Interacting Dark Energy

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Outline

1. Interacting Model
   - Phenomenological Model
2. Conclusions

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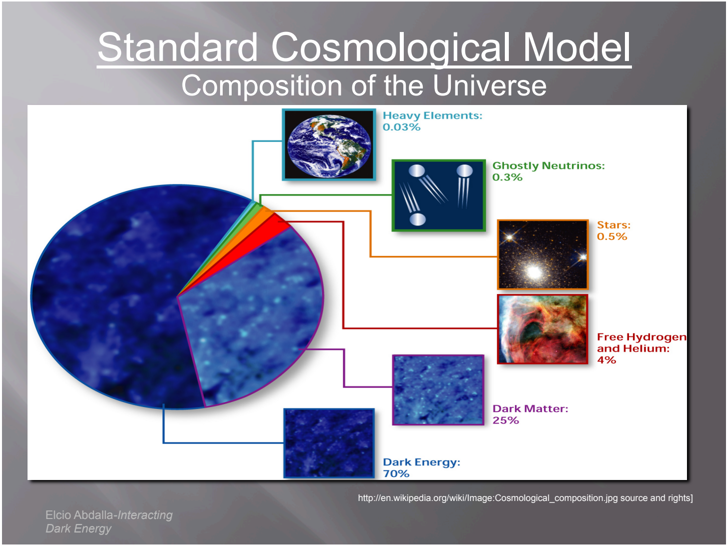
Standard Cosmological Model
Composition of the Universe
Heavy Elements: 0.03%
Ghostly Neutrinos: 0.3%
Stars: 0.5%
Free Hydrogen and Helium: 4%
Dark Matter: 25%
Dark Energy: 70%
http://en.wikipedia.org/wiki/Image:Cosmological_composition.jpg source and rights]
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Standard Cosmological Model

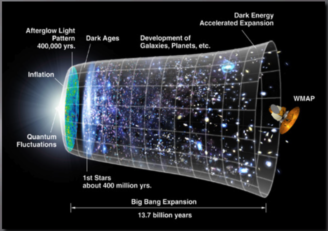
ΛCDM

- Cosmological principle

- General Relativity, valid during all evolution everywhere (+ SM of particles)

- Hot Big Bang

- Composed mainly by:
  - CDM: aglomerative matter;
  - Cosmological Constant (Λ): accelerates the late expansion



Source:NASA, WMAP Science Team.

Fits the observational data very well!!!

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But:

- Not capable of describing all the evolution;
- What is CDM? What is Λ?

Extending the SM: what if...

- The cosm. principle not valid all the time: inhomogeneous universe (LTB models, backreaction,...)
- Change GR, modified gravity: f(R), scalar-tensor theories, extra dim., …
- Not a cosm. Constant: 1) Dynamic Dark Energy (quintessence, phantom, Dilaton dark energy, K-essence,...)

  2) Interacting dark energy (Coupled DM, Chaplygin gas, coupled to neutrinos,...)

Many of these extensions can give the late acceleration!!!

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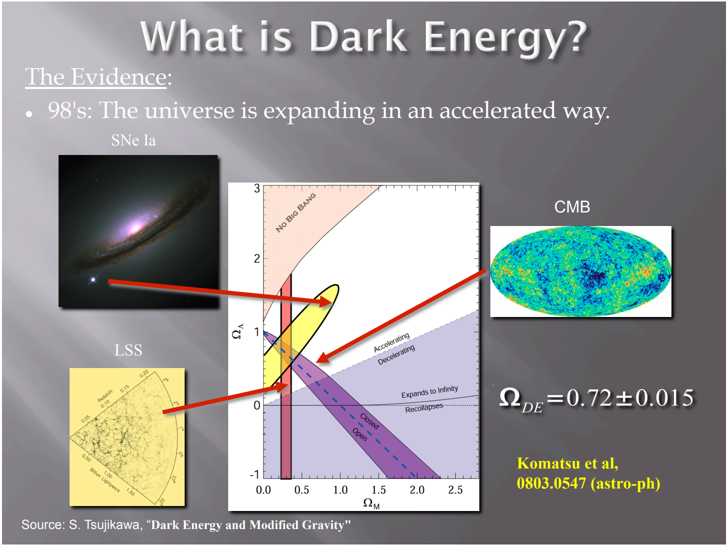
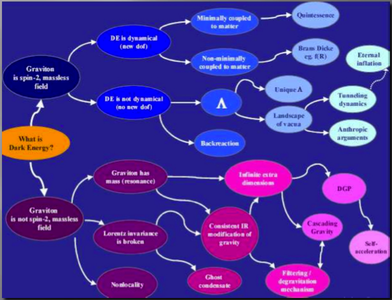

What is Dark Energy?
The Evidence:
• 98's: The universe is expanding in an accelerated way.
SNe Ia
LSS
CMB
No Big Bang
Accelerating
Decelerating
Expands to Infinity
Recollapses
Closed
Open
Ω_Λ
Ω_M
$\Omega_{DE}=0.72\pm0.015$
Komatsu et al,
0803.0547 (astro-ph)
Source: S. Tsujikawa, "Dark Energy and Modified Gravity"

What is causing the acceleration?

From the Friedmann equations:

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}(\rho + 3p) \quad \Rightarrow \quad p < -\frac{\rho}{3} \rightarrow \omega < -\frac{1}{3}$$



Source: de Rahm & Tolley, 2008

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Cosmological Dynamics

With this new dynamical quantity, cosmological viable models require a period where ρ_{scalar_field} is subdominant w.r.t. other components and dominates at late times.

We need to study the dynamics of a scalar field in the presence of a barotropic fluid ⇒ $\omega_m = p_m / \rho_m$

Friedmann equations:

$$H^2 = \frac{8\pi G}{3}(\rho_\phi + \rho_m)\,,$$
$$\dot{H} = -4\pi G(\rho_\phi + p_\phi + \rho_m + p_m)\,.$$

Energy Densities:

$$\dot{\rho}_\phi + 3H(1+\omega_\phi)\rho_\phi\,,$$
$$\dot{\rho}_m + 3H(1+\omega_m)\rho_m\,.$$

What we want: Analyze the dynamics of the model to find ***"scaling solutions"***

$$\rho_\phi / \rho_m = C$$

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Cosmological Constant Λ

- Simplest explanation: $\omega_{DE} = -1$ $\quad G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$
- Plagued by problems: Many...(but still ΛCDM best agreement w/ data)
  - Fine Tuning: if dark energy originates from vacuum energy density

Observations:

$$\Lambda \approx H_0^2 = (2.13\mathrm{h} \times 10^{-42}\, GeV)^2 \rightarrow \rho_\Lambda = \Lambda m_{pl}^2 / 8\pi \approx 10^{-47}\, GeV^4$$

Naive Field Theory estimation: evaluated by the sum of zero point energies of quantum fields with mas *m*

$$\rho_{vac} = \frac{1}{2(2\pi)^3} \int_0^\infty d^3k \sqrt{k^2 + m^2} = \frac{1}{4\pi^2} \int_0^\infty dk\, k^2 \sqrt{k^2 + m^2} \rightarrow \frac{k_{max}^4}{16\pi}$$

UV divergence

GR: $k_{max} = m_{pl}$ ⇒ $\rho_{vac} \approx 10^{74}\, GeV^4$

10^{121} Orders of magnitude!!!!

