Horndeskions,

a short biased introduction

- The simplest Galileon: DGP decoupling limit

  DGP model
  DGP decoupling limit

- Galileons and Generalized Galileons

  Flat space-time Galileon in 4D
  Flat space-time Galileon in arbitrary D
  Curved space-time Galileon
  Generalization to p-forms
  From k-essence to generalized Galileons
  Some previous and recent results and other approaches

- Some phenomenology

  Vainshtein mechanism and k-mouflage
  K-mouflaging MOND
  Late and primordial cosmology
  Time variation of G

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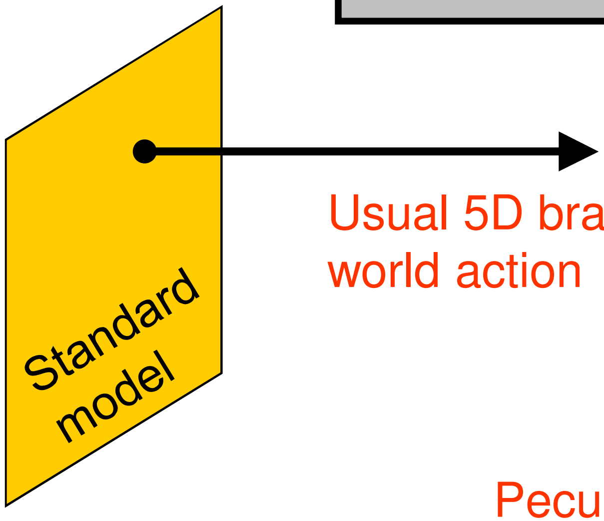
I. The simplest Galileon: DGP decoupling limit

I.1 The DGP model

Dvali, Gabadadze, Porrati, 2000



Usual 5D brane world action

$$S = M^3_{(5)} \int d^5x \sqrt{g} \left(\tilde{R} + \cdots \right) + \int_{\text{brane}} d^4x \sqrt{g} \mathcal{L}_{\text{matter}}$$

Peculiar to DGP model

$$+ M_P^2 \int_{\text{brane}} d^4x \sqrt{g} \left(R + \cdots \right)$$

- Brane localized kinetic term for the graviton
- Will generically be induced by quantum corrections

A special hierarchy between $M_{(5)}$ and M_P is required to make the model phenomenologically interesting

Leads to the e.o.m.

$$G^{(5)}_{AB} = \delta\left(\text{brane}\right) \frac{1}{M^3_{(5)}} \left[G_{\mu\nu} - \frac{1}{M_P^2} T_{\mu\nu} \right]$$

DGP model

- Phenomenological interest
  A new way to modify gravity at large distance, with a new type of phenomenology … The first framework where cosmic acceleration was proposed to be linked to a large distance modification of gravity (C.D. 2001; C.D., Dvali, Gababadze 2002)

  (Important to have such models, if only to disentangle what does and does not depend on the large distance dynamics of gravity in what we know about the Universe)

- Theoretical interest
  Consistent (?) non linear massive gravity …

- Intellectual interest
  Lead to many subsequent developments (massive gravity, Galileons, …)

Homogeneous cosmology of DGP model

One obtains the following modified Friedmann equations (C.D. 2001)

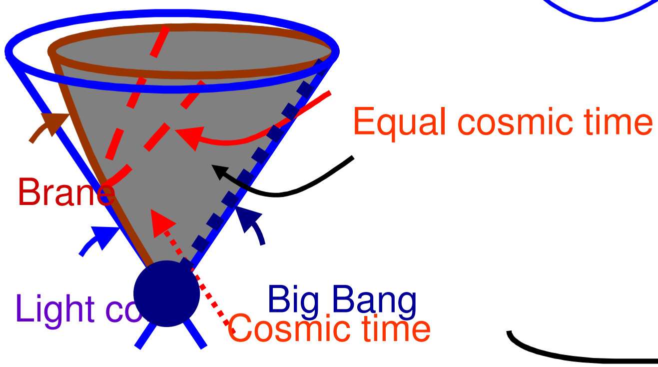
Two branches of solutions

Energy density of brane localized matter

$$\sqrt{H^2 + \frac{k}{a^2}} = \frac{\epsilon}{2r_c} + \sqrt{\frac{\rho_{(M)}}{3M_P^2} + \frac{1}{4r_c^2}}$$

$$\text{with } \epsilon = \pm 1$$

$$\dot{\rho} = -3H\left(P + \rho\right)$$

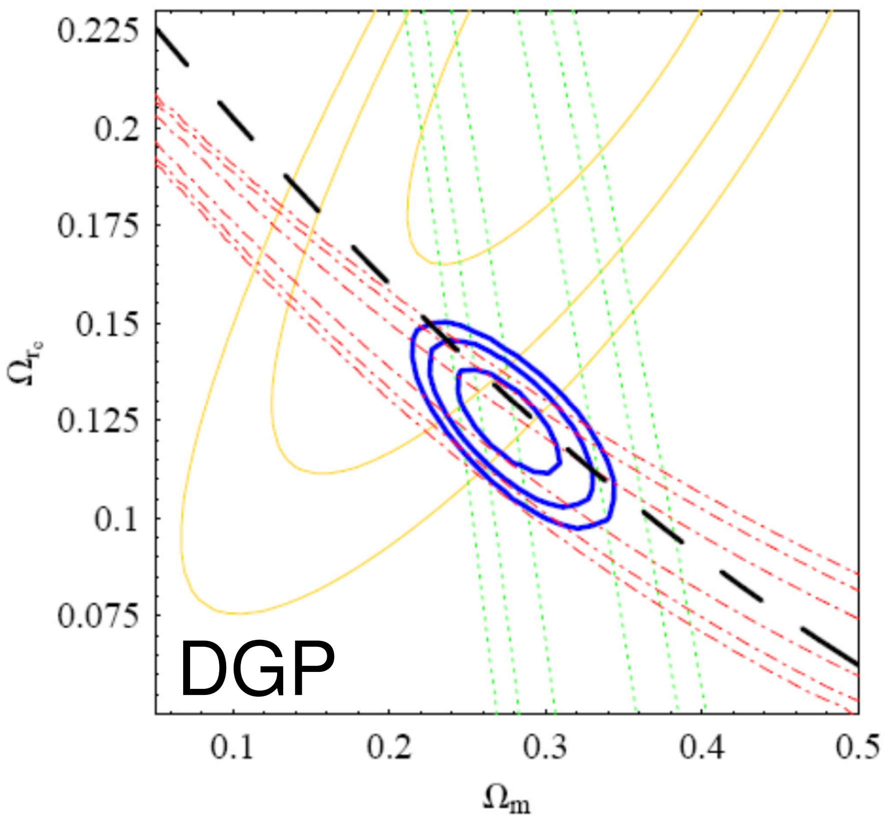


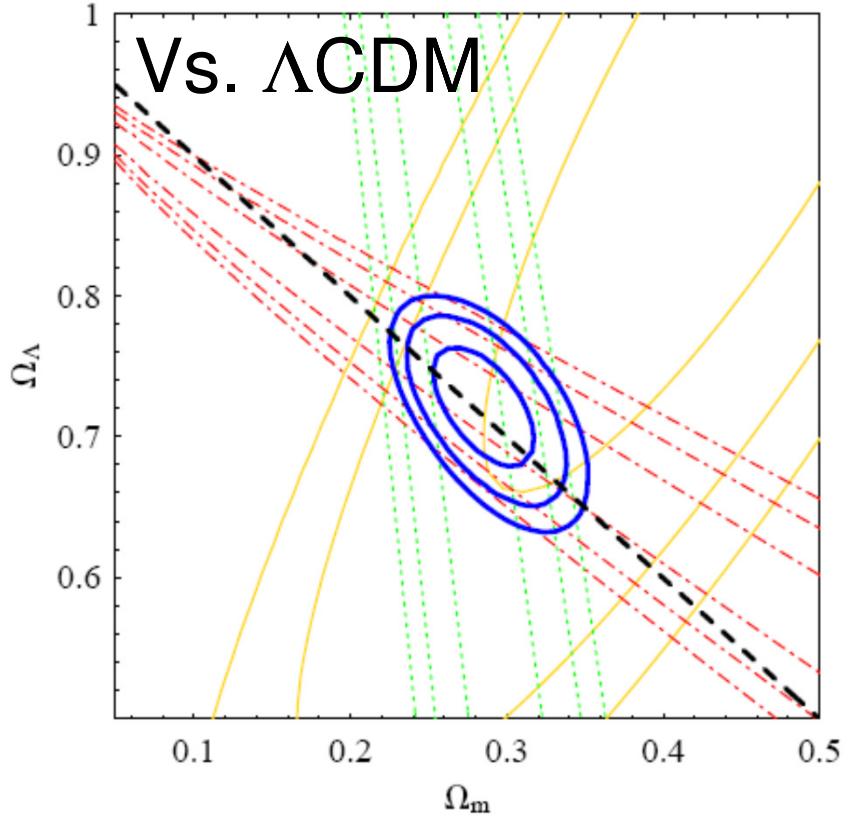
- Analogous to standard (4D) Friedmann equations

$$H^2 + \frac{k}{a^2} = \frac{\rho_{(M)}}{3M_P^2}$$

In the early Universe
(small Hubble radii $H^{-1} \ll r_c$)

- Deviations at late time (self-acceleration)





Maartens, Majerotto 2006
(see also Fairbairn, Goobar 2005;
Rydbeck, Fairbairn, Goobar 2007)

In the DGP model :

- Newtonian potential on the brane behaves as

$$V(r) \propto \frac{1}{r}$$ ← 4D behavior at small distances

$$V(r) \propto \frac{1}{r^2}$$ ← 5D behavior at large distances

- The crossover distance between the two regimes is given by

$$r_c = \frac{M_P^2}{2M_{(5)}^3}$$ ⇒ This enables to get a "4D looking" theory of gravity out of one which is not, without having to assume a compact (Kaluza-Klein) or "curved" (Randall-Sundrum) bulk.

