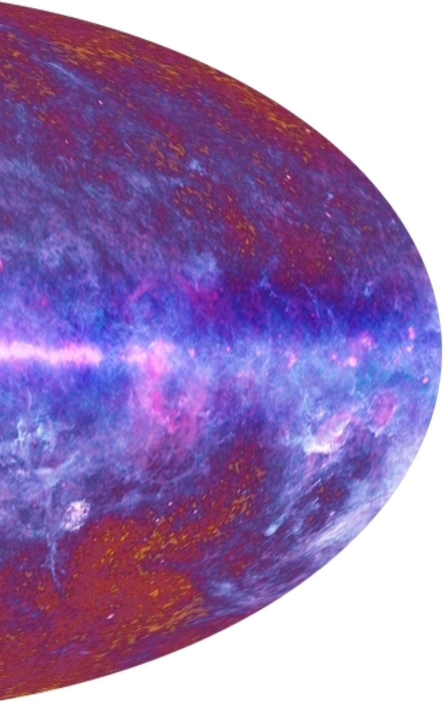

The Holographic Universe

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Introduction

- The notion of holography emerged from black hole physics as an answer to the question: ['t Hooft 1993]

  Why is the entropy of a black hole proportional to the area of its horizon rather than the volume it occupies?

Definition: Holography states that any theory which includes gravity can be described by a theory without gravity in one dimension less.

- Holography became a prominent research direction when precise holographic dualities were found in string theory (i.e. AdS/CFT).

  [Maldacena 1997, Gubser, Klebanov & Polyakov 1998, Witten 1998]

Holography for Cosmology

- The holographic dualities found in string theory involve spacetimes with a negative cosmological constant, but the general argument for holography is applicable to any theory of gravity.
- In particular, it should apply to our own universe.

Here we describe how to set up

a holographic framework for inflationary cosmology.

This framework will let us describe 4d inflationary cosmology in terms of a 3d quantum field theory (QFT) without gravity.

Motivations

Why set up a holographic description of inflation?

1. New perspective on problems of standard inflation, e.g. fine-tuning, initial conditions.
2. New methods and intuition for computing structure of cosmological perturbations, especially for higher order cosmological correlators (non-Gaussianity).
3. New models for the early universe based on a 3d perturbative QFT

$\Rightarrow$ today's lecture.

Necessary ingredients

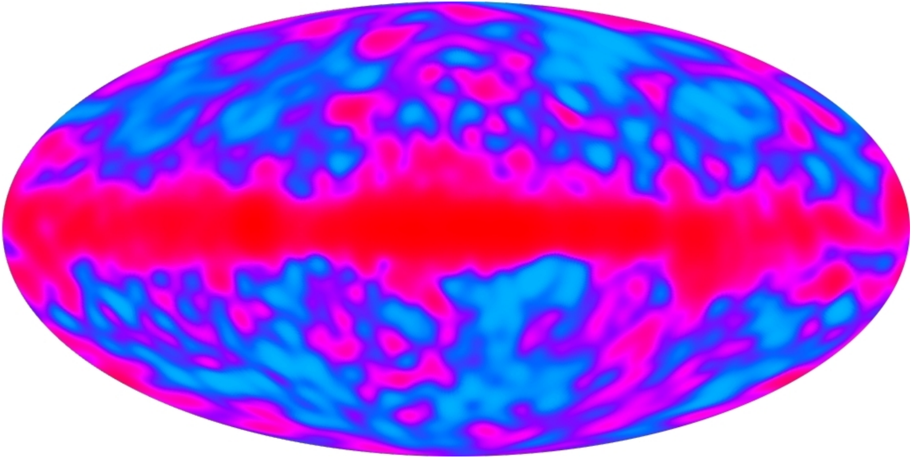
Any proposed holographic framework for cosmology should specify:

1. The precise nature of the dual QFT.

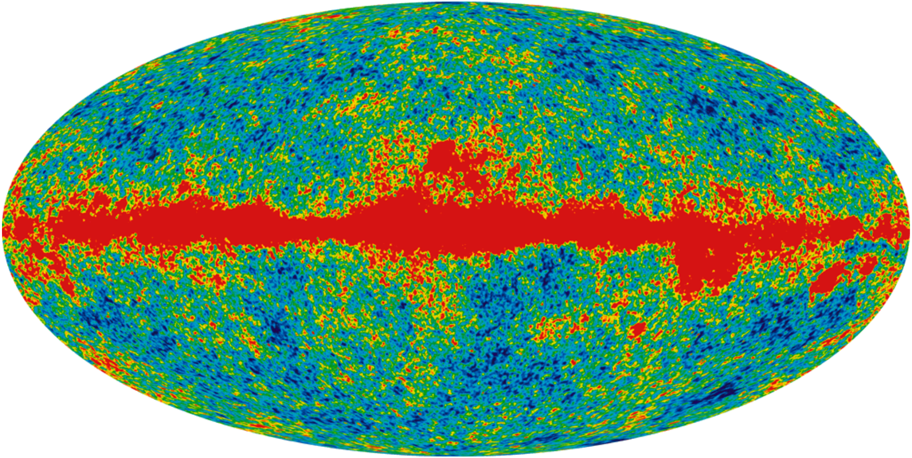
2. How to compute cosmological observables from the correlation functions of the dual QFT.

➡ Today, we'll focus on the primordial power spectrum, since we have observational data for this from the WMAP satellite and other CMB experiments.

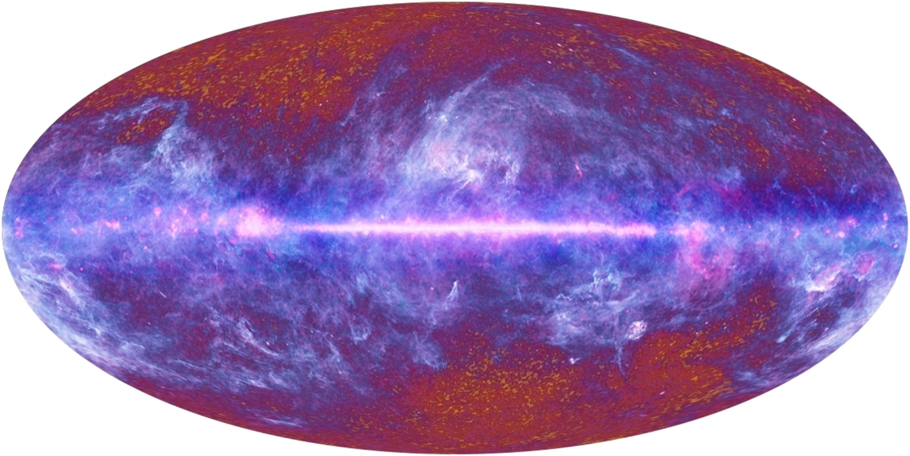
COBE (1989)



WMAP (2001)



Planck (2009)



Plan

1. The primordial power spectrum
2. Holography: a primer
3. Holography for cosmology
4. New holographic models for inflation
5. Testing the holographic power spectrum against WMAP data

References

Based on work with Kostas Skenderis:

- Cosmological 3-point correlators from holography [1104.3894]
- Holographic non-Gaussianity [1011.0452]
- Observational signatures of holographic models of inflation [1010.0244]
- The holographic universe* [1007.2007]
- Holography for cosmology [0907.5542]

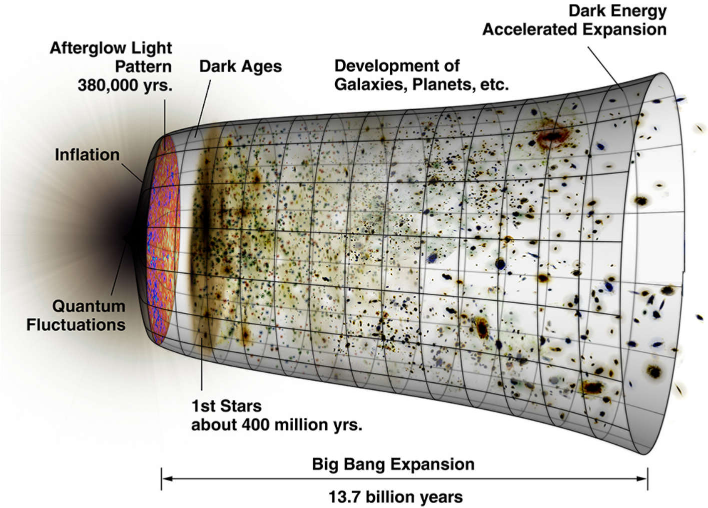
Work with Richard Easther, Raphael Flauger and KS:

- Constraining holographic inflation with WMAP [1104.2040]

New work to appear shortly with Adam Bzowski and KS.