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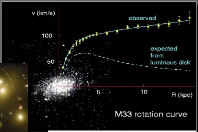
What is Dark Matter?

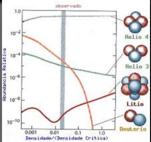
Evidences for Dark Matter

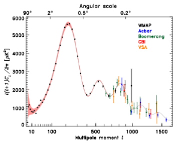
- Huge amount of evidences indicating that dark matter exists. One of the biggest unsolved, but very well measured, problems in physics.

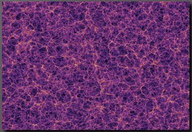
- Observations indicate that DM interacts mainly gravitationally.

- So far, we have no (non-contradictory) observations that DM was detected by any non-gravitational mechanism.









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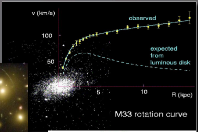
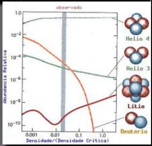
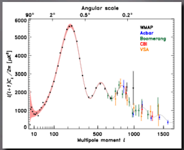
What is Dark Matter?

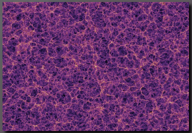
Evidences for Dark Matter

- Galaxy Rotation Curves
- Gravitational Lenses
- Anisotropies of the CMB
- Large Scale Structure









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What is Dark Matter?

What is it made of?

- We know only little about the nature of DM:
  - Cold (non-relativist-at leat the majority of it);
  - Stable;
  - Dark and non-collisional (no electric or color charge)

- No SM particle is compatible with that.

- Many possibilities, from the Planck/GUT "WIMPzilla" to ultra-light axions.

- Candidates in the form of *weakly interacting particles (WIMPS*) with masses from *GeV-TeV* stand out.

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Candidates

- Neutralinos (higgsino, bins, winos, singlinos)
- Axinos
- Gravitinos
- Sneutrinos
- Axions
- Sterile neutrinos
- 4th generation neutrinos
- Kaluza-Klein photons
- Kaluza-Klein gravitons
- Brane world dark matter/D-matter
- Little higgs dark matter
- Light scalars
- Superheavy states (ie. “WIMPzillas”)
- Self-interacting dark matter
- Super-WIMPs
- Asymmetric dark matter
- Q-balls (and other topological states)
- CHAMPs (charged massive particles)
- Cryptons, …

Supersymetric

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Supersymmetric Candidates

- SUSY happening around the electro-weak scale helps solving many problems:

  1) Hierarchy problem: cancels the quadratic divergences;
  2) Unification of the SM forces into a common scale;
  3) Gives candidates to DM, including WIMPS. Lightest SUSY particle (LSP) – stable and only destroyed trhough annihilation.

- These problems only solved together when the superparners are not heavier than TeV (although SUSY can be build witha wilder range of scales).

- Candidates to DM include WIMPS (neutralino, sneutrino) and stronger interacting candidates (gravitinos, axinos).

- Gravitinos and axinos even not being WIMPStake advantage of the WIMP miracle through the decaying into other sparticles.

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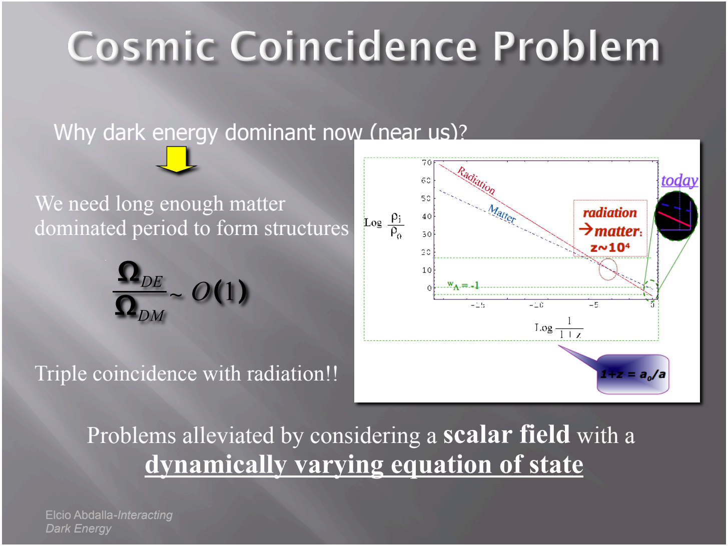
Cosmic Coincidence Problem
Why dark energy dominant now (near us)?
We need long enough matter dominated period to form structures
$\frac{\Omega_{DE}}{\Omega_{DM}} \sim O(1)$
Triple coincidence with radiation!!
Radiation
Matter
radiation
→matter:
z~10⁴
today
$w_\Lambda = -1$
$\text{Log} \frac{\rho_i}{\rho_0}$
$\text{Log} \frac{1}{1+z}$
70
60
50
40
30
20
10
0
-15
-10
-5
0
$1+z = a_0/a$
Problems alleviated by considering a scalar field with a dynamically varying equation of state
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Dynamical Dark Energy

- We want a dynamical component that evolves differently with the evolution of the universe.

- DE is an infrared phenomenon, so change physics to accommodate it in the Infrared.

- A Lagrangian formulation (every degree of freedom has a kinetic term).

- Examples:
  - Quintessence
  - K-essence
  - Scaling models
  - Tachyon field
  - Phantom (ghost) field
  - Dilaton dark energy
  - Chaplygin gas
  - ...