- But the tensorial structure of the graviton propagator is that of a massive graviton (gravity is mediated by a continuum of massive modes)

↳ Leads to the « van Dam-Veltman-Zakharov discontinuity » on Minkowski background (i.e. the fact that the linearized theory differs drastically – e.g. in light bending - from linearized GR at all scales)!

↳ the vDVZ discontinuity, is believed to disappear via the « Vainshtein mechanism » (taking into account of non linearities) **C.D.,Gabadadze, Dvali, Vainshtein, Gruzinov; Porrati; Lue; Lue & Starkman; Tanaka; Gabadadze, Iglesias;…**

I.2 DGP decoupling limit

Luty, Porrati, Rattazzi, 2003

A good description of many DGP key properties is given by the action

$$3\pi\Box\pi - \frac{1}{\Lambda^3}(\partial_\mu\pi\partial^\mu\pi)\Box\pi - \frac{2}{M_P}\pi T$$

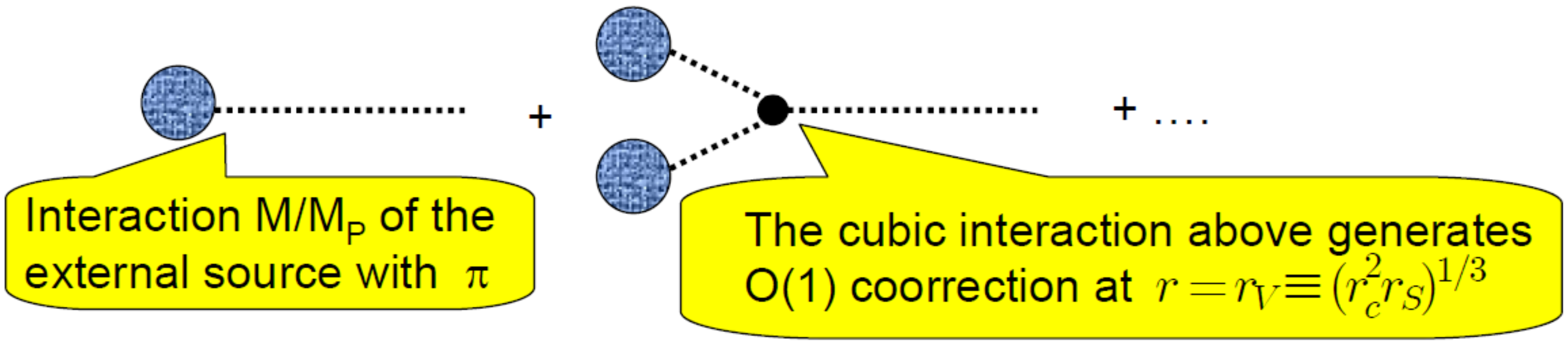
Scalar sector of the model

Energy scale $\Lambda = \left(r_c^2/M_P\right)^{1/3}$

Yielding the e.o.m. $\Box\pi - \frac{1}{3\Lambda^3}\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right] = \frac{T}{3M_P}$

Leads in vacuum to two branches of solutions, $\pi \sim 0$ and $\Box\pi \sim \Lambda^3$ representing the two branches of solutions of the original model…

Around massive body, the cubic self interaction of π becomes of the same order as the quadratic term at the « Vainshtein radius »



The action $$3\pi\Box\pi - \frac{1}{\Lambda^3}(\partial_\mu\pi\partial^\mu\pi)\Box\pi - \frac{2}{M_P}\pi T$$

Is obtained taking the « decoupling limit »

Luty, Porrati, Rattazzi;
Nicolis, Rattazzi

$$\begin{cases} M_P \to \infty \\ M_{(5)} \to \infty \\ \Lambda \ \text{fixed} \\ T/M_P \ \text{fixed} \end{cases}$$

It can be obtained from the (5D) « Hamiltonian » constraint

$$R = K^2 - K_{\mu\nu}^2$$

Where one substitutes the Israel junction condition

$$K = \frac{1}{6M_{(5)}^3}\left(T + M_P^2 R\right)$$

To obtain $$\frac{3}{r_c}K - K^2 + K_{\mu\nu}^2 = \frac{T}{M_P^2}$$

A last substitution $$K_{\mu\nu} = \frac{r_c}{M_P}\partial_\mu\partial_\nu\pi$$

Yields the e.o.m. for π deduced from above action :

$$\Box\pi - \frac{1}{3\Lambda^3}\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right] = \frac{T}{3M_P}$$

NB: second order e.o.m.

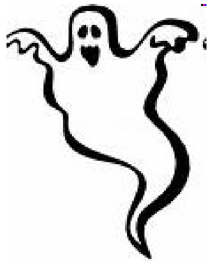
($\Longrightarrow$ No « Boulware-Deser » ghost, C.D., Rombouts, 2005)

Expanding around the vacuum solutions of

$$\Box\pi - \frac{1}{3\Lambda^3}\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right] = \frac{T}{3M_P}$$

$\pi \sim 0$ Positive energy fluctuations

$\Box\pi \sim \Lambda^3$ Negative energy fluctuations



i.e. widely discussed ghost of DGP self accelerating branch

Luty, Porrati, Rattazzi, 2003; Koyama, 2005; Gorbunov, Koyama, Sibiryakov, 2006; Charmousis, Gregory, Kaloper, Padilla, 2006. C.D, Gabadadze, Iglesias, 2006…

Note however that the background solution is at the scale Λ, believed (most of the time) to be the UV cutoff (Luty, Porrati, Rattazzi; Dvali; C.D.) . One should keep this in mind (often forgotten when addressing the DGP ghost and stability)!

II. Generalizations

II. 1 Flat space-time Galileon in 4 D

Galileon

Originally (Nicolis, Rattazzi, Trincherini 2009) defined in flat space-time as the most general scalar theory which has **(strictly) second order fields equations**

In 4D, there is only 4 non trivial such theories

(with $\pi_\mu = \partial_\mu \pi \quad \pi_{\mu\nu} = \partial_\mu \partial_\nu \pi$)

$$
\begin{aligned}
\mathcal{L}_{(2,0)} &= \pi_\mu \pi^\mu \\
\mathcal{L}_{(3,0)} &= \pi^\mu \pi_\mu \Box \pi \\
\mathcal{L}_{(4,0)} &= (\Box\pi)^2 (\pi_\mu \pi^\mu) - 2 (\Box\pi) (\pi_\mu \pi^{\mu\nu} \pi_\nu) \\
&\quad - (\pi_{\mu\nu} \pi^{\mu\nu}) (\pi_\rho \pi^\rho) + 2 (\pi_\mu \pi^{\mu\nu} \pi_{\nu\rho} \pi^\rho) \\
\mathcal{L}_{(5,0)} &= (\Box\pi)^3 (\pi_\mu \pi^\mu) - 3 (\Box\pi)^2 (\pi_\mu \pi^{\mu\nu} \pi_\nu) - 3 (\Box\pi) (\pi_{\mu\nu} \pi^{\mu\nu}) (\pi_\rho \pi^\rho) \\
&\quad + 6 (\Box\pi) (\pi_\mu \pi^{\mu\nu} \pi_{\nu\rho} \pi^\rho) + 2 \left(\pi_\mu{}^\nu \pi_\nu{}^\rho \pi_\rho{}^\mu\right) \left(\pi_\lambda \pi^\lambda\right) \\
&\quad + 3 (\pi_{\mu\nu} \pi^{\mu\nu}) \left(\pi_\rho \pi^{\rho\lambda} \pi_\lambda\right) - 6 \left(\pi_\mu \pi^{\mu\nu} \pi_{\nu\rho} \pi^{\rho\lambda} \pi_\lambda\right)
\end{aligned}
$$

Simple rewriting of those Lagrangians with epsilon tensors
(up to integrations by part):

(C.D., S.Deser, G.Esposito-Farese, 2009)

$$\mathcal{L}_{(2,0)} = \epsilon^{\mu_1\lambda_1\lambda_2\lambda_3}\epsilon^{\nu_1}{}_{\lambda_1\lambda_2\lambda_3}\pi_{\mu_1}\pi_{\nu_1}$$
$$\mathcal{L}_{(3,0)} = \epsilon^{\mu_1\mu_2\lambda_1\lambda_2}\epsilon^{\nu_1\nu_2}{}_{\lambda_1\lambda_2}\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_2\nu_2}$$
$$\mathcal{L}_{(4,0)} = \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}$$
$$\mathcal{L}_{(5,0)} = \epsilon^{\mu_1\mu_2\mu_3\mu_4}\epsilon^{\nu_1\nu_2\nu_3\nu_4}\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}\pi_{\mu_4\nu_4}$$

This leads to (exactly) second order field equations

Indeed, consider e.g.

