Canonical Quantum Cosmology, The Problem of Time and Decoherent Histories

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- T. Christodoulakis & PW: J. Phys.: Conf. Ser. **283**, 012041 (2011)
- PW: Int. J. Theor. Phys. **47**, 1512 (2008).
- J.J. Halliwell & PW: PRD **73,** 024011 (2006).

Contents

- Quantum Cosmology & Interpretation

- Introduce the Problem of Time and motivate the construction of timeless theories.

- Introduce the decoherent histories approach to Quantum Theory.

- Examine the decoherent histories analysis of the Problem of Time.

- Construct rep-invariant Class Operators & get probabilities and decoherent conditions.

- Give an example of an FRW quantum cosmology with a scalar field

- Summary & Conclusion

Quantum Cosmology and Interpretation

- Quantum Cosmology: Full gravity assuming certain symmetries to reduce the physical degrees of freedom

- Twofold interest:

  1. Toy model for full gravity (shares similar problems)

  2. May resolve open cosmological questions (singularity resolution)

- Interpretation:

  1. Define physical Hilbert space (take into account Diffeo's and inner product)

2. Physical wavefunction (respecting diffeos and thus solutions of the Wheeler DeWitt equation)

3. Observables:

   (a) self-adjoint

   (b) commute with the constraints

   (c) correspond to intuitive physical questions

Problem of Time

Diffeomorphism invariance in GR Vs
Fixed parameter time in Newtonian Physics.

- Time in Quantum Theory:
  - Not Observable
  - Appears as a parameter
  - Physical clocks run backwards in abstract Newtonian Time

- Time in General Relativity:
  - How does ‘change’ appears?
  - Time is locally defined
  - How to make it compatible with QT that is based on Newtonian Time?

GR as Constrained System

The "gauge" in general relativity is the invariance of the theory under diffeomorphisms, Diff($\mathcal{M}$), which breaks into:

(a)Spatial "three"-dimensional diffeomorphisms. Variables could be the 3-metric (geometrodynamics), or loop variables

(b) *Hamiltonian* constraint: $\widehat{H}|\psi\rangle = 0$

• Observables commute with constraints

$$i\hbar \frac{d\widehat{A}(t)}{dt} = [\widehat{H}, \widehat{A}(t)] = 0$$

for any $\hat{A}$ observable, due to the constraint:

$\boxed{\text{Any observable } \hat{A}\text{, is independent of time!}}$

• General feature of ANY theory that has vanishing Hamiltonian

Timeless Theories

• Need to construct a Quantum Theory that time does not have any fundamental role.

• Time "emerges" as a coarse grained property of the relative field configurations.

> All physical questions can be translated to questions about the possible relative configurations of the universe and its material content.

(a) Evolving Constants

(b) Partial Vs Complete Observables

(c) Decoherent Histories

The Decoherent Histories Approach to QT

An alternative formulation of Quantum Theory designed to deal with closed systems. Among other things it aims to

(a) Assign probabilities to histories of closed system.

(b) Deal with time-extended questions.

(c) Put space and time in equal footing. Time is no longer in a preferred position, since we are dealing with whole histories of the system (rather than single time propositions).

Due it these facts, it suits well for dealing with the problem of time.

Decoherent Histories: Non-relativistic QM

Copenhagen probabilities for sequential measurements:

$P(\alpha_{t_1}$ at t_1 and α_{t_2} at $t_2 \cdots\ \alpha_{t_n}$ at t_n; $\rho(t_0))=$

$\mathrm{Tr}(\alpha_{t_n}(t_n)\cdots\alpha_{t_1}(t_1)\rho(t_0)\alpha_{t_1}(t_1)\cdots\alpha_{t_n}(t_n))$

This is NOT probability for closed system, fails to satisfy the "additivity of disjoint regions of the sample space", due to interference.

Under certain conditions this probability CAN be assigned to histories of closed systems.

Class operator: $C_{\underline{\alpha}} = \alpha_{t_n}(t_n)\cdots\alpha_{t_1}(t_1)$

Decohernece Functional (measures interference):

$\mathcal{D}(\underline{\alpha}, \underline{\alpha}') = \mathrm{Tr}(C_{\underline{\alpha}} \rho C^\dagger_{\underline{\alpha}'})$

A set of histories $\{\underline{\alpha}_i\}$, that is *disjoint* and *exhaustive* is called *complete*.

Probabilities are assigned to a history α_i, provided it belongs to a complete set such that:

$\mathcal{D}(\underline{\alpha}_i, \underline{\alpha}_j) = 0$, for all $i \neq j$.

The probability is then $p(\underline{\alpha}_i) = \mathcal{D}(\underline{\alpha}_i, \underline{\alpha}_i)$.

- Typically, there exist more than one complete set that obeys the decoherence condition. There is some interpretational ambiguity.

- The above can be generalized (relativistic QT or quantum gravity). Construction of (a) Class Operators that correspond to physical questions, and (b) an inner product to define probabilities and the decoherence condition.