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Scalars as DE

Scalar filed minimally coupled to gravity:

$$S=\int d^4x\sqrt{-g}\,[\frac{-\epsilon}{2}g_{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)]$$

ε=1, ordinary
ε=-1, phantom

with potentials that lead to LATE time acceleration.

$$\rho_\phi=\frac{1}{2}\dot{\phi}^2+V(\phi),$$
$$p_\phi=\frac{1}{2}\dot{\phi}^2-V(\phi).$$

The system of quintessence plus matter is:

$$H^2=\frac{8\pi G}{3}\left[\epsilon\dot{\phi}^2/2+V(\phi)+\rho_m\right],$$
$$\dot{H}=\frac{-8\pi G}{3}\left[\epsilon\dot{\phi}^2+(1+\omega_m)\rho_m\right],$$
$$\epsilon\ddot{\phi}+3H\dot{\phi}+\frac{dV}{d\phi}=0\,.$$

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$$\rho_\phi / \rho_m = C$$

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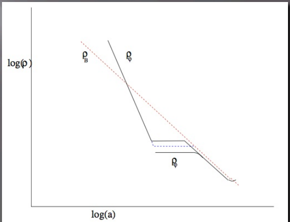
Scaling Solutions

- We want to find cosmological viable solutions, describing the period of decelerating expansion followed by accelerated expansion
- Energy density of the scalar field mimics the background fluid energy density - sub-dominant during radiation and matter dominating eras
- If the scaling solution is an attractor, for generic initial conditions, the field will eventually enter the scaling solution – alleviates fine tuning

- The system needs to exit from the scaling regime in order to give rise to an accelerated expansion
- Scaling solutions live on the border between acceleration and deceleration
- Depends on the form of the potential – this need to change to exit scaling solution.



M. Sami, “Models of Dark Energy”,

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Quintessence

$$H^2=\frac{8\pi G}{3}\left[\epsilon\dot{\phi}^2/2+V(\phi)+\rho_m\right],$$
$$\dot{H}=\frac{-8\pi G}{3}\left[\epsilon\dot{\phi}^2+(1+\omega_m)\rho_m\right],$$
$$\epsilon\ddot{\phi}+3H\dot{\phi}+\frac{dV}{d\phi}=0\ .$$

We construct an autonomous system by a change of variable of:

$$\frac{dx}{dN}=-3x+\frac{\sqrt{6}}{2}\epsilon\lambda y^2+\frac{3}{2}x\left[(1-\omega_m)\epsilon x^2+(1+\omega_m)(1-y^2)\right],$$
$$\frac{dy}{dN}=\frac{-\sqrt{6}}{2}\lambda x y+\frac{3}{2}y\left[(1-\omega_m)\epsilon x^2+(1+\omega_m)(1-y^2)\right],$$
$$\frac{d\lambda}{dN}=-\sqrt{6}\lambda^2(\Gamma-1)x\ .$$

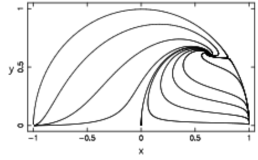
$$V(\phi)=V_0 e^{-\mathrm{X}^2\lambda\phi}$$

By doing the dynamical system analysis we find,
almost without fine-tuning of the initial conditions:

- Attractor/Stable solutions (figure)

- Tracking solutions(energydensity of DE eventually catches expansion at late times. up that of the fluid):
for varying $\lambda=\lambda(\phi)$
Problem: Potential too steep. No acceleration at late times.

- **Scaling solutions**: for λ constant.
Problem: No accelerated expansion for late Times. Does not leave the scaling sol..



Wetterich (1988);V. Sahni and A. A. Starobinsky, ;S. M. Carroll,(2001); T. Padmanabhan,(2003); P. J. E. Peebles and B. Ratra,(2000)

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Interacting Dark Energy

- Interaction possible since DE only detected gravitationally.
- Need a dynamical mechanism to make DE leave the scaling solution and give late acceleration.
- Aliviate coincidence problem.

Different ways of interacting:

- Coupling to gravity

[Boisseau etal 2000]
[Faraoni 2000]
[Perrotta, Baccigalupi 2002]
[Riazuelo Uzan 2002]
[Pettorino, Baccigalupi 2008]
and references therein

- Coupling to dark matter

[EA, Bin Wang 2004,...]
[Wetterich 1995]
[Amendola 2000, 2004]
[Mangano Miele Pettorino 2005]
[Quartin etal 2008]
[Bean etal 2008]

- Coupling to neutrinos

[Fardon etal 2004]
[Afshordi etal 2005]
[Brookfield etal 2007]
[Amendola etal 2007]
[Wetterich 2007]
[Mota, Pettorino, Robbers, Wetterich 2008]

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Source: Valeria Pettorino, ITP Heidelberg

Dynamical and Interacting Dark Energy
Dark Energy and Dark Matter

- Constitute 96% of the matter content of the universe.
- Only observed/detected gravitationally – could an interaction between DM and DE exists and still not measured?
- Could this interaction aliviate the coincidence problem?
- Is it compatible with observations?
- How to build these models?

DE/DM Interaction

Each component is not conserved alone anymore. Cosmological equations:

$$\dot{\rho}_m + 3H\rho_m = -Q,$$

$$\dot{\rho}_\phi + 3H(1+w_\phi)\rho_\phi = Q,$$

Many many models in the literature:

- Phenomenological (For a classification see [Koyama, Maartens, Song, 0907.2126])

  - Interaction depending on DM

  - Interaction depending on DE

  - Interaction depending on DM and DE

Constant coupling or Time varying coupling

Coupling must be small : constraints from observations!

In general no analytic solution!

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Tests to disclose the interaction (performed)

- Age Constraints
- Large angle CMB
- Clusters
- Corrections to Layser Irvine equations

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Tests to disclose the interaction (under construction)

CMB: WMAP7,9; Planck (E.A., Wang, Ferreira, Alencar,Xu)

Clusters

Redshift distortion

21 cm line and new BAO data

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Field Theory description

Understanding the interaction between DE and DM from Field Theory

S. Micheletti, E. Abdalla, B. Wang, PRD(09)

Two fields describing each of the dark components:
a fermionic field for DM, a bosonic field for the Dark Energy,

$$\mathcal{L} = \sqrt{-g}\{-V(\varphi)\sqrt{1-\alpha\partial^\mu\varphi\partial_\mu\varphi} + \frac{i}{2}[\bar{\Psi}\gamma^\mu\nabla_\mu\Psi - \bar{\Psi}\overleftarrow{\nabla}_\mu\gamma^\mu\Psi] - (M-\beta\varphi)\bar{\Psi}\Psi\},$$

where α is a constant with dimension MeV^{-4}. β a coupling between dark energy and dark matter fields. $V(\varphi)$ the tachyonic potential and g the determinant of the metric. For a Friedmann-Robertson-Walker cosmology $g_{\mu\nu} = diag(1, -a(t)^2, -a(t)^2, -a(t)^2)$

equation of motion for the scalar field to be

$$\ddot{\varphi} = -(1-\alpha\dot{\varphi}^2)\left[\frac{1}{\alpha}\frac{d\ln V(\varphi)}{d\varphi} + 3H\dot{\varphi} - \frac{\beta\bar{\Psi}\Psi}{\alpha V(\varphi)}\sqrt{1-\alpha\dot{\varphi}^2}\right].$$