$$\begin{aligned}\mathcal{L}_{(4,0)} &= \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\,\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\,\pi_{\mu_1}\,\pi_{\nu_1}\,\pi_{\mu_2\nu_2}\,\pi_{\mu_3\nu_3}\\ &\propto (\Box\pi)^2\,(\pi_\mu\,\pi^\mu) - 2\,(\Box\pi)\,(\pi_\mu\,\pi^{\mu\nu}\,\pi_\nu)\\ &\quad - (\pi_{\mu\nu}\,\pi^{\mu\nu})\,(\pi_\rho\,\pi^\rho) + 2\,(\pi_\mu\pi^{\mu\nu}\,\pi_{\nu\rho}\,\pi^\rho)\end{aligned}$$

Indeed, consider e.g.

$$\mathcal{L}_{(4,0)} = \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}$$

Varying this Lagrangian with respect to π yields (after integrating by part)

$$\delta\mathcal{L}_{(4,0)} \supset -\epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\delta\pi\partial_{\mu_1}\left\{\pi_{\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}\right\}$$

Second order derivative

Indeed, consider e.g.

$$\mathcal{L}_{(4,0)} = \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}$$

Varying this Lagrangian with respect to π yields (after integrating by part)

$$\delta\mathcal{L}_{(4,0)} \supset -\epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\delta\pi\partial_{\mu_1}\left\{\pi_{\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}\right\}$$

Third order derivative…

… killed by the contraction with epsilon tensor

Indeed, consider e.g.

$$\mathcal{L}_{(4,0)} = \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}$$

Varying this Lagrangian with respect to π yields (after integrating by part)

$$\delta\mathcal{L}_{(4,0)} \supset -\epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\delta\pi\partial_{\mu_1}\left\{\pi_{\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}\right\}$$

Similarly, one also have in the field equations

$$\delta\mathcal{L}_{(4,0)} \supset \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\delta\pi\partial_{\nu_2}\partial_{\mu_2}\left\{\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_3\nu_3}\right\}$$

Yields third and fourth order derivative… killed by the epsilon tensor

Hence the field equations are proportional to

$$\mathcal{E}_{(4,0)} = \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\pi_{\mu_1\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}$$

Which does only contain second derivatives

The field equations, containing only second derivatives,

$$\mathcal{E}_{(4,0)} = \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\pi_{\mu_1\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3} = 0$$

Have the « Galilean » symmetry

$$\pi \longrightarrow \pi + C + D_\mu x^\mu$$

They also can be written using 4 (and lower) dimensional determinants and are in fact **generalized Monge Ampere equation** of the form *det* (π_{ij}) = 0 (Monge Ampere equation have interesting integrability properties - Fairlie)

The field equations read

$$\sum_{\sigma\in S_n} \underbrace{\epsilon(\sigma) g^{\mu_{\sigma(1)}\nu_1} g^{\mu_{\sigma(2)}\nu_2} \dots g^{\mu_{\sigma(n)}\nu_n} \pi_{\nu_1\mu_1}\pi_{\nu_2\mu_2}\pi_{\nu_3\mu_3}\dots\pi_{\nu_n\mu_n}} = 0$$

Linear in second time derivative ($\Longrightarrow$ Good Cauchy problem ?).

II. 2 Flat space-time Galileon in arbitrary Dimension

In D dimensions, D non trivial Galileons can be defined as

$$\mathcal{L}_{(n+1,0)} = \sum_{\sigma \in S_n} \epsilon(\sigma) g^{\mu_{\sigma(1)}\nu_1} g^{\mu_{\sigma(2)}\nu_2} \dots g^{\mu_{\sigma(n)}\nu_n} (\pi_{\nu_1}\pi_{\mu_1})(\pi_{\nu_2\mu_2}\pi_{\nu_3\mu_3}\dots\pi_{\nu_n\mu_n}).$$

Only the Lagrangians with $D \geq n$ are non vanishing.

Using the tensors

$$\mathcal{A}_{(2n)}^{\mu_1\mu_2\dots\mu_{2n}} \equiv \frac{1}{(D-n)!} \varepsilon^{\mu_1\mu_3\mu_5\dots\mu_{2n-1}\;\nu_1\nu_2\dots\nu_{D-n}} \varepsilon^{\mu_2\mu_4\mu_6\dots\mu_{2n}}{}_{\nu_1\nu_2\dots\nu_{D-n}}$$

$\mathcal{A}_{(2n)}$ is antisymmetric (separately) in odd and even indices

Or to alleviate notations

$$\mathcal{A}_{(2n)}^{1234\dots} = \frac{1}{(D-n)!} \varepsilon^{135\dots\;\nu_1\nu_2\dots\nu_{D-n}} \varepsilon^{246\dots}{}_{\nu_1\nu_2\dots\nu_{D-n}}$$

One has

$$\mathcal{L}_{(n+1,0)} = -\mathcal{A}_{(2n)}(\pi_1\pi_2)(\pi_{34}\pi_{56}\pi_{78}\dots\pi_{\mu_{2n-1}\mu_{2n}})$$

All free indices are contracted with those of $\mathcal{A}_{(2n)}$

Up to total derivatives, the following Lagrangians are equivalent

$$\mathcal{L}_N^{\text{Gal},1} = \left(\mathcal{A}_{(2n+2)}^{\mu_1\ldots\mu_{n+1}\nu_1\ldots\nu_{n+1}}\pi_{\mu_{n+1}}\pi_{\nu_{n+1}}\right)\pi_{\mu_1\nu_1}\ldots\pi_{\mu_n\nu_n}$$

$$\mathcal{L}_N^{\text{Gal},2} = \left(\mathcal{A}_{(2n)}^{\mu_1\ldots\mu_n\nu_1\ldots\nu_n}\pi_{\mu_1}\pi_\lambda\pi^\lambda{}_{\nu_1}\right)\pi_{\mu_2\nu_2}\ldots\pi_{\mu_n\nu_n}$$

$$\mathcal{L}_N^{\text{Gal},3} = \left(\mathcal{A}_{(2n)}^{\mu_1\ldots\mu_n\nu_1\ldots\nu_n}\pi_\lambda\pi^\lambda\right)\pi_{\mu_1\nu_1}\ldots\pi_{\mu_n\nu_n}$$

One has the exact relation

$$(N-2)\mathcal{L}_N^{\text{Gal},2} = \mathcal{L}_N^{\text{Gal},3} - \mathcal{L}_N^{\text{Gal},1}$$

II. 3 Curved space-time Galileon

A naive covariantization leads to the loss of the distinctive properties of the Galileon

Indeed, consider now in curved space-time (with $\pi_\mu = \nabla_\mu \pi$ and $\pi_{\mu\nu} = \nabla_\mu \nabla_\nu \pi$)

$$\mathcal{L}_{(4,0)} = \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}$$

Variation yields in particular

$$\delta\mathcal{L}_{(4,0)} \supset \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\delta\pi\nabla_{\nu_2}\nabla_{\mu_2}\left\{\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_3\nu_3}\right\}$$

Third derivatives generate now Riemann tensors …

and fourth derivatives, derivatives of the Riemann

Indeed the (naively covariantized) $\mathcal{L}_{(4,0)}$ has the field equations

$$\begin{aligned}\mathcal{E} \quad \mathcal{E}_{(4,0)} = & -4(\Box\pi)^3 - 8\left(\pi_\mu{}^\nu\,\pi_\nu{}^\rho\,\pi_\rho{}^\mu\right) + 12(\Box\pi)(\pi_{\mu\nu}\pi^{\mu\nu}) - (\pi_\mu\,\pi^\mu)(\pi_\nu R^{;\nu}) \\ & +2(\pi_\mu\,\pi_\nu\,\pi_\rho R^{\mu\nu;\rho}) + 10(\Box\pi)(\pi_\mu R^{\mu\nu}\pi_\nu) - 8(\pi_\mu\,\pi^{\mu\nu}R_{\nu\rho}\pi^\rho) \\ & -2(\pi_\mu\,\pi^\mu)(\pi_{\nu\rho}R^{\nu\rho}) - 8(\pi_\mu\,\pi_\nu\,\pi_{\rho\sigma}R^{\mu\rho\nu\sigma}).\end{aligned}$$

Kinetic mixing $\quad {}^{\mu\nu})\,.$

Similarly, varying w.r.t. the metric

$$\mathcal{L}_{(4,0)} = \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\pi_{\mu_1}\pi_{\nu_1}\underbrace{\pi_{\mu_2\nu_2}}_{\cup\atop \partial g}\pi_{\mu_3\nu_3}$$

Yields third order derivatives of the scalar π in the energy momentum tensor

Do not vanish in flat space-time !

