Lorentz Violating Massive Gravity

Bertha Cuadros-Melgar

Department of Physics, National Technical University of Athens, Greece
&
Physics Department, University of Buenos Aires, Argentina

In collaboration with E. Papantonopoulos, M. Tsoukalas, V. Zamarias
[arXiv:1108.3771]

Motivation

♣ Hořava-Lifshitz gravity:

- Breaks dipheomorphism invariance explicitly.
- Preferred foliation splitting spacetime into space and time.
- It lets add higher order spatial derivatives of the metric $\longrightarrow$ improved UV behaviour, power-counting renormalisable theory.

♣ Problems:

- New scalar degree of freedom in addition to the massless graviton.
- Projectable version $\longrightarrow$ Unstable scalar or ghost.
- Non-projectable version $\longrightarrow$ Strong coupling, fast instabilities.
- Extended version $\longrightarrow$ For a certain UV scale, there is no strong coupling; however, limit $\lambda \longrightarrow 1$ is still problematic.

♣ Problems:

- New scalar degree of freedom in addition to the massless graviton.
- Projectable version $\longrightarrow$ Unstable scalar or ghost.
- Non-projectable version $\longrightarrow$ Strong coupling, fast instabilities.
- Extended version $\longrightarrow$ For a certain UV scale, there is no strong coupling; however, limit $\lambda \longrightarrow 1$ is still problematic.

♣ Our proposal: Add Lorentz violating 'mass' terms.

♣ Problems:

- New scalar degree of freedom in addition to the massless graviton.
- Projectable version $\longrightarrow$ Unstable scalar or ghost.
- Non-projectable version $\longrightarrow$ Strong coupling, fast instabilities.
- Extended version $\longrightarrow$ For a certain UV scale, there is no strong coupling; however, limit $\lambda \longrightarrow 1$ is still problematic.

♣ Our proposal: Add Lorentz violating 'mass' terms.

♣ Result: The theory in its scalar sector has normal quadratic and cubic Lagrangians, no ghosts, no tachyons, and no strong coupling.

Extended Hořava-Lifshitz Gravity + 'Massive' Term

The action of Extended HL Gravity is given by [**D.Blas, O.Pujolas, and S.Sibiryakov, Phys.Rev.Lett.104, 181302 (2010). A.Papazoglou and T.P.Sotiriou, Phys.Lett.B685, 197 (2010).**]

$$S = \frac{M_{Pl}^2}{2} \int d^3x dt N \sqrt{g} \left(K^{ij} K_{ij} - \lambda K^2 + R + \eta a_i a^i \right) , \qquad (1)$$

where

$$a_i \equiv \frac{\partial_i N}{N} \qquad (2)$$

Extended Hořava-Lifshitz Gravity + 'Massive' Term

The action of Extended HL Gravity is given by [**D.Blas, O.Pujolas, and S.Sibiryakov, Phys.Rev.Lett.104, 181302 (2010). A.Papazoglou and T.P.Sotiriou, Phys.Lett.B685, 197 (2010).**]

$$S = \frac{M_{Pl}^2}{2} \int d^3x dt N \sqrt{g} \left(K^{ij} K_{ij} - \lambda K^2 + R + \eta a_i a^i \right) , \tag{1}$$

where

$$a_i \equiv \frac{\partial_i N}{N} \tag{2}$$

We add a combination of metric fields which upon linearization over $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, is [**B.C-M., E.Papantonopoulos, M.Tsoukalas, V.Zamarias, arXiv:1108.3771**]

$$S_m = \frac{M_{Pl}^2}{4} \int d^3x dt N \sqrt{g} \left(c_0 h_{00} h^{00} - 2c_1 h_{0i} h^{0i} - c_2 h_{ij} h^{ij} + c_3 h_i^i h_j^j + 2c_4 h_0^0 h_i^i \right) . \tag{3}$$