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Field Theory Description

with $H = \frac{\dot{a}}{a}$. We also have

$$\frac{d(a^3 \Psi^\dagger \Psi)}{dt} = 0 \,,$$
$$\frac{d(a^3 \bar{\Psi} \Psi)}{dt} = 0 \,.$$

From the latter, $\bar{\Psi}\Psi = \frac{\bar{\Psi}_0 \Psi_0 a_0^3}{a^3}$. Moreover,

$$\rho_\varphi = \frac{V(\varphi)}{\sqrt{1 - \alpha\dot{\varphi}^2}} \,,$$
$$P_\varphi = -V(\varphi)\sqrt{1 - \alpha\dot{\varphi}^2} \,,$$
$$\rho_\Psi = M^* \bar{\Psi}\Psi$$
$$P_\Psi = 0 \,,$$

$$\dot{\rho_\varphi} + 3H\rho_\varphi(\omega_\varphi + 1) = \beta\dot{\varphi}\frac{\bar{\Psi}_0 \Psi_0 a_0^3}{a^3}$$
$$\dot{\rho_\Psi} + 3H\rho_\Psi = -\beta\dot{\varphi}\frac{\bar{\Psi}_0 \Psi_0 a_0^3}{a^3} \,.$$

$$\omega_\varphi \equiv P_\varphi/\rho_\varphi = -(1 - \alpha\dot{\varphi}^2).$$

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Under the conformal transformation $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$

Background dynamics

f(R) gravity → **Einstein frame**

Second order equation

$$\frac{d^2}{dx^2}F + \left(\frac{1}{2}\frac{d\ln E}{dx} - 1\right)\frac{dF}{dx} + \frac{d\ln E}{dx}F + \frac{3\Omega_m^0 e^{-3x}}{E} = 0 \quad ,$$

$$\dot{\rho}_m + 3H\rho_m = 0 \quad ,$$

Two first order equations

$$\frac{d\tilde{H}}{d\tilde{x}} = \frac{\tilde{R}}{6\tilde{H}} - 2\tilde{H} \quad ,$$

$$\frac{d\tilde{R}}{d\tilde{x}} + 36\Gamma\frac{d\Gamma}{d\tilde{x}} + 72\Gamma^2 = -3\kappa^2\tilde{\rho}_m\left(\frac{\Gamma}{\tilde{H}} + 1\right)$$

$$\frac{d\tilde{\rho}_m}{d\tilde{x}} + 3\tilde{\rho}_m = \frac{\Gamma}{\tilde{H}}\tilde{\rho}_m \quad ,$$

$$\frac{d\tilde{\rho}_d}{d\tilde{x}} + 3(1+\tilde{w})\tilde{\rho}_d = -\frac{\Gamma}{\tilde{H}}\tilde{\rho}_m$$

$$\frac{d\ln F}{d\tilde{t}} = -2\Gamma,$$

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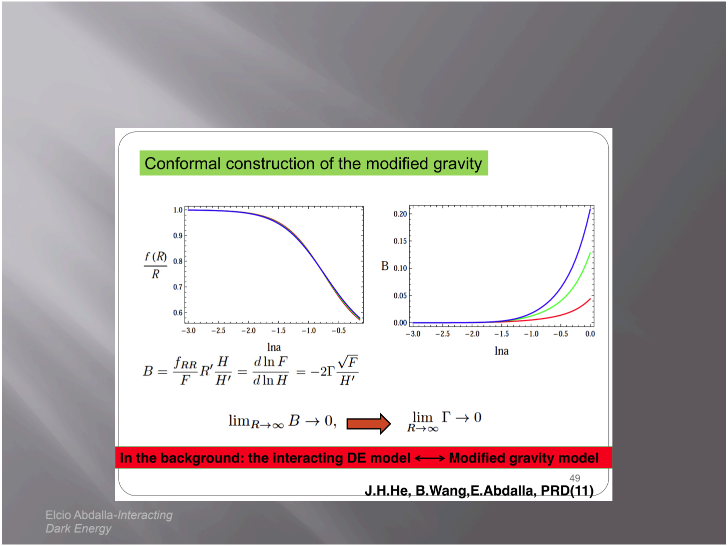
Conformal construction of the modified gravity
$\frac{f(R)}{R}$
lna
B
lna
$B = \frac{f_{RR}}{F} R' \frac{H}{H'} = \frac{d \ln F}{d \ln H} = -2\Gamma \frac{\sqrt{F}}{H'}$
$\lim_{R\to\infty} B \to 0,$ $\lim_{R\to\infty} \Gamma \to 0$
In the background: the interacting DE model ⟷ Modified gravity model
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J.H.He, B.Wang,E.Abdalla, PRD(11)
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J.H.He, B.Wang,E.Abdalla, PRD(11)

No interaction between DE&DM → Einstein Gravity

Interaction between DE&DM → Modified Gravity

Conformal transformation

Motion of particle with varying mass

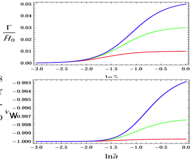
$$\Gamma = -\frac{d\ln\Omega}{d\tilde{t}} = \frac{1}{\tilde{m}}\frac{d\tilde{m}}{d\tilde{t}}$$

$$\frac{d\ln F}{d\tilde{t}} = -2\Gamma$$

freedom in choosing $\Gamma = \frac{\alpha}{\tilde{R}+\beta}$

$\lim_{\tilde{R}\longrightarrow\infty}\Gamma = 0$ and $\lim_{\tilde{R}\longrightarrow 0}\Gamma = \frac{\alpha}{\beta} < \infty$

Solving dynamics when there is interaction between DE&DM



We set the starting point at present, $\tilde{x} = 0$, with the initial conditions $\tilde{R}_0 = 12 - 3\tilde{\rho}^0_m - 18\Gamma_0^2$, $\tilde{\rho}^0_m = 3\tilde{\Omega}^0_m$ and $\tilde{H}_0 = 1$. In order to let our model fully return the standard Einstein gravity in the past, we need $F \to 1$, which is equivalent to setting the boundary condition $\lim_{\tilde{x}\longrightarrow -\infty}\varphi \to 0$.

