

# *Can Gravity be Localized ?*

C. Bachas, ENS - Paris

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based on : CB, J. Estes, *arXiv:1103.2800 [hep-th]*

B. Assel, CB, J. Estes, J. Gomis, *1106.4253 [hep-th]*  
*and in preparation*

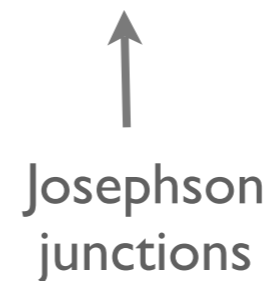
also: O. Aharony, L. Berdichevsky, M. Berkooz, I. Shamir, *arXiv:1106.1870 [hep-th]*

Fields can be localized on (extended) **solitons** in **QFT**:

there are many spin-0 and spin-1/2 examples



Can also localize abelian “spin-1” fields



Renormalizability requires  $D \leq 4 \implies d \leq 3$

which severely restrict the possibilities.

In **string theory**  $D=10$  and  $\exists$  UV completion  
so more room



can do *some*  
calculations without  
infinities

**non-abelian** spin-1 gauge fields localized on **D-branes**

but what about spin 2 ?

Einstein's theory is much harder to "tinker" with .....

*This is closely related to the questions:*

Can the graviton have mass?

Can it be a resonance?

Are sectors “hidden” from gravity possible ?

Other IR modifications of Einstein equations ?

*The subject has a long history, to which I will not  
try to do justice here .....*

see also Slava Mukhanov's talk

In Minkowski spacetime, the answer seems to be **NO**

An important obstruction is the **vDVZ discontinuity**

van Dam, Veltman, Zakharov '70

Notice that for the photon the answer is **YES**

Indeed, the particle data group quotes the experimental bound:

$$m_\gamma < 10^{-18} eV$$

range  $> 10^9 km \sim 1$  light hour **but could be finite!**

To understand the difference, consider the linearized Lagrangian for a **massive spin-1 particle**:

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{m^2}{2}A_\mu A^\mu + A_\mu j^\mu$$

Introducing a spurious field  $A_\mu = A'_\mu + \frac{1}{m}\partial_\mu\phi$  and taking  $m \rightarrow 0$  gives:

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}(\partial_\mu A'_\nu - \partial_\nu A'_\mu)^2 + A'_\mu j^\mu - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{m}\partial_\mu\phi j^\mu \\ &= \mathcal{L}_{\text{Maxwell}} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi\end{aligned}$$

The dangerous last term drops out, provided the *e-m* current is conserved, so that the extra scalar mode decouples.

Now repeat the exercise for a **massive spin-2 field**.

The (ghost-free) massive **Pauli - Fierz** Lagrangian is:



$$\mathcal{L}_{\text{PF}} = \mathcal{L}_{\text{EH}} - \frac{m^2}{2} (h^{\nu\lambda} h_{\nu\lambda} - (h^\rho{}_\rho)^2)$$

where

$$\mathcal{L}_{\text{EH}} = -\frac{1}{2} \partial_\mu h^{\nu\lambda} \partial^\mu h_{\nu\lambda} + \partial^\mu h^{\nu\lambda} \partial_\nu h_{\mu\lambda} - \partial_\mu h^{\mu\nu} \partial_\nu h^\lambda{}_\lambda + \frac{1}{2} \partial_\nu h^\lambda{}_\lambda \partial^\nu h^\rho{}_\rho + h_{\mu\nu} T^{\mu\nu}$$

with

$$\partial_\mu T^{\mu\nu} = 0$$

Introduce again compensators to restore gauge invariance:

$$h_{\mu\nu} = h'_{\mu\nu} + \frac{1}{m}(\partial_\mu A_\nu + \partial_\nu A_\mu) + \frac{2}{m^2}\partial_\mu\partial_\nu\phi$$

invariant under

$$\delta h_{\mu\nu} = \partial_\mu\xi_\nu + \partial_\nu\xi_\mu$$

$$\delta A_\mu = -m\xi_\mu + \partial_\mu\Lambda$$

$$\delta\phi = -m\Lambda$$

Inserting in  $\mathcal{L}_{\text{PF}}$  gives a free massless spin-1 field, and a **two-derivative** Lagrangian mixing  $\phi$  and  $h'_{\mu\nu}$ .