Studying the graviton scalar mode,

$$N = e^{\alpha(t,x)}, \quad N_i = \partial_i \beta(t,x), \quad g_{ij} = e^{2\zeta(t,x)}\delta_{ij}\,. \tag{4}$$

Studying the graviton scalar mode,

$$N = e^{\alpha(t,x)}, \quad N_i = \partial_i \beta(t,x), \quad g_{ij} = e^{2\zeta(t,x)}\delta_{ij} \,. \tag{4}$$

Adding (1) and (3) we obtain the quadratic Lagrangian,

$$\begin{aligned} \mathcal{L}_2 \;=\; & \frac{3}{2}(1-3\lambda)\dot{\zeta}^2 - (1-3\lambda)\dot{\zeta}\,\Delta\beta + \frac{1}{2}(1-\lambda)(\Delta\beta)^2 + (\partial_i\zeta)^2 - 2\alpha\,\Delta\zeta - \\ & -\frac{\eta}{2}\alpha\,\Delta\alpha + m_0^2\alpha^2 - \frac{1}{2}m_1^2\beta\,\Delta\beta - 3(m_2^2 - 3m_3^2)\zeta^2 + 6m_4^2\alpha\zeta \,. \end{aligned} \tag{5}$$

The momentum and hamiltonian constraints are

$$\beta = \frac{(1-3\lambda)}{(1-\lambda)\Delta - m_1^2}\dot{\zeta}\,, \tag{6}$$

$$\alpha = \frac{2(\Delta - 3m_4^2)}{-\eta\Delta + 2m_0^2}\zeta\,. \tag{7}$$

The momentum and hamiltonian constraints are

$$\beta = \frac{(1-3\lambda)}{(1-\lambda)\Delta - m_1^2}\dot{\zeta}\,, \qquad (6)$$

$$\alpha = \frac{2(\Delta - 3m_4^2)}{-\eta\Delta + 2m_0^2}\zeta\,. \qquad (7)$$

Thus,

$$\mathcal{L}_2 = \dot{\zeta}\left[\frac{1}{2}(1-3\lambda)\left(3 - \frac{(1-3\lambda)\Delta}{(1-\lambda)\Delta - m_1^2}\right)\right]\dot{\zeta} - -\zeta\left[\Delta + \frac{2(\Delta - 3m_4^2)^2}{-\eta\Delta + 2m_0^2}\right]\zeta - 3(m_2^2 - 3m_3^2)\zeta^2\,. \qquad (8)$$

The dispersion relation turns out to be

$$\omega^2 = \frac{Q(p^2)}{P(p^2)} + \frac{3(m_2^2 - 3m_3^2)}{P(p^2)}\,, \tag{9}$$

with

$$P(p^2) \quad = \quad \frac{1}{2}\,\frac{(1-3\lambda)(2p^2+3m_1^2)}{(1-\lambda)p^2+m_1^2}\,, \tag{10}$$

$$Q(p^2) \quad = \quad \frac{(2-\eta)p^4 - 2(m_0^2 - 6m_4^2)p^2 + 18m_4^4}{\eta p^2 + 2m_0^2}\,. \tag{11}$$