Decoherent Histories and the Problem of Time

Histories and Classical Timeless Questions:

> Does a (classical) trajectory cross a given region Δ of the configuration space? If it is the full trajectory, then this is indeed reparametrization invariant.

In the Quantum Case, we require also:

(i) Initial state has to obey: $\widehat{H}|\psi\rangle = 0$

(ii) Class operator: $[\widehat{C}_\alpha, H] = 0$

(iii) We have to use the *induced* (or Rieffel) inner product. (essentially an inner product defined on solutions of the constraint.)

Proposed Class Operator

> What is the probability that the system crosses region Δ of configuration space, with no reference in time.

Need to find a Class Operator (CO) that commutes with the Hamiltonian and gives (semi-classically) sensible results.

Since the classical reparametrization invariant object is full trajectory we consider the unphysical parameter time running from $-\infty$ to $+\infty$.

CO Crossing $\Delta = 1$- CO Always in $\bar{\Delta}$

$$C_{\bar{\Delta}} = \prod_{t=-\infty}^{t=+\infty} \bar{P}(t)$$

$$[C_{\bar{\Delta}}, H] = 0$$

$$C_{\bar{\Delta}} = \lim_{t'' \to \infty, t' \to -\infty} \exp(-iHt'') g_r(t'', t') \exp(iHt')$$

$$C_{\Delta} = \mathbf{1} - C_{\bar{\Delta}}$$

This expression resembles the arrival time problem in standard non-relativistic quantum mechanics (see J.J. Halliwell & E. Zafiris in PRD also PW in IJTP and recently Halliwell & Yearsley).

General No-Crossing Probabilities and D.Condition

Using the property of the restricted propagator

$$g_r^\dagger(t,t_0)g_r(t,t_0) = \bar{P} \tag{1}$$

we have (candidate) probability for not crossing:

$p_{\bar{\Delta}} = \langle\psi|C_{\bar{\Delta}}^\dagger C_{\bar{\Delta}}|\psi\rangle = \langle\psi|\bar{P}|\psi\rangle$

$p_{\Delta} = 1 - p_{\bar{\Delta}} = \langle\psi|P|\psi\rangle$

Provided we have *decoherence*, while as decoherence condition we get:

$$\lim_{t\to\infty, t_0\to-\infty} e^{iE(t-t_0)}\langle\psi|g_r(t,t_0)|\psi\rangle = \langle\psi|\bar{P}|\psi\rangle$$

which becomes the need to vanish on the boundary of the region, i.e.:

$\langle x|\psi\rangle = 0, \quad \forall x \in \partial\bar{\Delta}$

- The decoherence condition is <u>Ψ vanishes on the boundary</u> and in this case

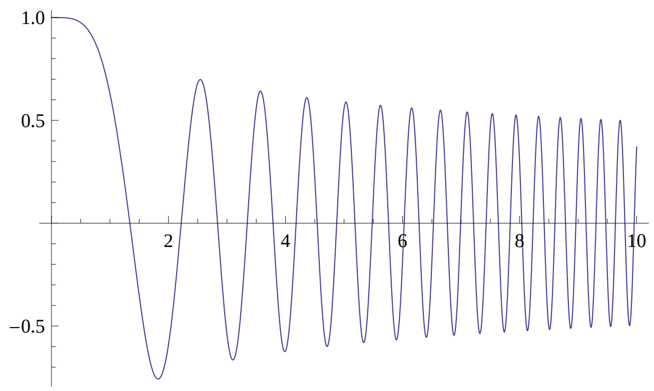
- We assign probability for crossing the region Δ the part of the wavefunction projected at the region in question

Model: FRW-QC

Homogeneous and isotropic universe with $k = 1$ (3-sphere). First (a) empty and then (b) with a scalar field with potential $V(\phi) = e^{2\phi}$. In both cases, we use quantum geometrodynamics. Loop QC analysis could be done in future work.**First case:**

$$\Psi''(\alpha)/4\alpha - \Psi'(\alpha)/8\alpha^2 + \alpha\Psi(\alpha) = 0$$

The solutions are Bessel functions we take one of them, $\Psi(\alpha) \propto \alpha^{3/4} J_{-\frac{2}{3}}(\alpha^2)$ with graph



And can ask the probability that it never crosses the region $\alpha > 6$. It coincides with a zero of the Bessel function and thus decoheres. The probability turns out to be $p_c \simeq 0.27$

Second case (with scalar field). Wheeler De-Witt equation

$$(2\alpha - \alpha^3 e^{2\phi})\Psi(\alpha,\phi) - \frac{1}{2\alpha}\frac{\partial^2\Psi(\alpha,\phi)}{\partial\alpha^2}$$
$$-\frac{1}{2\alpha^2}\frac{\partial\Psi(\alpha,\phi)}{\partial\alpha} + \frac{1}{2\alpha^3}\frac{\partial^2\Psi(\alpha,\phi)}{\partial\phi^2} = 0$$

