A Cosmological Model for 3rd Quantisation

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Outline

Third Quantisation

In quantum field theory one sometimes refers to *second quantisation*: The Hamiltonian (constraint) of a classical system is first viewed as an operator acting on a wavefunction (encoding the physics of a single classical system), which then also becomes an operator acting on multi-particle states.

$$p^2 + m^2 = 0 \quad \rightarrow \; \left(\Delta - m^2\right)\psi(x) = 0 \quad \rightarrow \; \left(\Delta - m^2\right)\hat{\Psi}(x) = 0.$$

In the field theory it is then natural to add interactions to the free part.

For the relativistic particle, one motivation for that is the difficulties of formulating a consistent one-particle theory (definition of positive definite inner product. . .)

Third quantisation is the same idea for gravity, where already the classical system is a field theory. There the Hamiltonian constraint is also quadratic in momenta. "Particles" would correspond to "universes".

Third Quantisation (II)

Possible motivations:

- Conceptual difficulties with the interpretation of the single-particle theory? (Kuchař)
- Possible mechanism for topology change (through interactions)
- *Group field theories* are QFTs that generate spinfoam amplitudes in their Feynman expansion. They can be viewed as a second quantisation of the geometry of an n-simplex (see talks by Sindoni, Carrozza).

Our goal will be to investigate all these issues in the context of (loop) *quantum cosmology*.

Outline

Minisuperspace Models

One restricts the classical system to homogeneous, isotropic universes, here spatially flat and with a massless scalar field, with constraint (Friedmann eq.)

$$C := \frac{2\pi G}{3}\frac{p_a^2}{a} - \frac{p_\phi^2}{2a^3}(\stackrel{!}{=} 0)$$

In quantisation à la LQC, one uses kinematical input from LQG to obtain (see other talks, $\nu \sim a^3$ corresponds to 3-volume)

$$\hat{C}\psi(\nu,\phi) = -B(\nu)(\Theta + \partial_\phi^2)\psi(\nu,\phi) = 0$$

which will become the free field equation in our field theory model. $\psi(\nu,\phi)$ will become a quantum field creating a "universe".

Outline

Definition of the Model

Assuming interactions local in ν and ϕ, as seems natural, our action is (for a real scalar field Ψ)

$$S[\Psi] = \frac{1}{2}\sum_{\nu}\int d\phi\,\Psi(\nu,\phi)\hat{C}\Psi(\nu,\phi) + \sum_{i=1}^{n}\frac{\lambda_i}{n!}\sum_{\nu}\int d\phi\,\Psi^i(\nu,\phi)$$

This restricts $\hat{C}$ to be self-adjoint w.r.t. the kinematical inner product of LQC; if for instance we try to enforce $B(\nu) = 1$ we have to take

$$\hat{C} = -\frac{1}{2}(\Theta + \Theta^\dagger) - \partial_\phi^2,$$

which in general leads to a different classical field equation.

The interaction term can be interpreted as generating topology change, or inhomogeneities within the same universe (work in progress).

Outline

Fock Space Interpretation

Originally, a multi-particle state $|\alpha_1; \ldots; \alpha_n\rangle$ would correspond to n disconnected homogeneous and isotropic universes.

However, such a picture is used in cosmology (*separate universe approach*) to describe a single universe to first order in a long wavelength expansion, where quantities are averaged over scales larger than the Hubble radius.

Then we also need to encode geometric (diffeomorphism-invariant) data giving the relative positions of different patches, etc. Use Fock space statistics for this??

Once this is achieved our model could make predictions for the presence or absence of inhomogeneities in the universe.

Outline

Physical Properties?

Questions we would like to investigate in the model:

- Role of the coupling constant(s) λ? Relation to group field theory coupling constant?
- Effective contributions to the Hamiltonian constraint (mean field approximation, one-loop effective action. . .)
- Many-particle states as describing one inhomogeneous universe $\to$ implications for GFT?
- Conceptual insights for GFT/general 3rd quantisation ideas?
- . . .

Outline

Summary

- we are proposing a model extending loop quantum cosmology by allowing for creation and annihilation of "universes", incorporating topology change
- in this model questions coming both from LQC and from GFTs could be analysed from a slightly different perspective
- physical predictions could include the creation of inhomogeneities in the universe, and effective contributions to the Hamiltonian constraint (acting as a cosmological constant? what would be its value? . . .)
- concrete conclusions in progress

Thank you!