❶ The primordial power spectrum

From quantum fluctuations to galaxies



Primordial perturbations

The primordial perturbations offer some of our best clues as to the fundamental physics underlying the big bang. Their form is surprisingly simple:

- Small amplitude: $\delta T/T \sim 10^{-5}$
- Adiabatic
- Nearly Gaussian
- Nearly scale-invariant

- Any proposed cosmological model must be able to account for these basic features.
- Any predicted deviations (*e.g.*, from Gaussianity) are likely to prove critical in distinguishing different models.

The primordial power spectrum

A Gaussian distribution is fully characterised by its 2-point function or power spectrum. Current observational data is consistent with a simple power-law form for the primordial power spectrum:

$$\Delta^2_{\mathcal{R}}(q) = \Delta^2_{\mathcal{R}}(q_0)\,(q/q_0)^{n_S - 1}$$

For a pivot point of $q_0 = 0.002\mathrm{Mpc}^{-1}$, the WMAP data yield

$$\Delta^2_{\mathcal{R}}(q_0) = (2.445 \pm 0.096) \times 10^{-9}, \qquad n_S - 1 = -0.040 \pm 0.013,$$

i.e., the scalar perturbations have small amplitude and are nearly scale invariant.

- These two small numbers should appear naturally in any theory that explains the data.

❷ Holography: a short primer

Holography

Holographic dualities relate gravitational theories to non-gravitational QFTs (typically large-N gauge theories) in one dimension less.

- To date, there is no holographic construction that works in general. Instead, we have explicit examples depending on the asymptotic form of the bulk metric.
- The properties of the dual theory depend on these asymptotics.

Holography

The best-understood examples originate from string theory via the decoupling limits of branes:

- D3, M5, etc. $\Rightarrow$ asymptotically AdS spacetimes $\Rightarrow$ dual to QFTs that become conformal in the UV.
- D2, D4, etc. $\Rightarrow$ asymptotically power-law spacetimes $\Rightarrow$ dual to QFTs with 'generalised conformal symmetry'.

[Kanitscheider, Skenderis & Taylor '08]

Today, we'll focus on this second class of examples. Although probably less familiar, they can be used to construct very simple holographic models of inflation.

The bulk spacetime

Poincaré invariance of the dual QFT dictates the bulk spacetime has the so-called 'domain-wall' form:

$$\mathrm{d}s^2 = \mathrm{d}r^2 + a^2(r)\mathrm{d}x^i\mathrm{d}x_i, \qquad \Phi = \phi(r)$$

- This is often referred to as a holographic RG flow:

  Radial evolution in bulk $\Leftrightarrow$ RG flow in QFT

- The relevant part of the bulk gravitational action is

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{g}\,[R - (\partial\Phi)^2 - 2\kappa^2 V(\Phi)].$$

- For asymptotically power-law spacetimes, we have

$$a \to r^n \quad \text{where } n > 1, \qquad \phi \to \sqrt{2n}\ln r \qquad \text{as } r \to \infty$$

The dual QFT

For aymptotically power-law spacetimes, the dual QFT takes the form

$$S = \frac{1}{g_{\rm YM}^2} \int {\rm d}^3x \,{\rm tr}\Big[\frac{1}{2}F^I_{ij}F^{Iij} + \frac{1}{2}(D\phi^J)^2 + \frac{1}{2}(D\chi^K)^2 + \bar{\psi}^L \not{D}\psi^L + \text{interactions}\Big]$$

i.e. 3d $SU(N)$ Yang-Mills theory with matter (minimal scalars ϕ, conformal scalars χ, fermions ψ) in the adjoint representation.

