Introduction to Massive Gravity II

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Sept. 25th 2013

7th Aegean Summer School, Paros, “Beyond Einstein's Theory of Gravity”

Summary (so far)

- 5d GR is a good starting point to obtain 4d theories of modified gravity (or gravity with a scalar field) which are free of ghost
- Has interesting Phenomenology & Cosmology

- Idea of the proof of the Absence of Ghost → yesterday
- Decoupling limit
- Open questions and open avenues

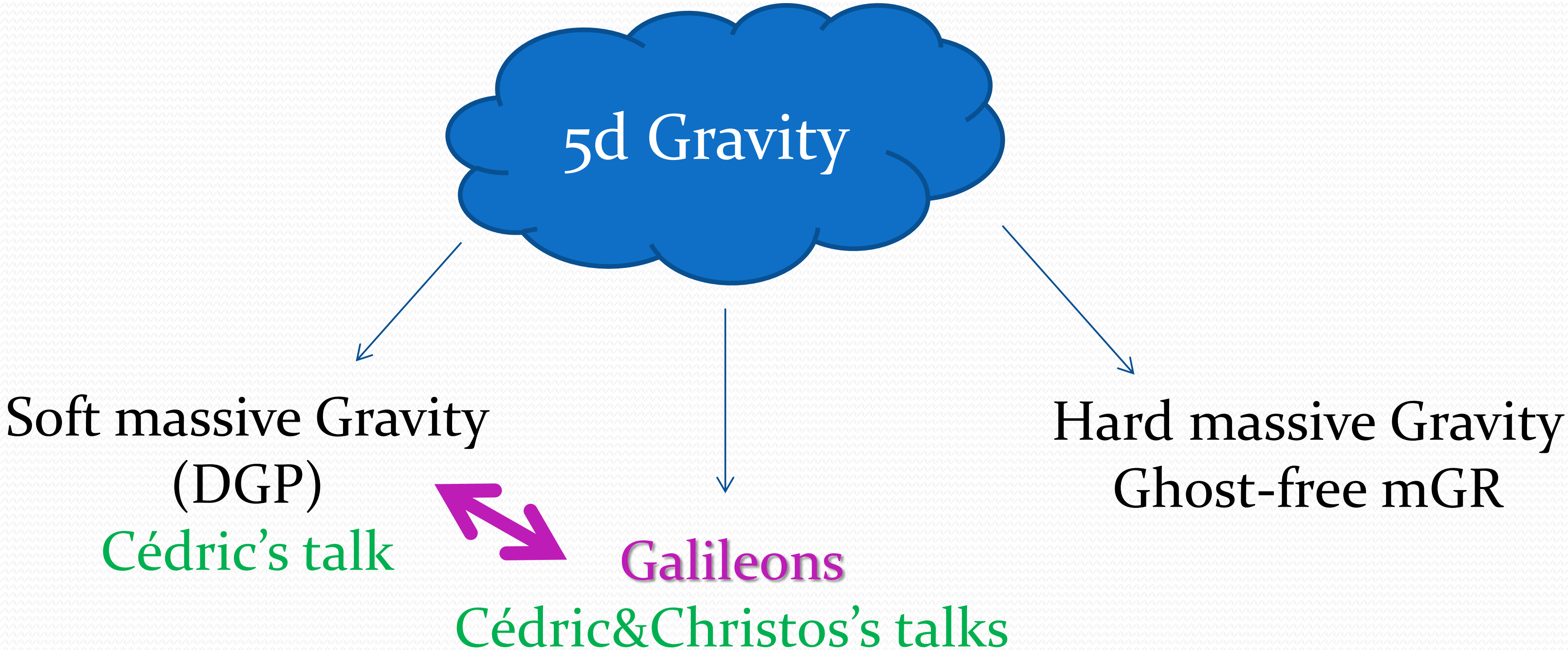
} → today

Quantum corrections
Superluminalities
Strong Coupling

↔ Galileon Duality

MG from Extra Dimensions

- Best way to obtain a sensible theory of Modified Gravity is to start with General Relativity



