Introduction to Massive Gravity I

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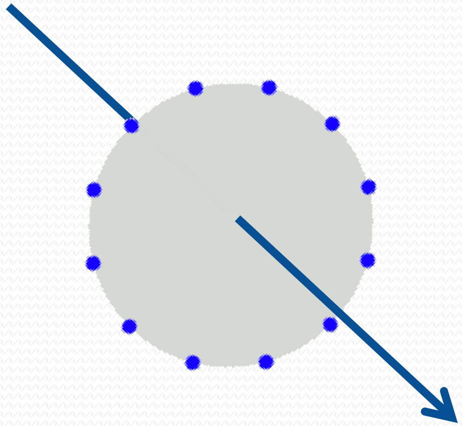
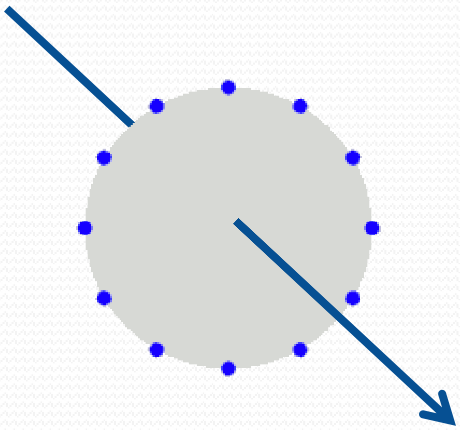
Sept. 24th 2013

7th Aegean Summer School, Paros, “Beyond Einstein's Theory of Gravity”

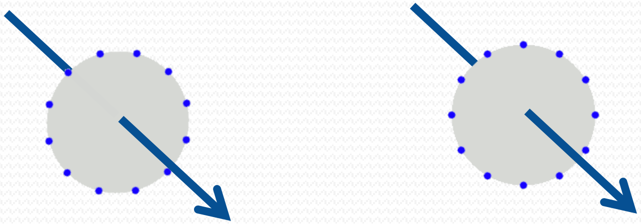
Gravitational Waves

- **GR: 2** polarizations

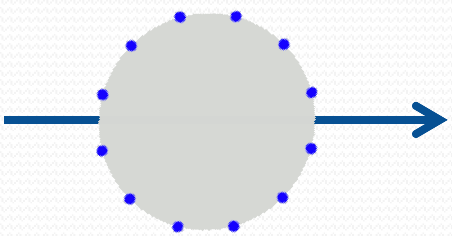




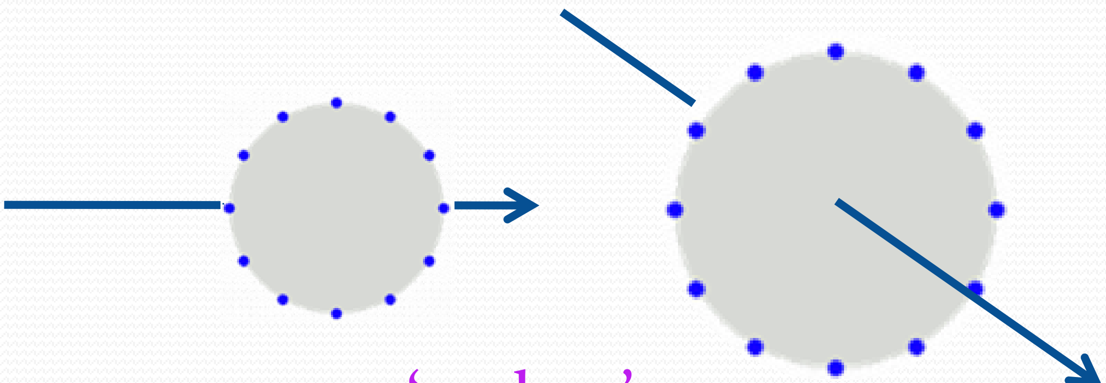
Gravitational Waves



- **GR: 2** polarizations
- In principle GW could have 4 other polarizations



2 ‘vectors’



2 ‘scalars’

Potential 4 `new polarizations’

Massive Gravity

- When breaking covariance, GW can in principle propagate up to 6 independent polarizations (in 4d)
- A massive spin-2 field in 4d has 2s+1=5 dofs
- The 6^{th} dof always comes in as a ghost.

$$2+4=6=5+1$$

Boulware & Deser, PRD6, 3368 (1972)

BD ghost in Massive Gravit

Why is the 6^{th} polarization
so bad ?...

+ see Thomas Sotiriou's talk (Monday)
Hamiltonian unbounded from below

Boulware & Deser, PRD6, 3368 (1972)

BD ghost in Massive Gravit

- *Example*: $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{6\Lambda^5}(\Box\phi)^3$

- About a non-trivial background, $\phi = \phi_0 + \delta\phi$

  for instance $\phi_0 = \frac{\Lambda^3}{8} B_0 \eta_{\mu\nu} x^\mu x^\nu$

Arkani-Hamed, Georgi & Schwartz, Annals Phys. 305, 96 (2003)
Deffayet & Rombouts, PRD 72, 044003 (2005)
Creminelli, Nicolis, Papucci & Trincherini, JHEP 0509, 003 (2005)

BD ghost in Massive Gravit

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- About a non-trivial background, $\phi = \phi_0 + \delta\phi$

  for instance $\phi_0 = \frac{\Lambda^3}{8} B_0 \eta_{\mu\nu} x^\mu x^\nu$

$$\mathcal{L} = \frac{1}{2}\delta\phi\left(1 + \frac{B_0}{\Lambda^2}\Box\right)\Box\,\delta\phi$$

BD ghost in Massive Gravity

- *Example*: $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{6\Lambda^5}(\Box\phi)^3$

- About a non-trivial background, $\phi = \phi_0 + \delta\phi$
- The associated propagator is

$$\mathcal{G}^{-1} = \frac{1}{\left(1 + \frac{B_0}{\Lambda^2}\Box\right)\Box} = \frac{1}{\Box} - \frac{1}{\Box + \Lambda^2}$$

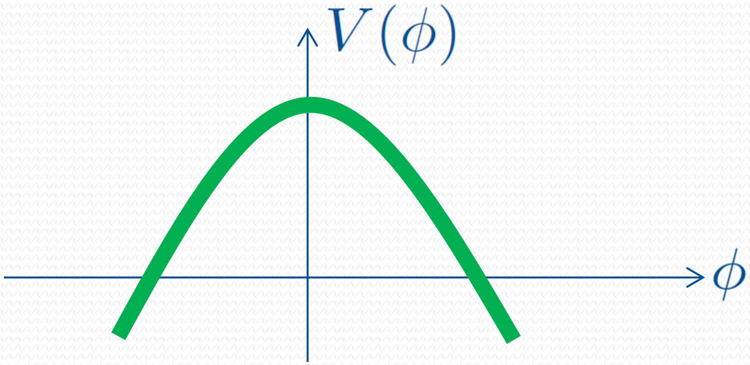
New dof which always comes in as a ghost
(wrong sign kinetic term)

Ostrogradsky, Mem. Acad. St. Petersbourg, VI 4, 385, 1850

Ghosts

- A ***ghost*** is a field with the wrong sign kinetic term $S_{\phi,\chi} = \int d^4x \left(-\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\chi)^2 - V(\phi,\chi) \right)$

- Different from a ***Tachyon***, which has an instability in the potential



$$S = \int d^4x \left(-\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 \right)$$

Scale of instability: m

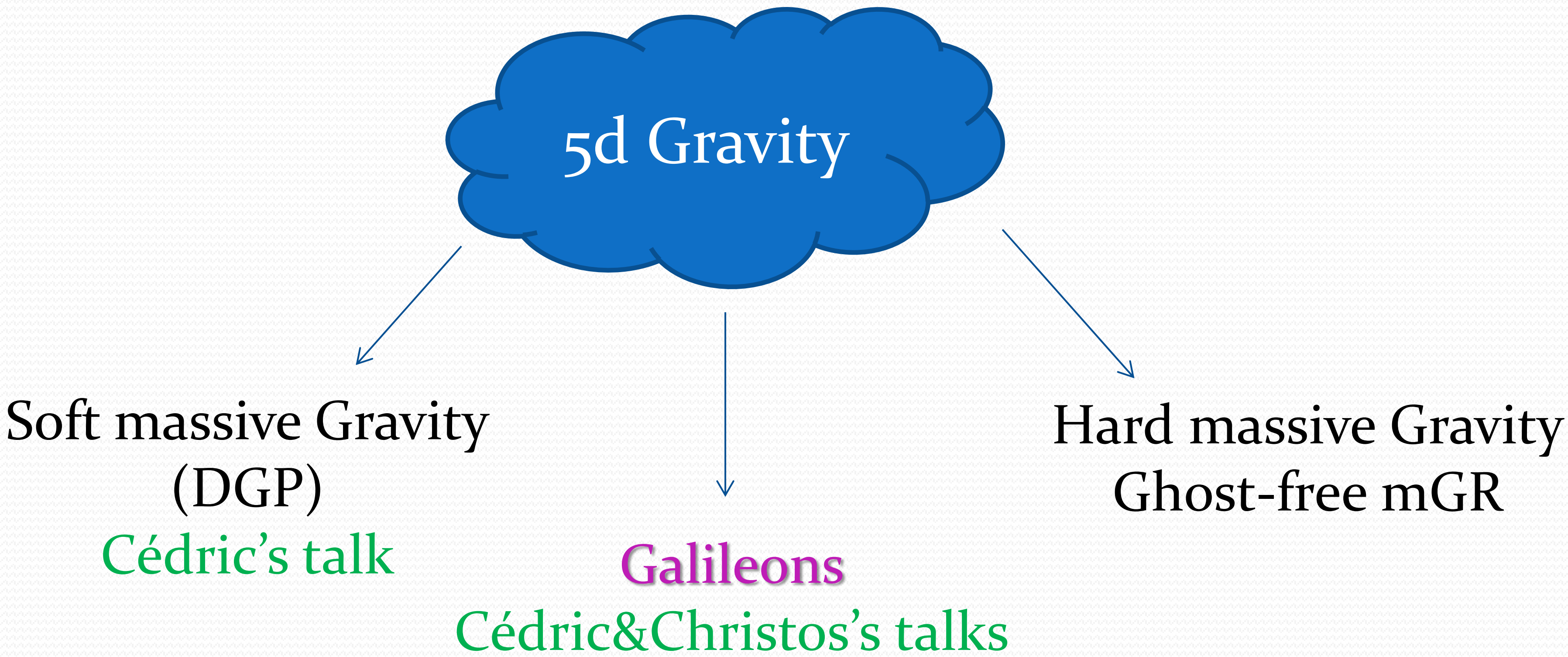
Ghosts

- A *ghost* is a field with the wrong sign kinetic term $S_{\phi,\chi} = \int \mathrm{d}^4x \left(-\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\chi)^2 - V(\phi,\chi) \right)$
- The scale associated with the instability of a ghost is the *momentum*