Kinetic Part of the Lagrangian

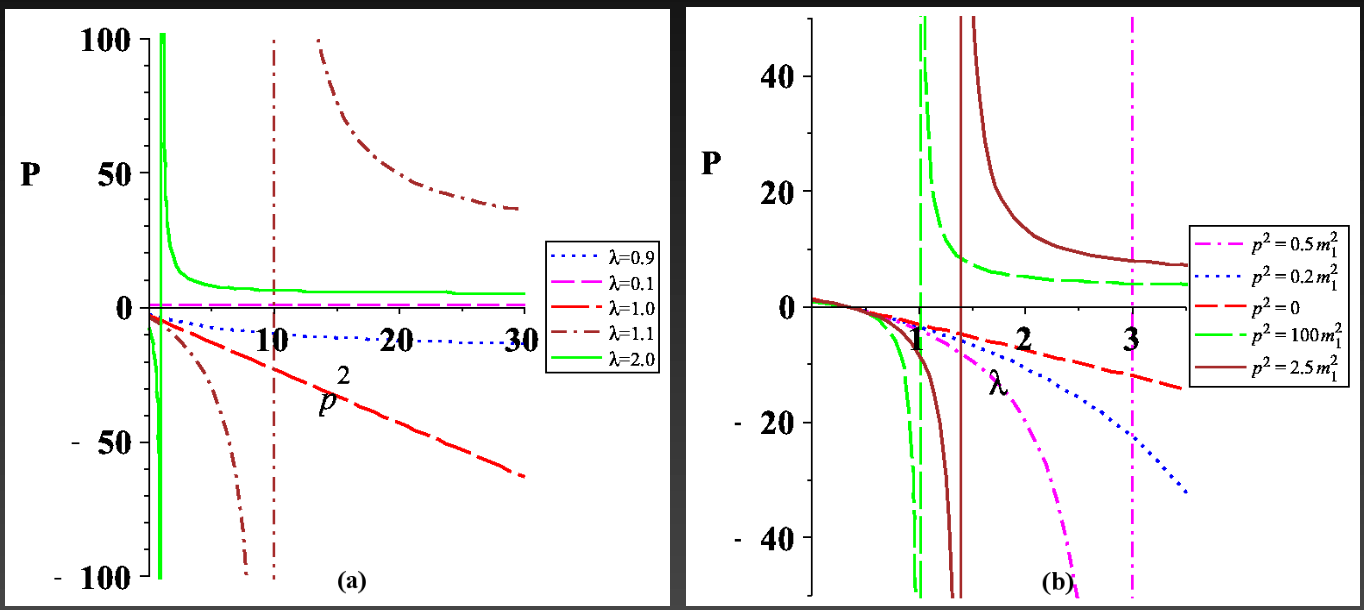

Figure 1: $P(p^2)$ as a function of p^2 and λ.

⋆ If $\frac{1}{3} < \lambda < 1$, $P < 0$. If $\lambda < \frac{1}{3}$, $P > 0$. For $\lambda > 1$, $P > 0$ for high p.