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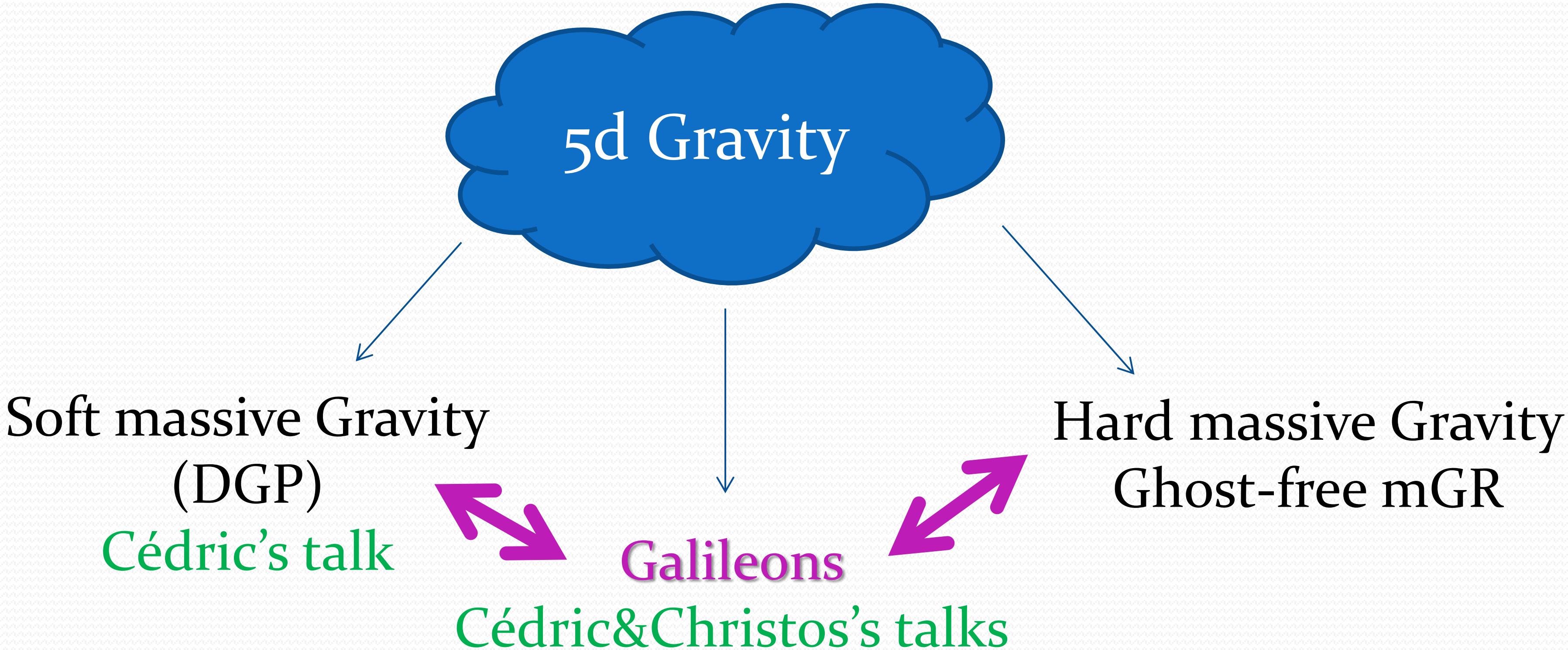

Fierz-Pauli Massive Gravity

$$\mathcal{U}_{\mathrm{FP}} = h_{\mu\nu}^2 - h^2$$

- Mass term for the fluctuations around flat space-time

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Fierz & Pauli, Proc.Roy.Soc.Lond.A **173**, 211 (1939)

Fierz-Pauli Massive Gravity

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- Transform under a change of coordinates,

$$x^\mu \;\rightarrow\; \tilde{x}^\mu(x)$$

$$g_{\mu\nu} \;\rightarrow\; \tilde{g}_{\mu\nu} = g_{\alpha\beta}\frac{\partial\tilde{x}^\alpha}{\partial x^\mu}\frac{\partial\tilde{x}^\beta}{\partial x^\nu}$$

Fierz-Pauli Massive Gravity

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- Transform under a change of coordinates,

$$x^\mu \;\rightarrow\; \tilde{x}^\mu(x) = x^\mu + \partial^\mu \xi$$

$$g_{\mu\nu} \;\rightarrow\; \tilde{g}_{\mu\nu} = g_{\alpha\beta} \left(\delta^\alpha_\mu + \partial^\alpha \partial_\mu \xi\right) \left(\delta^\beta_\nu + \partial^\beta \partial_\nu \xi\right)$$

$$g_{\mu\nu} - \eta_{\mu\nu} = h_{\mu\nu} \;\rightarrow\; h_{\mu\nu} + 2\partial_\mu \partial_\nu \xi + \partial_\mu \partial^\alpha \xi \partial_\nu \partial^\alpha \xi$$

Fierz-Pauli Massive Gravity

$$\mathcal{U}_{\rm FP} = H_{\mu\nu}^2 - H^2$$ invariant

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$$g_{\mu\nu} - \eta_{\mu\nu} = h_{\mu\nu} \;\to\; h_{\mu\nu} + 2\partial_\mu\partial_\nu\xi + \partial_\mu\partial^\alpha\xi\partial_\nu\partial^\alpha\xi$$

$$\pi \;\to\; \pi - \xi$$

$$H_{\mu\nu} = h_{\mu\nu} + 2\partial_\mu\partial_\nu\pi - \partial_\mu\partial_\alpha\pi\partial_\nu\partial^\alpha\pi$$

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$$H_{\mu\nu} = h_{\mu\nu} + 2\partial_\mu\partial_\nu\pi - \partial_\mu\partial_\alpha\pi\partial_\nu\partial^\alpha\pi$$

- The potential has higher derivatives…

$$\mathcal{U}_{\rm FP} = \underbrace{(\partial_\mu\partial_\nu\pi)^2 - (\Box\pi)^2}_{\text{Total derivative}}$$

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$$\mathcal{U}_{\rm FP} = \underbrace{(\partial_\mu\partial_\nu\pi)^2 - (\Box\pi)^2}_{\text{Total derivative}} + (\partial^2\pi)^3 + \cdots$$

Ghost reappears at the non-linear level

Deffayet & Rombouts, gr-qc/0505134

Ghost-free Massive Gravity

$$\mathcal{U}_{\rm FP} = H_{\mu\nu}^2 - H^2$$

Ghost-free Massive Gravity

$$\mathcal{U}_{\mathrm{GF}} = \mathcal{K}_{\mu\nu}^2 - \mathcal{K}^2$$

Ghost-free Massive Gravity

$$\mathcal{U}_{\rm GF} = \mathcal{K}^2_{\mu\nu} - \mathcal{K}^2$$

- Such that when neglecting $h_{\mu\nu}$, then $\mathcal{K}_{\mu\nu}|_{\rm dec} = \partial_\mu \partial_\nu \pi$

$$H_{\mu\nu} = h_{\mu\nu} + 2\partial_\mu\partial_\nu\pi - \partial_\mu\partial_\alpha\pi\partial_\nu\partial^\alpha\pi$$