- The parameters of the dual QFT are the number of colours N, the Yang-Mills coupling $g_{\rm YM}^2$, and the field content.
- In 3d, $g_{\rm YM}^2$ has mass dimensions so the theory is super-renormalisable.
- Perturbation theory may be organised in terms of the dimensionless effective 't Hooft coupling:

$$g_{\rm eff}^2 = g_{\rm YM}^2 N/q.$$

The holographic dictionary

- The QFT simplifies in the large-N limit:

$$N \gg 1, \qquad g_{\text{eff}}^2 \text{ fixed.}$$

In this limit only planar Feynman diagrams contribute. From the perspective of the gravitational theory, this limit suppresses gravitational loop corrections since

$$\kappa^2 \propto 1/N^2.$$

- When $g_{\text{eff}}^2 \gg 1$, stringy effects in the bulk are suppressed and ordinary Einstein gravity is a good approximation. If instead $g_{\text{eff}}^2 \ll 1$, the gravitational description is strongly coupled, and one has to resort to the full string theory.

$\Rightarrow$ Holographic dualities are strong/weak coupling dualities.

Holographic correlation functions

- ✦ Bulk gravitational fields are in 1-1 correspondence with local gauge-invariant operators in the boundary QFT:

  - ➢ The bulk metric corresponds to the stress-energy tensor T_{ij} of the boundary theory.
  - ➢ Bulk scalar fields correspond to boundary scalar operators, e.g. $\operatorname{tr} F_{ij}F^{ij}$.

- ✦ From the asymptotic behaviour of the bulk fields, one can read off the correlation functions of the boundary QFT.
- ✦ And vice versa: from the correlators of the boundary QFT one can reconstruct the bulk asymptotics.

❸ Holography for cosmology

The domain-wall/cosmology correspondence

The 'domain-wall' spacetimes describing holographic RG flows are closely related to cosmologies:

$$\mathrm{d}s^2 = \sigma\,\mathrm{d}z^2 + a^2(z)\mathrm{d}\vec{x}^2, \qquad \Phi = \varphi(z),$$

where $\sigma = +1$ for a (Euclidean) DW and $\sigma = -1$ for cosmology.

(We're assuming flat spatial slices for simplicity).

Recall in both cases the action is just that of a scalar field with potential $V(\Phi)$ minimally coupled to gravity.

➢ Examining the equations of motion for both the background solution and perturbations, one can show that any domain-wall solution can be mapped to a corresponding cosmological solution by the following analytic continuation:

Bulk analytic continuation

$$(\kappa^2 V)_{DW} = -(\kappa^2 V)_C, \qquad q_{DW} = -i q_C$$

Here $q = \sqrt{\vec{q}^{\,2}}$ is the magnitude of the perturbation 3-momentum.

- The sign in the continuation of q is fixed by mapping the cosmological Bunch-Davies vacuum to the holographic RG flow solution that is regular in the interior.
- Continuation also works at nonlinear order in perturbation theory, cf. cosmological non-Gaussianities.
- Asymptotically power-law domain-walls become asymptotically power-law inflating cosmologies:

$$\mathrm{d}s^2 \to -\mathrm{d}t^2 + t^{2n}\mathrm{d}\vec{x}^2, \qquad \varphi \to \sqrt{2n}\ln t \qquad \text{as } t \to \infty$$

In QFT language

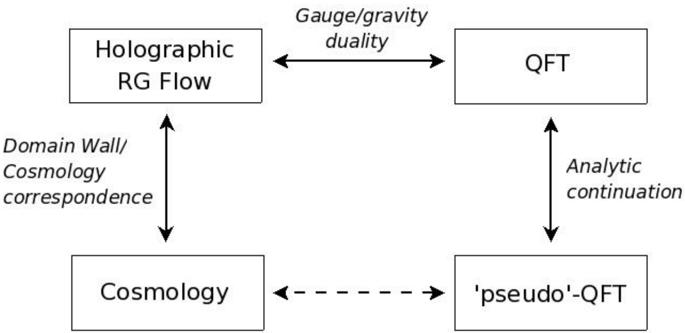
Via the holographic dictionary, this bulk analytic continuation may also be expressed as a continuation acting on the dual QFT:

$$N_{DW} = -iN_C, \qquad q_{DW} = -iq_C,$$

- The sign in the continuation of N is fixed so that $g^2_{\rm eff} = g^2_{\rm YM}N/q$ is invariant. (This is necessary because QFT amplitudes are non-analytic functions of $g^2_{\rm eff}$.)
- To compute tree-level cosmological correlators, we only need to continue the large-N correlators of the QFT dual to the DW spacetime:

$$\langle T(q)T(-q)\rangle = N^2_{DW}q^3_{DW}f(g^2_{\rm eff}) \to -iN^2_C q^3_C f(g^2_{\rm eff}).$$

