1+\frac{m}{2r}\right)^4 \left(dr^2 + r^2 d\Omega^2\right)$$

- Spherically symmetric
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$a(t) \to 1 \Rightarrow$ Schwarzschild

McVittie solution

$$ds^2 = -\frac{\left(1-\frac{m}{2a(t)r}\right)^2}{\left(1+\frac{m}{2a(t)r}\right)^2} dt^2 + a(t)^2 \left(1+\frac{m}{2a(t)r}\right)^4 \left(dr^2 + r^2 d\Omega^2\right)$$

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$$a(t) \rightarrow \mathrm{e}^{Ht} \Rightarrow$$

McVittie solution

$$ds^2 = -\frac{\left(1 - \frac{m}{2e^{Ht}r}\right)^2}{\left(1 + \frac{m}{2e^{Ht}r}\right)^2} dt^2 + e^{2Ht}\left(1 + \frac{m}{2e^{Ht}r}\right)^4 \left(dr^2 + r^2 d\Omega^2\right)$$

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$a(t) \rightarrow \mathrm{e}^{Ht} \Rightarrow$ Schwarzschild – de Sitter

McVittie solution

$$ds^2 = -\frac{\left(1 - \frac{m}{2a(t)r}\right)^2}{\left(1 + \frac{m}{2a(t)r}\right)^2} dt^2 + a(t)^2 \left(1 + \frac{m}{2a(t)r}\right)^4 \left(dr^2 + r^2 d\Omega^2\right)$$

- Spherically symmetric
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$$\left.\begin{array}{r} m \to 0 \\ \text{large} \quad r \end{array}\right\} \Rightarrow$$

McVittie solution

$$ds^2 = -dt^2 + a(t)^2 \left(dr^2 + r^2 d\Omega^2\right)$$

- Spherically symmetric
- Shear free
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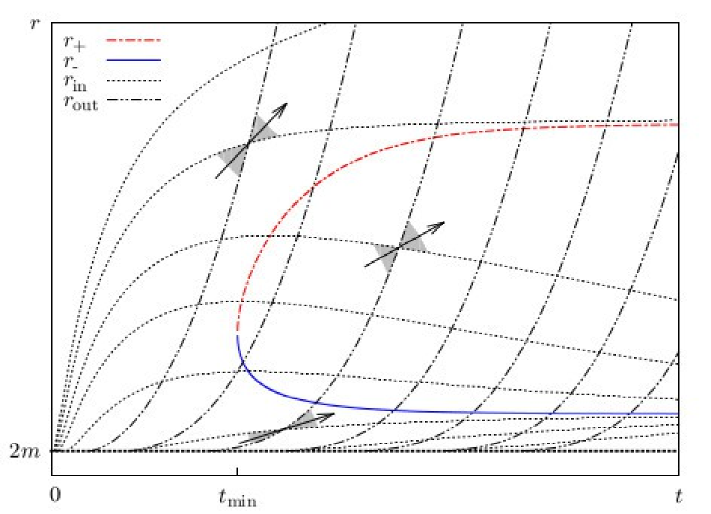
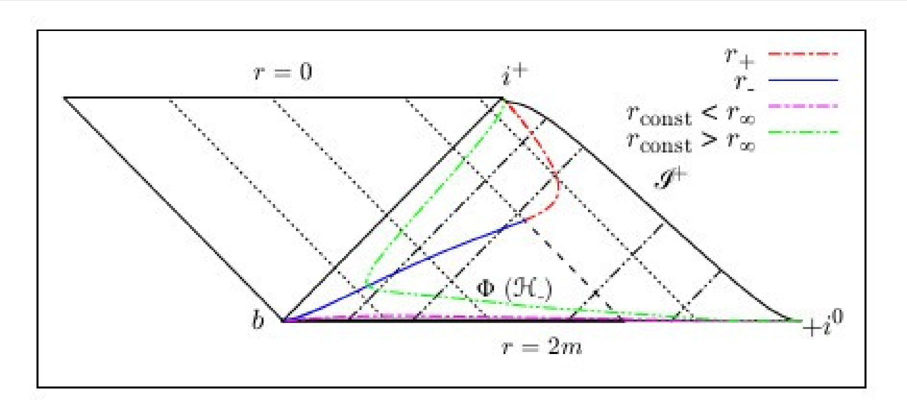
- Singularity

$$\left.\begin{array}{r} m \to 0 \\ \text{large} \quad r \end{array}\right\} \Rightarrow \quad \text{FLRW}$$