CdR, Gabadadze, 1007.0443
CdR, Gabadadze, Tolley, 1011.1232

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$2\partial_\mu\partial_\nu\pi \rightarrow \mathcal{K}_{\mu\nu}|_{\rm dec}$

$\partial_\mu\partial_\alpha\pi\partial_\nu\partial^\alpha\pi \rightarrow \mathcal{K}^2_{\mu\nu}|_{\rm dec}$

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$\mathcal{K}_{\mu\nu}|_{\rm dec}$ $\qquad$ $\mathcal{K}^2_{\mu\nu}|_{\rm dec}$

$$\begin{aligned}\mathcal{K}^\mu_\nu[H] &= \delta^\mu_\nu - \sqrt{\delta^\mu_\nu - H^\mu_\nu} \\ &= \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\eta_{\alpha\nu}}\end{aligned}$$

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Ghost-free Massive Gravity

$$\mathcal{U}_{\rm GF} = \mathcal{K}^2_{\mu\nu} - \mathcal{K}^2$$

- With $\mathcal{K}^\mu_\nu[H] \;=\; \delta^\mu_\nu - \sqrt{\delta^\mu_\nu - H^\mu_\nu}$

$$= \; \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\eta_{\alpha\nu}}$$

$\mathcal{K}_{\mu\nu}|_{\rm dec} = \partial_\mu \partial_\nu \pi$

- Has no ghosts at zero$^{\rm th}$ order in h

$$\mathcal{U}_{\rm GF} = \; [\Pi^2] - [\Pi]^2 + \frac{h_{\mu\nu}}{M_{\rm Pl}}(\partial\partial\pi) + \cdots$$

$\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi$

CdR, Gabadadze, 1007.0443
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Generalized Ghost-free MG

$$\mathcal{U}_{\rm GF} = \varepsilon^{\alpha\beta\gamma\delta}_{abcd}\left(\mathcal{K}^a_\alpha \mathcal{K}^b_\beta \delta^c_\gamma \delta^d_\delta + \alpha_3 \mathcal{K}^a_\alpha \mathcal{K}^b_\beta \mathcal{K}^c_\gamma \delta^d_\delta + \alpha_4 \mathcal{K}^a_\alpha \mathcal{K}^b_\beta \mathcal{K}^c_\gamma \mathcal{K}^d_\delta\right)$$

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- At zero[th] order in h $\qquad \mathcal{K}_{\mu\nu}|_{\rm dec} = \partial_\mu \partial_\nu \pi$

$$\begin{aligned}\mathcal{U}_{\rm GF} &= \varepsilon^{\alpha\beta\gamma\delta}_{abcd}\left(\Pi^a_\alpha \Pi^b_\beta \delta^c_\gamma \delta^d_\delta + \alpha_3 \Pi^a_\alpha \Pi^b_\beta \Pi^c_\gamma \delta^d_\delta + \alpha_4 \Pi^a_\alpha \Pi^b_\beta \Pi^c_\gamma \Pi^d_\delta\right) \\ &+ \varepsilon \frac{h_{\mu\nu}}{M_{\rm Pl}}(\partial\partial\pi + \cdots)\end{aligned}$$

$$\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi$$

CdR, Gabadadze, 1007.0443
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Ghost-free decoupling limit

- In the *decoupling limit,* $M_{\rm Pl} \to \infty$
  $m \to 0$

  keeping $\Lambda = \left(M_{\rm Pl} m^2\right)^{1/3}$ fixed

CdR, Gabadadze, 1007.0443

Ghost-free decoupling limit

- In the *decoupling limit,*

$$\mathcal{L} = -\frac{1}{2}h^{\mu\nu}(\hat{\mathcal{E}}h)_{\mu\nu} - h^{\mu\nu}\left(\partial^2\pi + \frac{(\partial^2\pi)^2}{\Lambda^3} + \cdots\right)$$

Ghost-free decoupling limit

- In the *decoupling limit,*

$$\mathcal{L} = -\frac{1}{2}\hat{h}^{\mu\nu}(\mathcal{E}\hat{h})_{\mu\nu} - \hat{h}^{\mu\nu}\left(X^{(1)}_{\mu\nu} + \frac{1+3\alpha_3}{\Lambda^3}X^{(2)}_{\mu\nu} + \frac{\alpha_3+4\alpha_4}{\Lambda^6}X^{(3)}_{\mu\nu}\right)$$

with

$$X^{(1)\mu}{}_{\mu'} = \epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu'\nu'\alpha\beta}\Pi^{\nu'}_{\nu}$$

$$X^{(2)\mu}{}_{\mu'} = \epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu'\nu'\alpha'\beta}\Pi^{\nu'}_{\nu}\Pi^{\alpha'}_{\alpha}$$

$$X^{(3)\mu}{}_{\mu'} = \epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu'\nu'\alpha'\beta'}\Pi^{\nu'}_{\nu}\Pi^{\alpha'}_{\alpha}\Pi^{\beta'}_{\beta}$$