PF was precisely devised for this !



Redefining fields to remove the mixing ( $h'_{\mu\nu} = h''_{\mu\nu} + \eta_{\mu\nu}\phi$ ) finally gives:

$$\mathcal{L}_{\text{PF}} = \mathcal{L}_{\text{EH}} - \frac{1}{2}F_{\mu\nu}F^{\mu\nu} - 3\partial_{\mu}\phi\partial^{\mu}\phi + \boxed{\phi T^{\rho}_{\rho}}$$

The residual coupling is different for light, than for massive matter;  
thus the Pauli-Fierz theory does not give Einstein's theory when  $m \rightarrow 0$

If we set Newton's law to its measured form,

light bending = 3/4 of measured effect

.... so however tiny the mass, it is ruled out !

The story looks more promising in AdS:

◆ The vDVZ discontinuity is absent if  $m_{\text{gr}} < 1/L_{\text{AdS}}$

Kogan - Mouslopoulos - Papazoglou;  
Porrati

◆  $\exists$  a simple “model”, possibly embed-able in string theory

Karch-Randall

◆ Supersymmetry can protect the required hierarchy

Of course, we don't seem to live in AdS spacetime !

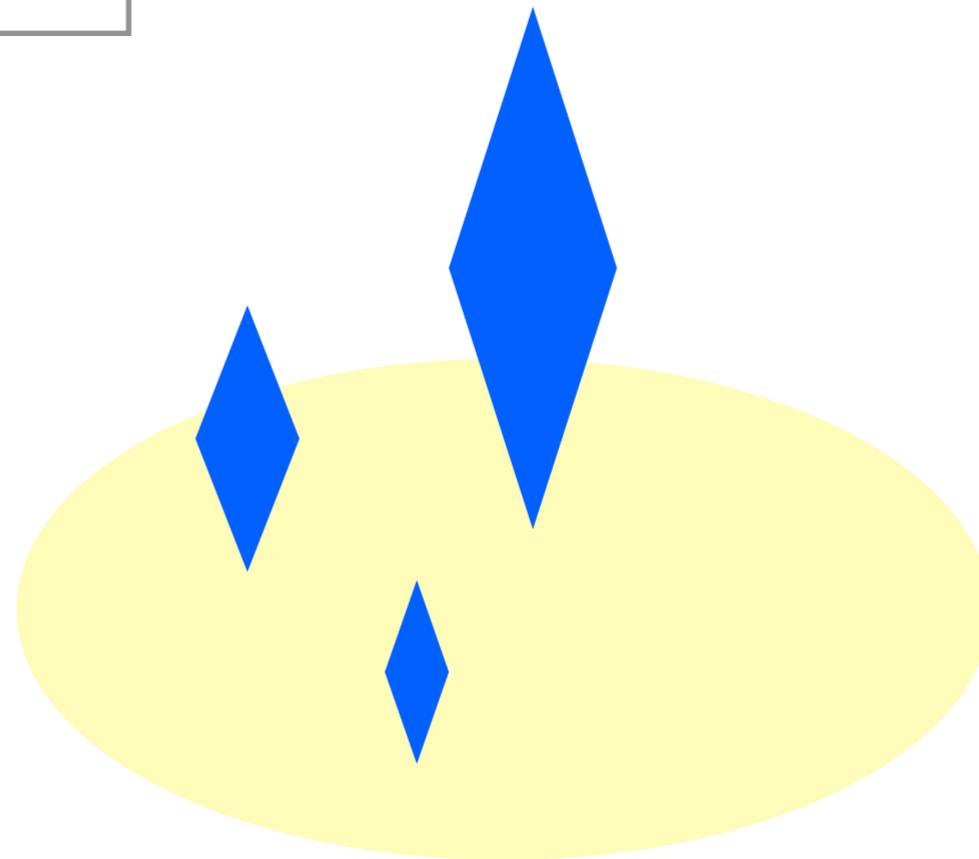
OK, take attitude that anything one can learn about IR gravity is interesting, and proceed.

