# Asymptotic Safety and the Gibbons-Hawking Term

A status report

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Martin Reuter, Daniel Becker Work in Progress



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Asymptotic Safety and the Gibbons-Hawking Term

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#### Outline

#### Classical motivation

The Einstein-Hilbert action The Gibbons-Hawking (York) action

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Functional renormalization group equation Asymptotic Safety

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# **Classical motivation**

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The Einstein-Hilbert action	

Conclusion

#### The Einstein-Hilbert action

The geometrical content of General Relativity is encoded in the **Einstein-Hilbert action**:

group

$$S_{\mathsf{EH}} = -\frac{1}{16\pi G_N} \int_{\mathcal{M}} \mathrm{d}^d x \sqrt{g} \, R \,, \qquad \qquad \partial \mathcal{M} = \emptyset$$

The principle of least action  $(\delta S_{EH}=0)$  requires that

$$0 = \frac{1}{16\pi G_N} \left( \int_{\mathcal{M}} \mathrm{d}^d x \sqrt{g} \left[ R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right] \delta g_{\mu\nu} \right)$$

for all  $\delta g_{\mu\nu}$ .

Hence, one deduces the Einstein field equations in vacuum:

$$G^{\mu\nu}\equiv \quad R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R=0$$

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The running Gibbons-Hawking term

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#### Functional renormalization group

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#### The Einstein-Hilbert action: $\partial \mathcal{M} \neq \emptyset$

The geometrical content of General Relativity is encoded in the **Einstein-Hilbert action**:

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for **all**  $\delta g_{\mu\nu}$ .

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# The Einstein-Hilbert action: $\partial \mathcal{M} \neq \emptyset$

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renormalization group

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$$0 = \frac{1}{16\pi G_N} \left( \int_{\mathcal{M}} \mathrm{d}^d x \sqrt{g} \, G^{\mu\nu} \delta g_{\mu\nu} + \int_{\partial \mathcal{M}} \mathrm{d}^{d-1} x \sqrt{H} \, H^{\alpha\beta} n^{\mu} D_{\mu} \delta g_{\alpha\beta} \right)$$

for all  $\delta g_{\mu\nu}$ , satisfying Dirichlet boundary conditions  $\delta g_{\mu\nu}|_{\partial\mathcal{M}} = 0$ .

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- $H_{\mu\nu} = g_{\mu\nu}|_{\partial\mathcal{M}}$  induced metric on  $\partial\mathcal{M}$
- $n^{\mu}$  normal vector on  $\partial \mathcal{M}$

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#### Functional renormalization group O

Conclusion

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for all  $\delta g_{\mu\nu}$ , satisfying Dirichlet boundary conditions  $\delta g_{\mu\nu}|_{\partial\mathcal{M}} = 0$ .

Hence, the boundary contribution obstructs the derivation of field equations as stationary points of  $S_{\rm EH}.$ 

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# The Gibbons-Hawking (York) term

Einstein field equation can be recovered by adding a boundary term to  $S_{\rm EH}$  which has the variation:

$$\delta S_{\rm GHY} = -\frac{1}{16\pi G_N} \int_{\partial \mathcal{M}} \mathrm{d}^{d-1} y \sqrt{H} \, H^{\alpha\beta} n^\mu D_\mu \delta g_{\alpha\beta}$$

Note the identity

$$+2 \cdot \delta K|_{\text{Dirichlet}} = H^{\alpha\beta} n^{\mu} D_{\mu} \delta g_{\alpha\beta} |_{\text{Dirichlet}} , \qquad \delta H_{\mu\nu} = 0 \,.$$

where

K<sub>µν</sub> = D<sub>µ</sub>n<sub>ν</sub> is the extrinsic curvature of ∂M
 and K = H<sup>µν</sup>K<sub>µν</sub> is its trace

Classical motivation	Functional renormalization group	The running Gibbons-Hawking term	Co
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The Gibbons-Hawking (York)	action		

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Einstein field equation can be recovered by adding a boundary term to  $S_{\rm EH}$  which has the variation:

$$\delta S_{\mathsf{GHY}} = \delta \left( -\frac{2}{16\pi G_N} \int_{\partial \mathcal{M}} \mathrm{d}^{d-1} x \sqrt{H} \, K \right)$$

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The Gibbon	s-Hawking (York) action		