* Identically conserved

* Similar structure as Galileon introduced by Cédric

* Galileon symmetry is trivial by construction

* No ghost

Ghost-free decoupling limit

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- *The helicity-2 and -0 modes can be "semi-diagonalized"*

$$\hat{h}_{\mu\nu} \to \bar{h}_{\mu\nu} + \pi\eta_{\mu\nu} + \frac{1+3\alpha_3}{\Lambda^3}\partial_\mu\pi\partial_\nu\pi$$

Ghost-free decoupling limit

- In the *decoupling limit,*

$$\mathcal{L} = -\frac{1}{2}\bar{h}^{\mu\nu}\left(\mathcal{E}\bar{h}\right)_{\mu\nu} - \frac{3}{2}(\partial\pi)^2 + \sum_{n=3}^{5}\frac{c_n}{\Lambda^3}\mathcal{L}_n^{\rm Gal}(\pi)$$

$$-\frac{\alpha_3 + 4\alpha_4}{\Lambda^6}\bar{h}^{\mu\nu}X^{(3)}_{\mu\nu} + \frac{1}{M_{\rm Pl}}\left(\bar{h}_{\mu\nu} + \pi\eta_{\mu\nu} + \frac{1+3\alpha_3}{\Lambda^3}\right)T^{\mu\nu}$$

- *The helicity-2 and -0 modes can be "semi-diagonalized"*

$$\hat{h}_{\mu\nu} \to \bar{h}_{\mu\nu} + \pi\eta_{\mu\nu} + \frac{1+3\alpha_3}{\Lambda^3}\partial_\mu\pi\partial_\nu\pi$$

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$$-\frac{\alpha_3+4\alpha_4}{\Lambda^6}\bar{h}^{\mu\nu}X^{(3)}_{\mu\nu} + \frac{1}{M_{\mathrm{Pl}}}\left(\bar{h}_{\mu\nu} + \pi\eta_{\mu\nu} + \frac{1+3\alpha_3}{\Lambda^3}\right)T^{\mu\nu}$$

- Galileon symmetry only after integrations by parts
  The real symmetry involves both h and π.
- No Vainshtein mechanism when $X^{(3)}_{\mu\nu}$ is present

Beyond the decoupling limit

- Beyond the DL, π is no longer a Galileon (nor covariant Galileon)…
- … *but we shouldn't expect it*
- Generally π is NOT a scalar. It is only a scalar under the Lorentz symmetry in DL
- Beyond the DL, π does not capture the physics of the helicity-0 mode

⟶ Need to work with all 4 Stückelberg fields

$$\mathcal{K} = \mathbb{I} - \sqrt{\mathbb{X}} \qquad \mathbb{X}^\mu_\nu = g^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}$$

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- Beyond the DL, π does not capture the physics of the helicity-0 mode
  - $\longrightarrow$ Need to work with all 4 Stückelberg fields
  - $\longrightarrow$ Or in ADM
  - $\longrightarrow$ Or many different languages…

Decoupling limits

- Decoupling limit of DGP: Galileon (cubic)
- Decoupling limit of Massive Gravity: Galileon (quintic)

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  (see Andrew Tolley's talk)

Decoupling limits

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- Decoupling limit of Massive Gravity: Galileon (quintic)
- Decoupling limit of BiGravity: Galileon
  (see Andrew Tolley's talk)
- Decoupling limit of New Massive Gravity:

  Galileon

New Massive Gravity

$$\mathcal{L}_{\rm 3d,NMG} = \frac{M_3}{2}\int \mathrm{d}^3x\sqrt{-g}\left(-R + \frac{1}{m^2}\left(R_{\mu\nu}^2 - \frac{3}{8}R^2\right)\right)$$

- Higher order Curvature terms
- By Ostrogadsky argument (Cf. yesterday's talk), we know that there will be 2 sets of fields hidden in $R_{\mu\nu}^2$
  - 1 ghost massless spin-2 field 0 polarizations
  - 1 healthy massive spin-2 field 2 polarizations
- Breaks one set of diff invariance, just like Bi-gravity breaks one set of diff invariance

See Paul Townsend's talks

Bi-Gravity/NMG/Massive Gravity

- Bi-Gravity has only 1 copy of diff (rather than 2)
  in 4d: 7 degrees of freedom
  in 3d: 2 degrees of freedom

- NMG has only 1 copy of diff (rather than 2)
  in 3d: 2 degrees of freedom

- Massive Gravity has 0 copy of diff (rather than 1)
  in 4d: 5 degrees of freedom
  in 3d: 2 degrees of freedom

NMG

$$\mathcal{L}_{3\text{d,NMG}} = \frac{M_3}{2} \int \mathrm{d}^3x \, \sqrt{-g} \left[-R - f^{\mu\nu} G_{\mu\nu} - \frac{1}{4} m^2 (f^{\mu\nu} f_{\mu\nu} - f^2) \right]$$

- Restore the 2nd copy of diff invariance with the Stü. fields

$$h_{\mu\nu} = \frac{\bar{h}_{\mu\nu}}{\sqrt{M_3}}, \qquad f_{\mu\nu} = \frac{\bar{f}_{\mu\nu}}{\sqrt{M_3}} + \nabla_\mu V_\nu + \nabla_\nu V_\mu$$

NMG

$$\mathcal{L}^{(\mathrm{dec})}_{\mathrm{3d,NMG}} = -\frac{1}{4}F^2_{\mu\nu} - 2(\partial\pi)^2 - \frac{1}{2}(\partial\pi)^2\Box\pi$$

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- Splitting the Stü. field into scalar and vector parts,

$$V_\mu = \frac{A_\mu}{\sqrt{M_3}m} + \frac{\nabla_\mu \pi}{\sqrt{M_3}m^2}$$

Decoupling limit of NMG

$$\mathcal{L}^{(\mathrm{dec})}_{\mathrm{3d,NMG}} = -\frac{1}{4}F^2_{\mu\nu} - 2(\partial\pi)^2 - \frac{1}{2}(\partial\pi)^2\Box\pi$$

- Cubic Galileon for the helicity-0 mode !
- And this includes all the interactions with the helicity-1 mode !