→ Arbitrarily fast instability already at the classical level

MG from Extra Dimensions

- Best way to obtain a sensible theory of Modified Gravity is to start with General Relativity



Galileons

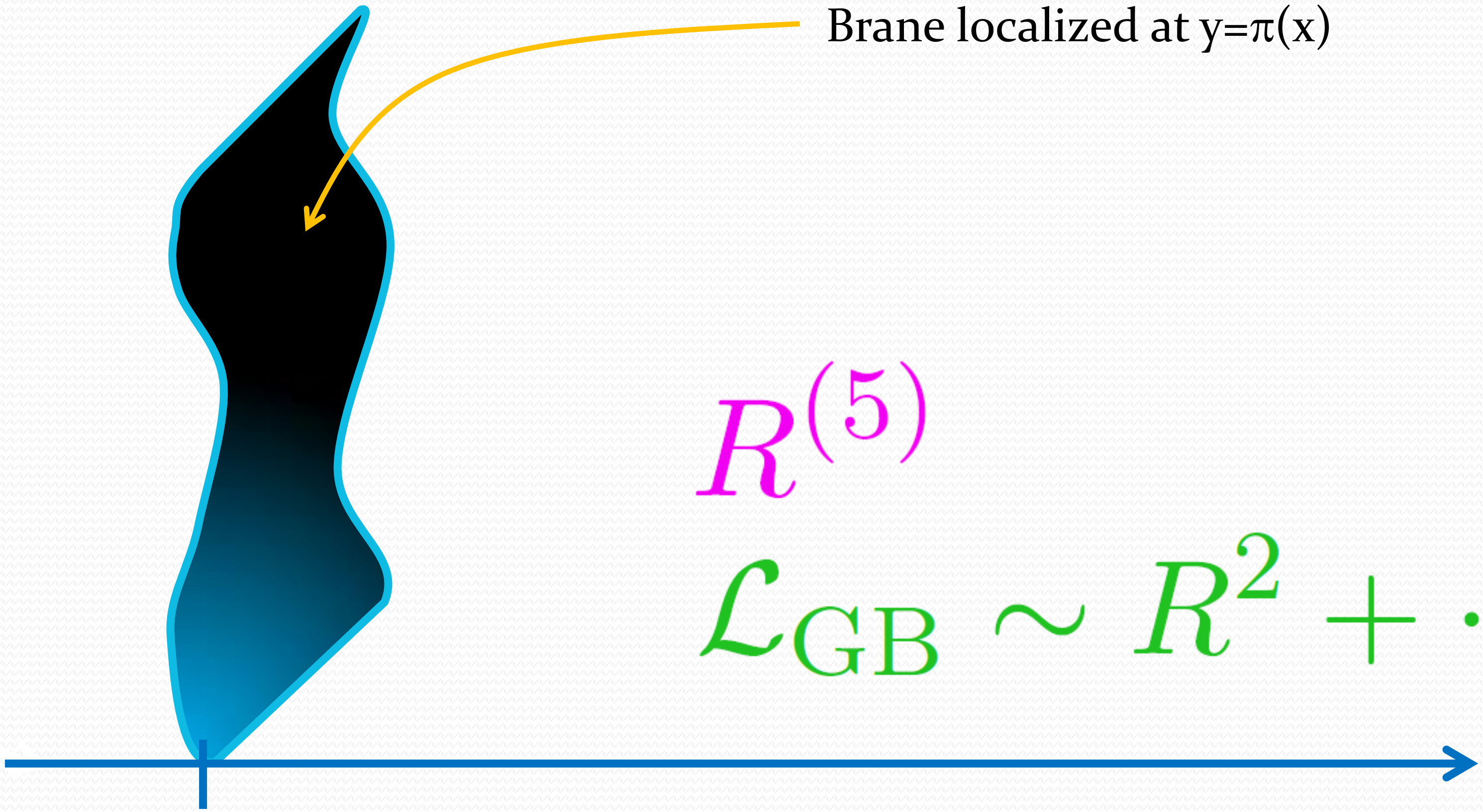
Lovelock Invariants in 5d

$$R^{(5)}$$

$$\mathcal{L}_{\rm GB} \sim R^2 + \cdots$$

CdR, Tolley, 1003.5917

Galileons



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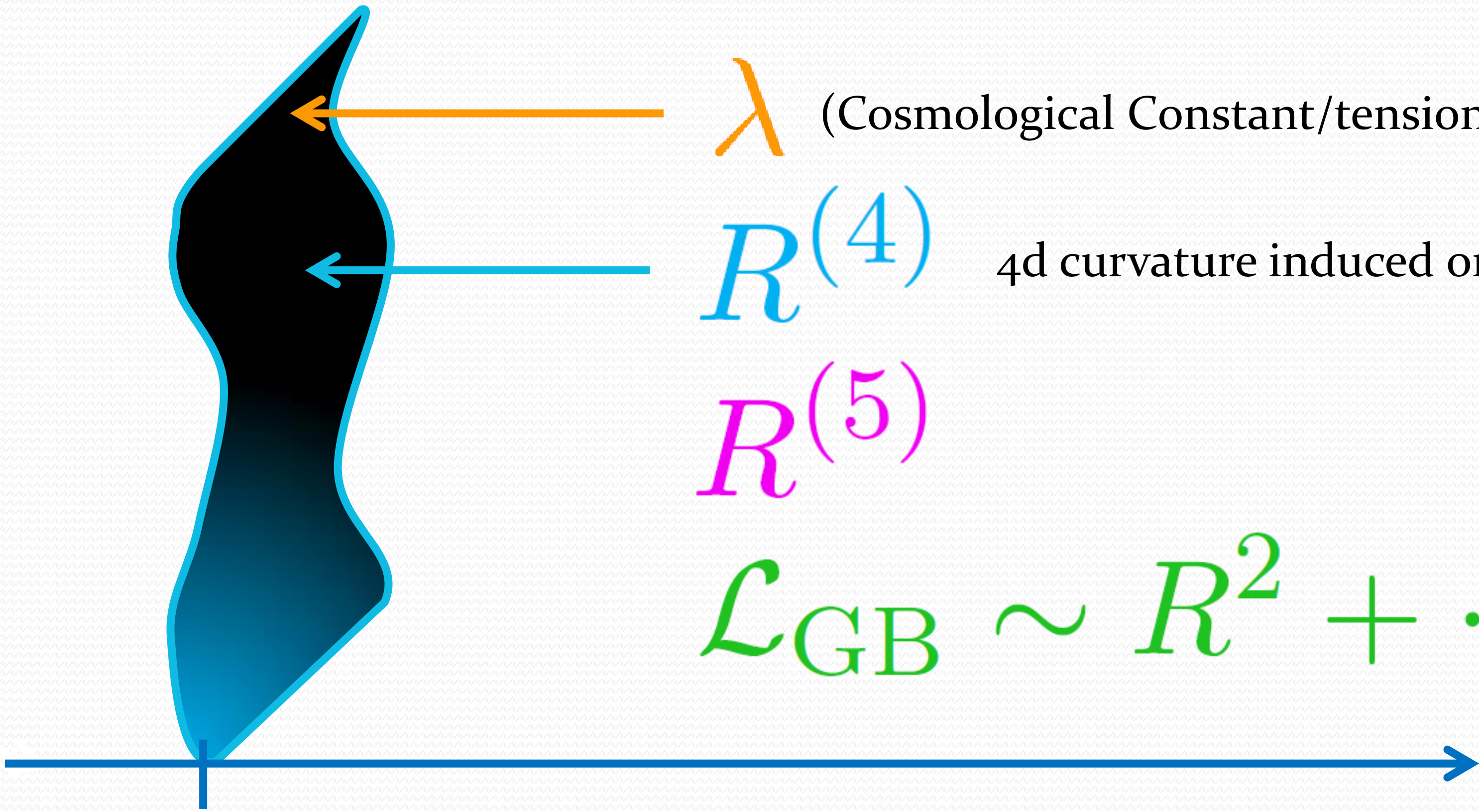


λ (Cosmological Constant/tension on the brane)

$R^{(4)}$ 4d curvature induced on the brane

$R^{(5)}$

$\mathcal{L}_{\rm GB} \sim R^2 + \cdots$

CdR, Tolley, 1003.5917

Galileons

Induced Metric

$$g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu \pi \partial_\nu \pi$$

(disformal coupling)

$\mathcal{L}_2 = (\partial\pi)^2$ From λ

$\mathcal{L}_3 = (\partial\pi)^2 \Box\pi$ From $R^{(5)}$

$\mathcal{L}_4 = (\partial\pi)^2((\Box\pi)^2 + \cdots)$ From $R^{(4)}$

$\mathcal{L}_5 = (\partial\pi)^2((\Box\pi)^3 + \cdots)$ From $\mathcal{L}_{\mathrm{GB}}$

CdR, Tolley, 1003.5917

Galileons

$$\mathcal{L}_2 = (\partial\pi)^2$$
$$\mathcal{L}_3 = (\partial\pi)^2\Box\pi$$
$$\mathcal{L}_4 = (\partial\pi)^2((\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2)$$
$$\mathcal{L}_5 = (\partial\pi)^2((\Box\pi)^3 + \cdots)$$

- Finite Class of Interactions
- That enjoy a shift and Galileon symmetry

$$\pi \to \pi + v_\mu x^\mu + c$$

- Have no ghost (2nd order eom)
- Enjoy a non-renormalization theorem

Nicolis, Rattazzi, Trincherini, 0811.2197 + see Cédric Deffayet's talk

Galileons

$$\mathcal{L}_2 = (\partial\pi)^2$$

$$\mathcal{L}_3 = (\partial\pi)^2\Box\pi$$

$)^2)$

- F
- T
- H
- E

Galileon scalar fields can play an important role on cosmological scales (eg. Candidate for dark energy)

Yet remain frozen on short distances via a Vainshtein mechanism

Nicolis, Rattazzi, Trincherini, 0811.2197

Covariant Galileons

$$g_{\mu\nu} = q_{\mu\nu} + \partial_\mu \pi \partial_\nu \pi$$

Same construction with curved 5d metric
Leads to covariant Galileons,
ie. curved space-time Galileons
see Cédric's talk (Monday).