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$$S_{\mathsf{EH-GHY}} = -\frac{1}{16\pi G_N} \left( \int_{\mathcal{M}} \mathrm{d}^d x \sqrt{g} \, R + 2 \oint_{\partial \mathcal{M}} \mathrm{d}^{d-1} y \sqrt{H} \, H^{\mu\nu} K_{\mu\nu} \right)$$

yields the Einstein equation as a stationary point in case of

- non-empty boundary  $\partial \mathcal{M} \neq \emptyset$
- Dirichlet boundary condition  $\delta g_{\mu\nu}|_{\partial\mathcal{M}} = 0$
- a relative coefficient of exactly +2



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**Classical motivation** 

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# Functional renormalization group

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Functional renormalization group eq	uation		

$$\left\{ \left. \Gamma_{k} \left[ \right. \right] \right|$$

Classical motivation	Functional renormalization group	The running Gibbons-Hawking term	Conclusion
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Functional renormalization group eq	uation		

 $\left\{ \Gamma_k \left[ g_{\mu\nu}, \bar{g}_{\mu\nu} \right] \right|$ 

Classical motivation	Functional renormalization group	The running Gibbons-Hawking term	Conclusion
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Functional renormalization group eq	uation		

 $\left\{ \left. \Gamma_{k}\left[g_{\mu\nu},\bar{g}_{\mu\nu},C^{\mu},\bar{C}_{\mu}\right]\right| \right.$ 

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... acts on theory space:

 $\left\{ \left. \Gamma_k\left[g_{\mu\nu}, \bar{g}_{\mu\nu}, C^{\mu}, \bar{C}_{\mu}\right] \right| \text{ invariant under diffeomorphisms} \right\}$ 

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#### Functional renormalization group equation

 $k\partial_k\Gamma_k$ 

exact, closed functional differential equation

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$$k\partial_k\Gamma_k = +\frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} \cdot k\partial_k R_k\right]$$

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#### Truncations

$$\Gamma_k = u_k^{(a)} \int \sqrt{g} + u_k^{(b)} \int \sqrt{g} R + u_k^{(c)} \int \sqrt{g} R^{\mu\nu} R_{\mu\nu} + \dots$$

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 $\{u_k^{(n)}\}$  coordinatize the **infinite** dimensional theory space

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**Truncations:** subspaces of span $\{u_k^{(n)}\}$ 

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#### Restricts the possible evolutions of $\Gamma_k$ by physical arguments

- Existence of a Non-Gaussian fixed point (NG-FP) Fundamental (non-trivial) theory in the UV
  - $\mathscr{S}_{UV} = \{ actions pulled into the NG-FP under the inverse flow <math>\}$ (inverse flow = increasing k)
- Finite dimensional UV-critical hypersurface  $\mathscr{S}_{UV}$  $\dim(\mathscr{S}_{UV}) \equiv n < \infty : \# \text{ of measurements needed to fix initial conditions}$

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# The running Gibbons-Hawking term

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#### EH-GHY-truncation

$\int_{\mathcal{M}} \sqrt{g}$	$\int_{\mathcal{M}} \sqrt{g}R$
$\int_{\partial \mathcal{M}} \sqrt{H}$	$\int_{\partial \mathcal{M}} \sqrt{H} K$

#### EH-GHY-truncation

$$\begin{split} \Gamma_k &= + u_k^{(a)} \int_{\mathcal{M}} \sqrt{g} + u_k^{(b)} \int_{\mathcal{M}} \sqrt{g} R \\ &+ u_k^{(c)} \int_{\partial \mathcal{M}} \sqrt{H} + u_k^{(d)} \int_{\partial \mathcal{M}} \sqrt{H} K \end{split}$$

#### **EH-GHY-truncation**

$$\Gamma_{\boldsymbol{k}} = + \frac{2\lambda_{k}k^{d}}{16\pi g_{k}} \int_{\mathcal{M}} \sqrt{g} - \frac{k^{d-2}}{16\pi g_{k}} \int_{\mathcal{M}} \sqrt{g}R \\ + \frac{2\lambda_{k}^{2}k^{d-1}}{16\pi g_{k}^{2}} \int_{\partial\mathcal{M}} \sqrt{H} - \frac{2k^{d-2}}{16\pi g_{k}^{2}} \int_{\partial\mathcal{M}} \sqrt{H}K$$

#### EH-GHY-truncation

$$\begin{split} \Gamma_{k} &= + \frac{2\lambda_{k}k^{d}}{16\pi g_{k}} \int_{\mathcal{M}} \sqrt{g} & - \frac{k^{d-2}}{16\pi g_{k}} \int_{\mathcal{M}} \sqrt{g}R \\ &+ \frac{2\lambda_{k}^{2}k^{d-1}}{16\pi g_{k}^{2}} \int_{\partial \mathcal{M}} \sqrt{H} & - \frac{2k^{d-2}}{16\pi g_{k}^{2}} \int_{\partial \mathcal{M}} \sqrt{H}K \end{split}$$

where  $g_k,\,g_k^\partial$  dimensionless Newton type couplings on  $\mathcal{M},\,\partial\mathcal{M}$ 