McVittie – causal structure

- $m = 2ar$ spacelike singularity (McV big bang)
- 2 apparent horizons, the inner one of which becomes
- $\begin{cases} t = \infty \\ r = r_m \end{cases}$ null surface at finite affine distance from every point in the bulk ...
- … corresponding to a S-dS horizon for $\lim_{t \to \infty} H = H_0 > 0$





Generalized McV – canonical coords

$$ds^2 = -\left(R^2 - r^2 H^2\right) dt^2 - 2\, r \frac{H}{R}\, dt\, dr + \frac{dr^2}{R^2} + r^2\, d\,\Omega^2$$

$$R[t, r] = \sqrt{1 - \frac{2\mathrm{m}(t)}{r}}$$

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$$m \rightarrow m(t)$$

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$$m \rightarrow m(t) \quad \Rightarrow \quad H \rightarrow H - M\left(1 - \frac{1}{R}\right)$$

$$H = \frac{\dot{a}}{a}, \quad M = \frac{\dot{m}}{m}$$

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$$m \to m(t) \qquad \Rightarrow \qquad H \to H - M\left(1 - \frac{1}{R}\right)$$

$$H = \frac{\dot{a}}{a}, \quad M = \frac{\dot{m}}{m}$$

$$ds^2 = -\left(R^2 - r^2\left(H - M + \frac{M}{R}\right)^2\right) dt^2 - 2\frac{r}{R}\left(H - M + \frac{M}{R}\right) dt\, dr + \frac{dr^2}{R^2} + r^2 d\Omega^2$$

GMcV – construction

- Multiple perfect fluids (→ phantom fluid)
- Imperfect fluid

$$G^t{}_r \propto \dot{m} \Rightarrow$$

Non comoving fluids
+
Ricci isotropy

$$G^r{}_r = G^\theta{}_\theta$$

→

phantom fluid
+
fine tuned cancellation

GMcV – imperfect fluid

$$T^{\mu\nu}=(\rho+p)u^{\mu}u^{\nu}+p\,g^{\mu\nu}-\zeta\,h^{\mu\nu}u^{\gamma}{}_{;\gamma}-\chi\,2\,h^{\gamma(\mu}u^{\nu)}q_{\gamma}$$

Eckart – Landau-Lifshitz model

$$q_{\mu}=\partial_{\mu}T+T\,u_{\mu;\gamma}u^{\gamma}$$

GMcV – imperfect fluid

$$T^{\mu\nu} = (\rho + p) u^{\mu} u^{\nu} + p\, g^{\mu\nu} - \zeta h^{\mu\nu} u^{\gamma}{}_{;\gamma} - \chi\, 2 h^{\gamma(\mu} u^{\nu)} q_{\gamma}$$

Einstein's equations

Eckart – Landau-Lifshitz model

$$q_{\mu} = \partial_{\mu} T + T u_{\mu;\gamma} u^{\gamma}$$

$$T = \left[T_{\infty}(t) + \frac{M}{4\pi\chi} \frac{\ln(R)}{R} \right]$$

GMcV – imperfect fluid

$$T^{\mu\nu} = (\rho + p) u^{\mu} u^{\nu} + p g^{\mu\nu} - \zeta h^{\mu\nu} u^{\gamma}{}_{;\gamma} - \chi 2 h^{\gamma(\mu} u^{\nu)} q_{\gamma}$$

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$$\rho = \frac{3}{8\pi} \left[\frac{\sqrt{2}\dot{m}}{r R \sqrt{R + 1 - \frac{m}{r}}} + H \right]^2$$

GMcV – imperfect fluid

$$T^{\mu\nu} = (\rho + p) u^\mu u^\nu + p\, g^{\mu\nu} - \zeta h^{\mu\nu} u^\gamma{}_{;\gamma} - \chi 2 h^{\gamma(\mu} u^{\nu)} q_\gamma$$

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$$\rho = \frac{3}{8\pi} \left[\frac{\sqrt{2}\,\dot{m}}{r R \sqrt{R + 1 - \frac{m}{r}}} + H \right]^2$$