Kinetic Part of the Lagrangian

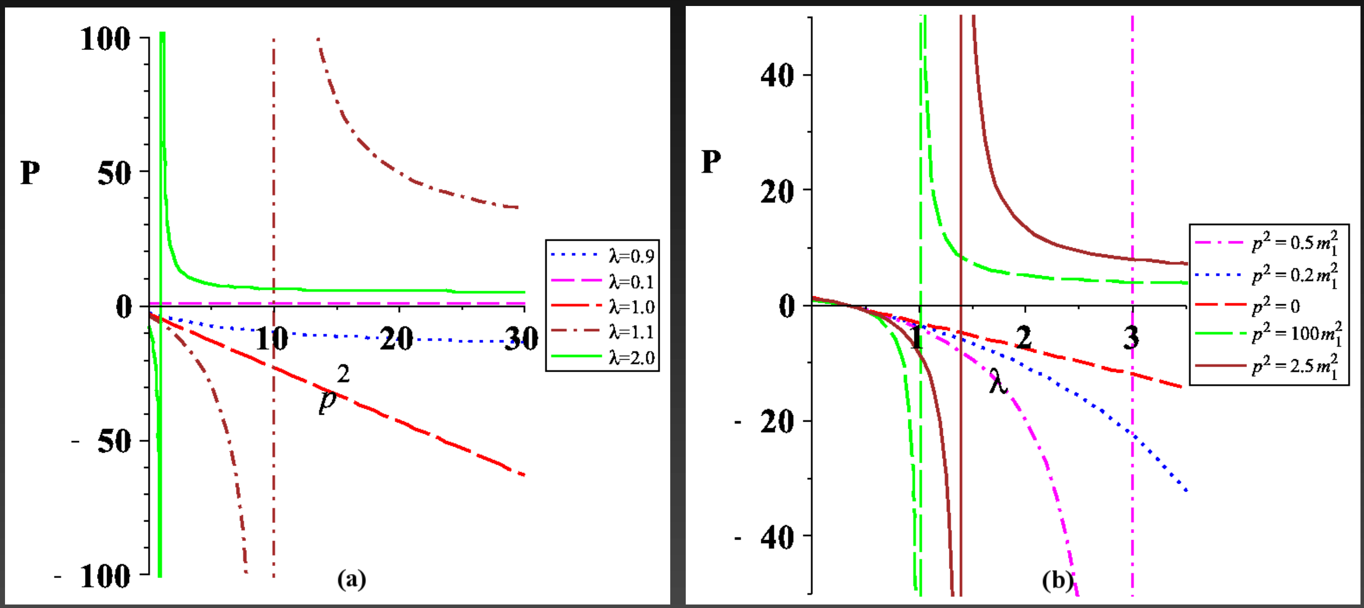

Figure 1: $P(p^2)$ as a function of p^2 and λ.

- ⋆ If $\frac{1}{3} < \lambda < 1$, $P < 0$. If $\lambda < \frac{1}{3}$, $P > 0$. For $\lambda > 1$, $P > 0$ for high p.
- ⋆ Experimental constraint: $0 < \lambda - 1 \lesssim 0.1$.

Dispersion Relation: ω^2

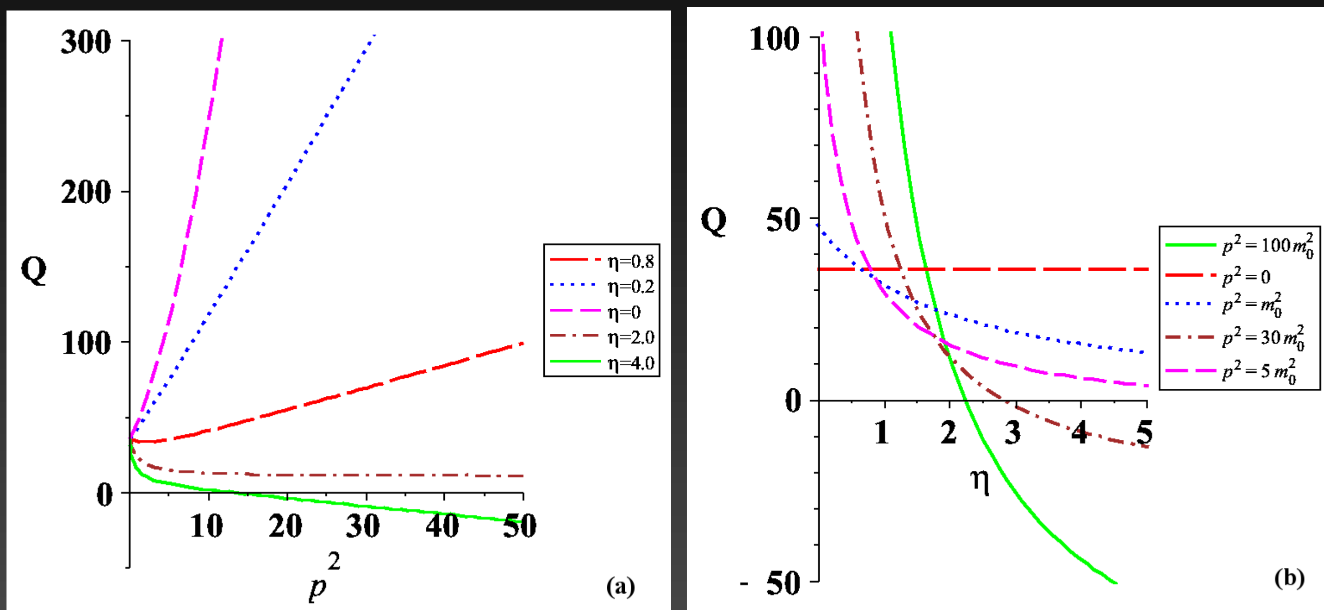

Figure 2: $Q(p^2)$ as a function of p^2 and η.

