Phenomenology of dRGT Massive Bi/Gravity

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Massive Gravity Vs Massive Bigravity

Massive deformation of General Relativity

$$\sqrt{g}\,R + V \xrightarrow{g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}} (\partial h)^2 + h_{\mu\nu}h^{\mu\nu} - h^2 + \ldots$$

- Non linear MG **needs** an extra rank-2 tensor $f_{\mu\nu}$
  (no non-trivial self coupling with only $g_{\mu\nu}$, i.e. $g_{\mu\alpha}g^{\alpha\nu} = \delta^\nu_\mu$)
- Non-derivative coupling $X^\mu_\nu = g^{\mu\alpha} f_{\alpha\nu}$ gives scalars $\tau_n = \mathrm{Tr}(X^n)$
  (e.g. for $f_{\mu\nu} = \eta_{\mu\nu}, \quad \tau_1 = 4 - h + h^{\mu\nu}h_{\mu\nu} + \ldots$)

$f_{\mu\nu}$ can be fixed or dynamical

Massive Gravity

Stuckelberg approach restores diff invariance

$$f_{\mu\nu} = \frac{\partial\phi^A}{\partial x^\mu}\frac{\partial\phi^B}{\partial x^\nu}\eta_{AB}$$

Unitary gauge: $\dfrac{\partial\phi^A}{\partial x^\mu} = \delta^A_\mu \Rightarrow f_{\mu\nu} = \eta_{\mu\nu}$

Massive Bigravity

$$M_P^2\left(\sqrt{g}\,R + V + \kappa\sqrt{f}\,\tilde{R}\right) + \sqrt{g}\,T$$

- $f_{\mu\nu}$ dynamically determined by the eom
- two gravitons, one massive and one massless, propagate
- $\kappa \to \infty$ freezes the dynamic of $f_{\mu\nu}$

de Rham - Gabadadze - Tolley model (2010)

$$X^{\mu}_{\nu} = g^{\mu\alpha} f_{\alpha\nu} \qquad Y^{\mu}_{\nu} = \left(\sqrt{X}\right)^{\mu}_{\nu} \qquad \tau_n = \mathrm{Tr}(Y^n)$$

$$V = \sum_{n=0}^{4} a_n \, V_n$$

$$V_0 = 1 \,, \qquad V_1 = \tau_1 \,, \qquad V_2 = \tau_1^2 - \tau_2 \,, \qquad V_3 = \tau_1^3 - 3\,\tau_1\,\tau_2 + 2\,\tau_3 \,,$$

$$V_4 = \tau_1^4 - 6\,\tau_1^2\,\tau_2 + 8\,\tau_1\,\tau_3 + 3\,\tau_2^2 - 6\,\tau_4 \,.$$

- 5 DoF propagate to all orders (no BD instability)
- Valid also for general $f_{\mu\nu}$ and for Bigravity (7 DoF)

Hassan – Rosen (2011)

Theoretically consistent $\longrightarrow$ study the phenomenology

- Issue: vDVZ discontinuity at linear level $\rightarrow$ spherically symmetric solutions $\&$ Vainshtein mechanism
- Cosmology
- Cosmological perturbations

Static Spherically Symmetric Solutions & Vainshtein

No Birkhoff theorem → no unicity of the solution: 2 classes of solution

- Type I: non bi-diagonal metrics → Schwarzschild de Sitter

Comelli, Nesti, Pilo, MC

$$ds^2 = -J(r)\,dt^2 + K(r)\,dr^2 + r^2\,d\Omega^2$$

$$d\tilde{s}^2 = -C(r)\,dt^2 + A(r)\,dr^2 + 2\;D(r)\,dt\,dr + B(r)\,d\Omega^2$$

$$J = 1 - \frac{2\,M_1}{r} + \Lambda_1\,r^2\,, \quad K = \frac{1}{J}, \quad B = \omega^2 r^2\,, \quad D^2 + A\,C = \omega^4\,,$$

$$C = \omega^2\left(1 - \frac{2\,M_2}{\kappa\,r} + \frac{\Lambda_2\,r^2}{\kappa}\right)\,, \quad A = \frac{2\,\omega^2\,J - C}{J^2}$$