$$\begin{aligned}
\mathcal{T}^{\mu\nu}_{(4,0)} = {} & (\pi^\mu\,\pi^\nu)\,\pi^\lambda\left(2\,\pi_{\lambda\rho}{}^{\rho} - \pi^{\rho}{}_{\rho\lambda}\right) \\
& + \left(\pi_\lambda\,\pi^\lambda\right)\pi^\mu\left(\pi_\rho{}^{\rho\nu} - \pi^{\nu\rho}{}_{\rho}\right) + \left(\pi_\lambda\,\pi^\lambda\right)\pi^\nu\left(\pi_\rho{}^{\rho\mu} - \pi^{\mu\rho}{}_{\rho}\right) \\
& - \pi^\lambda\,\pi^\rho\left(\pi^\mu\pi_{\lambda\rho}{}^{\nu} + \pi^\nu\pi_{\lambda\rho}{}^{\mu}\right) + \left(\pi_\lambda\,\pi^\lambda\right)\left(\pi_\rho\,\pi^{\mu\nu\rho}\right) \\
& + \left(\pi_\lambda\,\pi_\rho\,\pi_\sigma\,\pi^{\lambda\rho\sigma}\right)g^{\mu\nu} - \left(\pi_\lambda\,\pi^\lambda\right)\left(\pi_\rho\,\pi_\sigma{}^{\sigma\rho}\right)g^{\mu\nu} \\
& + (\pi^\mu\,\pi^\nu)\left[3\left(\pi_{\lambda\rho}\,\pi^{\lambda\rho}\right) - 2\,(\Box\pi)^2\right] + (\pi^{\mu\nu})\,\pi_\lambda\left(2\,\pi^{\lambda\rho}\,\pi_\rho + \pi^\lambda\,\Box\pi\right) \\
& + 3\,(\Box\pi)\,\pi_\lambda\left(\pi^{\lambda\mu}\,\pi^\nu + \pi^{\lambda\nu}\,\pi^\mu\right) - 4\,\pi_\lambda\,\pi^{\lambda\rho}\left(\pi_\rho{}^{\mu}\,\pi^\nu + \pi_\rho{}^{\nu}\,\pi^\mu\right) \\
& - 2\left(\pi_\lambda\,\pi^{\lambda\mu}\right)\left(\pi_\rho\,\pi^{\rho\nu}\right) - \frac{1}{2}\left(\pi_\lambda\,\pi^\lambda\right)\left[(\Box\pi)^2 + \left(\pi_{\rho\sigma}\,\pi^{\rho\sigma}\right)\right]g^{\mu\nu} \\
& + \pi_\lambda\,\pi_\rho\left[3\pi^{\lambda\sigma}\,\pi_\sigma{}^{\rho} - 2\,(\Box\pi)\,\pi^{\lambda\rho}\right]g^{\mu\nu}.
\end{aligned}$$

This can be cured by a non minimal coupling to the metric

Adding to

$$\mathcal{L}_{(4,0)} = (\Box\pi)^2 (\pi_\mu \pi^\mu) - 2 (\Box\pi) (\pi_\mu \pi^{\mu\nu} \pi_\nu) - (\pi_{\mu\nu} \pi^{\mu\nu}) (\pi_\rho \pi^\rho) + 2 (\pi_\mu \pi^{\mu\nu} \pi_{\nu\rho} \pi^\rho)$$

The Lagrangian

$$\mathcal{L}_{(4,1)} = \left(\pi_\lambda \pi^\lambda\right) \pi_\mu \left[R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R\right] \pi_\nu$$

Yields second order field equations for the scalar and the metric
(but loss of the « Galilean » symmetry)

e.g. one has now the energy momentum tensor

$$\begin{aligned}
\mathcal{T}_4^{\mu\nu} = {} & 4 (\Box\pi) \pi_\rho \left[\pi^\mu \pi^{\rho\nu} + \pi^\nu \pi^{\rho\mu}\right] - 2 (\Box\pi)^2 (\pi^\mu \pi^\nu) + 2 (\Box\pi) \left(\pi_\lambda \pi^\lambda\right) (\pi^{\mu\nu}) \\
& + 4 \left(\pi_\lambda \pi^{\lambda\rho} \pi_\rho\right) (\pi^{\mu\nu}) - 4 \left(\pi_\lambda \pi^{\lambda\mu}\right) (\pi_\rho \pi^{\rho\nu}) + 2 \left(\pi_{\lambda\rho} \pi^{\lambda\rho}\right) (\pi^\mu \pi^\nu) \\
& - 2 \left(\pi_\lambda \pi^\lambda\right) \left(\pi^\mu{}_\rho \pi^{\rho\nu}\right) - 4 \pi^\lambda \pi_{\lambda\rho} \left[\pi^{\rho\mu} \pi^\nu + \pi^{\rho\nu} \pi^\mu\right] - (\Box\pi)^2 \left(\pi_\lambda \pi^\lambda\right) g^{\mu\nu} \\
& - 4 (\Box\pi) \left(\pi_\lambda \pi^{\lambda\rho} \pi_\rho\right) g^{\mu\nu} + 4 \left(\pi_\lambda \pi^{\lambda\rho} \pi_{\rho\sigma} \pi^\sigma\right) g^{\mu\nu} \\
& + \left(\pi_\lambda \pi^\lambda\right) (\pi_{\rho\sigma} \pi^{\rho\sigma}) g^{\mu\nu} + \left(\pi_\lambda \pi^\lambda\right) (\pi^\mu \pi^\nu) R - \frac{1}{4} \left(\pi_\lambda \pi^\lambda\right) (\pi_\rho \pi^\rho) g^{\mu\nu} R \\
& - 2 \left(\pi_\lambda \pi^\lambda\right) \pi_\rho \left[R^{\rho\mu} \pi^\nu + R^{\rho\nu} \pi^\mu\right] + \frac{1}{2} \left(\pi_\lambda \pi^\lambda\right) (\pi_\rho \pi^\rho) R^{\mu\nu} \\
& + 2 \left(\pi_\lambda \pi^\lambda\right) (\pi_\rho R^{\rho\sigma} \pi_\sigma) g^{\mu\nu} - 2 \left(\pi_\lambda \pi^\lambda\right) (\pi_\rho \pi_\sigma R^{\mu\rho\nu\sigma}),
\end{aligned}$$

This can be generalized to arbitrary Galileons
(arbitrary number of fields and dimensions)

Introducing $\mathcal{L}_{(n+1,p)} = -\mathcal{A}_{(2n)}\pi_1\pi_2\mathcal{R}_{(p)}\mathcal{S}_{(q)}$

With
$$\mathcal{R}_{(p)} \equiv (\pi_\lambda\pi^\lambda)^p \prod_{i=1}^{i=p} R_{\mu_{4i-1}\,\mu_{4i+1}\,\mu_{4i}\,\mu_{4i+2}},$$
$$\mathcal{S}_{(q)} \equiv \prod_{i=0}^{i=q-1} \pi_{\mu_{2n-1-2i}\,\mu_{2n-2i}},$$

The action
$$I = \int d^Dx\sqrt{-g}\sum_{p=0}^{p_{\max}} \mathcal{C}_{(n+1,p)}\mathcal{L}_{(n+1,p)}$$

with
$$\mathcal{C}_{(n+1,p)} = \left(-\frac{1}{8}\right)^p \frac{(n-1)!}{(n-1-2p)!\,(p!)^2} = \left(-\frac{1}{8}\right)^p \binom{n-1}{2p}\binom{2p}{p}$$

Yields second order field equations.

Heuristically, one needs to replace sucessively pairs of twice differentiated π by Riemanns

This can be understood as follows. Considers e.g.

$$\mathcal{L}_{(5,0)} = -\varepsilon^{\mu_1\mu_3\mu_5\mu_7}\,\varepsilon^{\mu_2\mu_4\mu_6\mu_8}\,\pi_{\mu_1}\pi_{\mu_2}\,\pi_{\mu_3\mu_4}\pi_{\mu_5\mu_6}\pi_{\mu_7\mu_8} = -\mathcal{A}_{(8)}\,\pi_1\pi_2\,\pi_{34}\pi_{56}\pi_{78}.$$

⇓ Vary w.r.t. π

$$\delta_\pi\mathcal{L}_{(5,0)} = -2\mathcal{A}_{(8)}\delta\pi_1\pi_2\pi_{34}\pi_{56}\pi_{78} - 3\mathcal{A}_{(8)}\pi_1\pi_2\delta\pi_{34}\pi_{56}\pi_{78}$$

⇓ Only « dangerous » term (i.e. term leading to higher derivatives)

$$\delta_\pi\mathcal{L}_{(5,0)} \sim -3\mathcal{A}_{(8)}\pi_1\pi_2\delta\pi_{34}\pi_{56}\pi_{78}.$$

⇓ Integrating by part

$$\delta_\pi\mathcal{L}_{(5,0)} \sim -3\times 2\,\delta\pi\,\mathcal{A}_{(8)}\pi_1\pi_2\pi_{5643}\pi_{78}$$

⇓ Using the antisymmetry of $\mathcal{A}_{(8)}$ and the Riemann Bianchi identity

$$\begin{aligned}\delta_\pi\mathcal{L}_{(5,0)} &\sim -3\,\delta\pi\,\mathcal{A}_{(8)}\pi_1\pi_2\left(\pi_{5643}-\pi_{5463}\right)\pi_{78}\\ &\sim -3\,\delta\pi\,\mathcal{A}_{(8)}\pi_1\pi_2\pi^\lambda R_{465\lambda;3}\pi_{78}\\ &\sim -\frac{3}{2}\,\delta\pi\,\mathcal{A}_{(8)}\pi_1\pi_2\pi^\lambda\left(R_{465\lambda;3}+R_{46\lambda3;5}\right)\pi_{78}\\ &\sim \frac{3}{2}\,\delta\pi\,\mathcal{A}_{(8)}\pi_1\pi_2\pi^\lambda R_{3546;\lambda}\pi_{78},\end{aligned}$$

↪ Can be cancelled by varying $\mathcal{L}_{(5,1)} = \frac{3}{4}\mathcal{A}_{(8)}\pi_1\pi_2\left(\pi_\lambda\pi^\lambda\right)R_{3546}\,\pi_{78}.$