#### Truncation ansatz

#### EH-GHY-truncation

$$\begin{split} \Gamma_{k} &= + \frac{2\lambda_{k}k^{d}}{16\pi g_{k}} \int_{\mathcal{M}} \sqrt{g} & - \frac{k^{d-2}}{16\pi g_{k}} \int_{\mathcal{M}} \sqrt{g}R \\ &+ \frac{2\lambda_{k}^{0}k^{d-1}}{16\pi g_{k}^{0}} \int_{\partial \mathcal{M}} \sqrt{H} & - \frac{2k^{d-2}}{16\pi g_{k}^{0}} \int_{\partial \mathcal{M}} \sqrt{H}K \end{split}$$

where  $\lambda_k, \, \lambda_k^\partial$  dimensionless cosmological type couplings on  $\mathcal{M}, \, \partial \mathcal{M}$ 

#### EH-GHY-truncation

$$\Gamma_{k} = + \frac{2\lambda_{k}k^{d}}{16\pi g_{k}} \int_{\mathcal{M}} \sqrt{g} - \frac{k^{d-2}}{16\pi g_{k}} \int_{\mathcal{M}} \sqrt{g}R$$
$$+ \frac{2\lambda_{k}^{2}k^{d-1}}{16\pi g_{k}^{2}} \int_{\partial\mathcal{M}} \sqrt{H} - \frac{2k^{d-2}}{16\pi g_{k}^{2}} \int_{\partial\mathcal{M}} \sqrt{H}K$$

Dirichlet boundary conditions  $\delta g_{\mu\nu}|_{\partial\mathcal{M}} =$  fixed, i.e.

 $(\text{metric fluctuations})|_{\partial \mathcal{M}} = 0 \,.$ 

#### **EH-GHY-truncation**

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#### Results

#### **EH-GHY-truncation**

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# gk<br/> $\lambda_k$ NG fixed point $g_k$ <br/> $\lambda_k$ 0.707<br/>0.193 $\psi$ <br/> $g_k^{\partial}$ <br/> $\lambda_k^{\partial}$ -2.292<br/> $\psi$ <br/> $\lambda_k^{\partial}$

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Asymptotic Safety and the Gibbons-Hawking Term

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Running action in EH-GHY truncation:

$$\Gamma_k = \frac{k^2}{16\pi} \left( \frac{1}{g_k} \int_{\mathcal{M}} \sqrt{g} R + \frac{2}{g_k^{\partial}} \int_{\partial \mathcal{M}} \sqrt{H} K \right) + \dots$$

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Beta-functions of Newton-type couplings:

#### The running Gibbons-Hawking term

Running action in EH-GHY truncation: mismatch,  $\partial_k g_k \neq \partial_k g_k^{\partial}$ 

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#### Example: d = 4, near G-FP $(g=\lambda=0)$

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Example: d = 4, near G-FP  $(g=\lambda=0)$ 

First approximation to the scale dependence of  $g_k$ ,  $g_k^{\partial}$ :

$$g_k = g_0 \left( 1 - \frac{11}{6\pi} g_0 \cdot k^2 \right) , \qquad g_k^{\partial} = g_0^{\partial} \left( 1 + \frac{1}{6\pi} g_0^{\partial} \cdot k^2 \right)$$

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sical motivation	Functional renormalization group	The running Gibbons-Hawking term

Running action in EH-GHY truncation: mismatch,  $\partial_k g_k \neq \partial_k g_k^{\partial}$ 

$$\Gamma_k = \frac{k^2}{16\pi g_0} \left( \frac{1}{1 - ck^2} \int_{\mathcal{M}} \sqrt{g} \, R + \frac{2}{1 + \tilde{c}k^2} \int_{\partial \mathcal{M}} \sqrt{H} \, K \right) + \dots \quad c, \tilde{c} > 0$$

'Correct' relative coefficient at most at one scale , k=0, say.  $(g_0=g_0^\partial)$ 

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Conclusion

#### Classical general relativity

- Gibbons-Hawking-(York) term needed for  $\partial \mathcal{M} \neq \emptyset$
- Relative coefficient has to be +2

- Einstein-Hilbert subsystem is uneffected by boundary contribution
- Truncation shows a Non-Gaussian fixed point
- Couplings for EH and GHY show different scale dependence
- Well defined variational principle at one scale only, with the standard FRGE . . .

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