$\tilde{\Omega}^0_m = 0.3, \tilde{\Omega}^0_d = 0.7, \alpha/\tilde{H}_0^3 = 1.2$

$\Gamma_0/\tilde{H}_0 = 0.01, \Gamma_0/\tilde{H}_0 = 0.03$, and $\Gamma_0/\tilde{H}_0 = 0.05$

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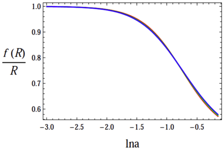
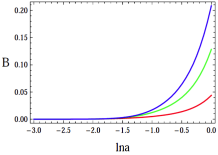
Conformal construction of the modified gravity





$$B = \frac{f_{RR}}{F} R' \frac{H}{H'} = \frac{d \ln F}{d \ln H} = -2\Gamma \frac{\sqrt{F}}{H'}$$

$$\lim_{R\to\infty} B \to 0, \quad \Longrightarrow \quad \lim_{R\to\infty} \Gamma \to 0$$

In the background: the interacting DE model ⟷ Modified gravity model

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Elcio Abdalla-*Interacting Dark Energy*

- Field Theory Models:
  - Tachyonic scalar field DE interacting with fermionic Dirac DM with Yukawa coupling and power-law potential– [S. Micheletti, EA, Bin Wang 2009].
  - Canonical Scalar field DE interacting with fermionic Dirac DM with exponential potential and varying fermion mass

$$m(\phi)=m_0 e^{-\kappa\beta(\phi)\phi}$$

[Wetterich 1995], [Amendola 2000], [Baldi 2009]

And the model we are interested in studying here:

- Canonical Scalar field DE interacting with fermionic DM (neutralino or gravitino in future work) interacting with an Yukawa coupling.

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Our model:

The Lagragian is given by:

$$\mathscr{L}(x)=\sqrt{-g}\{g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi+V(\phi)+\tfrac{i}{2}[\bar\psi\gamma^\mu D_\mu\psi-D_\mu\bar\psi\gamma^\mu\psi]-(M-g\phi)\bar\psi\psi\}$$

where we considered a spin ½ field, that could the neutralino with broken SUSY and *g* is the interaction constant.

- We chose the Yukawa coupling, being the simplest renormalizable interaction we could have between the fields.

The cosmological equations from this system are given by:

$$H^2=\frac{1}{3M_{pl}^2}\left[\frac{\dot\varphi^2}{2}+V(\varphi)+(M-g\varphi)\frac{\bar\Psi_0\Psi_0}{a^3}\right],$$
$$\dot H=-\frac{1}{3M_{pl}^2}\left[\dot\varphi^2+(M-g\varphi)\frac{\bar\Psi_0\Psi_0}{a^3}\right],$$
$$\ddot\varphi=-\left[3H\dot\varphi+V'-g\frac{\bar\Psi_0\Psi_0}{a^3}\right].$$

where we used current conservation to get $\frac{d(a^3\bar\Psi\Psi)}{dt}=0\Longrightarrow\bar\Psi\Psi=\frac{\Psi_0\Psi_0}{a^3}.$

For the gravitino, spin 3/2, we get a similar set of equations.

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- The set of cosmological equations obtainned with the inclusion of the interaction can be very complicated and non-solvable.
- We use many different techniques to solve this system, although some depend on the others:

  - Numerical methods;
  - Phase Space Analysis;
  - Other methods that reduce in a smart way the number of varibles, ...

Here, we will show two techniques used to study the previous model and test its validity:

- The First Order formalism;
- Phase Space Analysis (work in progress);

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Autonomous system, fixed points and stability

We need to examine the dynamical system properties in order to find viable cosmological solutions in the parameter space.

$$\dot{x} = f(x, y, t),$$
$$\dot{y} = g(x, y, t).$$

Cosmology: autonomous system – f and g do not depend explicitly on time

Analysis:

Fixed Points: (x_c, y_c) are *fixed points* or *critical points* if

$$(f, g)_{x_c, y_c} = 0$$

Attractor: $(x(t), y(t)) \to (x_c, y_c), \quad for\ t \to \infty$

Stability around the fixed points: Consider small perturbations around the fixed points

$$x = x_c + \delta x, \quad y = y_c + \delta y$$

Expanding f and g in Taylor expansion around the fixed points, we can write the linearized system

$$\frac{d}{dN}\begin{pmatrix}\delta x\\ \delta y\end{pmatrix} = M\begin{pmatrix}\delta x\\ \delta y\end{pmatrix}$$

Dark Energy

Models with coupled quintessence

- To exit the attractor scaling solution, one needs to change the slope of the potential.
- As an example, if we couple dark energy with the background fluid (that can be dark matter, for example), it is possible to get this accelerated expansion even with steep potentials.

$$\dot{\rho}_\phi + 3H(1+\omega_\phi)\rho_\phi = -Q\rho_m\phi,$$
$$\dot{\rho}_m + 3H(1+\omega_m)\rho_m = Q\rho_m\phi,$$
$$\dot{H} = \frac{-1}{2}\left[(1+\omega_m)\rho_\phi + (1+\omega_m)\rho_m\right],$$
$$H^2 = (\rho_\phi + \rho_m)/3 \,.$$

<u>Autonomous system:</u>

$$\frac{dx}{dN} = -3x + \frac{\sqrt{6}}{2}\lambda c y^2 + \frac{3}{2}x\left[(1-\omega_m)x^2 + (1+\omega_m)(1-c\,y^2)\right] - \frac{\sqrt{6}Q}{2}(1-x^2-y^2),$$
$$\frac{dy}{dN} = \frac{-\sqrt{6}}{2}\lambda \% x\, y + \frac{3}{2}y\left[(1-\omega_m)x^2 + (1+\omega_m)(1-cy^2)\right].$$

Luca Amendola, Miguel Quartin, Shinji Tsujikawa, and Ioav Waga, arXiv: astro-ph/0605428

Phase Space Analysis
(work in progress)

- For the model of canonical scalar field DE coupled to fermionic spin ½ DM
- Work in progress:

- We have been doing the analysis in the background;

- Need to find the right change of variables;

- -a nonrenormalizable action seems unavoidable; the only stable scalar potential without fine tuning are exponential or single power law

- Future work: Analysis of the stability of the perturbations in this class of models.

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Phenomenological Model

The interaction in the dark sector can be introduced through the energy-momentum tensor

$$\nabla_\mu T^{\mu\nu}_{(\lambda)} = Q^{\nu}_{(\lambda)}, \tag{1}$$

such that

$$\sum_\lambda Q^{\nu}_{(\lambda)} = 0. \tag{2}$$

Therefore, considering dark energy and dark matter as perfect fluids with energy-momentum tensor

$$T^{\mu\nu} = (\rho + P)U^\mu U^\nu + P g^{\mu\nu}, \tag{3}$$

we obtain in a FLRW background the conservation equations

$$\dot{\rho}_{dm} + 3H\rho_{dm} = Q^0 \quad (4)$$

$$\dot{\rho}_{de} + 3H\rho_{de}(1 + \omega_{de}) = -Q^0. \quad (5)$$

So, given an interaction, we can solve the homogeneous and isotropic Friedman equation

$$H^2(t) = \frac{8\pi G}{3}\rho(t) \quad (6)$$

Beyond homogeneity and isotropy, to study LSS and CMB anisotropies we introduce linear perturbations.