Note that the extra term $\mathcal{L}_{(5,1)} = \frac{3}{4}\mathcal{A}_{(8)}\pi_1\pi_2\left(\pi_\lambda\pi^\lambda\right) R_{3546}\,\pi_{78}.$

Does not generate unwanted derivatives of the curvature thanks to Bianchi identity

Indeed one has $\begin{cases} \mathcal{A}_{(8)} R_{3546;7} = 0 \\ \mathcal{A}_{(8)} R_{3546;8} = 0 \end{cases}$

Similarly to $\begin{cases} \mathcal{A}_{(8)}\partial_7\pi_{35} = 0 \\ \mathcal{A}_{(8)}\partial_8\pi_{46} = 0 \end{cases}$

Yields an easy generalization to p-forms

II. 4 Generalization to p-forms

C.D., S.Deser, G.Esposito-Farese,
arXiv 1007.5278 [gr-qc] (PRD)

E.g. consider

$$I = \int d^D x \, \varepsilon^{\mu\nu\ldots} \varepsilon^{\alpha\beta\ldots} \, \omega_{\mu\nu\ldots} \omega_{\alpha\beta\ldots} (\partial_\rho \omega_{\gamma\delta\ldots} \, \ldots)(\partial_\epsilon \omega_{\sigma\tau\ldots} \, \ldots)$$

With $A_{\mu\nu\ldots}$ a *p*-form of field strength $\omega_{\lambda\mu\nu\ldots} = \partial_{[\lambda} A_{\mu\nu\ldots]}$

In the field equations, Bianchi identities annihilate any $\partial_\mu \partial_{[\alpha} \omega_{\beta\gamma\ldots]}$

E.o.m. are (purely) second order

E.g. for a 2-form

$$I = \int d^7 x \, \varepsilon^{\mu\nu\rho\sigma\tau\varphi\chi} \varepsilon^{\alpha\beta\gamma\delta\epsilon\zeta\eta} \, \omega_{\mu\nu\rho} \, \omega_{\alpha\beta\gamma} \, \partial_\sigma \omega_{\delta\epsilon\zeta} \, \partial_\eta \omega_{\tau\varphi\chi}$$

Note that one must go to 7 dimensions (in general one has $D \geq 2p+3$) and that this construction does not work for odd p (the field equations vanish identically ⇨ open question: vector Galileon ?)

This can be generalized to multi p-forms (different species) in which case one can have [odd p]-forms

(labels different p-forms)

$$I = \int d^D x \, \varepsilon^{\mu\nu\dots} \varepsilon^{\alpha\beta\dots} \, \omega^a_{\mu\nu\dots} \omega^b_{\alpha\beta\dots} (\partial_\rho \omega^c_{\gamma\delta\dots} \dots)(\partial_\epsilon \omega^d_{\sigma\tau\dots} \dots)$$

(NB: this can/must also be « covariantized » using previously introduced technique)

One simple example: bi-galileon, e.g.

$$I = \int d^D x \epsilon^{\alpha\beta\dots} \epsilon^{\mu\nu\dots} \pi_\alpha \varphi_\mu \left(\pi_{\beta\gamma}\dots\right) \left(\varphi_{\nu\rho}\dots\right)$$

with $\begin{cases} \pi_\alpha = \partial_\alpha \pi, & \pi_{\beta\gamma} = \partial_\beta \partial_\gamma \pi \\ \varphi_\mu = \partial_\mu \varphi, & \varphi_{\nu\rho} = \partial_\nu \partial_\rho \varphi \end{cases}$

π and φ being two scalar fields

Padilla, Saffin, Zhou (bi-galileon);
Hinterbichler,Trodden, Wesley (multi-scalar galileons)

II. 5 From k-essence to generalized Galileons

C.D. Xian Gao, Daniele Steer, George Zahariade
arXiv:1103.3260 [hep-th] (PRD)

What is the most general scalar theory which has (not necessarily exactly) second order field equations in **flat space** ?

Specifically we looked for the most general scalar theory such that (in flat space-time)

i/ Its **Lagrangian contains derivatives of order 2** or less of the scalar field π

ii/ Its **Lagrangian is polynomial in second derivatives of** π (can be relaxed: Padilla, Sivanesan; Sivanesan)

iii/ The **field equations are of order 2 or lower** in derivatives

(NB: those hypothesis cover k-essence, simple Galileons ,…)

Answer: the most general such theory is given by a linear combination of the Lagrangians $\mathcal{L}_n\{f\}$

Free function of π and X

defined by

$$\mathcal{L}_n\{f\} = f(\pi, X) \times \mathcal{L}^{\text{Gal},3}_{N=n+2},$$
$$= f(\pi, X) \times \left(X \mathcal{A}_{(2n)}^{\mu_1 \ldots \mu_n \nu_1 \ldots \nu_n} \pi_{\mu_1 \nu_1} \ldots \pi_{\mu_n \nu_n} \right)$$

where $X \equiv \pi_\mu \pi^\mu$

Sketch of the proof

Consider « cycles »
containing second derivatives

$$\left\{ \begin{array}{l} [i] \equiv \pi^{\mu_1}{}_{\mu_2}\pi^{\mu_2}{}_{\mu_3}\pi^{\mu_3}{}_{\mu_4}\cdots\pi^{\mu_i}{}_{\mu_1} \\ \langle i \rangle \equiv \pi_{\mu_1}\pi^{\mu_1}{}_{\mu_2}\pi^{\mu_2}{}_{\mu_3}\pi^{\mu_3}{}_{\mu_4}\cdots\pi^{\mu_i}{}_{\mu_{i+1}}\pi^{\mu_{i+1}} \end{array} \right.$$

then

$$\left\{ \begin{array}{l} \begin{bmatrix} p_1 & p_2 & \cdots & p_r \\ 1 & 2 & \cdots & r \end{bmatrix} = [1]^{p_1}[2]^{p_2}\cdots[r]^{p_r} \\ \left\langle \begin{matrix} q_1 & q_2 & \cdots & q_s \\ 1 & 2 & \cdots & s \end{matrix} \right\rangle = \langle 1\rangle^{q_1}\langle 2\rangle^{q_2}\cdots\langle s\rangle^{q_s} \end{array} \right.$$

The most general theory we look for is made out of linear combinations of

$$\mathcal{L}^{p_1,p_2,\cdots,p_r}_{q_1,q_2,\cdots,q_s} = f(\pi, X) \times \begin{bmatrix} p_1 & p_2 & \cdots & p_r \\ 1 & 2 & \cdots & r \end{bmatrix} \left\langle \begin{matrix} q_1 & q_2 & \cdots & q_s \\ 1 & 2 & \cdots & s \end{matrix} \right\rangle$$

Recursion relation leads to the previously decribed theory

Our most general flat space-time theories can easily be « covariantized » using the previously described technology

The covariantized theory is given by a linear combination of the Lagrangians

$$\mathcal{L}_{n,p}\{f\} = \mathcal{P}_{(p)}^{\mu_1\mu_2\cdots\mu_n\nu_1\nu_2\cdots\nu_n}\mathcal{R}_{(p)}\mathcal{S}_{(q\equiv n-2p)}$$

with

$$\begin{cases} \mathcal{S}_{(q\equiv n-2p)} \equiv \prod_{p}^{q-1} \pi_{\mu_{n-i}\nu_{n-i}} \\ \mathcal{R}_{(p)} \equiv \prod_{i=1}^{p} R_{\mu_{2i-1}\mu_{2i}\nu_{2i-1}\nu_{2i}} \\ \mathcal{P}_{(p)} \equiv \int_{X_0}^{X} dX_1 \int_{X_0}^{X_1} dX_2 \cdots \int_{X_0}^{X_{p-1}} dX_p\, \mathcal{T}_{(2n)}^{\mu_1\mu_2\cdots\mu_n\nu_1\nu_2\cdots\nu_n}(\pi, X_1) \\ \mathcal{T}_{(2n)} = \mathcal{T}_{(2n)}(\pi, X) \\ \qquad = f(\pi, X) \times X\mathcal{A}_{(2n)} \end{cases}$$

Specifically the covariantized theory is given by

$$\mathcal{L}_n^{\rm cov}\{f\} = \sum_{p=0}^{\lfloor \frac{n}{2} \rfloor} \mathcal{C}_{n,p}\mathcal{L}_{n,p}\{f\} \quad \text{with} \quad \mathcal{C}_{n,p} = \left(-\frac{1}{8}\right)^p \frac{n!}{(n-2p)!p!}$$

II. 6 Some previous and recent results and other approaches

• Flat space-time Galileons and flat-space time generalized Galileons (in the shift symmetric case) have been obtained previously by Fairlie, Govaerts and Morozov (1992) by the « Euler hierarchy » construction :

Start from a set of arbitrary functions $F_\ell = F_\ell(\pi^\mu)$

Then define the recursion relation $W_{\ell+1} = -\hat{\mathcal{E}} F_{\ell+1} W_\ell$

$\hat{\mathcal{E}}$ being the Euler-Lagrange operator (and $W_0=1$)

$$\hat{\mathcal{E}} = \left[\frac{\partial}{\partial \pi} - \partial_\mu \left(\frac{\partial}{\partial \pi_\mu} \right) + \partial_\mu \partial_\nu \left(\frac{\partial}{\partial \pi_{\mu\nu}} \right) \right]$$