- ⋆ For $0 < \eta \leq 2$, $Q > 0$.

Dispersion Relation: ω^2

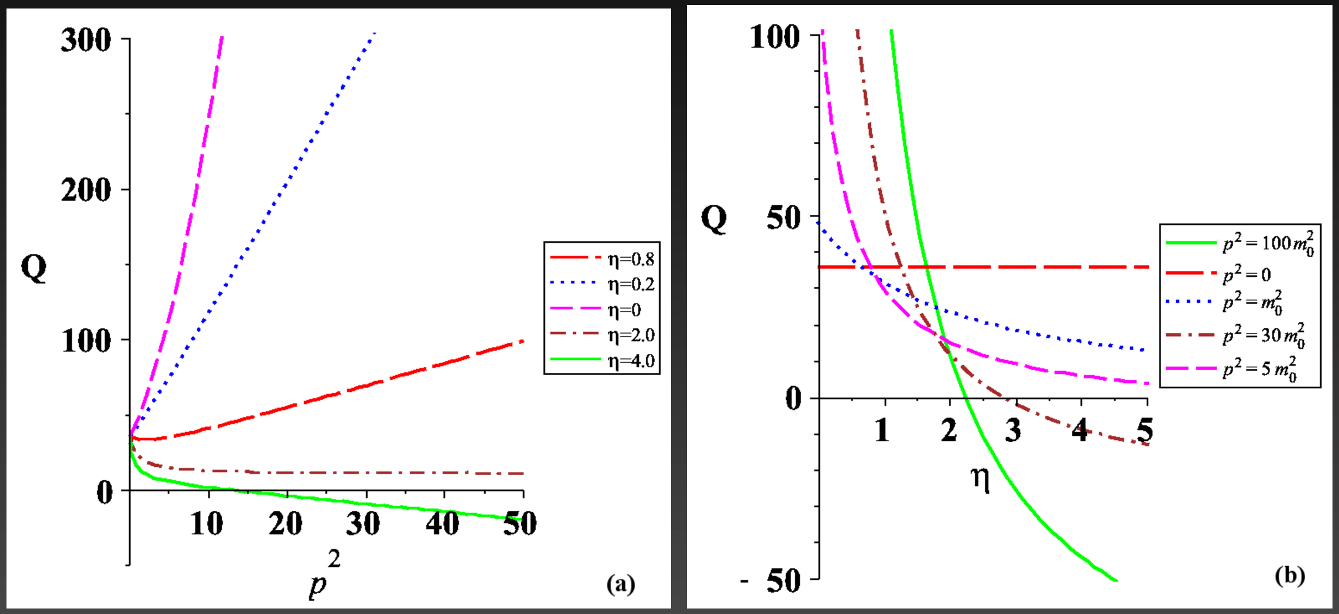

Figure 2: $Q(p^2)$ as a function of p^2 and η.

- For $0 < \eta \leq 2$, $Q > 0$.
- When $\eta > 2$, $Q < 0$ for high momenta. As η grows, $Q > 0$ as long as $p^2 \sim m_0^2$.

ζ Rest mass

$$M_\zeta^2 = \frac{2}{3\lambda - 1}\left[-3\frac{m_4^4}{m_0^2} - (m_2^2 - 3m_3^2)\right]. \tag{12}$$

ζ Rest mass

$$M_\zeta^2 = \frac{2}{3\lambda - 1}\left[-3\frac{m_4^4}{m_0^2} - (m_2^2 - 3m_3^2)\right]. \qquad (12)$$

⋆ If $m_2^2 - 3m_3^2 \geq 0 \Rightarrow \omega^2 > 0$. Not compatible.