Open Questions in MG

- Viable Cosmological Solutions & Phenomenology
- Quantum Corrections
  - is the small mass technically natural ?
  - how does the special structure of the potential gets affected by QC ?
- Superluminalities
- Strong Coupling

Tuning of the Mass

- We typically consider the graviton mass to be

$$m \sim 10^{-32}\mathrm{eV} \sim 10^{-60} M_{\mathrm{Pl}}$$

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- We typically consider the graviton mass to be

$$m \sim 10^{-32}\text{eV} \sim 10^{-60} M_{\text{Pl}}$$

same tuning as for the CC problem !

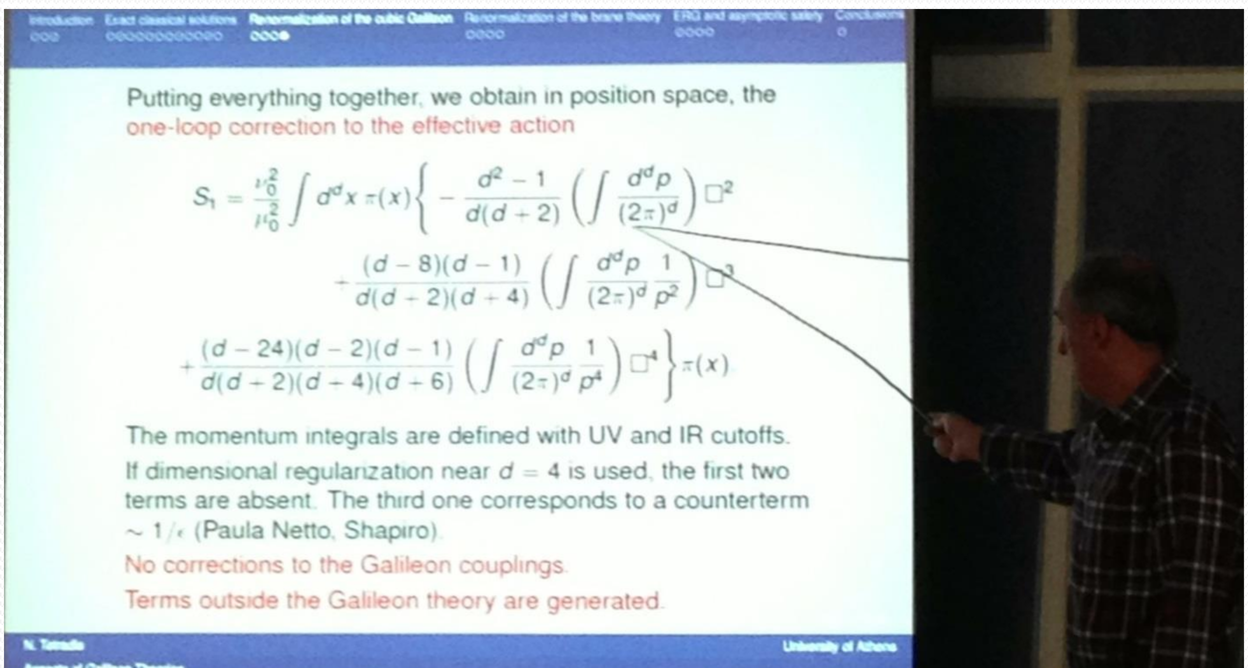
- However the graviton mass is stable against quantum corrections

→ Technically natural tuning, unlike the tuning of the CC to zero

Non-renormalization

- In the DL, we recover a Galileon theory which enjoys a non-renormalization theorem

See Cédric Deffayet's talk
+ Nikolaos Tetradis's talk



Non-renormalization

- In the DL, we recover a Galileon theory which enjoys a non-renormalization theorem

→ graviton mass does not get renormalized in the DL

- Beyond the DL, the graviton mass gets renormalized by

$$\delta m^2 \sim m^2 \left(\frac{m}{M_{\rm Pl}}\right)^{2/3}$$

→ Technically natural tuning, unlike the tuning of the CC to zero

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Buchbinder, Pereira & Shapiro, PLB712 (2012) 104, [arXiv:1201.3145]
Netto & Shapiro

Ghost-free Massive Gravity

- Structure of mass term is essential to avoid BD ghost

$$\mathcal{L}_{\rm mGR} = M_{\rm Pl}^2 \left(R + m^2 \left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)\right)$$

Boulware & Deser, PRD 6, 3368 (1972)
CdR & Gabadadze, PRD 82, 044020 (2010)
CdR, Gabadadze & Tolley, PRL 106, 231101 (2011)

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$$\mathcal{L}_{\rm mGR} = M_{\rm Pl}^2 \left(R + m^2 \left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)\right)$$

- We expect the structure to detune the potential

$$\frac{m^4}{M_{\rm Pl}^2} h^2 \longrightarrow \frac{(\partial^2 \pi)^2}{M_{\rm Pl}^2} \longrightarrow m_{\rm gh}^2 \sim M_{\rm Pl}^2$$