Hence W_ℓ is the field equation of the Lagrangian $\mathcal{L}_\ell = F_\ell W_{\ell-1}$ (« Euler hierarchy »)

⟹ The hierarchy stops after at most D steps

⟹ Choosing $F_k = \pi^\mu \pi_\mu /2$, one has $\mathcal{L}_\ell = \frac{1}{2} X W_{\ell-1} = \frac{1}{2} \mathcal{L}^{\mathrm{Gal},3}_{\ell+1}$

• Horndeski (1972) obtained the most general scalar tensor theory in 4D which has second order field equation for the scalar and the metric

Using our notations it is given by

$$\begin{aligned}
\mathcal{L}_H \; = \; & -\mathcal{A}_{(3)}^{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3}\left(\kappa_1 R_{\mu_1\mu_2\nu_1\nu_2}\pi_{\mu_3\nu_3} - \frac{4}{3}\kappa_{1,X}\pi_{\mu_1\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}\right) \\
& -\mathcal{A}_{(3)}^{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3}\left(\kappa_3 R_{\mu_1\mu_2\nu_1\nu_2}\pi_{\mu_3}\pi_{\nu_3} - 4\kappa_{3,X}\pi_{\mu_1\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3}\pi_{\nu_3}\right) \\
& -\mathcal{A}_{(2)}^{\mu_1\mu_2\nu_1\nu_2}\left(F R_{\mu_1\mu_2\nu_1\nu_2} - 4F_{,X}\pi_{\mu_1\nu_1}\pi_{\mu_2\nu_2}\right) \\
& -2\kappa_8\mathcal{A}_{(2)}^{\mu_1\mu_2\nu_1\nu_2}\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_2\nu_2} \\
& -3\left(2F_{,\pi} + X\kappa_8\right)X + \kappa_9,
\end{aligned}$$

and is parametrized by four free functions of X and π : $\kappa_1,\ \kappa_3,\ \kappa_8,\ \kappa_9$
and one constraint $F_{,X} = \kappa_{1,\pi} - \kappa_3 - 2X\kappa_{3,X}$

⇨ First it is clear that the flat space-time restriction of Horndeski theory must be included in our generalized flat-space time Galileon

⇨ Conversely, our covariantized generalized Galileons must be included into Horndeski theory

$$
\begin{aligned}
\mathcal{L}_H \;=\; & -\mathcal{A}_{(3)}^{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3}\left(\kappa_1 R_{\mu_1\mu_2\nu_1\nu_2}\pi_{\mu_3\nu_3} - \frac{4}{3}\kappa_{1,X}\pi_{\mu_1\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}\right) \\
& -\mathcal{A}_{(3)}^{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3}\left(\kappa_3 R_{\mu_1\mu_2\nu_1\nu_2}\pi_{\mu_3}\pi_{\nu_3} - 4\kappa_{3,X}\pi_{\mu_1\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3}\pi_{\nu_3}\right) \\
& -\mathcal{A}_{(2)}^{\mu_1\mu_2\nu_1\nu_2}\left(F R_{\mu_1\mu_2\nu_1\nu_2} - 4F_{,X}\pi_{\mu_1\nu_1}\pi_{\mu_2\nu_2}\right) \\
& -2\kappa_8\mathcal{A}_{(2)}^{\mu_1\mu_2\nu_1\nu_2}\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_2\nu_2} \\
& -3\left(2F_{,\pi} + X\kappa_8\right)X + \kappa_9, \qquad\qquad F_{,X} = \kappa_{1,\pi} - \kappa_3 - 2X\kappa_{3,X}
\end{aligned}
$$

In fact, one can show that the two sets of theories (Horndeski and covariantized generalized Galileons) are identical in 4D (even though they start from different hypothesis)

$$
\mathcal{L}_H = \sum_{0}^{3} \mathcal{L}_n^{\text{cov}}\{f_n\}
$$

$$
\begin{aligned}
X f_0(\pi, X) &= -\kappa_9(\pi, X) - \frac{X}{2}\int dX\,\left(2\kappa_8 - 4\kappa_{3,\pi}\right)_{,\pi}, \\
X f_1(\pi, X) &= X\left(4\kappa_{3,\pi} + \kappa_8\right) - \frac{1}{2}\int dX\,\left(2\kappa_8 - 4\kappa_{3,\pi}\right) + 6F_{,\pi} \\
X f_2(\pi, X) &= 4\left(F + X\kappa_3\right)_{,X}, \\
X f_3(\pi, X) &= \frac{4}{3}\kappa_{1,X}.
\end{aligned}
$$

C.D. Xian Gao, Daniele Steer, George Zahariade
arXiv:1103.3260 [hep-th] (PRD)

T.Kobayashi, M.Yamaguchi, J. Yokoyama
arXiv 1105.5723 [hep-th]

• Galileons and generalized Galileons have also been obtained more recently by other constructions

Kaluza-Klein compactifications of Lovelock Gravity

Van Acoleyen, Van Doorsselaere 1102.0847 [gr-qc]

Brane world constructions

De Rahm, Tolley 1003.5917 [hep-th]
Padilla, Saffin, Zhou 1007.5424 [hep-th]
Hinterbichler, Trodden, Wesley 1008.1305 [hep-th]

• Galileons can be supersymmetrized but stability issues

Khoury, Lehners, Ovrut 1103.0003 [hep-th]
Koehn, Lehners, Ovrut 1302.0840 [hep-th]
Farakos, Germani, Kehagias 1306.2961 [hep-th]

- Pure Galileons interactions obey a non renormalization theorem

Luty, Porrati, Rattazzi hep-th/0303116
Hinterbichler, Trodden, Wesley 1008.1305 [hep-th]

- Recently discussed Galileons duality

De Rham, Fasiello, Tolley 1308.2702 [hep-th]

$$\tilde{x}^\mu = x^\mu + \pi^\mu(x)$$

invert

$$x^\mu = \tilde{x}^\mu + \tilde{\pi}^\mu(\tilde{x})$$

Some linear combination of $\mathcal{L}_{(n,0)}\left(\pi(x)\right)$

equivalent

Some linear combination of $\mathcal{L}_{(n,0)}\left(\tilde{\pi}(\tilde{x})\right)$

III. Some phenomenology

III. 1 Vainshtein mechanism and k-mouflage

A (new) way to hide the scalar of a scalar-tensor (S-T) theory

$$S = M_P^2 \int d^4x \sqrt{-g} \left(\frac{R}{2} + \frac{\gamma}{2} m^2 \phi R + m^2 H(\phi) \right) + S_m$$

Standard piece (S-T) in the Jordan frame

Matter is minimally coupled to the metric $g_{\mu\nu}$

Derivative self-interactions

$$H(\phi)_{DGP} = m^2 \Box\phi \, \phi_{;\mu}\phi^{;\mu}$$

$$H(\phi)_{Gal} = m^2 \left(\phi_{;\lambda}\,\phi^{;\lambda}\right) \left[2\left(\Box\phi\right)^2 - 2\left(\phi_{;\mu\nu}\,\phi^{;\mu\nu}\right)\right]$$

$$H(\phi)_{CovGal} = m^2 \left(\phi_{;\lambda}\,\phi^{;\lambda}\right) \left[2\left(\Box\phi\right)^2 - 2\left(\phi_{;\mu\nu}\,\phi^{;\mu\nu}\right) - \frac{1}{2}\left(\phi_{;\mu}\,\phi^{;\mu}\right) R\right]$$

$\vdots$

$$S = M_P^2 \int d^4x \sqrt{-g} \left(\frac{R}{2} + \frac{\gamma}{2} m^2 \phi R + m^2 H(\phi) \right) + S_m$$