CdR, Tolley, 1003.5917

Deffayet, Esposito-Farese & Vikman, PRD 79, 084003 (2009)
Deffayet, Deser & Esposito-Farese, PRD 80, 064015 (2009)

Conformal Galileons

$$g_{\mu\nu} = e^{-2\pi/\ell}\eta_{\mu\nu} + \partial_\mu\pi\partial_\nu\pi$$

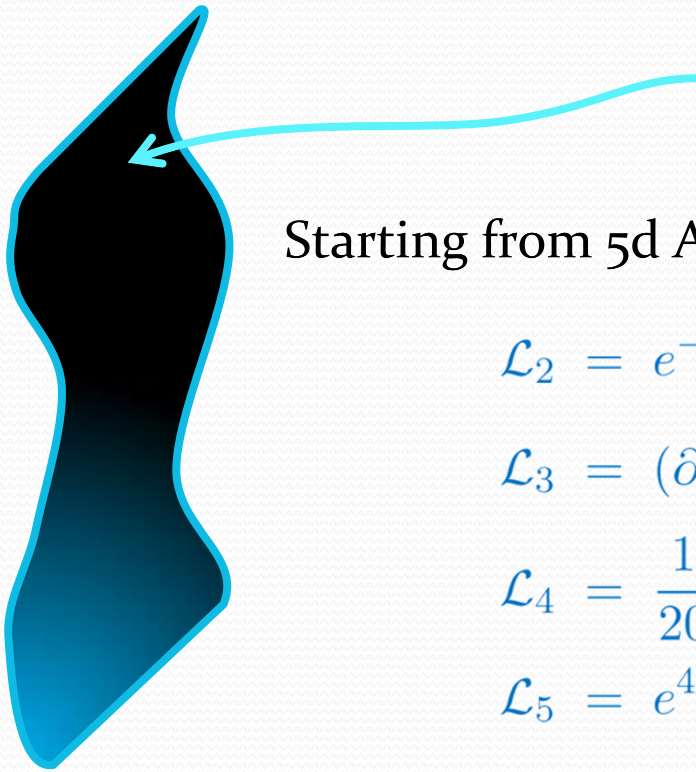
Starting from 5d AdS, we get the conformal Galileon

$$\begin{aligned}
\mathcal{L}_2 &= e^{-2\hat\pi}(\partial\hat\pi)^2 \\
\mathcal{L}_3 &= (\partial\hat\pi)^2\Box\hat\pi - \frac{1}{2}(\partial\hat\pi)^4 \\
\mathcal{L}_4 &= \frac{1}{20}e^{2\hat\pi}(\partial\hat\pi)^2\left(10([\hat\Pi]^2 - [\hat\Pi^2]) + 4((\partial\hat\pi)^2\Box\hat\pi - [\partial\right. \\
\mathcal{L}_5 &= e^{4\hat\pi}(\partial\hat\pi)^2\left([\hat\Pi]^3 + \cdots\right)
\end{aligned}$$

CdR, Tolley, 1003.5917

Nicolis, Rattazzi, Trincherini, 0811.2197

Conformal Galileons



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\mathcal{L}_5 &= e^{4\hat{\pi}}(\partial\hat{\pi})^2\left([\hat{\Pi}]^3 + \cdots\right)
\end{aligned}$$

Invariant under

$$\delta\pi = c + v_\mu x^\mu + \partial_\mu\pi\left(\frac{\ell}{2}(e^{2\pi/\ell} - 1)v^\mu + \frac{1}{2\ell}v^\mu x^2 - \frac{1}{\ell}(v.x)x^\mu\right)$$

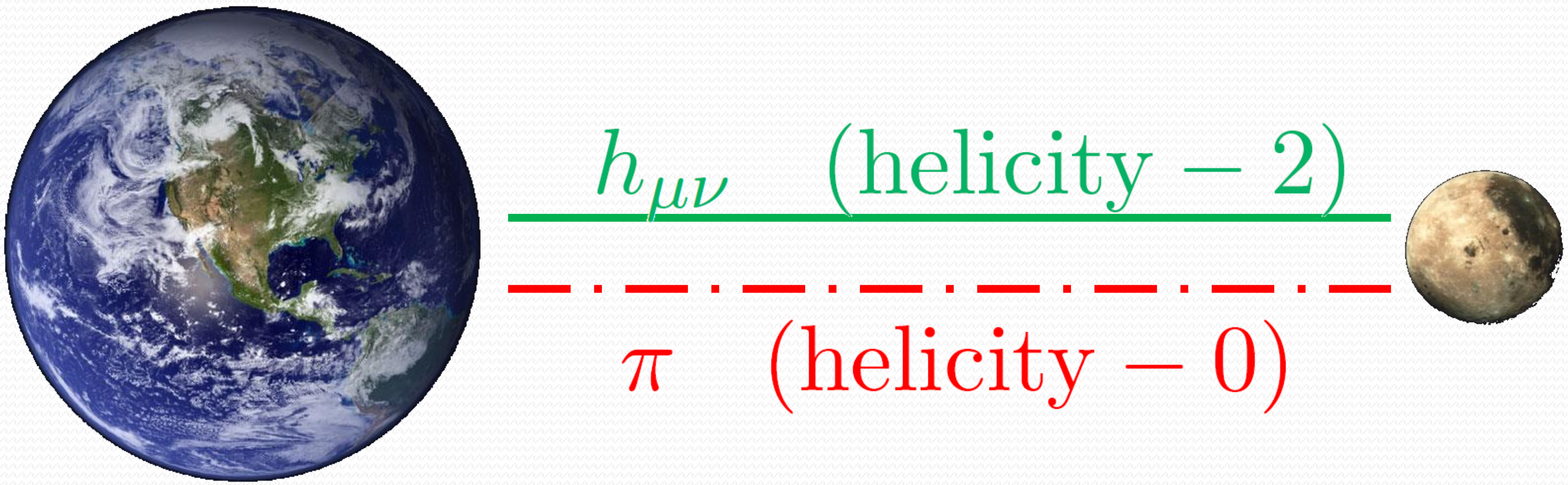
Conformal Galileons

Conformal Galileons arise in the IR when flowing from CFT's in the UV

Actors for Galileon Genesis
(see Alex Vikman's talk)