Framework



Holographic RG Flow
Gauge/gravity duality
QFT
Domain Wall/ Cosmology correspondence
Analytic continuation
Cosmology
'pseudo'-QFT

Holographic power spectrum

Using the standard prescription to compute holographic correlation functions, along with the domain-wall/cosmology correspondence, we arrive at the following holographic formula for the cosmological power spectrum:

$$\Delta^2_{\mathcal{R}}(q) = \frac{-q^3}{4\pi^2 \mathrm{Im}\langle T(-iq)T(iq)\rangle},$$

where $T \equiv T^i_i$ is the trace of the stress-energy tensor of the dual QFT. (Note one takes the analytic continuation before taking the imaginary part.)

Similar formulae can be worked out for other cosmological observables.
(e.g. tensor power spectrum, 3-point functions)

4 New holographic models of inflation

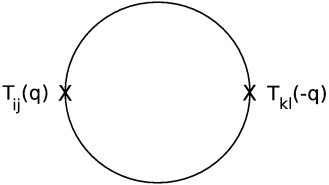
New holographic models

- Ordinary inflationary models are described by Einstein gravity hence the dual QFT is strongly coupled.
- New models arise when we consider the dual QFT at weak coupling, but still at large N.
- In these models, the very early universe is in a non-geometric phase. This phase should have a string theory description in terms of a strongly coupled sigma model. Here we use holography to reconstruct the late-time asymptotic geometry that emerges.
- The end of this phase is the beginning of conventional hot big bang cosmology.

➢ To make predictions, we do perturbative QFT calculations and use our holographic formulae.

QFT calculations

To compute the holographic power spectrum, we need to compute the 2-pt function of T_{ij}. The leading order contribution is at 1-loop:



$$\langle T(q)T(-q)\rangle \sim N^2 q^3$$

The answer follows from general considerations:

- The stress tensor has dimension 3 in three dimensions.
- 1-loop amplitudes are independent of g^2_{YM}.
- There is a factor of N^2 from the trace over gauge indices.

QFT calculations

Including higher-loop corrections now, the answer takes the form

$$\langle T(q)T(-q)\rangle = N^2 q^3 f(g_{\text{eff}}^2),$$

where

$$f(g_{\text{eff}}^2) = f_0(1 - f_1 g_{\text{eff}}^2 \ln g_{\text{eff}}^2 + f_2 g_{\text{eff}}^2 + O(g_{\text{eff}}^4)).$$

Here

- f_0 is fixed by our 1-loop calculation.
- f_1 requires a 2-loop calculation.
- f_2 is related to a physical scale generated by infrared effects $q_{\text{IR}} \sim g_{\text{YM}}^2$.

[Jackiw, Templeton (1981); Appelquist, Pisarski (1981)].

(Provided one probes the theory at scales large compared to the IR scale we can neglect this term however.)

Holographic power spectrum

Plugging this result into our holographic formula,

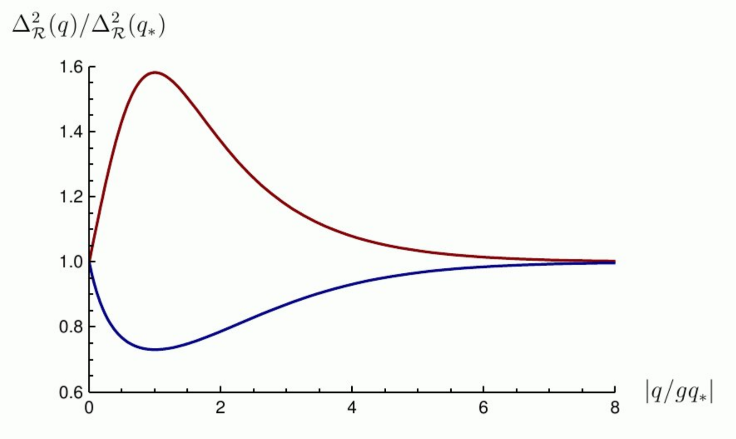
$$\langle T(q)T(-q)\rangle = N^2 q^3 f(g_{\rm eff}^2) \quad \Rightarrow \quad \Delta_{\mathcal{R}}^2(q) = \frac{1}{4\pi^2 N^2}\frac{1}{f(g_{\rm eff}^2)}.$$