## KK reduction for spin 2

Interested in *warped*-(A)dS geometries,

$$\widehat{ds}^2 = e^{2A(y)} \bar{g}_{\mu\nu}(x) dx^\mu dx^\nu + \hat{g}_{ab}(y) dy^a dy^b$$

$$\begin{aligned} \bar{\mathcal{M}}_4 &= \text{AdS}_4, \mathbb{M}_4, \text{dS}_4 \\ k &= -1, 0, 1 \end{aligned}$$



Consider (consistent reduction to) metric perturbations

$$ds^2 = e^{2A} (\bar{g}_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu + \hat{g}_{ab} dy^a dy^b ,$$

with  $h_{\mu\nu}(x, y) = h_{\mu\nu}^{[tt]}(x) \psi(y)$

where  $(\bar{\square}_x^{(2)} - \lambda) h_{\mu\nu}^{[tt]} = 0$  and  $\bar{\nabla}^\mu h_{\mu\nu}^{[tt]} = \bar{g}^{\mu\nu} h_{\mu\nu}^{[tt]} = 0$  .

Pauli-Fierz  $(\lambda = m^2 + 2k)$

Linearizing the Einstein equations  $R_{MN} - \frac{1}{2}g_{MN}R = T_{MN}$

leads to the Schrodinger problem :

$$-\frac{e^{-2A}}{\sqrt{[\hat{g}]}} (\partial_a \sqrt{[\hat{g}]} \hat{g}^{ab} e^{4A} \partial_b) \psi = m^2 \psi$$

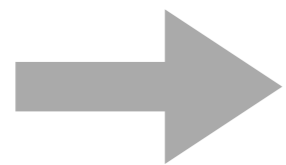
This is equivalent to a *scalar-Laplace* equation in d dimensions :

$$\frac{1}{\sqrt{\hat{g}}} (\partial_M \sqrt{\hat{g}} \hat{g}^{MN} \partial_N) h_{\mu\nu}(x, y) = 0 .$$

**Important:** the linearized equation **depends only on the geometry**, not on the detailed matter-field backgrounds.

Csaki, Erlich, Hollowood, Shirman

CB, JE



**Localization of spin-2 can only come from geometry**

The wavefunction norm is

$$\|\psi\|^2 \equiv \int d^{d-4}y \sqrt{[\hat{g}]} e^{2A} |\psi|^2$$

The would-be massless graviton has  $\psi(y) = \text{constant}$

It is normalizable **iff** the transverse volume is finite

Why can't the warp factor "help"?

When it does, *infinity* is an apparent horizon, so

- geometry should be made **geodesically complete**
- or need to supplement the *quantum* theory with **boundary conditions at the horizon** ("IR brane")

In the cases

$$\mathcal{M}_4 = \mathbb{M}_4 \quad \text{or} \quad \text{dS}_4$$

the energy conditions show (at least for codim = 1) that the warp factor  $A$  is monotonic, so it cannot turn around to form an **effective “graviton trap”**

But for  $\mathcal{M}_4 = \text{AdS}_4$  localization, and a tiny AdS graviton mass, cannot be ruled out in this way.

The Karch-Randall model illustrates this point.



## Karch-Randall model

Starting point is 5D Einstein action plus a thin 3-brane

$$I_{\text{KR}} = -\frac{1}{2\kappa_5^2} \int d^4x dy \sqrt{g} \left( R + \frac{12}{L^2} \right) + \lambda \int d^4x \sqrt{[g]_4} ,$$

The solution is:

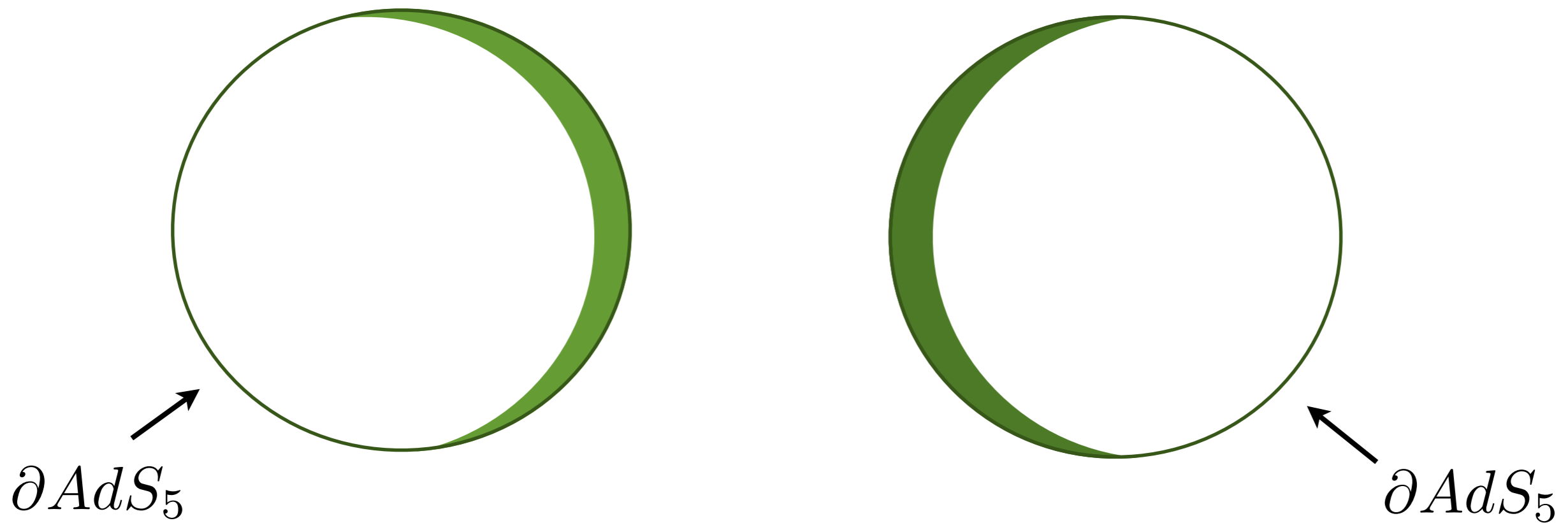
$$ds^2 = L^2 \cosh^2 \left( \frac{y_0 - |y|}{L} \right) \bar{g}_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad \text{where} \quad y_0 = L \operatorname{arctanh} \left( \frac{\kappa_5^2 \lambda L}{6} \right)$$

It describes two (large) slices of AdS<sub>5</sub> glued along a AdS<sub>4</sub> brane