The general perturbed scalar metric can be written as

$$ds^2 = a^2 \left[-(1+2\psi) d\eta^2 + 2\partial_i B d\eta dx^i + (1+2\phi)\delta_{ij} dx^i dx^j + D_{ij} E dx^i dx^j \right], \quad (7)$$

where

$$D_{ij} = \left(\partial_i \partial_j - \frac{1}{3}\delta_{ij}\nabla^2 \right). \quad (8)$$

And the energy-momentum tensor is given by

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + \delta T^{\mu\nu} \tag{9}$$

where

$$\rho = \rho_{(0)} + \delta\rho \tag{10}$$
$$P = P_{(0)} + \delta P \tag{11}$$
$$U^{\mu} = U^{\mu}_{(0)} + \delta U^{\mu} \tag{12}$$

Using the gauge synchronous $\psi = B = 0$ the density contrast and peculiar velocity evolve as

$$\delta'_{dm} = -3k\phi' + 3\mathcal{H}\lambda_2 \frac{\rho_{de}}{\rho_{dm}}(\delta_{de} - \delta_{dm}) \quad (13)$$

$$\delta'_{de} = 3\mathcal{H}\omega\delta_{de} + 3\mathcal{H}\lambda_1 \frac{\rho_{dm}}{\rho_{de}}(\delta_{de} - \delta_{dm}) - (1 + \omega_{de})kV_{de} - 3\mathcal{H}c_e^2\delta_{de} - 3\mathcal{H}(c_e^2 - c_a^2)\left[3\mathcal{H}(1 + \omega_{de}) + 3\mathcal{H}\left(\lambda_1 \frac{\rho_{dm}}{\rho_{de}} + \lambda_2\right)\right]\frac{V_{de}}{k} - 3(1 + \omega_{de})k\phi' \quad (14)$$

$$V'_{de} = -\mathcal{H}(1 - 3c_e^2)V_{de} + \frac{3\mathcal{H}}{1 + \omega_{de}}(1 + c_e^2)\left(\lambda_1 \frac{\rho_{dm}}{\rho_{de}} + \lambda_2\right)V_{de} + \frac{kc_e^2\delta_{de}}{1 + \omega_{de}}. \quad (15)$$

Where we are taking a phenomenological model with interaction

$$aQ^0_{dm} = 3\mathcal{H}(\lambda_1\rho_{dm} + \lambda_2\rho_{de}) = -aQ^0_{de}, \tag{16}$$

and perturbed dark energy pressure

$$\delta P_{de} = c_e^2\delta_{de}\rho_{de} + (c_e^2 - c_a^2)\left[\frac{3\mathcal{H}(1+\omega_{de})V_{de}\rho_{de}}{k} - a^2 Q^0_{de}\frac{V_{de}}{k}\right]. \tag{17}$$

The perturbed Einstein equations give

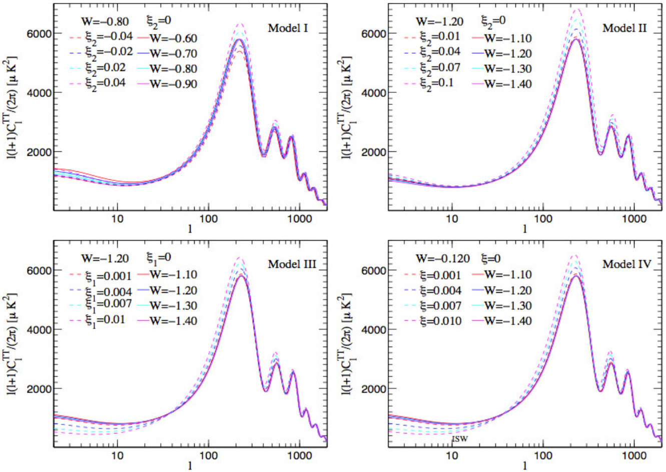
$$k^2\eta - \frac{1}{2}\frac{\dot{a}}{a}\dot{h} = -4\pi G a^2 \delta\rho, \tag{18}$$

$$k^2\dot{\eta} = -4\pi G a^2 (\rho_{(0)} + P_{(0)}) k^2 V, \tag{19}$$

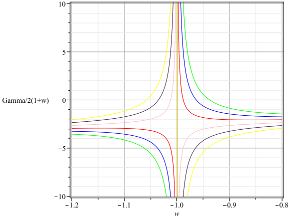
$$\ddot{h} + 2\frac{\dot{a}}{a}\dot{h} - 2k^2\eta = -8\pi G a^2 \delta P, \tag{20}$$

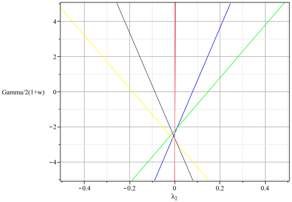
$$\ddot{h} + 6\ddot{\eta} + 2\frac{\dot{a}}{a}(\dot{h} + 6\dot{\eta}) - 2k^2\eta = -16\pi G a^2 P_{(0)} \Pi \tag{21}$$

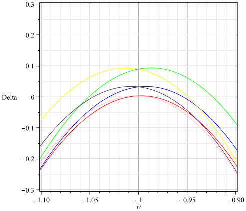
With these equations we can calculate the CMB and matter power spectra

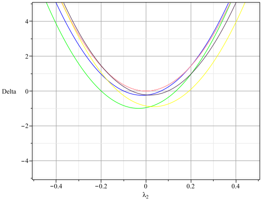


To calculate the CMB and matter power spectra and to compare with observations, we have to take into account that some regions of the parameters space can lead to instabilities.