Also display a Vainshtein mechanism

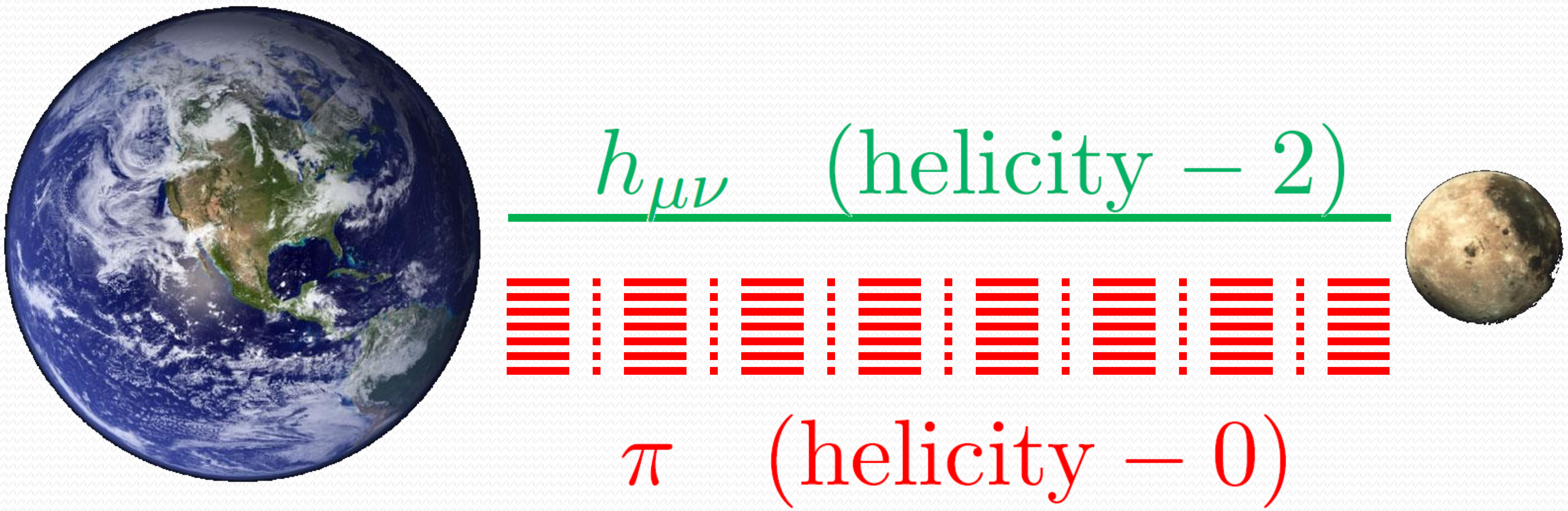
Vainshtein Mechanism



Origin of the vDVZ discontinuity

van Dam & Veltman, Nucl.Phys.B 22, 397 (1970)
Zakharov, JETP Lett.12 (1970) 312

Vainshtein Mechanism

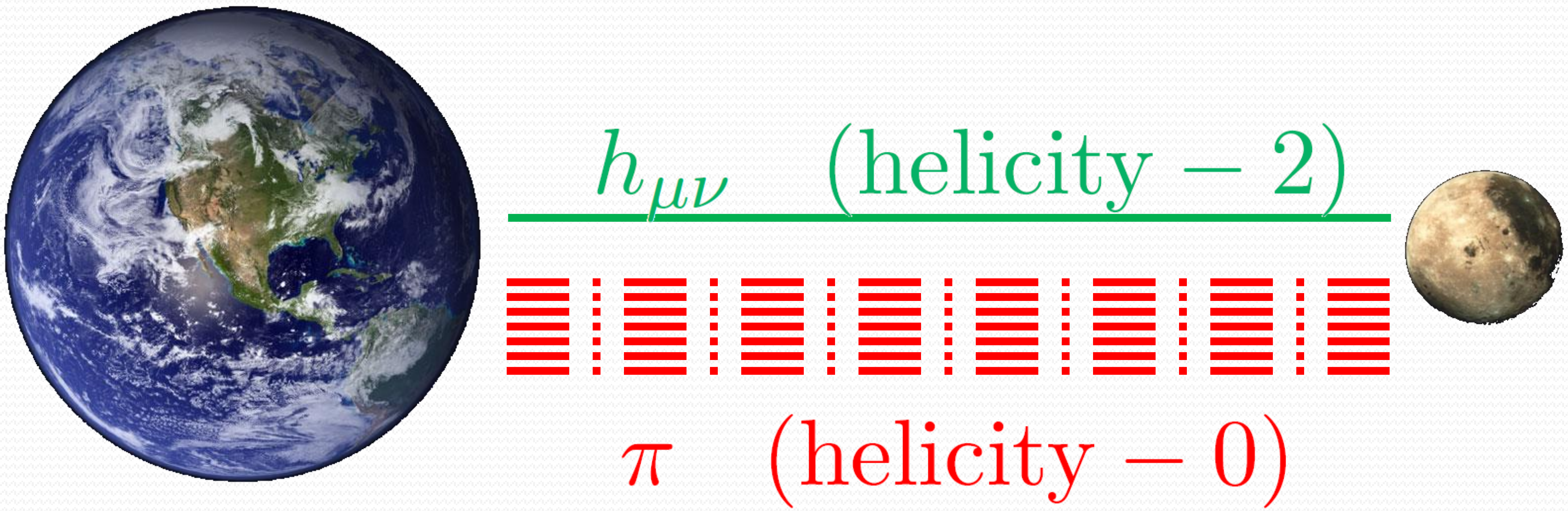


The non-linearities (kinetic interactions) are essential **to screen** the helicity-0 mode

Babichev & Deffayet, 1304.7240
+ see Eugeny Babichev's talk (Monday)

Vainshtein, PLB**39**, 393 (1972)

Vainshtein Mechanism



The interactions for the helicity-0 mode are important at a very low energy scale, $\Lambda \ll M_{\mathrm{Pl}}$

Vainshtein Mechanism

- Close to a source the interactions are important

$$\Box\pi + \frac{1}{\Lambda^3}\left((\Box\pi)^2 + \cdots\right) = -T$$

- Perturbations end up being weakly coupled to matter,

$$\pi = \pi_0 + \delta\pi$$

$$\underbrace{\left(1 + \frac{\Box\pi_0}{\Lambda^3} + \cdots\right)}_{Z\left(\partial^2\pi_0\right) \gg 1}\Box\delta\pi = -\frac{1}{M_{\rm Pl}}\delta T$$

Vainshtein Mechanism

- Close to a source the interactions are important

$$\Box\pi + \frac{1}{\Lambda^3}\left((\Box\pi)^2 + \cdots\right) = -T$$

- Perturbations end up being weakly coupled to matter,

$$\pi = \pi_0 + \widehat{\delta\pi}/\sqrt{Z}$$

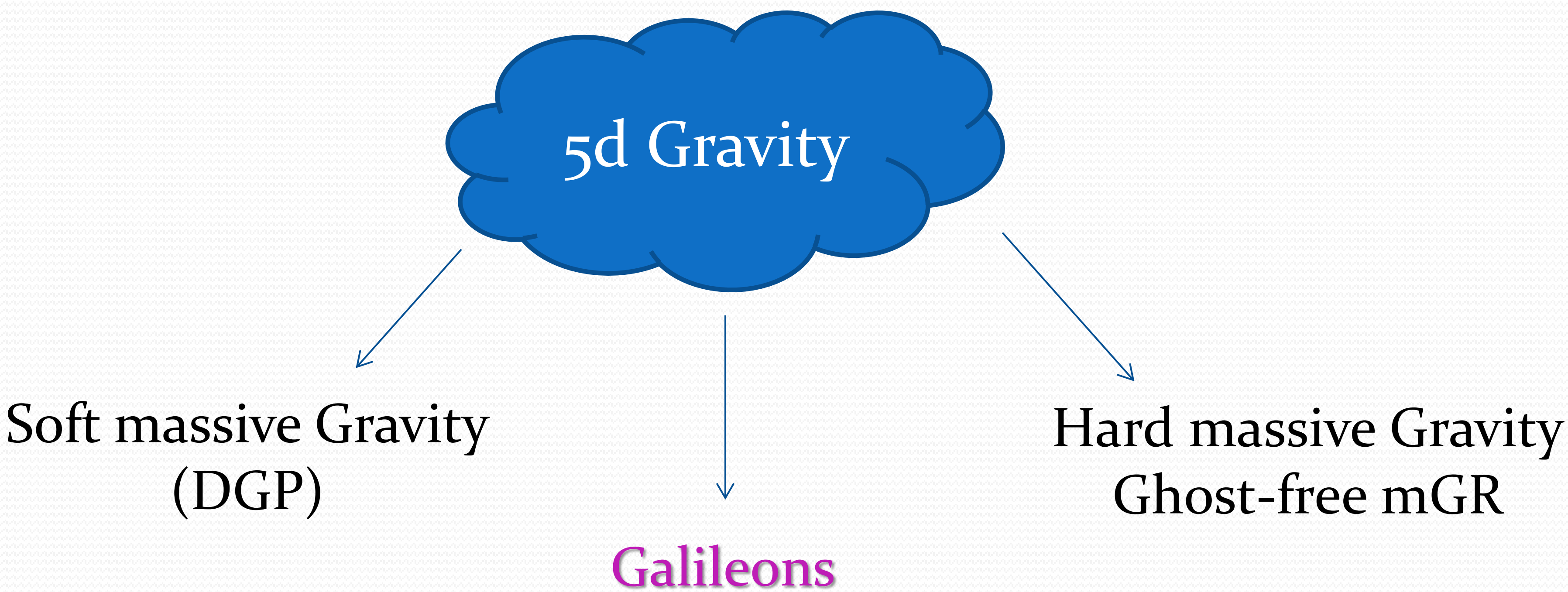
$$\Box\widehat{\delta\pi} = -\frac{1}{M_{\rm Pl}\sqrt{Z}}\delta T$$

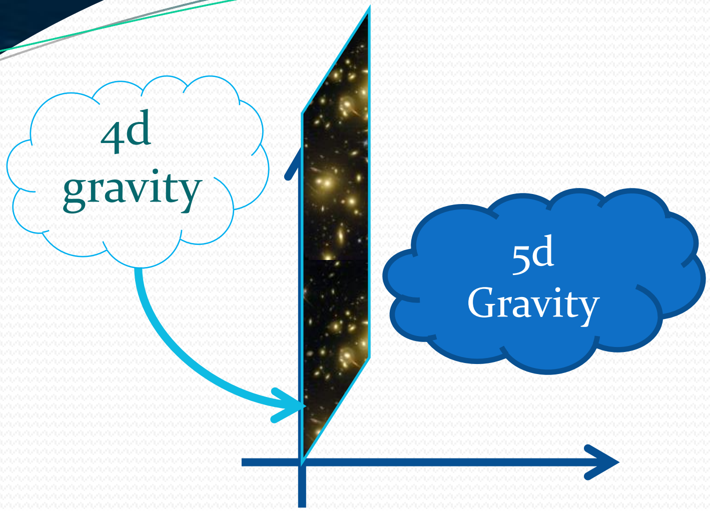
$$\underbrace{\qquad}_{Z\left(\partial^2\pi_0\right) \gg 1}$$

$$Z = 1 + \frac{\partial^2\pi_0}{\Lambda^3} + \cdots \gg 1$$

MG from Extra Dimensions

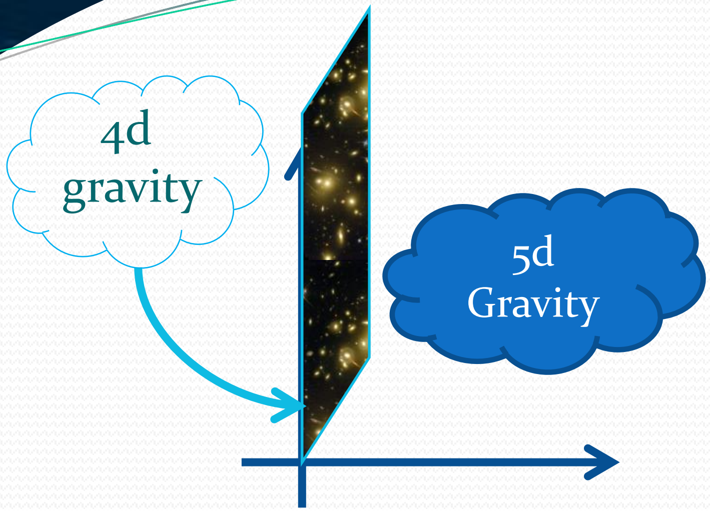
- Best way to obtain a sensible theory of Modified Gravity is to start with General Relativity