  - $J\,K = 1 \longrightarrow$ no vDVZ discontinuity
  - for asymptotically flat solutions, the graviton mass is zero
  - no Yukawa modifications

Take the limit $\kappa \to \infty$ and change coordinates to bring the second metric in a diagonal Minkowski form

$$f_{\mu\nu} = \eta_{\mu\nu}\,, \qquad ds^2 = -J(r)\,dt^2 + K(r)\,dr^2 + 2\;D(r)\,dt\,dr + B(r)\,d\Omega^2$$

$$B(r) = \omega^2 r^2\,, \qquad J = \omega^2 - \frac{2\,M}{r} + \Lambda\,r^2\,, \qquad J + K = 2\,\omega^2\,, \qquad D^2 + J\,K = \omega^4$$

Koyama, Niz, Tasinato

Static Spherically Symmetric Solutions & Vainshtein

No Birkhoff theorem → no unicity of the solution: 2 classes of solution

- Type II: bi-diagonal metrics (for small density source ρ)

Babichev, MC

$$ds^2 = -e^{\nu} dt^2 + e^{\lambda} dr^2 + r^2 d\Omega^2$$

$$d\tilde{s}^2 = -e^{n} dt^2 + e^{l} \, (r + r\mu)'^2 \, dr^2 + (r + r\mu)^2 d\Omega^2$$

$$\rho \ll 1 \qquad \Rightarrow \qquad \{\lambda,\, \nu,\, l,\, n\} \ll 1, \text{ retain all the non-linearities in } \mu$$

r	$R_\odot < r \ll r_V$	$r_V \ll r \ll 1/m$	$r > 1/m$
μ	$\text{const} + \frac{m^2 r^3}{r_S}$	$-\frac{r_S\,\kappa}{3\,m^2\,r^3\,(1+\kappa)}$	$-\frac{r_S}{r} e^{-m\,r\sqrt{1+1/\kappa}}$
λ	$\frac{r_S}{r} - m^2\,r^2$	$\frac{r_S\,(3+2\,\kappa)}{3\,r\,(1+\kappa)}$	$\frac{r_S}{r}\left(\frac{1}{\kappa} + e^{-m\,r\sqrt{1+1/\kappa}}\right)$
ν	$-\frac{r_S}{r} + m^2\,r^2$	$-\frac{r_S\,(3+4\,\kappa)}{3\,r\,(1+\kappa)}$	$-\frac{r_S}{r}\left(\frac{1}{\kappa} + e^{-m\,r\sqrt{1+1/\kappa}}\right)$
l	$-\frac{m^2\,r^2}{\kappa}$	$\frac{r_S}{3\,r\,(1+\kappa)}$	$\frac{r_S}{\kappa\,r}\left(1 - e^{-m\,r\sqrt{1+1/\kappa}}\right)$
n	$\frac{m^2\,r^2}{\kappa}$	$\frac{r_S}{3\,r\,(1+\kappa)}$	$-\frac{r_S}{\kappa\,r}\left(1 - e^{-m\,r\sqrt{1+1/\kappa}}\right)$

Vainshtein mechanism works well also in bigravity

$\kappa \to \infty$ reproduces solutions found with fixed $f_{\mu\nu} = \eta_{\mu\nu}$

