Chiral vacuum fluctuations in quantum gravity

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Outline

Testing Quantum Gravity

- QG thought to have significant effects only above the Planck scale ($\sim 1.22 \times 10^{19}$ GeV)
- Highest energies achieved by LHC $\sim$16 orders of magnitude smaller
  $\rightarrow$ No hope of directly detecting QG effects
- However, CMB measurements provide us with information about the very early Universe...

Cosmological perturbations

- Inflationary epoch: Quantum fluctuations in inflaton field lead to fluctuations in the metric

- Three types of perturbation:
  - scalar (density perturbation $\rightarrow$ structure formation)
  - vector (vorticity, decay with time)
  - **tensor** (gravitational waves)

- These perturbations are seeded by a quantum fluctuation $\rightarrow$ QG effects might survive until today

The idea

- Use Ashtekar formalism of Loop Quantum Gravity (LQG) within cosmological perturbation theory
- Two aims:
  - Show that formalism is consistent with well-established results in Cosmology
  - See whether any novelties arise that might be linked to QG effects
- Note: We're not trying to perturbatively quantise gravity; we're just testing (non-perturbative) LQG it in the regime of cosmological P.T.

Ashtekar formalism

- Based on the Palatini-Kibble formulation of GR with an additional topological term

$$S = -\frac{1}{2 l_P^2} \int \Sigma^{IJ} \wedge \left(F_{IJ} + \frac{1}{\gamma} {}^* F_{IJ} \right)$$

where $\Sigma^{IJ} = e^I \wedge e^J$ and γ is the Immirzi parameter

- The Ashtekar canonical variables are

$$A_a^i = \Gamma_a^i + \gamma \Gamma_a^{0i}$$

$$E_i^a = \det\left(e_b^j\right) e_i^a$$

$\rightarrow$ form a canonical pair:
$\{A_a^i(\mathbf{x}), E_j^b(\mathbf{y})\} = \gamma l_P^2 \delta_a^b \delta_j^i \delta(\mathbf{x} - \mathbf{y})$

Perturbation Theory

- Consider tensor perturbations around de Sitter background:

$$ds^2 = a^2[-d\eta^2 + (\delta_{ab} + h_{ab})dx^a dx^b]$$

  where scale factor $a = -\frac{1}{H\eta}$, $\eta < 0$ and h_{ab} is a TT tensor perturbation

- Using a Fourier space expansion we can define a variable

$$v(\mathbf{k}, \eta) = \sqrt{\frac{\epsilon^{ij}\bar{\epsilon}_{ij}}{32\pi G}}\, a h(\mathbf{k}, \eta)$$

- Solving Hamilton's equation for the perturbation this leads to

$$\boxed{v'' + \left(k^2 - \frac{2}{\eta^2}\right) v = 0}$$

Our approach

- Start with Hamiltonian formulation of Ashtekar gravity
- Expand Ashtekar variables in terms of perturbations around de Sitter
  $\rightarrow$ Rederive e.o.m. for metric tensor perturbations given by v in Cosmology
- Find the corresponding Fourier space Hamiltonian (2nd order in perturbations) to set up quantum theory

Consider case $\gamma = \pm i$ in the following (SD/ASD)

Ashtekar Hamiltonian

$$\mathcal{H} = \frac{1}{2l_P^2}\int d^3x N E_i^a E_j^b \epsilon_{ijk}(F_{ab}^k + H^2\epsilon_{abc}E_k^c)$$

(+ boundary term)

- In de Sitter with perturbations, Ashtekar's variables are

$$A_a^i = \gamma H a \delta_a^i + \frac{a_a^i}{a}$$

$$E_i^a = a^2\delta_i^a - a\delta e_i^a$$

- Dynamics given by 2nd order Hamiltonian in terms of 1st order variables

The perturbed Hamiltonian

- After taking a few subtleties into account... (see our paper!)