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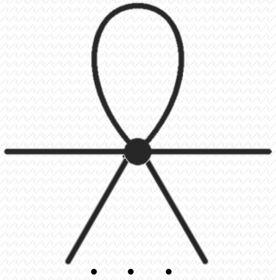

$$\frac{m^4}{M_{\rm Pl}^{n+2}} h^{n+2} \longrightarrow \frac{(\partial^2\pi)^2}{M_{\rm Pl}^2}\left(\frac{\partial^2\pi_0}{\Lambda^3}\right)^n$$

$$m_{\rm gh}^2 \sim M_{\rm Pl}^2 \left(\frac{\partial^2\pi_0}{\Lambda^3}\right)^{-n}$$

CdR, Heisenberg & Ribeiro, 1307.7169

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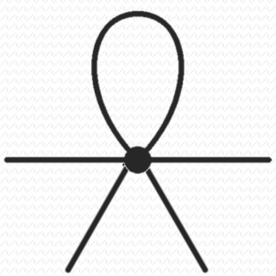


$$\frac{m^4}{M_{\rm Pl}^{n+2}} h^{n+2} \longrightarrow \frac{(\partial^2 \pi)^2}{M_{\rm Pl}^2} \left(\frac{\partial^2 \pi_0}{\Lambda^3}\right)^n$$

$$m_{\rm gh}^2 \sim M_{\rm Pl}^2 \underbrace{\left(\frac{\partial^2 \pi_0}{\Lambda^3}\right)^{-n}}_{\ll 1}$$

CdR, Heisenberg & Ribeiro, 1307.7169

1-loop Effective Action

- The 1-loop effective action is itself redressed

$$\mathcal{L}_{\rm eff} = \frac{1}{M_{\rm Pl}^2} \frac{1 + c_1 \frac{\partial^2 \pi_0}{\Lambda^3}}{1 + c_2 \frac{\partial^2 \pi_0}{\Lambda^3}} \left(\partial^2 \pi\right)^2$$

- The detuning of the potential is never a problem at that level

$$m_{\rm gh}^2 \gtrsim M_{\rm Pl}^2$$

- Even on top of large background configurations

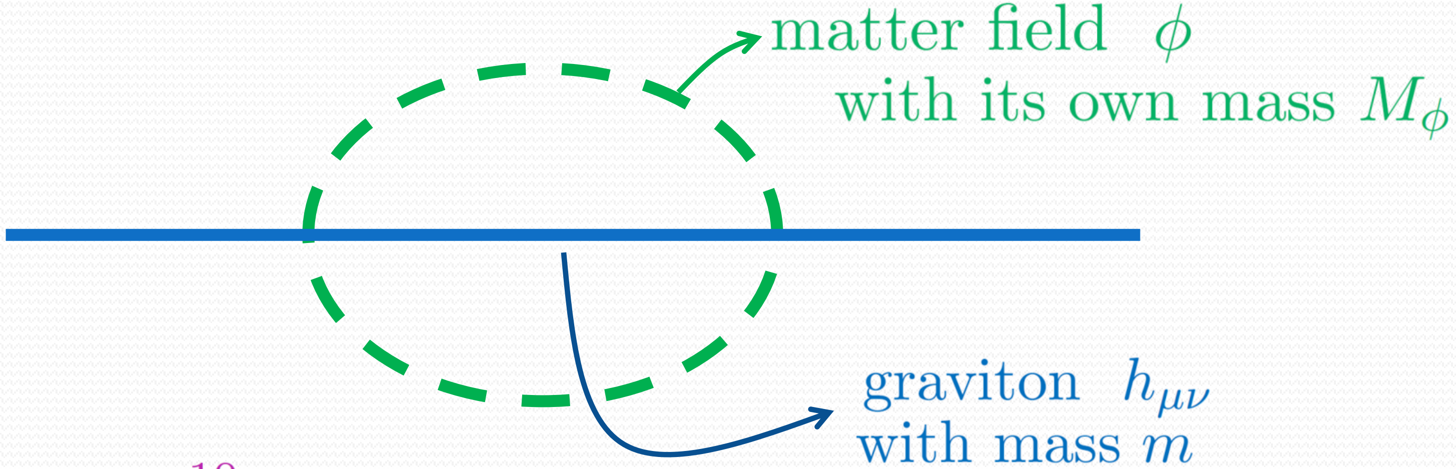
$$\frac{\partial^2 \pi_0}{\Lambda^3} \gg 1$$

CdR, Heisenberg & Ribeiro, 1307.7169

Beyond 1-loop…

- Beyond 1-loop the question is still open…

- At higher order in loops, loops can mix virtual matter fields and graviton fields



Could have a mixing $\frac{M_\phi^{10}}{m^4 M_{\rm Pl}^4}[h^2]$ which could be fatal…

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Galileons in trouble (?)

$$\mathcal{L} = -\frac{1}{2}(\partial\pi)^2 + \frac{1}{\Lambda^3}(\partial\pi)^2\Box\pi$$

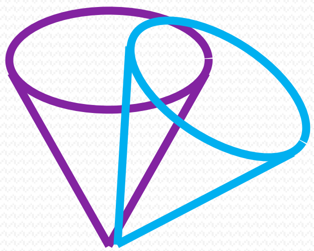
Perturbations about a background

$$\begin{aligned} \pi &= \pi_0 + \pi \\ Z^{\mu\nu} &= \eta^{\mu\nu} + \frac{1}{\Lambda^3}(\partial^\mu\partial^\nu\pi_0 - \Box\pi_0\eta^{\mu\nu}) \end{aligned}$$

$$Z^{\mu\nu} = \eta^{\mu\nu} + \frac{1}{\Lambda^3}(\partial^\mu \partial^\nu \pi_0 - \Box \pi_0 \eta^{\mu\nu})$$