Considering these instabilities we use the following priors

Parameters	Prior			
$\Omega_b h^2$	$[0.005, 0.1]$			
$\Omega_c h^2$	$[0.001, 0.5]$			
100θ	$[0.5, 10]$			
τ	$[0.01, 0.8]$			
n_s	$[0.9, 1.1]$			
$log(10^{10} A_s)$	$[2.7, 4]$			
	Model I	Model II	Model III	Model IV
ω	$[-1, -0.1]$	$[-2.5, -1]$	$[-2.0, -1]$	$[-2.0, -1]$
ξ	$[-0.4, 0]$	$[0, 0.4]$	$[0, 0.02]$	$[0, 0.02]$

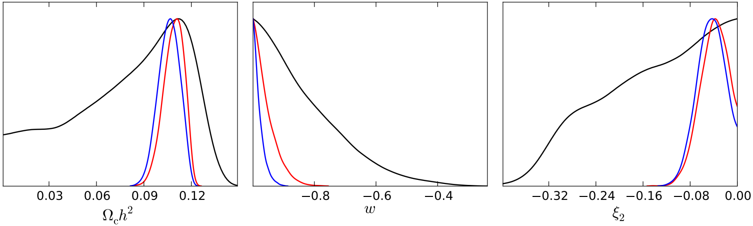


Tabela : Cosmological parameters - Model I.

Parameter	Planck		Planck+BAO		Planck+BAO+SNIa+H0	
	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits
$\Omega_b h^2$	0.02053	0.02033 ± 0.00028	0.02044	0.02064 ± 0.00025	0.02074	$0.02074^{+0.00023}_{-0.00025}$
$\Omega_c h^2$	0.114	$0.080^{+0.048}_{-0.021}$	0.1149	$0.1088^{+0.0077}_{-0.0054}$	0.1099	$0.1058^{+0.0074}_{-0.0063}$
H_0	63.8	$63.4^{+5.3}_{-3.8}$	66.6	$65.8^{+1.2}_{-1.0}$	67.1	67.1 ± 0.8
w	-0.976	< -0.799	-0.999	< -0.949	-0.995	< -0.976
ξ_2	-0.048	> -0.186	-0.017	$-0.041^{+0.029}_{-0.019}$	-0.033	$-0.045^{+0.027}_{-0.022}$
τ	0.066	0.076 ± 0.012	0.085	$0.080^{+0.011}_{-0.013}$	0.090	$0.081^{+0.012}_{-0.013}$
n_s	0.9257	$0.9331^{+0.0078}_{-0.0077}$	0.9420	0.9426 ± 0.0062	0.9445	$0.9449^{+0.0059}_{-0.0060}$
$\ln(10^{10} A_s)$	3.044	$3.064^{+0.024}_{-0.025}$	3.068	$3.062^{+0.023}_{-0.024}$	3.081	$3.061^{+0.024}_{-0.025}$

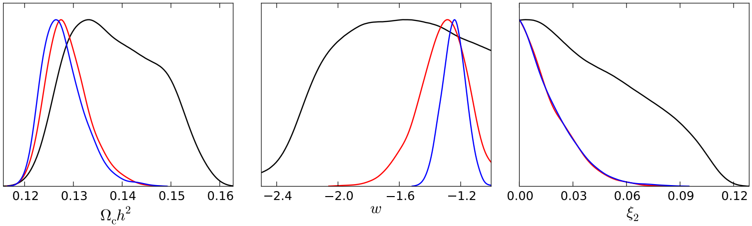


Tabela : Cosmological parameters - Model II.

Parameter	Planck		Planck+BAO		Planck+BAO+SNIa+H0	
	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits
Ω_b	0.01997	$0.02032^{+0.00027}_{-0.00028}$	0.02047	0.02043 ± 0.00025	0.02044	$0.02052^{+0.00024}_{-0.00026}$
Ω_c	0.1396	$0.1386^{+0.0079}_{-0.0111}$	0.1226	$0.1289^{+0.0033}_{-0.0050}$	0.1274	$0.1280^{+0.0029}_{-0.0052}$
H_0	83.0	$80.2^{+9.4}_{-14.3}$	71.8	$72.9^{+2.6}_{-4.0}$	71.9	$71.4^{+1.6}_{-1.5}$
w	-1.83	$-1.63^{+0.47}_{-0.36}$	-1.24	$-1.33^{+0.19}_{-0.13}$	-1.28	-1.25 ± 0.08
ξ_2	0.036	< 0.056	0.00005	< 0.020	0.010	< 0.021
τ	0.069	$0.077^{+0.011}_{-0.012}$	0.075	$0.079^{+0.011}_{-0.013}$	0.080	$0.080^{+0.011}_{-0.013}$
n_s	0.9235	$0.9319^{+0.0072}_{-0.0081}$	0.9314	0.9359 ± 0.0066	0.9323	$0.9380^{+0.0061}_{-0.0060}$
$\ln(10^{10} A_s)$	3.058	3.068 ± 0.023	3.054	3.066 ± 0.024	3.069	$3.066^{+0.023}_{-0.025}$

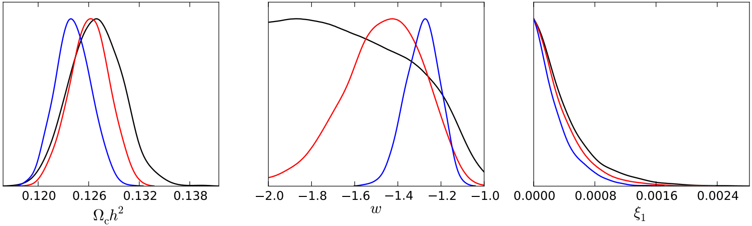


Tabela : Cosmological parameters - Model III.

Parameter	Planck		Planck+BAO		Planck+BAO+SNIa+H0	
	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits
Ω_b	0.02034	$0.0204^{+0.00028}_{-0.00029}$	0.02052	$0.02041^{+0.00026}_{-0.00028}$	0.02031	0.02055 ± 0.00026
Ω_c	0.1257	0.1269 ± 0.0031	0.1247	$0.1261^{+0.0023}_{-0.0024}$	0.1247	0.1242 ± 0.0021
H_0	83.3	$78.0^{+7.9}_{-8.2}$	75.4	$75.0^{+3.3}_{-4.3}$	71.4	$71.5^{+1.5}_{-1.6}$
w	-1.74	< -1.46	-1.44	$-1.47^{+0.22}_{-0.15}$	-1.30	$-1.29^{+0.09}_{-0.08}$
ξ_1	0.00007	< 0.00046	0.00024	< 0.00040	0.00005	< 0.00033
τ	0.070	$0.078^{+0.011}_{-0.013}$	0.080	$0.078^{+0.011}_{-0.013}$	0.073	$0.079^{+0.011}_{-0.013}$
n_s	0.9307	0.9316 ± 0.0073	0.9330	$0.9331^{+0.0064}_{-0.0063}$	0.9324	$0.9366^{+0.0063}_{-0.0064}$
$\ln(10^{10} A_s)$	3.050	$3.067^{+0.022}_{-0.025}$	3.065	$3.067^{+0.023}_{-0.024}$	3.050	$3.064^{+0.022}_{-0.026}$