Koyama, Niz, Tasinato
Chkareuli, Pirtskhalava

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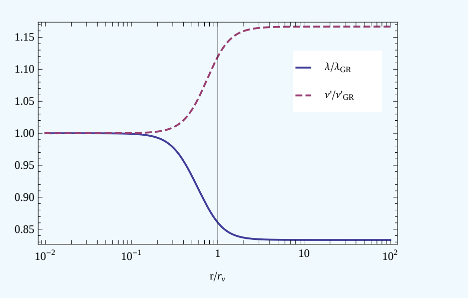
$\rho \ll 1 \qquad \Rightarrow \qquad \{\lambda,\, \nu,\, l,\, n\} \ll 1$, retain all the non-linearities in μ

r	$R_\odot < r \ll r_V$	$r_V \ll r \ll 1/m$
μ	const $+ \dfrac{m^2 r^3}{r_S}$	$-\dfrac{r_S\,\kappa}{3\,m^2\,r^3\,(1+\kappa)}$
λ	$\dfrac{r_S}{r} - m^2\,r^2$	$\dfrac{r_S\,(3+2\,\kappa)}{3\,r\,(1+\kappa)}$
ν	$-\dfrac{r_S}{r} + m^2\,r^2$	$-\dfrac{r_S\,(3+4\,\kappa)}{3\,r\,(1+\kappa)}$
l	$-\dfrac{m^2\,r^2}{\kappa}$	$\dfrac{r_S}{3\,r\,(1+\kappa)}$
n	$\dfrac{m^2\,r^2}{\kappa}$	$\dfrac{r_S}{3\,r\,(1+\kappa)}$



Vainshtein mechanism works well also in bigravity

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Cosmological Solutions & Perturbations

Consider both metrics homogeneous and isotropic

Comelli, Nesti, Pilo, MC
Volkov
von Strauss et al.

$$ds^2 = a^2(t)\left(-dt^2 + \frac{dr^2}{1-k_1 r^2} + r^2\, d\Omega^2\right)$$

$$\tilde{ds}^2 = \omega^2(t)\left[-c^2(t)\, dt^2 + \frac{dr^2}{1-k_2 r^2} + r^2\, d\Omega^2\right]$$

$k_1 \neq k_2 \Rightarrow H = 0$, so take $k_1 = k_2 = k$
Bianchi gives two branches of solution:

- Branch I: $\underbrace{\omega/a = \text{const.}}_{H_\omega/H = \text{const.}^{-1}} \quad \longrightarrow$ FRW + CC

$$H^2 = \frac{8\pi G}{3}\,\rho_m - \frac{k}{a^2} + m^2\Lambda_1 \qquad \frac{H_\omega^2}{c^2} = -\frac{k}{\omega^2} + \frac{m^2\Lambda_2}{\kappa}$$

  - the condition $\omega/a =$ const. at quadratic level around bi-flat solution gives zero graviton mass
  - linear perturbations for scalars and vectors are stuck to GR, and both the tensors propagate.
    $8-(4+1)=3$ DoF (1S+1V) are probably strongly coupled

Comelli, Pilo, MC

$\kappa \to \infty$ forces $k < 0 \quad \Rightarrow$ No spatially flat Universe

D'Amico et al.
Gumrukcuoglu, Lin, Mukohyama
De Felice, Gumrukcuoglu, Mukohyama

Non-linear perturbations are unstable

Possible solutions: anisotropy, quasi-dilaton, mass-varying...

Cosmological Solutions & Perturbations

Consider both metrics homogeneous, isotropic and flat

Comelli, Nesti, Pilo, MC
Volkov
von Strauss et al.

$$ds^2 = a^2(t)\left(-dt^2 + dr^2 + r^2\, d\Omega^2\right)$$
$$\tilde{ds}^2 = \omega^2(t)\left[-c^2(t)\, dt^2 + dr^2 + r^2\, d\Omega^2\right] \qquad \xi \equiv \frac{\omega}{a}$$

Bianchi gives two branches of solution:

- Branch II: $c = \frac{H_\omega}{H}\xi \qquad \longrightarrow \qquad H^2 = \frac{m^2}{\kappa}\rho_{\text{eff}}(\xi) \qquad F(\xi) = \frac{G\,\rho_m}{m^2}$