Galileons in trouble (?)

$$\mathcal{L} = -\frac{1}{2} Z^{\mu\nu} \partial_\mu \pi \partial_\nu \pi + \frac{1}{\Lambda^3}(\partial \pi)^2 \Box \pi$$



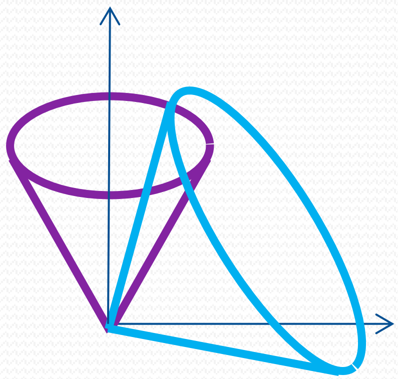
- Change the kinetic structure !!!
- Fluctuations see a different effective metric
- Fluctuations can propagate faster than light !

Adams, Arkani-Hamed, Dubovsky, Nicolis & Rattazzi, JHEP 0610 (2006) 014
Hinterbichler, Nicolis & Porrati, JHEP 0909, 089 (2009)
Burrage, CdR, Heisenberg & Tolley, JCAP 1207, 004 (2012)
Deser & Waldron, PRL 110, 111101 (2013)

$$Z^{\mu\nu} = \eta^{\mu\nu} + \frac{1}{\Lambda^3}(\partial^\mu\partial^\nu\pi_0 - \Box\pi_0\eta^{\mu\nu})$$

Acausality ?

$$\mathcal{L} = -\frac{1}{2}Z^{\mu\nu}\partial_\mu\pi\partial_\nu\pi + \frac{1}{\Lambda^3}(\partial\pi)^2\Box\pi$$

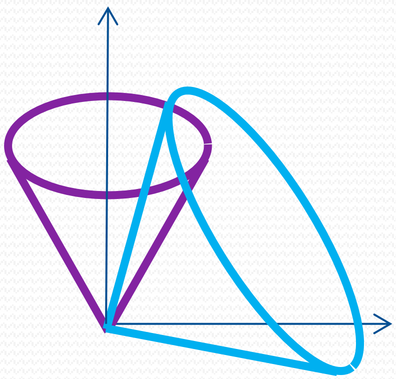


- Can we widen the cone such that so as to allow acausality ?
- Starting from a healthy configuration, this means that we must necessarily go through a region where $Z^{00} = 0$

$$Z^{\mu\nu} = \eta^{\mu\nu} + \frac{1}{\Lambda^3}(\partial^\mu \partial^\nu \pi_0 - \Box \pi_0 \eta^{\mu\nu})$$

Acausality ?

$$\mathcal{L} = -\frac{1}{2} Z^{\mu\nu} \partial_\mu \pi \partial_\nu \pi + \frac{1}{\Lambda^3} (\partial \pi)^2 \Box \pi$$



- Can we widen the cone such that so as to allow acausality ?
- Starting from a healthy configuration, this means that we must necessarily go through a region where $Z^{00} = 0$
- But then one cannot remain within the regime of validity of the EFT…

$$Z^{\mu\nu} = \eta^{\mu\nu} + \frac{1}{\Lambda^3}(\partial^\mu\partial^\nu\pi_0 - \Box\pi_0\eta^{\mu\nu})$$

Galileons in trouble (?)

$$\mathcal{L} = -\frac{1}{2}(\partial\hat{\pi})^2 + \frac{1}{\Lambda^3 Z^{3/2}}(\partial\hat{\pi})^2\Box\hat{\pi}$$

- Strong coupling scale gets redressed:

$$\Lambda_* = \Lambda Z^{1/2} \sim \mathrm{cm}^{-1}$$

At the surface of the Earth (only from the effect of the Earth)

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- After that ? …  who knows…

Galileons in trouble (?)

Strong Coupling Issues & Superluminalities

Ubiquitous to Galileon theories

& to massive gravity (?)

Yet these theories are dual to perfectly fine theories

(eg. Free theory)

Dual Galileon

$$\mathcal{L} = -\frac{1}{2}(\partial\pi)^2 + \frac{g^2}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{1}{M_{\rm Pl}}\pi J_\pi$$

In 3d

Dual Galileon

$$\mathcal{L} = -\frac{1}{2}(\partial\pi)^2 + \frac{g^2}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{1}{M_{\rm Pl}}\pi J_\pi$$

In 3d

$$x \;\to\; \tilde{x} = \partial\pi(x)$$

$$\partial_x\pi \;\to\; \partial_{\tilde{x}}\rho(\tilde{x}) = x(\tilde{x})$$

$$\mathcal{L}_{\rm dual} = -\frac{g^2}{2}(\partial\rho)^2(\tilde{x}) + \frac{1}{\Lambda^3}(\partial\rho)^2\Box\rho(\tilde{x}) + \frac{1}{M_{\rm Pl}}\rho J_\rho$$

Curtright & Fairlie, 1212.6972
CdR, Fasiello & Tolley, 1308.2702
+ in preparation