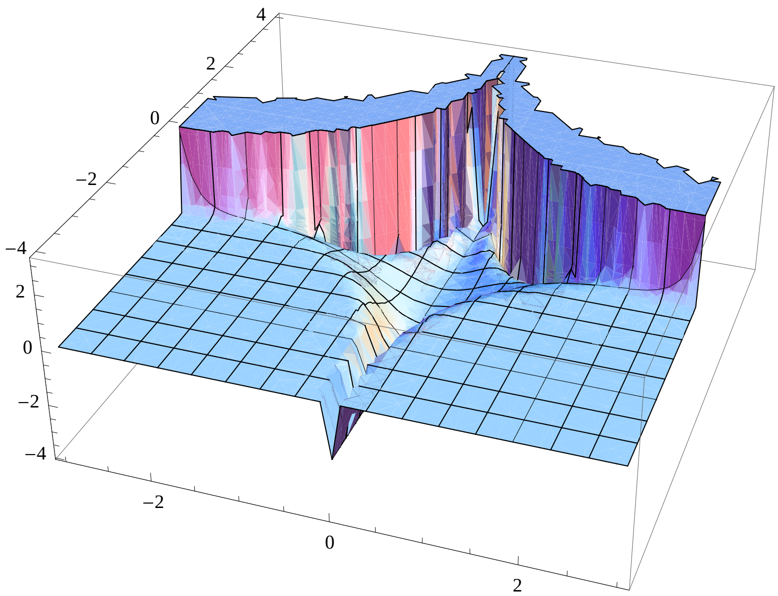
The general solution is

$$\Psi(\alpha,\phi) = C_2 \exp \frac{\alpha^2 e^{-2\phi}(-4C_1^2 - 4e^{4\phi} + 6e^{6\phi}\alpha^2)}{8C_1}$$

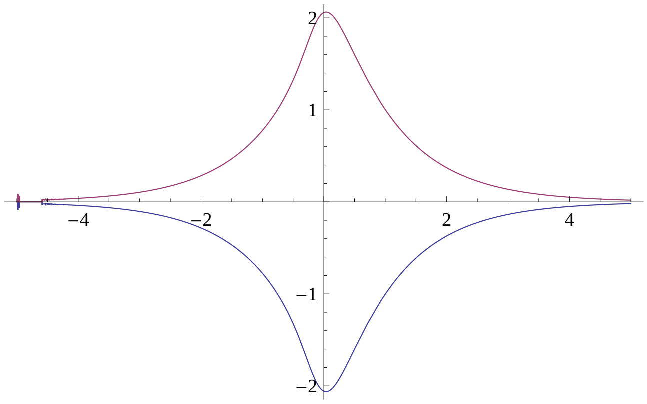
Consider one solution (note that it is NOT normalizable in the normal inner product, and we need to use an inner product on solutions)

$$\Psi(\alpha,\phi) = \exp\frac{\alpha^2 e^{-2\phi}(-4^2 - 4e^{4\phi} + 6e^{6\phi}\alpha^2)}{8} - \exp\frac{\alpha^2 e^{-2\phi}(-4\cdot 1.5^2 - 4e^{4\phi} + 6e^{6\phi}\alpha^2)}{12}$$

Which looks like



We can ask which is the probability that the universe never crosses the region defined by:



Since our solution vanish at the above boundary, we have decoherence and we can assign the crossing probability (note that the integral $|\Psi(\alpha, \phi)|^2$ at the region is no-zero, however in the induced inner product it results to zero probability) $p_c = 0$.

Other solutions (involving superpositions) or systems with more degrees of freedom (e.g. Bianchi Cosmologies, different matter content), result to non-trivial questions.

Choose questions like "value(s) of α when $\phi = 15$ or when $\phi > 15$". This corresponds to an observable that projects at the range of ϕ in question. The resulting operator does NOT commute with Hamiltonian. Fails only on the boundary of the region.

If one restricted attention to a single solution (not formally allowed at this approach) that vanishes at this boundary, he would recover exactly our result.

- We followed the general prescription for constructing class operators that was described earlier.

- For particular models-examples it is possible to construct meaningful class operators in different ways (e.g. D. Craig and P. Singh, PRD82, 123526 (2010)) For those models it is possible to answer questions involving the existence of singularity.

-In the model examined, the class operators constructed was not decoherent for the question involving the existence of singularity.

Summary & Conclusions

• We examined the DH analysis of timeless QT. We got Class Operators that respect the Hamiltonian constraint.

• They consisted of a general enough set of physical questions of the type:

"Which is the prob that it crosses a region in configuration space with no reference in time"

• We have got an easy but restrictive decoherence condition "The initial state has to vanish on the boundary of the region considered"

• The probabilities for those histories are easily calculated.

• We considered as an example the case of FRW Quantum Cosmology with scalar field. Given a solution of the Wheeler-DeWitt equation we found questions that can be answered and compared with other works.