&

$$p = \dots$$

GMcV – constraints

Additional constraints:

i. $\dot{m}(t)$ does not change sign

– AND

ii. $\dot{a}(t)$ and $\dot{m}(t)$ have same sign

$$\begin{cases} \dot{m}(t) > 0 \\ \dot{a}(t) > 0 \end{cases}$$

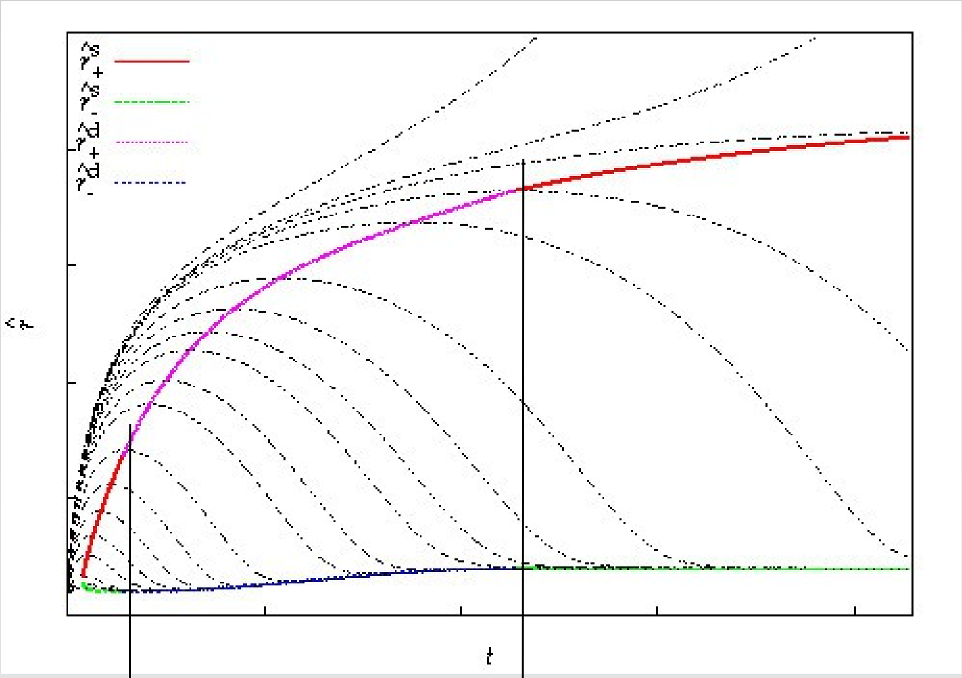
Accreting BH in an expanding background

Remaining freedom encoded in

$$a(t), \quad m(t), \quad T_\infty(t)$$

GMcV – an explicit model

Are these solutions still describing a BH in a cosmological background?