Infinite extra dimension
Confine Gravity on a brane
DGP model

Dvali Gabadadze Porrati, 2000



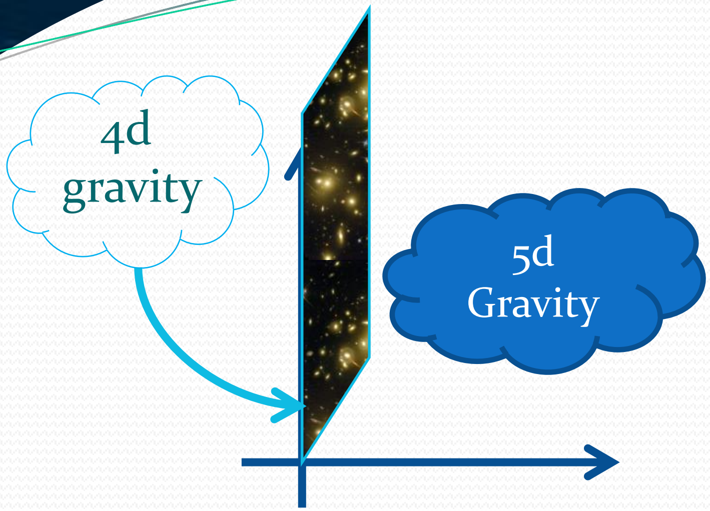
Effective Friedman equation on the brane

$$M_{\rm Pl}^2 H^2 \pm M_5^3 H = \rho$$

Brane bending mode behaves as
a cubic Galileon in the DL (Cédric's talk)

$$\mathcal{L}_3 = (\partial\pi)^2 \Box\pi$$

Dvali Gabadadze Porrati, 2000
Deffayet, PLB 502 (2001) 199
Luty, Porrati, Rattazzi, 2003



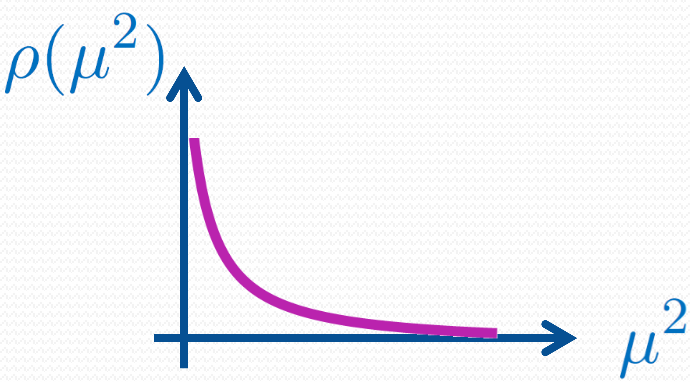
Effective Friedman equation on the brane

$$M_{\rm Pl}^2 H^2 \pm M_5^3 H = \rho$$

$$m_{\rm eff}^2(k) = mk$$

Effectively massive 4d graviton

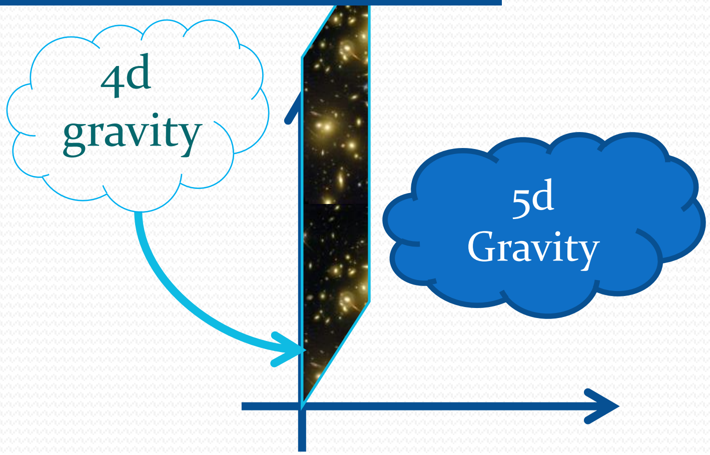
$$\left(\Box - m\sqrt{-\Box}\right) h_{\mu\nu} = \frac{1}{M_{\rm Pl}} T_{\mu\nu}$$



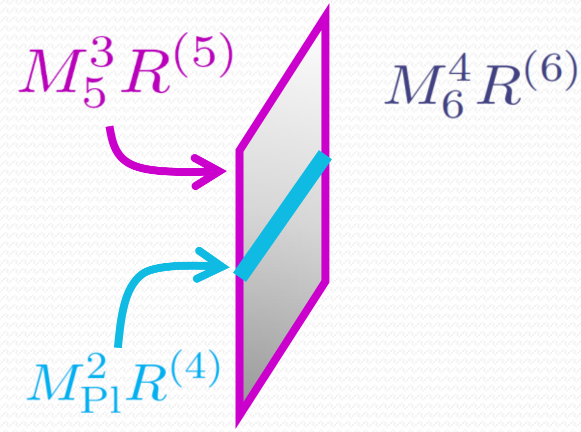
$$m = M_5^3 / M_4^2$$

Dvali Gabadadze Porrati, 2000
Deffayet, PLB 502 (2001) 199

PRL 100, 251603 (2008)
JCAP 0802, 011 (2008)
PRL 103, 161601 (2009)

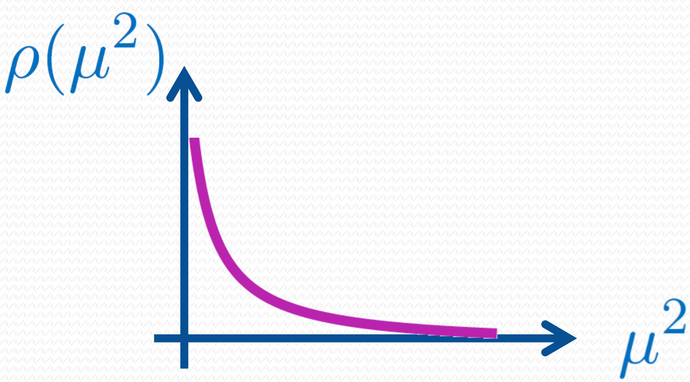


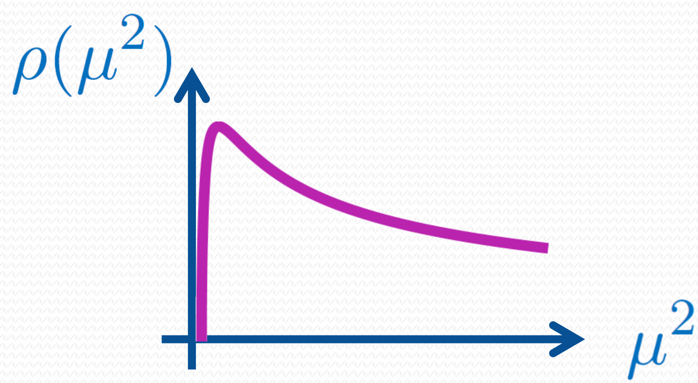
Higher-dimensional Extension



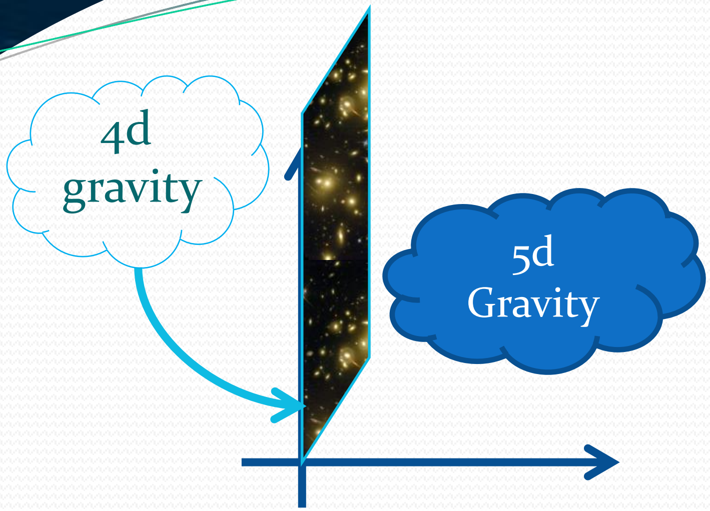
$$m^2_{\rm eff}(k) = mk$$

$$m^2_{\rm eff}(k) = m^2 \log\left(k^2 + m^2\right)$$

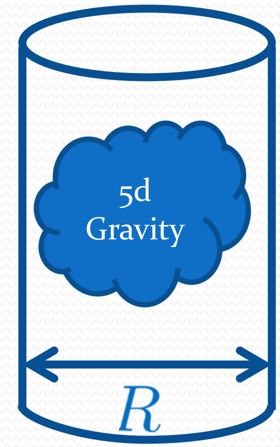




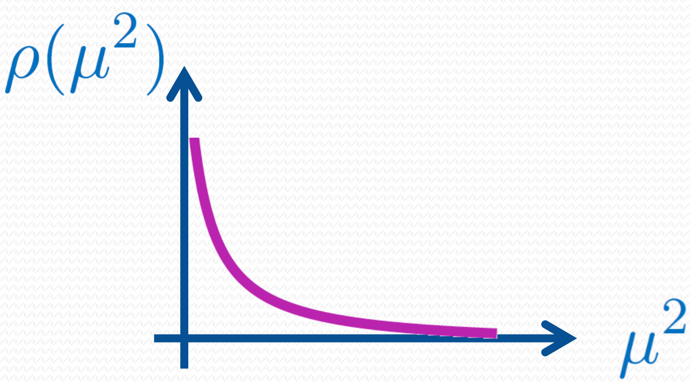
$$m = M_5^3 / M_4^2$$



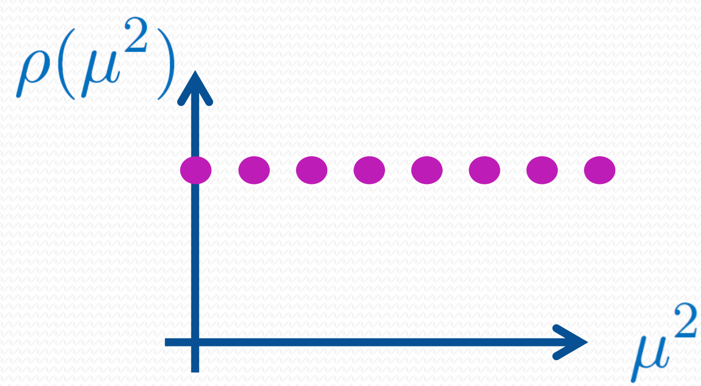
Infinite extra dimension
Confine Gravity on a brane
DGP model