ζ Rest mass

$$M_\zeta^2 = \frac{2}{3\lambda - 1}\left[-3\frac{m_4^4}{m_0^2} - (m_2^2 - 3m_3^2)\right] . \qquad (12)$$

- ⋆ If $m_2^2 - 3m_3^2 \geq 0 \Rightarrow \omega^2 > 0$. Not compatible.
- ⋆ If $m_2^2 - 3m_3^2 < 0$, we need $Q > 3(3m_3^2 - m_2^2)$. And $3m_3^2 - m_2^2 > 3\frac{m_4^4}{m_0^2}$.

Cubic Part of the Lagrangian

Adding a cubic term,

$$\mathcal{L}_m^c = N\sqrt{g}\left(\xi_1 h_{0i} h^{0i} h_0^0 + \xi_2 h_{0i} h^{0i} h_k^k + \xi_3 h_{ij} h^{0j} h_0^i\right) \tag{13}$$

Cubic Part of the Lagrangian

Adding a cubic term,

$$\mathcal{L}_m^c = N\sqrt{g}\left(\xi_1 h_{0i}h^{0i}h_0^0 + \xi_2 h_{0i}h^{0i}h_k^k + \xi_3 h_{ij}h^{0j}h_0^i\right) \tag{13}$$

Then,

$$\mathcal{L} = \mathcal{L}_{EHL} + \mathcal{L}_m + \mathcal{L}_m^c\,. \tag{14}$$

Cubic Part of the Lagrangian

Adding a cubic term,

$$\mathcal{L}_m^c = N\sqrt{g}\left(\xi_1 h_{0i} h^{0i} h_0^0 + \xi_2 h_{0i} h^{0i} h_k^k + \xi_3 h_{ij} h^{0j} h_0^i\right) \tag{13}$$

Then,

$$\mathcal{L} = \mathcal{L}_{EHL} + \mathcal{L}_m + \mathcal{L}_m^c \,. \tag{14}$$

Substituting momentum and hamiltonian constraints, our result shows the following operators,

$$\frac{(1-3\lambda)\,\mathcal{O}}{(1-\lambda)\Delta - m_1^2}\,, \quad \frac{(\Delta - 3m_4^2)\,\mathcal{O}}{-\eta\Delta + 2m_0^2}\,. \tag{15}$$

Clearly, when $\lambda \longrightarrow 1$, (14) is regular $\Rightarrow$ Theory does not have a strong coupling problem at the cubic order.

Summary

♣ We began with the extended Hořava-Lifshitz gravity model and added a Lorentz violating 'mass' term.

Summary

♣ We began with the extended Hořava-Lifshitz gravity model and added a Lorentz violating 'mass' term.

♣ Analysing the scalar sector of the graviton we found that under certain conditions on the parameters ('mass', η and λ couplings) the quadratic Lagrangian displays no ghosts or tachyons.

Summary

♣ We began with the extended Hořava-Lifshitz gravity model and added a Lorentz violating 'mass' term.

♣ Analysing the scalar sector of the graviton we found that under certain conditions on the parameters ('mass', η and λ couplings) the quadratic Lagrangian displays no ghosts or tachyons.

♣ The experimental constraint pushes λ very near to 1.

Summary

♣ We began with the extended Hořava-Lifshitz gravity model and added a Lorentz violating 'mass' term.

♣ Analysing the scalar sector of the graviton we found that under certain conditions on the parameters ('mass', η and λ couplings) the quadratic Lagrangian displays no ghosts or tachyons.

♣ The experimental constraint pushes λ very near to 1.

♣ The cubic Lagrangian has no strong coupling problem when taking the limit $\lambda \to 1$.