$$m = m_1 \qquad m = m(t) \qquad m = m_2$$

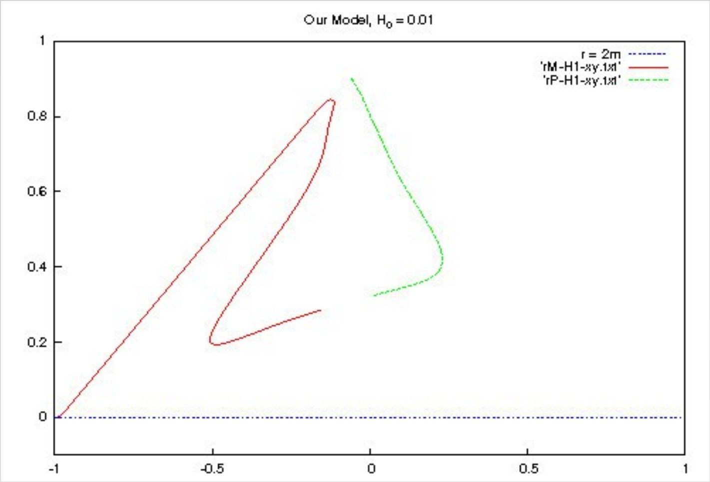
Patching McV and GMcV

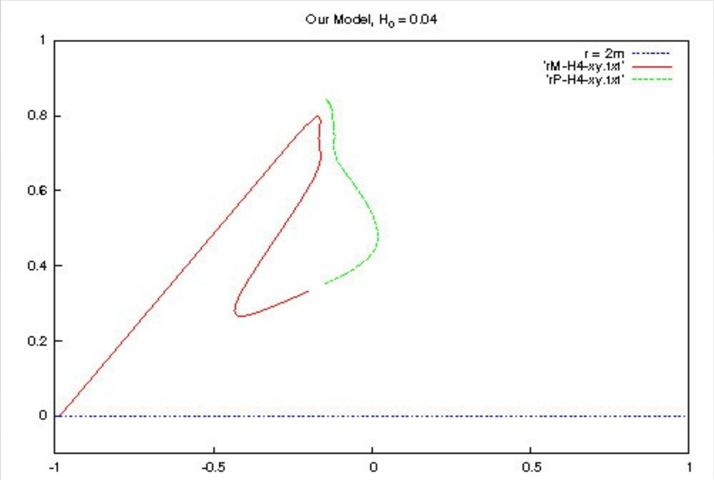
$$m(t) \sim t - \sin(t)$$

smooth connection
and smooth derivatives
→
continuous pressure and
energy density

GMcV – apparent horizons

Apparent horizons behavior in conformal coordinates for different choice of the free parameters