$$\mathcal{H}_{eff} = \frac{1}{2l_P^2}\int d^3x[-a_{ij}a_{ij} - 2\epsilon_{ijk}(\partial_j\delta e_{li})a_{kl} \\ -2H^2a^2\delta e_{ij}\delta e_{ij}]$$

- Using Hamilton's equations and torsion free condition

$$a_{ij} = \epsilon_{ikl}\partial_k\delta e_{lj} + \gamma\delta e'_{ij}$$

we can derive

$$\boxed{\delta e''_{ij} - \left(\partial^2 + \frac{2}{\eta^2}\right)\delta e_{ij} = 0}$$

Fourier space expansion

We need to write the perturbation variables in Fourier space:

- Positive and negative frequency states
- Graviton and anti-graviton states, independent before reality conditions imposed
- Right and left-hand helicities, corresponding to two polarisations of graviton

Fourier space expansion

Satisfied by

$$\begin{aligned}\delta e_{ij} &= \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} \sum_r \epsilon^r_{ij}(\mathbf{k})\Psi_e(k,\eta)e_{r+}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} \\ &+\epsilon^{r\star}_{ij}(\mathbf{k})\Psi^\star_e(k,\eta)e^\dagger_{r-}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}} \\ a_{ij} &= \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} \sum_r \epsilon^r_{ij}(\mathbf{k})\Psi^{r+}_a(k,\eta)a_{r+}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} \\ &+\epsilon^{r\star}_{ij}(\mathbf{k})\Psi^{r-\star}_a(k,\eta)a^\dagger_{r-}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}\end{aligned}$$

where ϵ^r_{ij} are polarisation tensors and, within the horizon ($|k\eta| \gg 1$)

$$\Psi(k,\eta) \sim e^{-ik\eta}$$

Torsion free condition implies

$$\Psi^{rp}_a = (r - ip\gamma)k\Psi_e$$

The algebra

Writing $\tilde{a}_{rp} = a_{rp}\Psi_a^{rp}$ and $\tilde{e}_{rp} = e_{rp}\Psi_e$

- Commutation relations

$$[\tilde{a}_{rp}(\mathbf{k}), \tilde{e}^{\dagger}_{sq}(\mathbf{k}')] = -i\gamma p \frac{l_P^2}{2}\delta_{rs}\delta_{p\bar{q}}\delta(\mathbf{k}-\mathbf{k}')$$

- Reality conditions

$$\tilde{e}_{r+}(\mathbf{k}) = \tilde{e}_{r-}(\mathbf{k})$$

$$\begin{aligned}
\tilde{a}_{r+}(\mathbf{k},\eta) + \tilde{a}_{r-}(\mathbf{k},\eta) &= 2rk\tilde{e}_{r+}(\mathbf{k},\eta) \\
\tilde{a}^{\dagger}_{r+}(\mathbf{k},\eta) + \tilde{a}^{\dagger}_{r-}(\mathbf{k},\eta) &= 2rk\tilde{e}^{\dagger}_{r-}(\mathbf{k},\eta)
\end{aligned}$$

Quantum Hamiltonian

Write Fourier space Hamiltonian in terms of graviton creation and annihilation operators $g^{\dagger}_{rp}$ and g_{rp}:

- Combinations of connection and metric operators
- Before reality conditions imposed, half of the modes unphysical

For $\gamma = i$, on-shell we obtain

$$\mathcal{H}^{ph}_{eff} \approx \frac{1}{l_P^2} \int d\mathbf{k} \, (g_{L-} g^{\dagger}_{L-} + g^{\dagger}_{R+} g_{R+})$$

where e.g.

$$g_{L-}(\mathbf{k}) = -\tilde{a}_{L+}(\mathbf{k}) + 2kr\tilde{e}_{L+}(\mathbf{k}) \approx \tilde{a}_{L-}(\mathbf{k})$$

- Only right handed graviton normal ordered $\rightarrow$ only left produces vacuum energy