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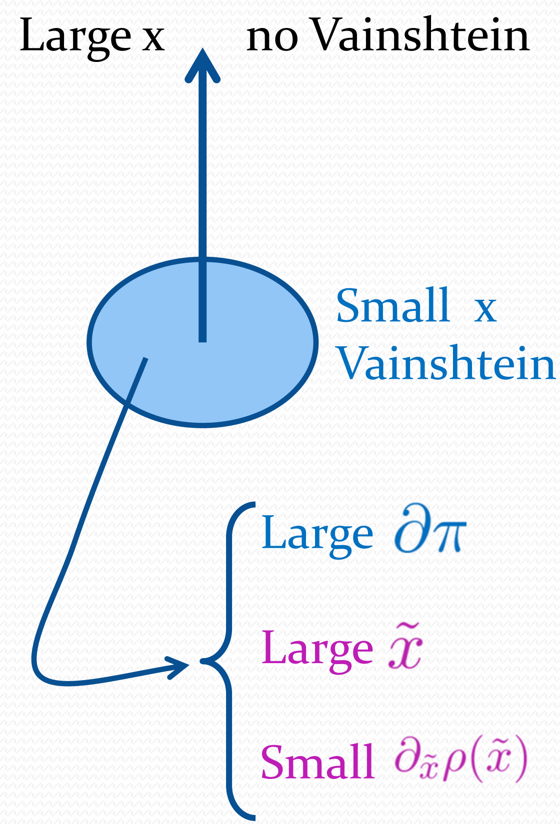

Vainshtein region in original theory maps to weak region in dual theory.

Canonical normalization of ρ: $\rho = \frac{\hat{\rho}}{g}$

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Strong coupling

$$x \;\to\; \tilde{x} = \partial\pi(x)$$

$$\partial_x\pi \;\to\; \partial_{\tilde{x}}\rho(\tilde{x}) = x(\tilde{x})$$

$$\mathcal{L}_{\rm dual} = -\frac{1}{2}(\partial\rho)^2(\tilde{x}) + \frac{1}{g\Lambda^3}(\partial\rho)^2\Box\rho(\tilde{x}) + \frac{1}{gM_{\rm Pl}}\rho\tilde{J}_\rho(\tilde{x})$$

Weak coupling

$$\tilde{J}_\rho(\tilde{x}) = \frac{\partial^2\rho}{g\Lambda^3}J_\pi\left(\frac{\partial\rho}{g\Lambda^3}(\tilde{x})\right)$$

Strong Coupling in the Dual theory

- The dual theory is under control as long as

$$\Delta \tilde{x} > \Lambda^{-1}$$

- Back to the original theory, this implies

$$\Delta x > \frac{(\Lambda x)^{3/2}}{(M/M_{\rm Pl})^{1/2}} \Lambda^{-1} \sim 10^{-8} {\rm cm}$$

For the Earth

CdR, Fasiello & Tolley, 1308.2702
+ in preparation

Other eg. of duality

$$\mathcal{L} = -\frac{1}{2}(\partial\pi)^2 - \frac{1}{6}\mathcal{L}_3^{(\mathrm{Gal})}(\pi) - \frac{1}{8}\mathcal{L}_4^{(\mathrm{Gal})}(\pi) - \frac{1}{5}\mathcal{L}_5^{(\mathrm{Gal})}(\pi)$$

In 4d

$$\begin{aligned}\tilde{x}^a &= x^a + \frac{\partial^a \pi}{\Lambda^3}\\ x^a &= \tilde{x}^a + \frac{\tilde{\partial}^a \rho(\tilde{x})}{\Lambda^3}\end{aligned}$$

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In 4d

$$\tilde{x}^a = x^a + \frac{\partial^a \pi}{\Lambda^3}$$

$$x^a = \tilde{x}^a + \frac{\tilde{\partial}^a \rho(\tilde{x})}{\Lambda^3}$$

$$\mathcal{L}_{\mathrm{dual}} = -\frac{1}{2}(\partial\rho)^2(\tilde{x})$$

Other eg. of duality

- The original theory admits superluminal propagation

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plane wave solutions $\pi = F(x-t)$

- Speed of fluctuations: $c_s = 1$ $c_s = \frac{1-F''}{1+F''}$

Superluminal propagation for $F'' < 0$

Other eg. of duality

- The original theory admits superluminal propagation

$$\mathcal{L} = -\frac{1}{2}(\partial\pi)^2 - \frac{1}{6}\mathcal{L}_3^{(\mathrm{Gal})}(\pi) - \frac{1}{8}\mathcal{L}_4^{(\mathrm{Gal})}(\pi) - \frac{1}{5}\mathcal{L}_5^{(\mathrm{Gal})}(\pi)$$

- Yet the dual theory is just free !

$$\mathcal{L}_{\mathrm{dual}} = -\frac{1}{2}(\partial\rho)^2(\tilde{x})$$

- Trivially causal, unitary, UV complete,…

Something to think about…

Outlook

- Massive Gravity is a specific framework to study IR modifications of Gravity
- It could play a role for
  - the late-time acceleration of the Universe
  - the cosmological constant problem
- We now have the theoretical formalism to describe a stable theory of massive gravity
- It behaves a scalar-tensor Galileon theory in some limit, hand in hand with a Vainshtein mechanism

Outlook

- Galileon theories are plagued with both

  Superluminalities & Strong Issues

- These issues could be tackled by afield redefinition
  DUAL GALILEON ?

- **Massive Gravity requires tuning**
  The graviton mass is technically natural
  At 1-loop the potential does not get strongly distabilized
  At higher loops, it still need to be explored…