Generates O(1) correction to GR by a scalar exchange

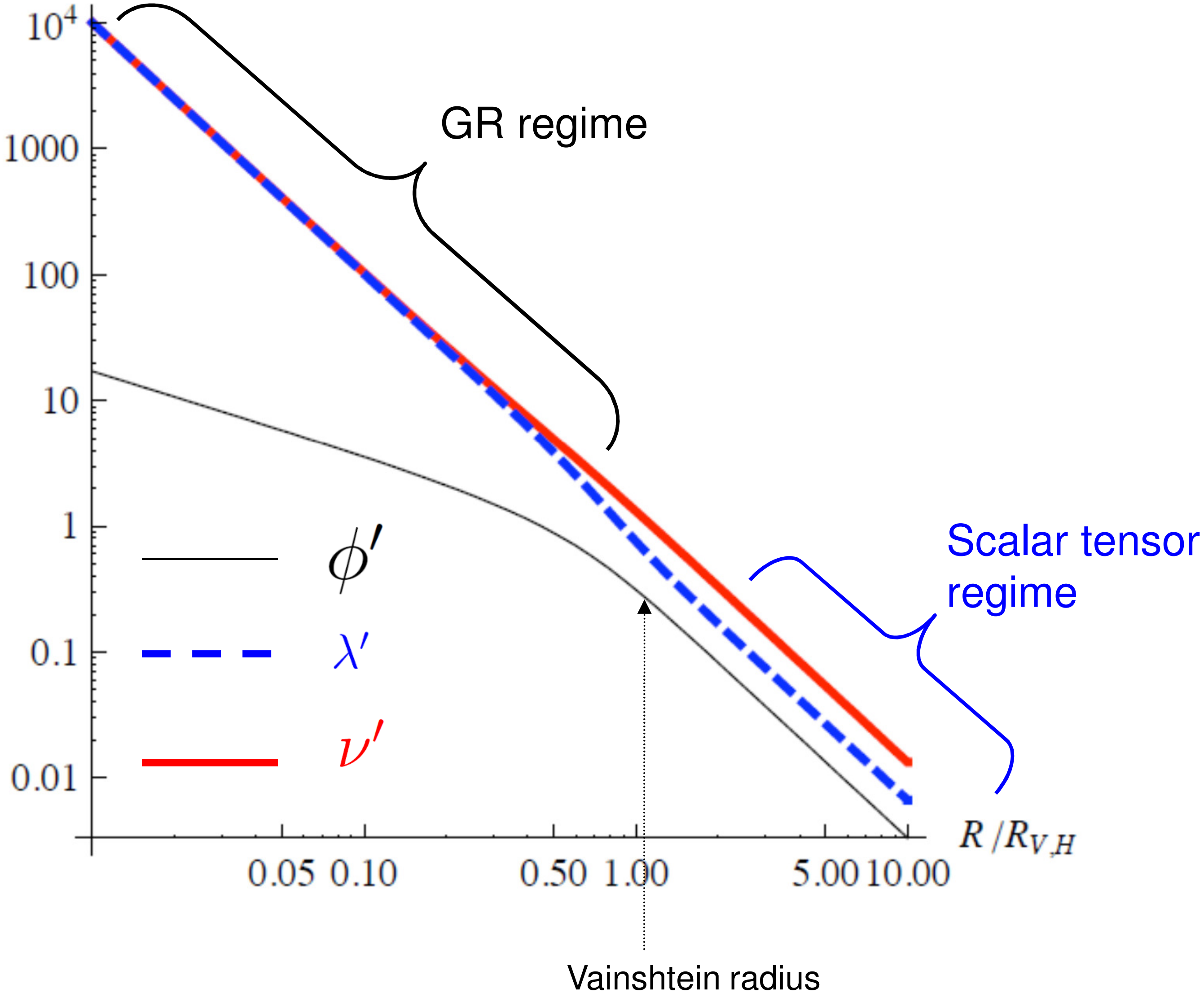
Standard piece (S-T) in the Jordan frame

Matter is minimally coupled to the metric $g_{\mu\nu}$

The derivative self- interactions can screen the effect of the scalar at distances below the « Vainshtein Radius » R_V
(Vainshtein mechanism or « k-Mouflage »)

E.g. for a static and spherically symmetric solution

$$g_{\mu\nu}dx^\mu dx^\nu = -e^{\nu(R)}dt^2 + e^{\lambda(R)}dR^2 + R^2 d\Omega^2$$



Babichev, C.D., Ziour 2009

This can be used to screen other interactions than GR

E.g. : A simple (well maybe not so !) model for MOND

Babichev, C.D., Esposito-Farese 1106.2538 (PRD)

MOND can be obtained by considering a scalar with the non standard kinetic term

$$\mathcal{L}_{\rm MOND} = -\frac{c^2}{3a_0} s\sqrt{|s|} \quad \text{with} \quad s \equiv g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}$$

And matter coupled « disformally » to the metric

$$\tilde{g}_{\mu\nu} \approx e^{2\varphi} g_{\mu\nu} + B(\varphi, s)\varphi_{,\mu}\varphi_{,\nu}$$

(using some appropriately chosen function *B*)

Or $$\tilde{g}_{\mu\nu} \equiv e^{-2\varphi} g_{\mu\nu} - 2\sinh(2\varphi) U_\mu U_\nu$$

(using some time like vector field U)

The recovery of GR at « small distances » (rather large accelerations) requires usually the introduction of a very tuned « interpolating function » (as well as difficulties with the vector field)

The screening of MOND effects at small distances can be rather obtained by a suitable k-Mouflage

$$S = \frac{c^3}{4\pi G}\int d^4x\sqrt{-g}\left(\frac{R}{4} + \mathcal{L}_{\rm standard} + \mathcal{L}_{\rm MOND} + \mathcal{L}_{\rm Galileon}\right) + S_{\rm matter}\left[\psi_{\rm matter};\tilde{g}_{\mu\nu}\right],$$

« standard » MOND piece (TeVeS)

$$\begin{cases} \mathcal{L}_{\rm standard} = -\frac{\epsilon}{2}\, s = -\frac{\epsilon}{2}(\partial_\lambda\varphi)^2, \\ \mathcal{L}_{\rm MOND} = -\frac{c^2}{3a_0} s\sqrt{|s|}, \end{cases}$$

New ingredient :

$$\mathcal{L}_{\rm Galileon} = -\frac{k}{3}\varepsilon^{\alpha\beta\gamma\delta}\varepsilon^{\mu\nu\rho\sigma}\varphi_{,\alpha}\varphi_{,\mu}\varphi_{;\beta\nu}R_{\gamma\delta\rho\sigma}$$

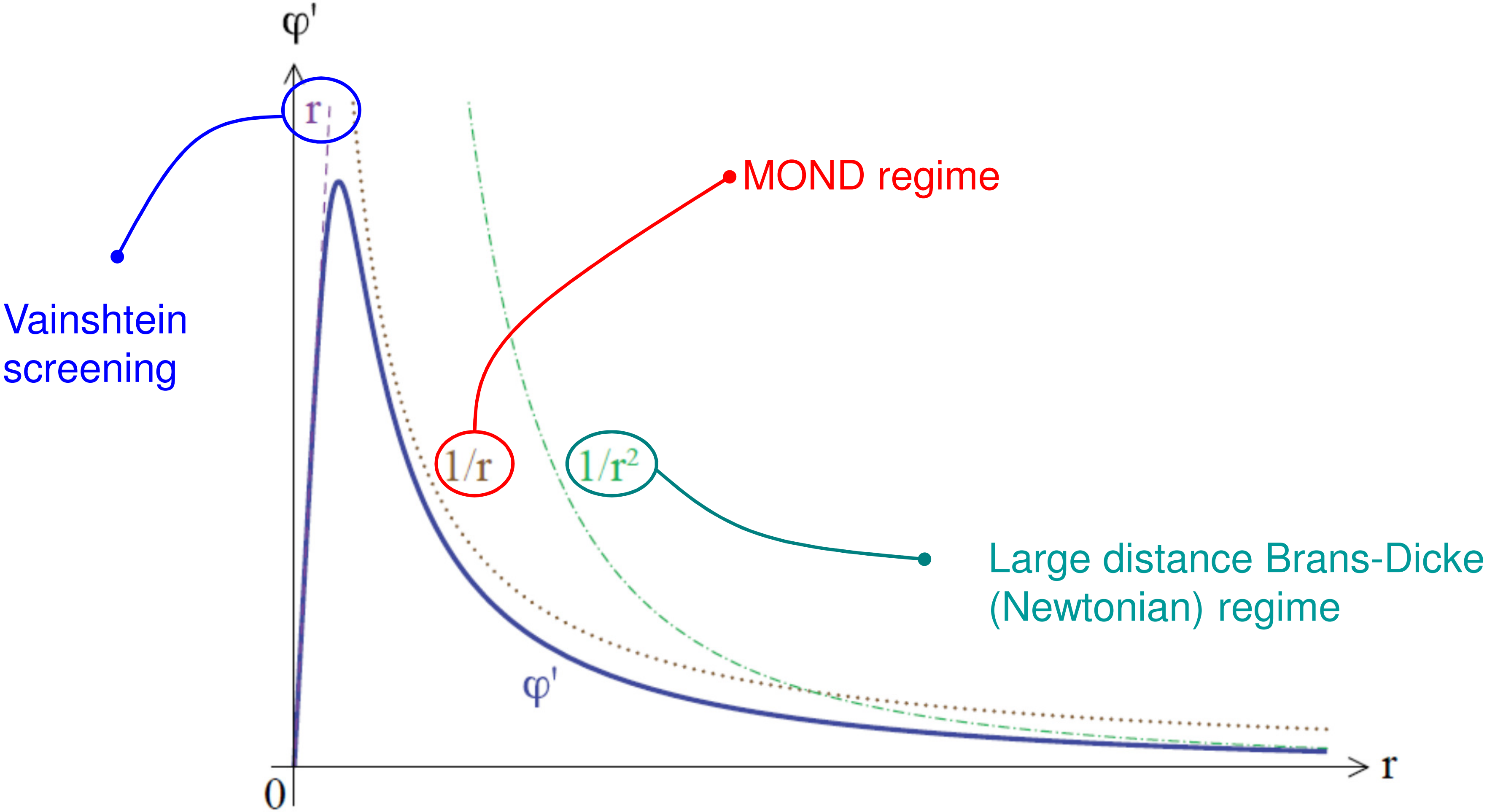
$$\mathcal{L}_{\text{Galileon}} = -\frac{k}{3}\varepsilon^{\alpha\beta\gamma\delta}\varepsilon^{\mu\nu\rho\sigma}\varphi_{,\alpha}\varphi_{,\mu}\varphi_{;\beta\nu}R_{\gamma\delta\rho\sigma}$$

- Covariant version of a « generalized » Galileon
- This simple Lagrangian has second order e.o.m.