Vacuum fluctuations

CMB experiments measure 2-point correlation functions describing vacuum fluctuations:

$$\langle 0|A_r^\dagger(\mathbf{k})A_r(\mathbf{k}')|0\rangle = P_r(k)\delta(\mathbf{k}-\mathbf{k}')$$

where

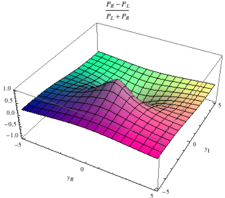
$$A_r(\mathbf{k}) = a_{r+}(\mathbf{k})e^{-ik\cdot x} + a_{r-}^\dagger(\mathbf{k})e^{ik\cdot x}$$

For $\gamma = i$,

$$\begin{aligned}\langle 0|A_R^{ph\dagger}(\mathbf{k})A_R^{ph}(\mathbf{k}')|0\rangle &= 0\\ \langle 0|A_L^{ph\dagger}(\mathbf{k})A_L^{ph}(\mathbf{k}')|0\rangle &\neq 0\end{aligned}$$

In general,

$$\frac{P_R - P_L}{P_R + P_L} = -\frac{2\gamma_I}{1+|\gamma|^2}$$



The Ground State

Use graviton operators to set up a Hilbert space representation

- Annihilation and creation operators define Fock space
- Can define inner product; particle states normalisable

Known ground state of quantum gravity: Kodama state

$$\Phi = \mathcal{N} \exp\left(\frac{i\gamma}{2 l_P^2 H^2} S_{CS}\right)$$

where S_{CS} is the Chern-Simons form

- Solves full Hamiltonian constraint
- Not an eigenfunction of dynamical Hamiltonian to second order $\Rightarrow$ cannot describe gravitons!

Conclusion

- Discovered a potentially measurable quantum gravity effect
- Ordering issues have to be addressed
- Need to compare to other mechanisms that produce chirality
- Currently working on a connection representation of graviton states

Defining a Hilbert space

- Graviton commutation relations:

$$[g_{rp}(\mathbf{k}), g^\dagger_{sq}(\mathbf{k}')] = -i\gamma l_P^2 (pr) k \delta_{rs}\delta_{pq}\delta(\mathbf{k}-\mathbf{k}')$$

- Choose a holomorphic representation:
  $g^\dagger_{rp}\Phi(z) = z_{rp}\Phi(z) \Rightarrow g_{rp}\Phi = -i\gamma l_P^2 (pr) k \frac{\partial\Phi}{\partial z_{rp}}$

- Inner product defined as

$$\langle\Phi_1|\Phi_2\rangle = \int dz d\bar{z} e^{\mu(z,\bar{z})} \bar{\Phi}_1(\bar{z})\Phi_2(z)$$

where

$$\mu(z,\bar{z}) = \int d\mathbf{k} \sum_{rp} \frac{pr}{ik\gamma l_P^2} z_{rp}(\mathbf{k})\bar{z}_{rp}(\mathbf{k})$$

- Ground and particle states given by $\Phi_0 = \langle z|0\rangle = 1$, $\Phi_n \propto (g^\dagger_{rp})^n \Psi_0 = z^n_{rp}$

The Kodama state

A ground state of quantum gravity (solves all constraints)

$$\Phi = \mathcal{N} \exp\left(\frac{i\gamma}{2 l_P^2 H^2} S_{CS}\right)$$

where S_{CS} is the Chern-Simons form

- It solves the Hamiltonian constraint, $\mathcal{H} = EE\mathcal{S}$, where $\mathcal{S} = B + H^2 E$ ("self dual" operator)
- Dynamical part of Hamiltonian constraint to second order:

$$^2_1\mathcal{H} = 2(^0E)(^1_1E)(^1_1\mathcal{S}) + (^1_1E)(^1_1E)(^0\mathcal{S}) + (^0E)(^0E)(^2_1\mathcal{S})$$

- But perturbed Kodama state $^2_1\Phi$ annihilated by ^1_1S $\Rightarrow$ Kodama state cannot describe gravitons!