Lovelock brane cosmology

Miguel Cruz
in colaboration with Rubén Cordero, Alberto Molgado and Efraín Rojas
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Outline

We consider the following action

$$S[X] = \underbrace{\int d^4 \ \sqrt{-g}\left(-\Lambda + \ K + \frac{}{2}\mathcal{R}\right)}_{\text{First brane Lovelock invariants}} : \qquad (1)$$

Important:

- This action is invariant under reparameterizations of the worldvolume.
- We consider the Minkowski spacetime as background.
- We have the codimension-1 case.

The induced geometry on the brane

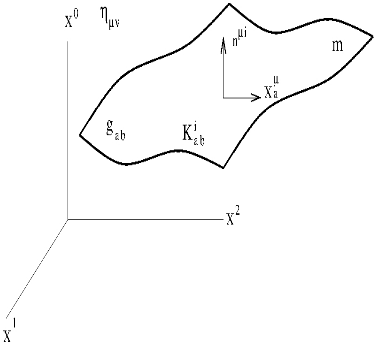
Induced metric on m (First fundamental form):

$$g_{ab} = \ {}_{\mu\nu} X^{\mu}_{a} X^{\nu}_{b}; \qquad (2)$$

and the extrinsic curvature (Second fundamental form)

$$K^{i}_{ab} = -n^{i}_{\mu} \nabla_{a} X^{\mu}_{b} = -n^{i}_{\mu} \nabla_{a} \nabla_{b} X^{\mu}: \qquad (3)$$

R. Capovilla y J. Guven, gr-qc/9411060



By a variational procedure we obtain

$$G_{ab}K^{ab} - \quad \mathcal{R} + \Lambda K = 0: \tag{4}$$

We have:

- G_{ab} is the brane Einstein tensor.
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In this configuration, $S = 6\ \ ^2 \int d\ \ L$, where the Lagrangian can be written as

$$L = \frac{a\dot{t}}{N^3}(a\ddot{a}\dot{t} - a\dot{a}\ddot{t} + N^2\dot{t}) - Na^3\bar{\Lambda}^2 + \frac{a^3\,\bar{}}{N^2}(\dot{t}\ddot{a} - \dot{a}\ddot{t}) + 3a^2\,\bar{}\,\dot{t}: \quad (5)$$

We expect a conserved quantity. Certainly, L can be split as $L = L_b + L_d$ as follows:

$$L_b = \frac{d}{d}\left[\frac{a^2\dot{a}}{N} + a^3\,\bar{}\,\mathrm{arctanh}\left(\frac{\dot{a}}{\dot{t}}\right)\right]; \quad (6)$$

and

$$L_d = -\frac{a\dot{a}^2}{N} + aN(1 - a^2\bar{\Lambda}^2) + 3a^2\,\bar{}\left[\dot{t} - \dot{a}\,\mathrm{arctanh}\left(\frac{\dot{a}}{\dot{t}}\right)\right]: \quad (7)$$

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- Following the Ostrogradski construction, we identify that the phase space by: $\left\{ t; p_t; a; p_a; \dot{t}; P_t; \dot{a}; P_a \right\}$.

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The system has First and Second-class constraints:

$$\mathcal{F}_1 = P \cdot \dot{X} \approx 0; \quad (8)$$

$$\mathcal{F}_2 = \left(\frac{N\Theta}{a^2\Phi} \right) '_2 + H_0 \approx 0; \quad (9)$$

$$\mathcal{S}_1 = N(P \cdot n) - \frac{a^2}{N}(\dot{t} + {}^{-}Na) = '_2 \approx 0; \quad (10)$$

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N &:= \sqrt{\dot{t}^2 - \dot{a}^2}; \\
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Then, we can write

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\mathcal{F}_1 &= N\Pi_N, \\
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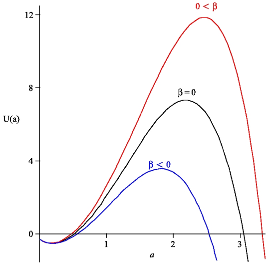
Wheeler-DeWitt equation

- We assume then that Ψ is represented in the usual manner as $\Psi(a;t) := \quad (a)e^{-i\Omega t}$.
- $\left[-\frac{\partial^2}{\partial a^2} + U(a)\right] \quad (a) = 0.$
- Where:



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Turning points for the potential:

$$a_l \simeq \Omega, \quad (19)$$

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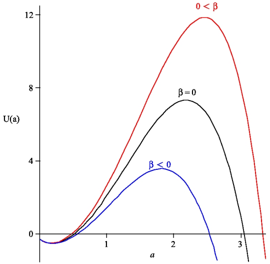

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$$\mathcal{P} \sim \exp\left(-2\int_{a_l}^{a_r} |\sqrt{U(a)}|da\right); \tag{21}$$

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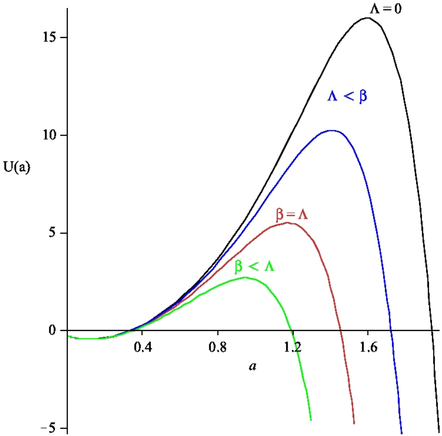
The Wheeler-DeWitt potential



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