Finite-size extra dimension
KK or deconstruction
Hard massive Gravity
(or multi-gravity)



$$m = M_5^3/M_4^2$$



$$m = 1/R$$

MG from Extra Dimensions

- In both cases (DGP and hard mass gravity),

5 dof from massless spin-2 in 5d

"ensures"

5 dof for massive spin-2 in 4d
(no BD ghost)

If you know where to start from…

MG from Extra Dimensions

- In both cases (DGP and hard mass gravity),

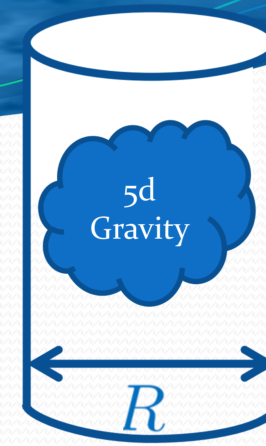
  5 dof from massless spin-2 in 5d

  "ensures"

  5 dof for massive spin-2 in 4d
  (no BD ghost)

- In both cases, the scalar dof behaves as a Galileon scalar field (in some limit)
- Screened via Vainshtein mechanism

Deconstructing Gravity



- **Start with 5d gravity** in the Einstein Cartan form

5d metric: $$g_{\alpha\beta}(x,y) = e^A_\alpha(x,y) e^B_\beta(x,y) \eta_{AB}$$

Connection – set by the torsionless condition

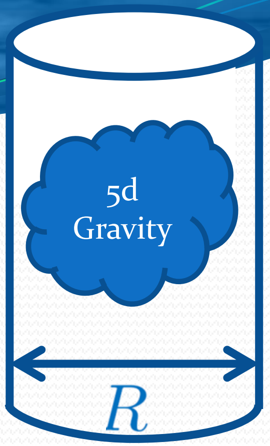
$$\omega_\alpha^{AB} = \frac{1}{2} e_\alpha^C \left(O^{AB}{}_C - O_C{}^{AB} - O^B{}_C{}^A\right)$$

$$O^{AB}{}_C = 2 e^{A\alpha} e^{B\beta} \partial_{[\alpha} e_{\beta]C}$$

Then 5d Curvature 2-form is

$$\mathcal{R}^{AB} = \mathrm{d}\omega^{AB} + \omega^A{}_C \wedge \omega^{CB}$$

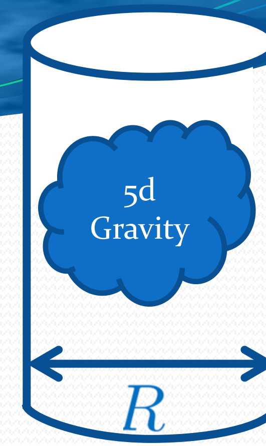
Deconstructing Gravity



- **Start with 5d gravity** in the Einstein Cartan form

$$S_{\rm EH}^{(5)} = \frac{M_5^3}{2}\int \mathrm{d}^4x\mathrm{d}y\sqrt{-g}\,R^{(5)}[g]$$
$$= \frac{M_5^3}{2\times 3!}\int \epsilon_{ABCDE}\,\mathcal{R}^{AB}\wedge e^C\wedge e^D\wedge e^E$$

Deconstructing Gravity



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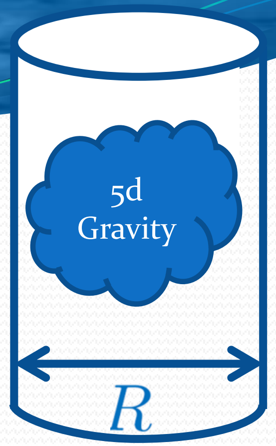
$$= \frac{M_5^3}{2 \times 3!} \int \epsilon_{ABCDE}\, \mathcal{R}^{AB} \wedge e^C \wedge e^D \wedge e^E$$

- Before proceeding, we should set our 5d gauge

Deconstructing Gravity



- 5d Symmetries in the Vielbein

  10 Lorentz gauge freedom

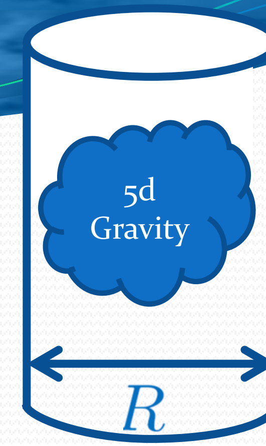
  5 spacetime diffeomorphism freedom

- Fix the 5d gauge by imposing

$$e^A = e_y^A \mathrm{d}y + e_\mu^A \mathrm{d}x^\mu = \begin{pmatrix} e_\mu^a \mathrm{d}x^\mu \\ \mathrm{d}y \end{pmatrix}$$

$e_y^a = 0,\ \ e_\mu^5 = 0,\ \ e_y^5 = 1 \longrightarrow$ 9 gauge fixing

Deconstructing Gravity



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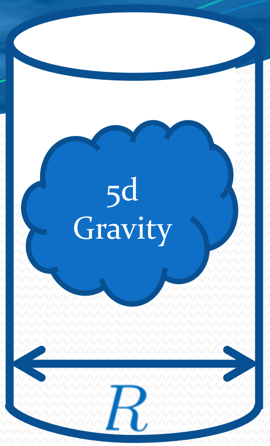
$$e_y^a = 0, \ \ e_\mu^5 = 0, \ \ e_y^5 = 1 \longrightarrow$$ 9 gauge fixing

$$\omega_y^{ab} = e^{\mu[a} \partial_y e_\mu^{b]} = 0 \longrightarrow$$ 6 gauge fixing

$\longrightarrow$ Symmetric vielbein

Deconstructing Gravity

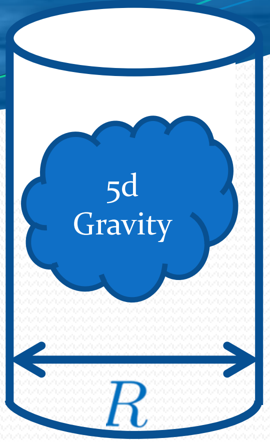


- **Start with 5d gravity** in the our gauge

$$S_{\mathrm{EH}}^{(5)} = \frac{M_5^3}{2}\int \mathrm{d}^4x\mathrm{d}y\sqrt{-g}\,R^{(5)}[g]$$

$$= \frac{M_5^3}{2\times 3!}\int \epsilon_{ABCDE}\,\mathcal{R}^{AB}\wedge e^C\wedge e^D\wedge e^E$$

Deconstructing Gravity



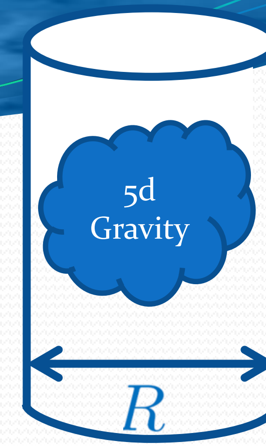
- **Start with 5d gravity** in the our gauge

$$[\mathbb{X}] := \mathrm{Tr}\,(\mathbb{X})$$

$$S^{(5)}_{\mathrm{EH}} = \frac{M_5^3}{2}\int \mathrm{d}^4x\mathrm{d}y\sqrt{-g}\,\left(R[g] + [K]^2 - [K^2]\right)$$
$$= \frac{M_5^3}{4}\int \epsilon_{abcd}\left(R^{ab} + K^a \wedge K^b\right)\wedge e^c \wedge e^d \wedge \mathrm{d}y$$

With extrinsic curvature: $K^\mu_\nu(x,y) = \frac{1}{2}g^{\mu\alpha}(x,y)\partial_y g_{\alpha\nu}(x,y)$

Deconstructing Gravity

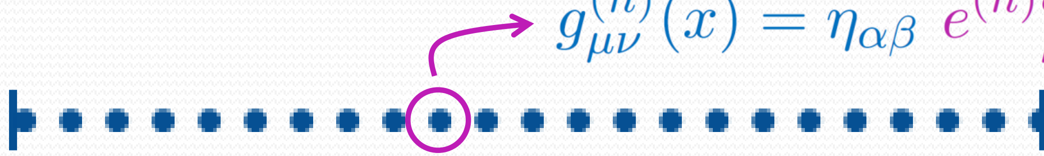


- **Start with 5d gravity** (in our gauge)

$$g_{AB}(x,y) = \eta_{ab} e^a_A(x,y) e^b_B(x,y)$$

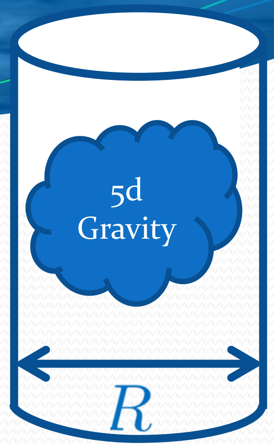
- Discretize it along the extra dimension

$$g^{(n)}_{\mu\nu}(x) = \eta_{\alpha\beta}\; e^{(n)\alpha}_{\;\mu}(x) e^{(n)\beta}_{\;\nu}(x)$$



Deconstruction: Arkani-Hamed, Cohen & Georgi, PRL 86, 4757 (2001)

Deconstructing Gravity



- 5d metric and extrinsic curvature (after gauge fixing)

$$g^{(n)}_{\mu\nu}(x) = \eta_{\alpha\beta}\; e^{(n)\alpha}_{\ \ \mu}(x) e^{(n)\beta}_{\ \ \nu}(x)$$

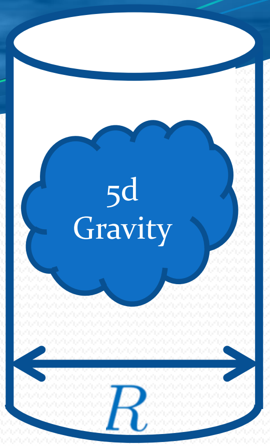
$$K^{\mu}_{\ \nu} \sim g^{\mu\alpha}\partial_y g_{\alpha\nu} \sim e^{-1}\partial_y e$$