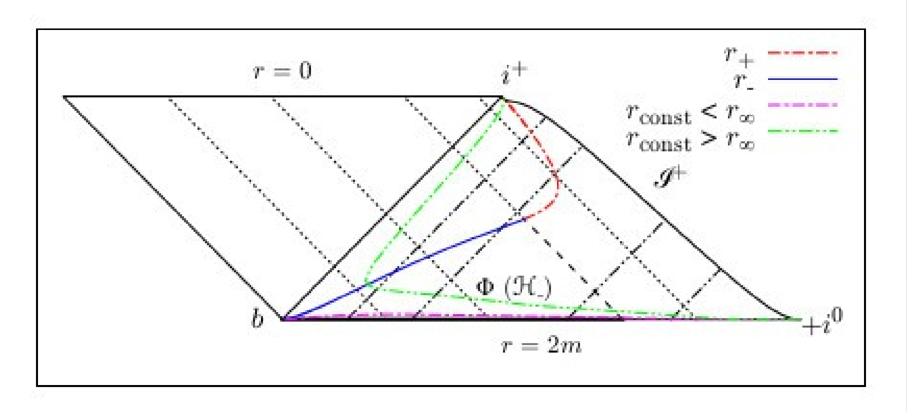


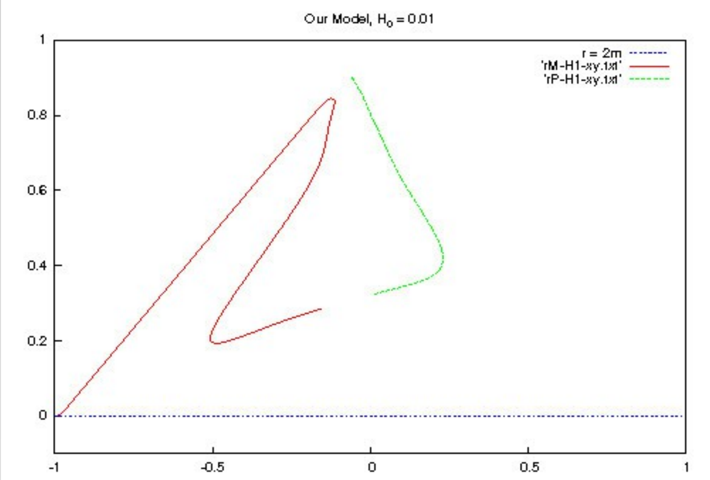


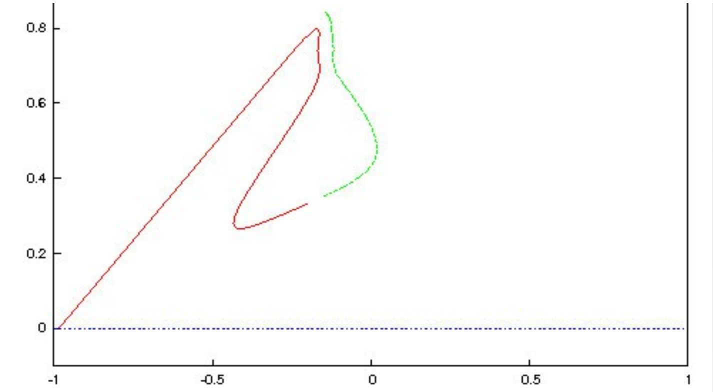
Increasing H_0

GMcV – apparent horizons

Apparent horizons behavior in confo[...]
choice of the free parameters



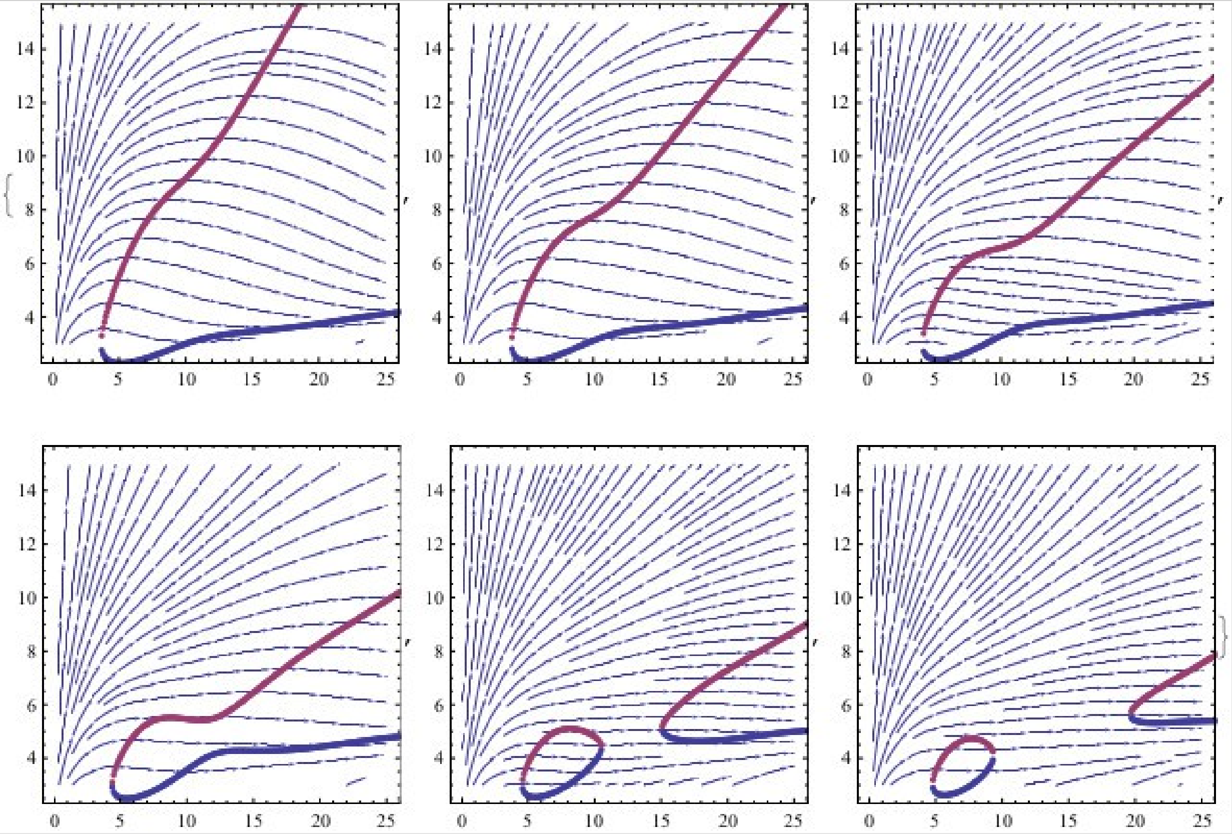




Increasing H_0

GMcV – pushing the boundaries

Even models smoothly patched to McV can produce unexpected results and need to be investigated



Future works

- Studying the most general conditions for which the GMcV solution can be interpreted as an accreting BH in an expanding background.
- Thermodynamics of McVittie and GmcV: Eckart – Landau-Lifshitz → Israel – Stewart

Future works

- Connection to fields: extension of the MTZ solution (a conformally coupled scalar field and a vector sourcing S–dS spacetime).

$$S_{MTZ}=\int d^4x\sqrt{-g}\left[\frac{R-2\Lambda}{16\pi}-\frac{1}{2}\nabla^{\mu}\varphi\nabla_{\mu}\varphi-\frac{1}{12}R\varphi^2-\alpha\varphi^4-\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}\right]$$

Conclusions

- Imperfect fluid can source GMcV, $m \to m(t)$ describing a BH in an expanding background with mass parameter evolving with time.
- "Sandwich" solutions satisfy all the constraints and accommodate a wide class of possibilities.
- Possibly a mass-varying solution for both GR and modified gravity.