with radius

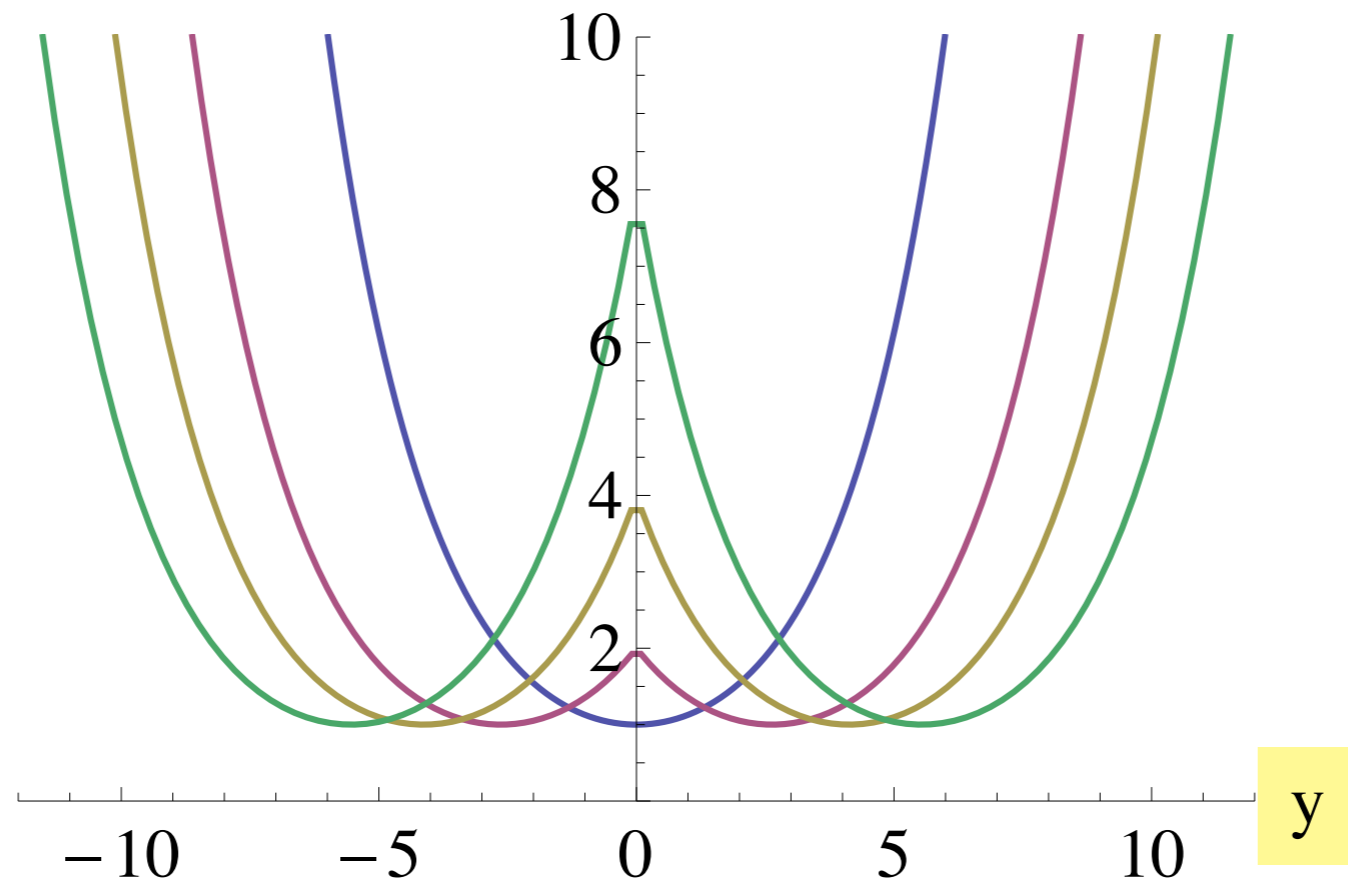
$$\ell^2 = e^{2A(0)} = L^2 \cosh^2 \left( \frac{y_0}{L} \right) .$$

One can tune  $\lambda L$  so that  $\frac{\ell}{L} \gg 1$



Cut away green slices, then glue the white ones in a symmetric fashion. Gives two 4D boundaries glued across two 3D defects (domain walls).

$$e^A$$



Warp factor 
$$e^{2A} = L^2 \cosh^2 \left( \frac{y_0 - |y|}{L} \right)$$

as  $\ell/L$  is gradually tuned up

4D parameters:  $8\pi G_N \simeq \kappa_5^2/L$  ← *as in usual KK*

$$V_{\text{Newton}} + \Delta V \simeq -\frac{G_N m_1 m_2}{r} \left(1 + \gamma \frac{L^2}{r^2} + \dots\right)$$

so  $\frac{\ell}{L} \sim 10^{31} - 10^{62}$

↑  
*unlike standard KK*

Spectrum : - a nearly-constant, nearly massless mode  $m_0^2 \simeq \frac{3L^2}{2\ell^2}$