The holographic power spectrum is therefore

$$\Delta_{\mathcal{R}}^2(q) = \Delta_{\mathcal{R}}^2 \frac{1}{1 + (gq_*/q)\ln|q/gq_*|}.$$

- $\Delta_{\mathcal{R}}^2 = 1/(4\pi^2 N^2 f_0)$ so small amplitude $\Delta_{\mathcal{R}}^2 \sim 10^{-9}$ implies $N \sim 10^4$, consistent with 't Hooft large-N limit.
- The parameter g is defined by $f_1 g_{\rm eff}^2 = gq_*/q$, where q_* is the cosmological pivot scale. For gravity to be strongly coupled over CMB wavelengths, we require $g_{\rm eff}^2 \ll 1$. This means $(gq_*/q) \ll 1$ hence spectrum is automatically near scale invariant.

Holographic power spectrum



Red curve $g < 0$, blue curve $g > 0$. Perturbative calculation only reliable for large momenta $q/gq_* \gg 1$ far from peak/trough. At very high momenta spectrum rapidly becomes scale invariant (asymptotic freedom).

❺ Testing the holographic power spectrum

Holographic model vs ΛCDM

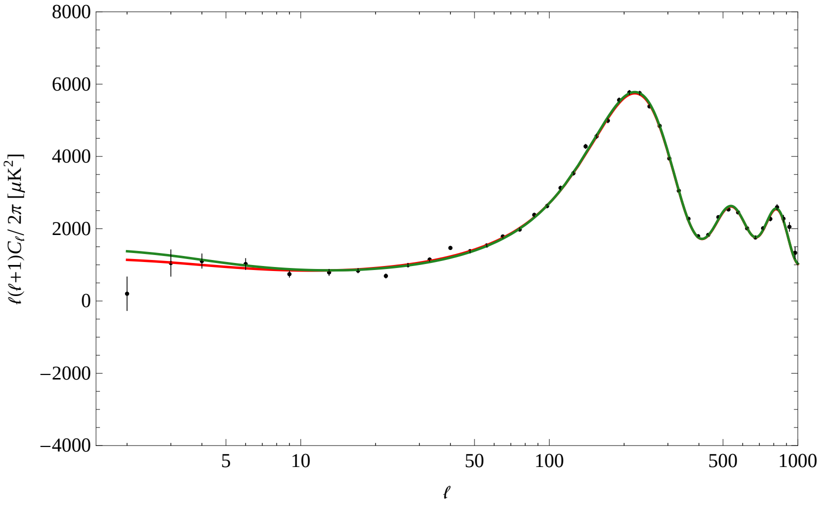
We undertook a custom fit of the holographic model to the current cosmological data, using the empirical power-law ΛCDM model to provide a comparison.

[Dias (2011); Easther, Flauger, PM, Skenderis (2011)]

- ΛCDM power spectrum: $\Delta^2_{\mathcal{R}}(q) = \Delta^2_{\mathcal{R}}(q_0)\,(q/q_0)^{n_S-1}$.
- Both models have six parameters: five of these ($\Omega_b h^2$, $\Omega_c h^2$, h, $\Delta^2_{\mathcal{R}}$) are common to both and were found to lie within one standard deviation of each other.
- The sixth parameter is the tilt n_s (ΛCDM) or the coupling g (holographic model). The WMAP7 data favour a slightly red spectrum with a best-fit:

$$g = (-1.27 \pm 0.93) \times 10^{-3}, \qquad q_* = 0.05\,\mathrm{Mpc}^{-1}.$$

Best-fit angular power spectra



Red $=$ ΛDCM, green $=$ holographic model.

Holographic model vs ΛCDM

The difference in best-fit log likelihoods are:

	Holographic Model	ΛCDM	$\Delta \ln \mathcal{L}_{\text{best}}$
WMAP7	3735.5	3734.3	1.2
WMAP+BAO+H_0	3737.3	3735.7	1.6
WMAP+CMB	3815.0	3812.5	2.5

ΛCDM therefore provides a somewhat better fit, i.e. a higher probability of obtaining the data given the *optimal choice* of model parameters.