- Discretization

$$e(x,y) \ \rightarrow \ e^{(n)}(x)$$

$$\partial_y e(x,y) \ \rightarrow \ m\left(e^{(n+1)}(x) - e^{(n)}(x)\right)$$

Deconstructing Gravity



- 5d metric and extrinsic curvature (after gauge fixing)

$$g^{(n)}_{\mu\nu}(x) = \eta_{\alpha\beta}\, e^{(n)\alpha}_{\ \ \mu}(x) e^{(n)\beta}_{\ \ \nu}(x)$$

$$K^{\mu}_{\nu} \sim g^{\mu\alpha}\partial_y g_{\alpha\nu} \sim e^{-1}\partial_y e \to m(e^{(n)})^{-1}(e^{(n+1)} - e^{(n)})$$

- Discretization

$$e(x,y) \ \to \ e^{(n)}(x)$$

$$\partial_y e(x,y) \ \to \ m\left(e^{(n+1)}(x) - e^{(n)}(x)\right)$$

$$K^{\mu}_{\nu} \to -m\left(\delta^{\mu}_{\nu} - \sqrt{(g^{(n)})^{\mu\alpha}\, g^{(n+1)}_{\alpha\nu}}\right) \equiv -m\mathcal{K}^{\mu}_{\nu}[g^{(n)}, g^{(n+1)}]$$

Ghost-free Massive Gravity

- Deconstructing 5d GR naturally leads to a 4d Ghost-free multi-gravity theory

$$[\mathbb{X}] := \mathrm{Tr}\,(\mathbb{X})$$

$$\mathcal{L} = M_5^3 \int_0^R \mathrm{d}y \left({}^{(4)}R + [K]^2 - [K^2] \right)$$

Ghost-free Massive Gravity

- Deconstructing 5d GR naturally leads to a **4d Ghost-free** multi-gravity theory

$$[\mathbb{X}] := \mathrm{Tr}\,(\mathbb{X})$$

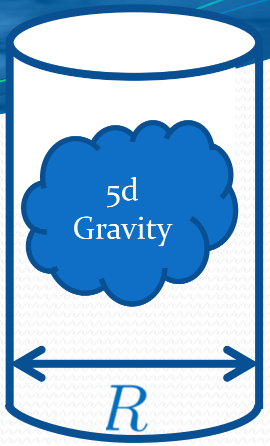
$$\mathcal{L} = M_5^3 \int_0^R \mathrm{d}y \left({}^{(4)}R + [K]^2 - [K^2] \right)$$

$$\mathcal{L} = \frac{M_5^3}{m} \sum_{n=1}^{N} \left({}^{(4)}R_n + m^2 \left([\mathcal{K}_{n,n+1}]^2 - [\mathcal{K}_{n,n+1}^2] \right) \right)$$

$$M_4^2 = M_{\rm Pl}^2 / N$$

CdR, Matas & Tolley, 1308.4136

Hinterbichler & Rosen, JHEP, 1207, 047 (2012)



Generalized Mass Term

- Going back to the vielbein,

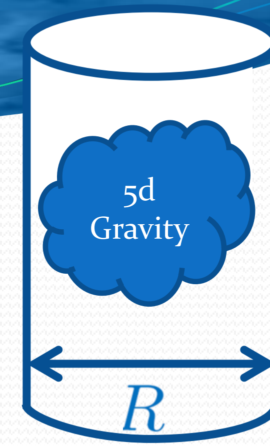
$$\sqrt{-g}\left([K^2]-[K]^2\right) = \epsilon_{abcd}\, e^a \wedge e^b \wedge \partial_y e^c \wedge \partial_y e^d$$

- Discretization

$$\sqrt{-g}\left([K^2]-[K]^2\right) \;\to\; m^2 \epsilon_{abcd} e^a_n \wedge e^b_n \wedge (e^c_{n+1}-e^c_n) \wedge (e^d_{n+1}-e^d_n)$$

$$\equiv m^2\sqrt{-g}\left([\mathcal{K}^2]-[\mathcal{K}]^2\right)$$

Generalized Mass Term



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- Discretization

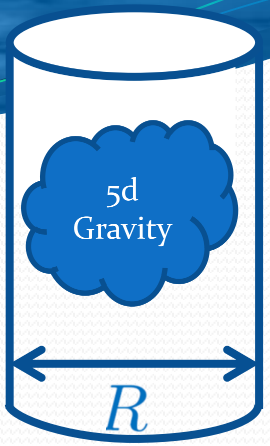
$$\sqrt{-g}\left([K^2]-[K]^2\right)\;\rightarrow\; m^2\epsilon_{abcd}(w_1 e^a_n+(1-w_1)e^a_{n+1})\wedge(w_2 e^b_n+(1-w_2)e^b_{n+1})$$

$$\wedge(e^c_{n+1}-e^c_n)\wedge(e^d_{n+1}-e^d_n)$$

$$\equiv m^2\sqrt{-g}\left(\mathcal{L}_2(\mathcal{K})+(w_1+w_2)\mathcal{L}_3(\mathcal{K})+w_1w_2\mathcal{L}_4(\mathcal{K})\right)$$

2-parameter family of potential for a massive graviton in 4d

Generalized Mass Term



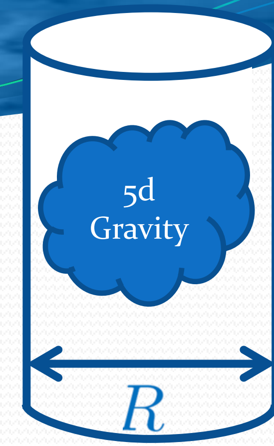
- 2 parameter family of potential for MG,

$$m^2\sqrt{-g}\left(\mathcal{L}_2(\mathcal{K})+(w_1+w_2)\mathcal{L}_3(\mathcal{K})+w_1 w_2\mathcal{L}_4(\mathcal{K})\right)$$

$$\begin{aligned}2\mathcal{L}_2[\mathcal{K}] &= \varepsilon^{\mu\nu\alpha\beta}\varepsilon_{\mu'\nu'\alpha\beta}\mathcal{K}^{\mu'}_{\nu}\mathcal{K}^{\nu'}_{\nu}\\ \mathcal{L}_3[\mathcal{K}] &= \varepsilon^{\mu\nu\alpha\beta}\varepsilon_{\mu'\nu'\alpha'\beta}\mathcal{K}^{\mu'}_{\nu}\mathcal{K}^{\nu'}_{\nu}\mathcal{K}^{\alpha'}_{\alpha}\\ \mathcal{L}_4[\mathcal{K}] &= \varepsilon^{\mu\nu\alpha\beta}\varepsilon_{\mu'\nu'\alpha'\beta'}\mathcal{K}^{\mu'}_{\nu}\mathcal{K}^{\nu'}_{\nu}\mathcal{K}^{\alpha'}_{\alpha}\mathcal{K}^{\beta'}_{\beta}\end{aligned}$$

Very similar structure as Galileons as introduced in Cédric Deffayet's talk

Generalized Mass Term



- 2 parameter family of potential for MG,

$$m^2\sqrt{-g}\,\left(\mathcal{L}_2(\mathcal{K}) + (w_1+w_2)\mathcal{L}_3(\mathcal{K}) + w_1 w_2 \mathcal{L}_4(\mathcal{K})\right)$$

$$2\mathcal{L}_2[\mathcal{K}] = \varepsilon^{\mu\nu\alpha\beta}\varepsilon_{\mu'\nu'\alpha\beta}\mathcal{K}^{\mu'}_{\nu}\mathcal{K}^{\nu'}_{\nu}$$

$$\mathcal{L}_3[\mathcal{K}] = \varepsilon^{\mu\nu\alpha\beta}\varepsilon \ldots \mathcal{K}^{\mu'}\mathcal{K}^{\nu'}\mathcal{K}^{\alpha'}$$

$$\mathcal{L}_4[\mathcal{K}]$$

Very similar structure as Galileons as introduced

$$\mathcal{L}_2[\mathcal{K}] = ([\mathcal{K}^2] - [\mathcal{K}]^2)$$

$$\mathcal{L}_3[\mathcal{K}] = ([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3])$$

$$\mathcal{L}_4[\mathcal{K}] = ([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4])$$

Ghost-free Massive Gravity

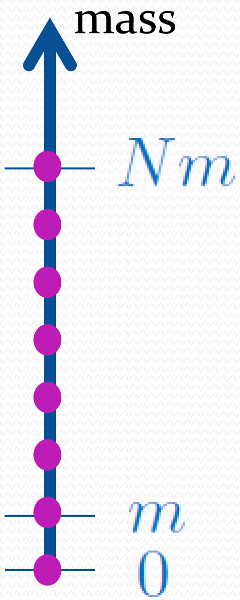
- Deconstructing 5d GR in a gauge-fixed way (5d lapse=1) leads to Ghost-free multi-gravity
- Truncated theory is consistent
- 5 dofs with strong coupling scale $\Lambda = (M_{\rm Pl} m^2)^{1/3}$
- Scalar mode decouples in the massless limit

never recover 5d GR in the limit $m \to 0$, $R \to \infty$

CdR, Matas & Tolley, 1308.4136

Ghost-free Massive Gravity

- Strong coupling is avoided if the lapse remains dynamical before discretization



- Truncated theory is NOT consistent (ghost at the scale of the highest mode)
- Deconstruction is EQUIVALENT to KK (after non-linear field redefinition)

CdR, Matas & Tolley, 1308.4136

Bi-Gravity

- Discretization with only 2 sights

$$\mathcal{L}_{g,f} = M_g^2\sqrt{-g}R[g] + M_f^2\sqrt{-f}R[f]$$

2 massless spin-2 field

2+2 = 4 degrees of freedom

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \left(\sqrt{g^{-1}f}\right)^\mu_\nu$$

Bi-Gravity

- Discretization with only 2 sights

$$\begin{aligned}\mathcal{L}_{g,f} &= M_g^2\sqrt{-g}R[g] + M_f^2\sqrt{-f}R[f] \\ &+ m^2 M_f M_g \sqrt{-g}\left(\mathcal{L}_0 + \sum_{n=2}^{4}\mathcal{L}_n(\mathcal{K}[g,f])\right)\end{aligned}$$

Interaction between the two metrics f, g

Hassan & Rosen, JHEP, 1202, 126 (2012)