Note that other terms also provide (not quite as) efficient screenings , such as the covariant $\mathcal{L}_5$ given by

$$\varepsilon^{\alpha\beta\gamma\delta}\varepsilon^{\mu\nu\rho\sigma}\varphi_{,\alpha}\varphi_{,\mu}\varphi_{;\beta\nu}\left[\varphi_{;\gamma\rho}\varphi_{;\delta\sigma} - \frac{3}{4}(\varphi_{,\lambda})^2 R_{\gamma\delta\rho\sigma}\right]$$

This yields the following profile for φ'



III. 2 Self acceleration, homogeneous cosmology

Consider a Scalar Tensor theory in the Einstein frame, Matter is coupled to the metric $\tilde{g}_{\mu\nu} = \mathcal{A}^2(\varphi) g_{\mu\nu}$ where $g_{\mu\nu}$ has a standard Einstein-Hilbert action.

Expanding $\tilde{g}_{\mu\nu}$ around a flat space time as $\tilde{g}_{\mu\nu} \sim \eta_{\mu\nu} \left[1 + \pi\left(x^\rho\right)\right]$

De Sitter space-time can be defined locally as an expansion around Minkowski of the form

$$d\tilde{s}^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu \sim \Big(1 + \underbrace{H^2 x^\rho x_\rho}_{\pi\left(x^\rho\right)} + \cdots \Big) \eta_{\mu\nu} dx^\mu dx^\nu$$

Quadratic form of the coordinates …

… and one of the original motivations for the Galileons

That there is such a solution in vacuum (self- acceleration) will be garanteed if the field equations are of the DGP decoupling limit type

$$\Box\pi - \frac{1}{3\Lambda^3} \underbrace{\left[\left(\Box\pi\right)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right]}_{} = \frac{T}{3M_P}$$

Or, any pure second order operator

Hence, a linear combination of the Galileons

$$
\begin{aligned}
\mathcal{L}_{(2,0)} &= \pi_\mu \pi^\mu \\
\mathcal{L}_{(3,0)} &= \pi^\mu \pi_\mu \Box\pi \\
\mathcal{L}_{(4,0)} &= (\Box\pi)^2 \left(\pi_\mu\, \pi^\mu\right) - 2\left(\Box\pi\right)\left(\pi_\mu\, \pi^{\mu\nu}\, \pi_\nu\right) \\
&\quad - \left(\pi_{\mu\nu}\, \pi^{\mu\nu}\right)\left(\pi_\rho\, \pi^\rho\right) + 2\left(\pi_\mu \pi^{\mu\nu}\, \pi_{\nu\rho}\, \pi^\rho\right) \\
\mathcal{L}_{(5,0)} &= (\Box\pi)^3 \left(\pi_\mu\, \pi^\mu\right) - 3\left(\Box\pi\right)^2 \left(\pi_\mu\, \pi^{\mu\nu}\, \pi_\nu\right) - 3\left(\Box\pi\right)\left(\pi_{\mu\nu}\, \pi^{\mu\nu}\right)\left(\pi_\rho\, \pi^\rho\right) \\
&\quad +6\left(\Box\pi\right)\left(\pi_\mu \pi^{\mu\nu}\, \pi_{\nu\rho}\, \pi^\rho\right) + 2\left(\pi_\mu{}^\nu\, \pi_\nu{}^\rho\, \pi_\rho{}^\mu\right)\left(\pi_\lambda\, \pi^\lambda\right) \\
&\quad +3\left(\pi_{\mu\nu}\, \pi^{\mu\nu}\right)\left(\pi_\rho\, \pi^{\rho\lambda}\, \pi_\lambda\right) - 6\left(\pi_\mu\, \pi^{\mu\nu}\, \pi_{\nu\rho}\, \pi^{\rho\lambda}\, \pi_\lambda\right)
\end{aligned}
$$

should yield branches of self-accelerating solutions

Nicolis, Rattazzi, Trincherini

Many works have studied application to late and early cosmology, where the Galileon drives the cosmological expansion

Chow, Khoury 0905.1325; de Rahm, Heisenberg 1106.3312, de Rahm, Tolley; 1003.5917; Creminelli, Nicolis, Trincherini, 1007.0027; Padilla, Saffin, Zhou 1007.5424; C.D., Pujolas, Sawicki, Vikman, 1008.0048; Hinterbichler, Trodden, Wesley; 1008.1305; Mizuno, Koyama, 1009.0677; Kobayashi, Yamaguchi, Yokoyama, 1105.5723; Charmousis, Copeland, Padilla, Saffin, 1106.2000; Perreault Levasseur, Brandenberger, David, 1105.5649; Renaux-Petel, Mizuno, Koyama, 1108.0305; Gao, Steer, 1107.2642;…

III. 3 Vainshtein mechanism and cosmology (see also E. Babichev talk)

Babichev, C.D., Esposito-Farese 1107.1569 [gr-qc]

Consider a Scalar Tensor theory with derivative self interaction in the scalar sector (here in the Einstein frame)

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left(R + \mathcal{L}_s + \mathcal{L}_{NL} \right) + S_m \left[\tilde{g}_{\mu\nu}, \psi_m \right]$$

where $\tilde{g}_{\mu\nu} = \mathcal{A}^2(\varphi) g_{\mu\nu}$

Standard kinetic term for the scalar φ ($\mathcal{L}_s$)

Derivative self-interactions ($\mathcal{L}_{NL}$)

If one considers shift-symmetric theories, the fields equations take the form

$$\nabla_\mu \left(\nabla^\mu \varphi + J^\mu_{NL} \right) = -\alpha(\varphi) M_P^{-2} T^{(m)}$$

where $\alpha(\varphi) \equiv d \ln(\mathcal{A}) / d\varphi$

α measures the coupling of the scalar to matter, ($\alpha \sim 1$ to get deviations from GR of order 1)

Consider the cosmological evolution of φ

Obtained by solving $\underbrace{\ddot{\varphi}_{\rm cosm} + 3H\dot{\varphi}_{\rm cosm}} - \nabla_0\left(J^0_{\rm NL}\right) = \alpha(\varphi) M_{\rm P}^{-2} T^{(\rm m)}$

In most of the cases (for cosmology), this dominates over the NL piece for cosmology

Yields
$|\dot{\varphi}_{\rm cosm}| \sim \alpha H$ when the scalar field is subdominant (i.e. when $\rho_\varphi \ll \rho_{\rm m}$)

$|\dot{\varphi}_{\rm cosm}| \sim H_0$ when the scalar field dominates (say today) (i.e. when $\rho_\varphi \gg \rho_{\rm m}$)

Appoximatively, one has then today $\varphi_{cosm}(t) \sim \varphi_{cosm}(t_0) + \dot{\varphi}_{cosm}(t_0) \times t + \cdots$

with $\begin{cases} \dot{\varphi}_{cosm}(t_0) \sim \alpha H_0 \\ \dot{\varphi}_{cosm}(t_0) \sim H_0 \end{cases}$

This provides the boundary condition (at spatial infinity) for the solution corresponding to localized sources

For a localized source, one can consider the following ansatz

$$\varphi(t,r) = \varphi(r) + \underbrace{\dot{\varphi}_{\rm cosm}(t_0)t + \varphi_{\rm cosm}(t_0)}$$

Passes through the field equations if one assumes the theory to be shift symmetric

Inserting this in the field equations

$$\nabla_\mu \left(\nabla^\mu \varphi + J^\mu_{\rm NL}\right) = -\alpha(\varphi) M_{\rm P}^{-2} T^{(\rm m)}$$

This yields an ODE for φ(r) where the only remnant of cosmology (possibly) appears in the form of a constant

Whatever the solution for φ(r)
(which features the Vainshtein recovery mechanism),

the time derivatives of the solution is given by $\dot{\varphi}_{cosm}(t_0)$

But, the time derivative of φ also enters into the time derivative of the Newton constant

One finds $$\left|\dot{G}/G\right| \approx 2\alpha\dot{\varphi}_{\rm cosm}(t)$$

with $$\begin{cases} \dot{\varphi}_{cosm}(t_0) \sim \alpha H_0 \\ \dot{\varphi}_{cosm}(t_0) \sim H_0 \end{cases}$$

However, the most stringent bound on $\left|\dot{G}/G\right|$ is

$$|\dot{G}/G| \, < \, 1.3 \times 10^{-12}\,{\rm yr}^{-1} \quad \Leftrightarrow \quad |\dot{G}/G| \, < \, 0.02 H_0$$

Incompatible with a gravitationnally coupled φ

Ways out ?

- It may be that the ansatz

$$\varphi(t,r) = \varphi(r) + \dot{\varphi}_{\rm cosm}(t_0)t + \varphi_{\rm cosm}(t_0)$$

Leads to a solution φ(*r*) in the field equations (for a static source) that is singular or unstable ?

And that the real solution appropriate for cosmology is more complicated and features some kind of Vainshtein screening of the time evolution of G …

• Freeze the cosmological evolution of φ today (i.e. by giving it a mass)… but then not interesting for dark energy.

• Do something like DGP: large distance behavious is not captured by a 4D theory

Conclusions

Galileons, father and sons

↪ Lead to a (re)discovery of a whole family of scalar-tensor theories with various interesting theoretical and phenomenological aspects:

- Uniqueness theorems
- Non renormalization theorems
- Vainshtein mechanism and k-mouflaging
- Self-acceleration and self-tuning
- Application to early cosmology
  (e.g. « Galilean genesis » thanks to stable NEC violation)
- Links with massive gravity, classicalization

⋮

↪ Several aspects still needed to be explored / understood / cured ?

(phenomenology, $\dot{G}$,UV completion, superluminal propagation, duality …)