To perform model comparison one should compute the Bayesian evidence, i.e. the probability of the model given the data and the prior probability distribution for the model's parameters.

$$E = \int \mathrm{d}\alpha_M P(\alpha_M) \mathcal{L}(\alpha_M)$$

Bayesian Evidence

The specification of priors is important:

- For the holographic model there is a natural choice of prior (require perturbation theory valid for all CMB scales).
- Difficult to fairly assign prior for tilt n_s in ΛCDM: we tried two choices

  (i) $0.92 < n_s < 1.0,$ (ii) $0.9 < n_s < 1.1.$

  The first is near optimal for ΛCDM; the second is symmetric about $n_s = 1$ (since we don't tell the holographic model the sign of the tilt).

For choice (i) we find weak evidence in favour of ΛCDM ($\Delta \ln E \sim 1.2$ to 1.8).

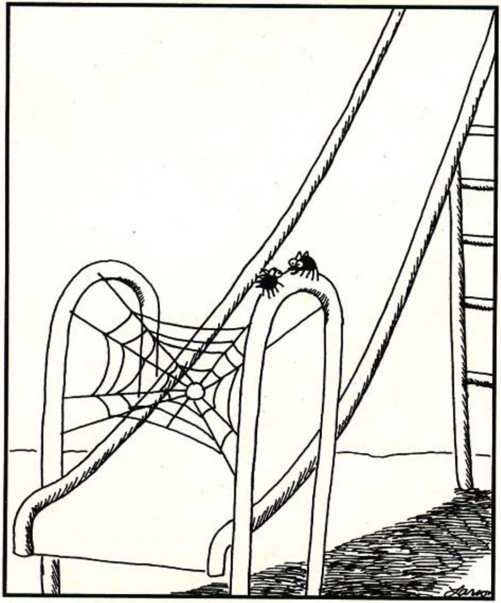
For choice (ii) the evidence is inconclusive ($\Delta \ln E \lesssim 1$).

We conclude that more data is required (Planck satellite), as well as a better theoretical understanding of 2-loop and IR effects, to permit more scale dependence.

Conclusions

1. Standard inflation is holographic: all cosmological observables such as power spectra can be expressed in terms of analytic continuations of correlation functions of a strongly coupled dual QFT.
2. There are new holographic models based on weakly coupled QFT. These describe a universe that started in a non-geometric strongly coupled phase. The dual QFT describing this phase is well-behaved in both the UV and the IR $\rightarrow$ a complete description?
3. These models are compatible with the WMAP7 data. Some hints that further theoretical work needed to understand higher-order effects.
4. Planck might allow a definitive test of the holographic power spectrum within 2 years!

Outlook



"If we pull this off, we'll eat like kings."

Tensor-to-scalar ratio

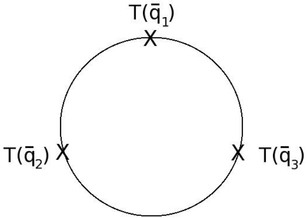
- Holographic model predicts

$$r = \frac{\Delta_T^2}{\Delta_S^2} = 32\left(\frac{\mathcal{N}_A + \mathcal{N}_\phi}{\mathcal{N}_A + \mathcal{N}_\phi + \mathcal{N}_\chi + \mathcal{N}_\psi}\right)$$

where $\mathcal{N}_A = \#$ gauge fields, $\mathcal{N}_\phi = \#$ minimally coupled scalars, $\mathcal{N}_\chi = \#$ conformally coupled scalars, $\mathcal{N}_\psi = \#$ fermions.

- An upper bound on r translates into a constraint on the field content of the dual QFT.
- r is not parametrically suppressed as in slow-roll inflation, nor does it satisfy the slow-roll consistency condition $r = -8n_T$.

Non-Gaussianity



Evaluating the QFT 3-pt function, our holographic formula predicts a scalar bispectrum of exactly the equilateral form with

$$f_{NL}^{equil} = 5/36.$$

[1011.0452]

- This result is independent of all details of the theory, such as QFT field content.
- Too small for direct detection by Planck, but observation of larger f_{NL} would exclude model.
- For tensor bispectra see [1104.3894] and work to appear with Bzowski, Skenderis.