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \left(\sqrt{g^{-1}f}\right)^\mu_\nu$$

Bi-Gravity

- Discretization with only 2 sights

$$\mathcal{L}_{g,f} = M_g^2\sqrt{-g}R[g] + M_f^2\sqrt{-f}R[f] + m^2 M_f M_g \sqrt{-g}\left(\mathcal{L}_0 + \sum_{n=2}^{4}\mathcal{L}_n(\mathcal{K}[g,f])\right)$$

1 massless spin-2 field
1 massive spin-2 field

Only 1 copy of diff
would need 4 Stückelberg fields to restore the 2nd diff
(only 3 independent Stückelberg fields)

2+5 = 7 degrees of freedom

Hassan & Rosen, JHEP, 1202, 126 (2012)

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \left(\sqrt{g^{-1}f}\right)^\mu_\nu$$

Bi-Gravity

- Discretization with only 2 sights

$$\begin{aligned}\mathcal{L}_{g,f} \; &= \; M_g^2\sqrt{-g}R[g] + M_f^2\sqrt{-f}R[f] \\ &+ \; m^2 M_f M_g \sqrt{-g}\left(\mathcal{L}_0 + \sum_{n=2}^{4}\mathcal{L}_n(\mathcal{K}[g,f])\right) \\ &+ \; \sqrt{-g}\mathcal{L}(\psi_g, g_{\mu\nu}) + \sqrt{-f}\mathcal{L}(\psi_f, f_{\mu\nu}) + (m^2\psi_g\psi_f)\end{aligned}$$

- Proven to be ghost-free

Hassan & Rosen, JHEP, 1202, 126 (2012)

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \left(\sqrt{g^{-1}f}\right)^\mu_\nu$$

Massive Gravity

- Take the scaling $M_f \longrightarrow \infty$ which freezes the metric $f_{\mu\nu}$
- Obtain MG on the fixed reference metric $f_{\mu\nu}$

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Massive Gravity

- Take the scaling $M_f \longrightarrow \infty$ which freezes the metric $f_{\mu\nu}$
- Obtain MG on the fixed reference metric $f_{\mu\nu}$

$$\mathcal{L}_{\mathrm{mGR}} = M_{\mathrm{Pl}}^2 \sqrt{-g} \left(R[g] + m^2 \sum_{n=2}^{4} \mathcal{L}_n(\mathcal{K}[g,f]) \right)$$

- Proven to be ghost-free (see also tomorrow's talk)
  5 degrees of freedom in graviton

CdR, Gabadadze, Tolley, PRL, 106, 231101 (2011)

Summary (so far)

- 5d GR is a good starting point to obtain 4d theories of modified gravity (or gravity with a scalar field) which are free of ghost

Galileon → DBI-Galileon, Covariant Galileon, Conformal Galileon

DGP

Multi-gravity

Summary (so far)

- 5d GR is a good starting point to obtain 4d theories of modified gravity (or gravity with a scalar field) which are free of ghost

Galileon

DGP → Higher dimensional extension (Cascading Gravity)

Multi-gravity

Summary (so far)

- 5d GR is a good starting point to obtain 4d theories of modified gravity (or gravity with a scalar field) which are free of ghost

Galileon

DGP

Multi-gravity → Bi-gravity

Multi-gravity → Massive Gravity on an arbitrary reference metric

Summary (so far)

- 5d GR is a good starting point to obtain 4d theories of modified gravity (or gravity with a scalar field) which are free of ghost
- Has interesting Phenomenology & Cosmology

Very constrained!

see talks by

Deffayet Babichev Vikman	}	Monday
Charmousis Tsujikawa	}	Tuesday
Tolley Volkov	}	Wednesday
Crisostomi Babichev Scargill	}	Thursday

...

Summary (so far)

- 5d GR is a good starting point to obtain 4d theories of modified gravity (or gravity with a scalar field) which are free of ghost
- Has interesting Phenomenology & Cosmology

- General proof of the Absence of Ghost → now
- Decoupling limit → tomorrow
- Open questions and open avenues

Quantum corrections
Superluminalities
Strong Coupling
↔
Galileon Duality

2. ADM in GR

- The ghost of massive gravity was originally pointed out by Boulware and Deser, using the ADM decomposition

$$\mathrm{d}s^2 = -N_0^2 \mathrm{d}t^2 + \gamma_{ij} \left(\mathrm{d}x^i + N^i \mathrm{d}t\right) \left(\mathrm{d}x^j + N^j \mathrm{d}t\right)$$

- In GR, both the lapse and shifts play the role of Lagrange multipliers, propagating 4 constraints

Arnowitt, Deser & Misner, Gen.Rel.Grav., 40, 1997, 2027 (2008), arXiv:gr-qc/0405109
Boulware & Deser, PRD6, 3368 (1972)

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$$\mathcal{H} = N_0 R^0(\gamma, P_\gamma) + N_i R^i(\gamma, P_\gamma)$$

$$6 \times 2$$

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symmetry

$$6 \times 2 - 4$$

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symmetry

$$6 \times 2 - 4 - 4 = 4 = 2 \times 2 \text{ dof in field space}$$

constraints

2. BD Ghost in ADM

- In massive gravity, both the lapse and shifts enter non-linearly

$$\mathcal{H} = N_0 R^0(\gamma, P_\gamma) + N_i R^i(\gamma, P_\gamma) + m^2\, \mathcal{U}(N_0, N_i, \gamma, P_\gamma)$$

- The Fierz-Pauli combination, ensures that the lapse remains linear at the quadratic order,

$$\begin{aligned} \mathcal{U}(h) &= h_{\mu\nu}^2 - h^2 \\ &= \cancel{\delta N^2} + \delta N h_{ii} + \cdots \end{aligned}$$

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symmetry

$6 \times 2 - \cancel{4} - \cancel{4} =$ 6 dof propagating non-linearly

constraints

Boulware & Deser,1972
Creminelli et. al. hep-th/0505147

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symmetry

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constraints

5+1 dof ⟶  ghost ...

Boulware & Deser,1972
Creminelli et. al. hep-th/0505147

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Is that really the right criteria ???

2. BD Ghost in ADM

- Whether or not there is a constraint,

$$\mathcal{H} = N_0 R^0(\gamma, P_\gamma) + N_i R^i(\gamma, P_\gamma) + m^2\, \mathcal{U}(N_0, N_i, \gamma, P_\gamma)$$

simply depends on the Hessian, $L_{\mu\nu} = \dfrac{\partial^2 \mathcal{U}}{\partial N^\mu \partial N^\nu}$

$$\text{constraint} \quad \Longleftrightarrow \quad \det L = 0$$

CdR, Gabadadze, Tolley, PRL, 106, 231101 (2011)

2d case

As an instructive example, we can take

$$\mathcal{H} = N_0 R_0 + N^i R_i - m^2 \underbrace{\sqrt{(1+N_0)^2 - N_i^2}}_{1 + N_0 + \frac{1}{2}N_i^2 + \frac{1}{2}N_0 N_i^2 - \frac{1}{2}N_0^2 N_i^2 + \cdots}$$

CdR, Gabadadze, Tolley, PRL, 106, 231101 (2011)

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Despite being non-linear in the lapse, there is a constraint:

$$\det L_{\mu\nu} = \#\det \left(\begin{array}{c|c} -N_i^2 & (1+N_0)N_i \\ \hline (1+N_0)N_j & -((1+N_0)^2 - N_i^2)\delta_{ij} - N_i N_j \end{array} \right) = 0$$

CdR, Gabadadze, Tolley, PRL, 106, 231101 (2011)

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We could have simply redefined the shift to make the constraint transparent:

$$N_i = n_i(1 + N_0)$$

$$\mathcal{H} = N_0 R_0 + (1 + N_0)\left(n^i R_i - m^2\sqrt{1 - n_i^2}\right)$$

CdR, Gabadadze, Tolley, PRL, 106, 231101 (2011)

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$$N_i = n_i(1+N_0)$$

$$\mathcal{H} = N_0 R_0 + (1+N_0)\underbrace{\left(n^i R_i - m^2\sqrt{1-n_i^2}\right)}_{\text{Constraint } C_1}$$

2d case

- The lapse propagates a constraint $C_1 = 0$

- Which does not commute with the Hamiltonian

$$\{\mathcal{H}, C_1\} = C_2$$

imposes a secondary constraint $C_2 = 0$.

- This removes the 2 dof in phase space which were responsible for the ghost.

CdR, Gabadadze, Tolley, PRL, 106, 231101 (2011)

No Ghost in the Full theory

The same has been proven to remain true to all orders in the theory of massive gravity in 4d

Hassan & Rosen, 1106.3344

No Ghost in the Full theory

- The full Hamiltonian is

$$\mathcal{H} = \underbrace{N_0 R^0 + N_i R^i}_{\text{Invariant under global Lorentz transformations}} - m^2 \underbrace{\left(\mathcal{K}^2_{\mu\nu} - \mathcal{K}^2\right)}_{\text{Invariant under local Lorentz transformations}}$$

No Ghost in the Full theory

- The full Hamiltonian is

$$\mathcal{H} = N_0 R^0 + N_i R^i - m^2 \left(\mathcal{K}_{\mu\nu}^2 - \mathcal{K}^2\right)$$

Can always choose a Lorentz frame such that at a given point $\bar{x}$

$$N_i(\bar{x}) = 0$$

And so

$$\mathcal{K}_{\mu\nu}^2 - \mathcal{K}^2 = N_0 \; k_0(\gamma_{ij}) + k_1(\gamma_{ij})$$

No Ghost in the Full theory

- The full Hamiltonian is

$$\mathcal{H} = N_0 R^0 + N_i R^i - m^2 \left(\mathcal{K}_{\mu\nu}^2 - \mathcal{K}^2 \right)$$

For any given point $\bar{x}$, we can always make a global Lorentz transform such that at that point the constraint is manifest

Summary

- 5d GR is a good starting point to obtain 4d theories of modified gravity (or gravity with a scalar field) which are free of ghost
- Has interesting Phenomenology & Cosmology

- General proof of the Absence of Ghost
- Decoupling limit → tomorrow
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Quantum corrections
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