  Fixed point for $\rho_m \to 0 \quad \Rightarrow \quad \xi =$ const. $\quad c = 1$ late time de Sitter phase

  At early time $\frac{G\,\rho_m}{m^2} \gg 1 \quad \longrightarrow$ viable cosmology **only** for small $\xi \sim \frac{m^2}{G\,\rho_m}$

$$H^2 = \frac{8\,\pi\, G\,\rho_m}{3}\left[1 + f(\xi)\right] \qquad f(\xi) \sim \xi \ll 1 \qquad c = 4 + 3\,w$$

  - all the 7 + 1 DoF propagate at linear level
  - perturbations around the de Sitter phase are fine
  - early time perturbations are problematic in the scalar sector inside the horizon: gradient instability in the second metric scalar

Comelli, Pilo, MC

$\kappa \to \infty \quad \Rightarrow \quad H = 0$

since $\kappa \to \infty$ fixes $c = \frac{\omega\, H_\omega}{\sqrt{-k}}$, this branch does not exist with fixed $f_{\mu\nu}$

Conclusions

- Spherically symmetric solutions are quite similar in Massive Gravity and Bigravity
  - both have a branch where the effective modification reduces to a cosmological constant term, so we have Schwarzschild de Sitter solutions
  - both have a branch where inside the Vainshtein radius we recover GR solutions (so it is ok for Solar System tests) and asymptotically the Yukawa decay
- Cosmological solutions instead are very different in Massive Gravity and Bigravity
  - massive gravity has only one branch of solution of the type FRW + CC that allows just open Universe
    - this solution is unstable (ghost) at non-linear level
    - one has to rely on anisotropy or extra-fields to avoid instabilities
  - massive bigravity has also another branch with viable cosmological background solutions
    - gradient instability in linear perturbations: perturbativity problem
    - cosmological Vainshtein mechanism?
- Both Massive gravity and Bigravity are problematic in the cosmological setup, even if the bigravity formulation seems to work better

Thank you

Cosmological perturbations: Equations

$$GR(\Phi_1) + a^2 m^2 f(\xi) \; F(\Phi_1, \Phi_2) = G\, a^2\, \delta\rho$$

$$a^2\, m^2\, f(\xi) \approx a^2\, m^2\, \xi \approx a^2\, \frac{m^4}{G\,\rho} \overset{m \to 0}{\to} 0$$

GR

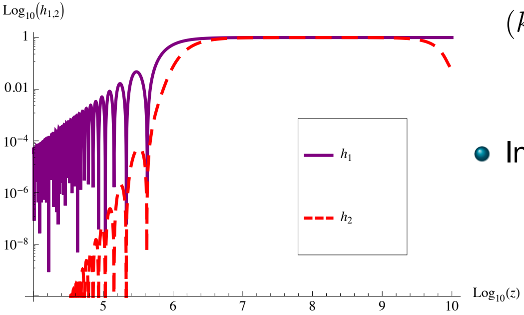
$$GR(\Phi_2) + \frac{m^2 a^2 f(\xi)}{\kappa\, \xi^2} \; F(\Phi_1, \Phi_2) = 0$$

$$\frac{m^2\, a^2\, f(\xi)}{\kappa\, \xi^2} \approx \frac{m^2\, a^2}{\xi} \approx a^2\, G\, \rho \overset{m \to 0}{\to} \text{finite}$$

Nontrivial structure in k

Cosmological perturbations: Tensor Sector

RD era:



- Outside horizon $(k\,t \ll 1)$

$$h_2 = h_1$$

- Inside horizon $(k\,t \gg 1)$

$$h_1 \sim \frac{1}{t}$$

$$h_2 \sim \frac{1}{t^5}$$

Cosmological perturbations: Scalar Sector

RD era:

- Outside horizon $(k\,t \ll 1)$

$$\Phi_1 \sim \text{const.}$$

$$\Phi_2'' + \frac{14}{t}\,\Phi_2' + \frac{37\ \Phi_2 - 39\ \Phi_1}{t^2} = 0 \quad \Rightarrow \quad \Phi_2 \sim \frac{39}{37}\,\Phi_1 = \text{const.}$$

Both scalar perturbations are frozen

- Inside horizon $(k\,t \gg 1)$

$$\Phi_1 \sim \frac{1}{k^2\,t^2}\ \cos(kt)$$

$$\Phi_2'' + \frac{10}{t}\Phi_2' \boxed{-\frac{5\,k^2}{3}}\ \Phi_2 - \frac{36}{k^2\,t^3}\Phi_1' - \frac{15}{t^2}\Phi_1 = 0 \quad \Rightarrow \quad \Phi_2 \sim \frac{1}{(k\,t)^{1/2}}\,e^{+(\frac{5}{3})^{1/2}\,k\,t} + \mathcal{O}(\Phi_1)$$

Gradient instability

dS phase perturbations

- Scalar sector

$$\Phi'' + 2\mathcal{H}\Phi' \left[\frac{2k^4}{9a^2\mathcal{H}^2 m_\Phi^2 + k^4 - 18\mathcal{H}^4} - 1\right] + \frac{1}{3}\Phi\left[\frac{4\left(k^6 - 3k^4\mathcal{H}^2\right)}{9a^2\mathcal{H}^2 m_\Phi^2 + k^4 - 18\mathcal{H}^4} + 3a^2 m_\Phi^2 - k^2 - 6\mathcal{H}^2\right] = 0$$

- Vector sector

$$\mathcal{V}_i'' + \frac{2\,\mathcal{H}\,\mathcal{V}_i'\left(2\,k^2 + a^2\,m_\Phi^2\right)}{k^2 + a^2\,m_\Phi^2} + \mathcal{V}_i\left(k^2 + a^2\,m_\Phi^2\right) = 0$$

- Tensor sector

$$h_{+\,ij}^{TT\;\prime\prime} + 2\,\mathcal{H}\,h_{+\,ij}^{TT\;\prime} + k^2\,h_{+\,ij}^{TT} = 0$$

$$h_{-\,ij}^{TT\;\prime\prime} + 2\,\mathcal{H}\,h_{-\,ij}^{TT\;\prime} + \left(k^2 + a^2\,m_\Phi^2\right)\,h_{-\,ij}^{TT} + a^2\,m_\Phi^2\,\frac{\left(\xi^2\kappa - 1\right)}{\left(\xi^2\kappa + 1\right)}h_{+\,ij}^{TT} = 0$$

Perturbations in RD era at leading order

- Scalar sector

$$\Phi_1'' + \frac{4}{t}\,\Phi_1' + \frac{k^2}{3}\,\Phi_1 = 0$$

$$\Phi_2'' + \frac{10x^2+42}{t(x^2+3)}\Phi_2' + \frac{-5x^6-15x^4+333x^2+999}{3t^2(x^2+3)^2}\Phi_2 - \frac{36}{t(x^2+3)}\Phi_1' - \frac{3(5x^2+39)}{t^2(x^2+3)}\Phi_1 = 0$$

$x = k\,t$

- Vector sector

$$\mathcal{V}'' + \frac{8\,k^2\,t^2+50}{t(k^2\,t^2+5)}\mathcal{V}' + \frac{3}{t^2}(k^2\,t^2+5)\,\mathcal{V} - \frac{48\,k^2\,t^2+320}{k^2\,t^3\,(k^2\,t^2+5)}\delta v = 0$$

- Tensor sector

$$h_1'' + \frac{2}{t}\,h_1' + k^2\,h_1 = 0$$

$$h_2'' + \frac{10}{t}h_2' + 25k^2h_2 + \frac{15}{t^2}\,(h_1 - h_2) = 0$$