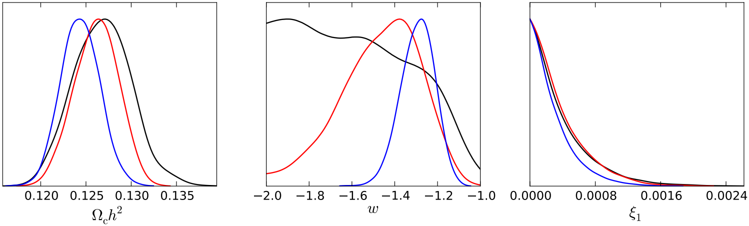
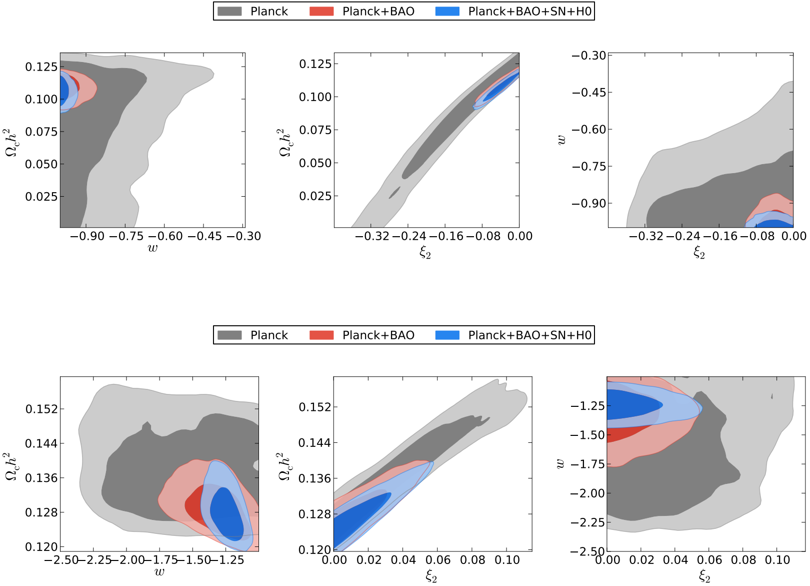
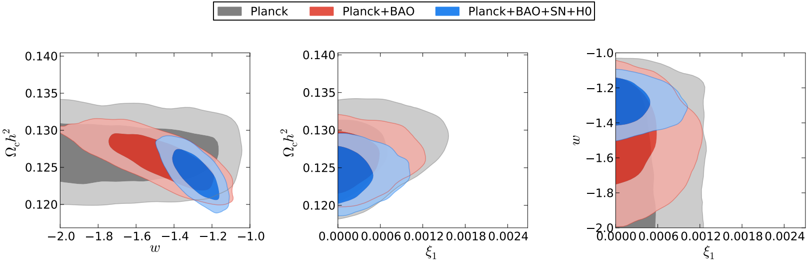


Tabela : Cosmological parameters - Model IV.

Parameter	Planck		Planck+BAO		Planck+BAO+SNIa+H0	
	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits
Ω_b	0.02017	0.02040 ± 0.00028	0.0203	$0.02043^{+0.00025}_{-0.00028}$	0.02035	0.02055 ± 0.00026
Ω_c	0.1266	$0.1268^{+0.0031}_{-0.0032}$	0.1252	$0.1261^{+0.0026}_{-0.0024}$	0.1248	0.1244 ± 0.0021
H_0	74.2	$78.0^{+8.0}_{-9.0}$	72.0	$74.9^{+3.1}_{-4.5}$	72.0	$71.5^{+1.4}_{-1.6}$
w	-1.45	< -1.45	-1.33	$-1.47^{+0.22}_{-0.15}$	-1.32	$-1.29^{+0.09}_{-0.07}$
ξ_1	0.00012	< 0.00043	6×10^{-6}	< 0.00042	0.00015	< 0.00033
τ	0.081	$0.078^{+0.012}_{-0.013}$	0.074	0.078 ± 0.011	0.075	$0.079^{+0.012}_{-0.013}$
n_s	0.9311	$0.9317^{+0.0075}_{-0.0073}$	0.9338	$0.9332^{+0.0065}_{-0.0064}$	0.9306	0.9367 ± 0.0060
$\ln(10^{10} A_s)$	3.074	$3.067^{+0.024}_{-0.025}$	3.058	$3.066^{+0.022}_{-0.023}$	3.055	3.064 ± 0.024


Planck
Planck+BAO
Planck+BAO+SN+H0
0.125
0.100
0.075
0.050
0.025
$\Omega_c h^2$
−0.90 −0.75 −0.60 −0.45 −0.30
w
−0.32 −0.24 −0.16 −0.08 0.00
ξ_2
−0.30
−0.45
−0.60
−0.75
−0.90
Planck
Planck+BAO
Planck+BAO+SN+H0
0.152
0.144
0.136
0.128
0.120
−2.50 −2.25 −2.00 −1.75 −1.50 −1.25
0.00 0.02 0.04 0.06 0.08 0.10
−1.25
−1.50
−1.75
−2.00
−2.25
−2.50


Planck
Planck+BAO
Planck+BAO+SN+H0
$\Omega_c h^2$
w
ξ_1
0.140
0.135
0.130
0.125
0.120
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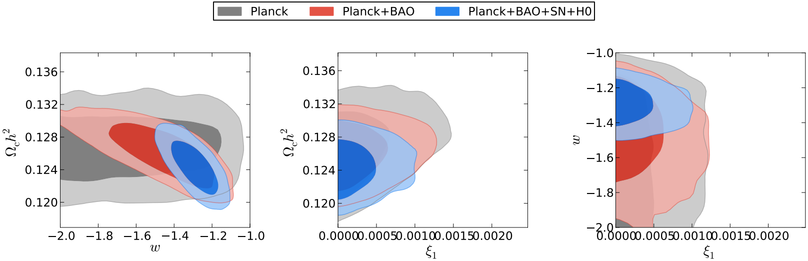
0.0024


Planck
Planck+BAO
Planck+BAO+SN+H0
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w
ξ_1
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−2.0
−1.8
−1.6
−1.4
−1.2
−1.0
0.0000
0.0005
0.0010
0.0015
0.0020

Conclusions

- About 95% of the content of the universe is unknown;
- The ΛCDM model is the simplest one coherent with the data, but it suffers from theoretical problems;
- Motivated by the *Coincidence Problem*, we can think that there is an interaction between dark energy and dark matter;
- Following the fluid equations we can construct a phenomenological interacting model, or we can introduce a more fundamental one from field theory.
- Until now these models can offer good agreement with the data.