- two towers of AdS5 modes

$$m^2 \simeq (2n + 1)(2n + 4) \quad n = 0, 1, \dots$$

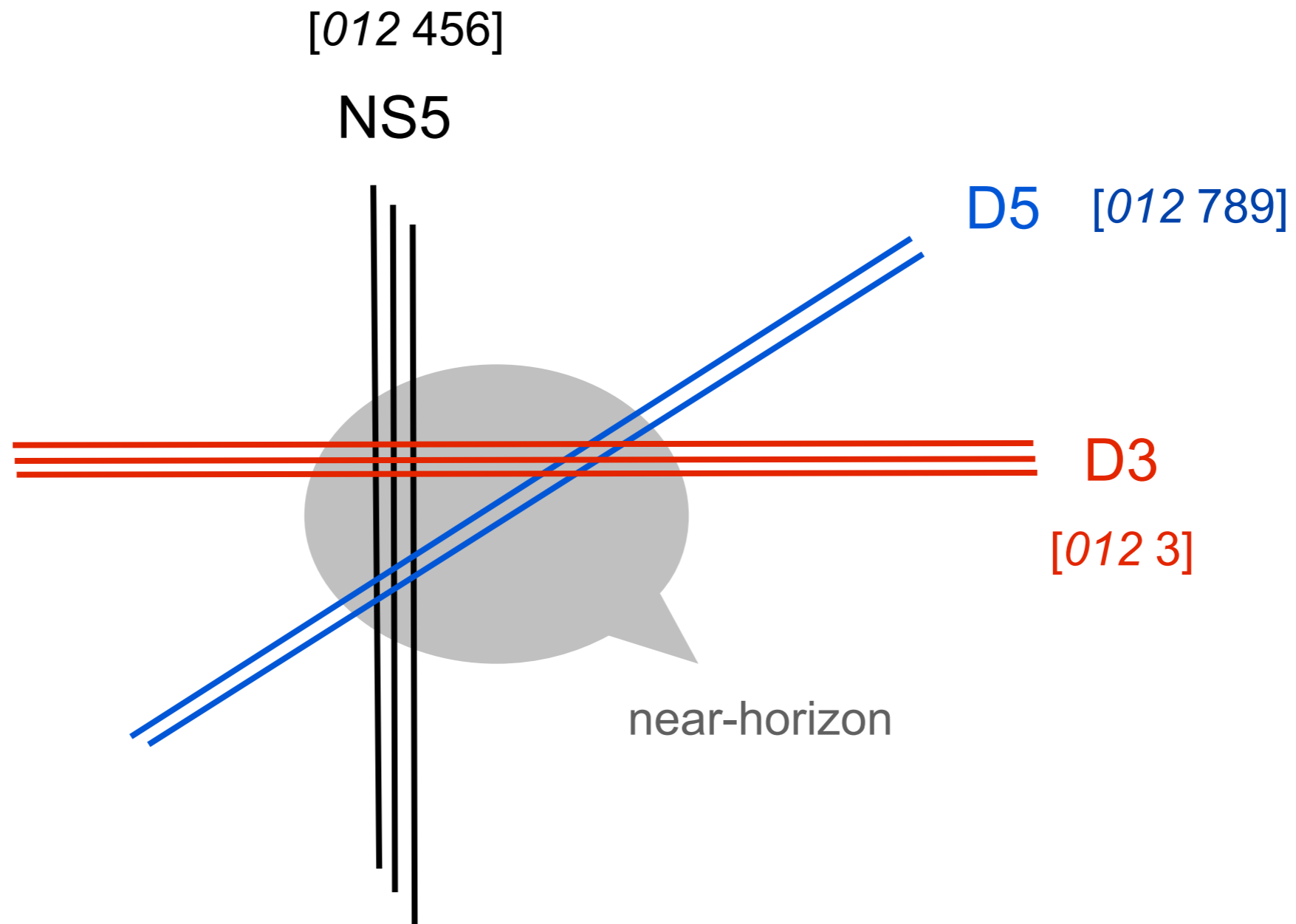
These masses are expressed in units of the AdS<sub>4</sub> radius  
so states with  $m^2 \simeq o(1)$  mediate long-range interactions.

What “saves the day” is that the AdS<sub>5</sub> states live at the  
bottom of the warp-factor well . Their wavefunctions are  
**exponentially suppressed at the brane position**

Furthermore,  $\int \psi_0 \psi^\dagger \psi \neq \text{universal}$

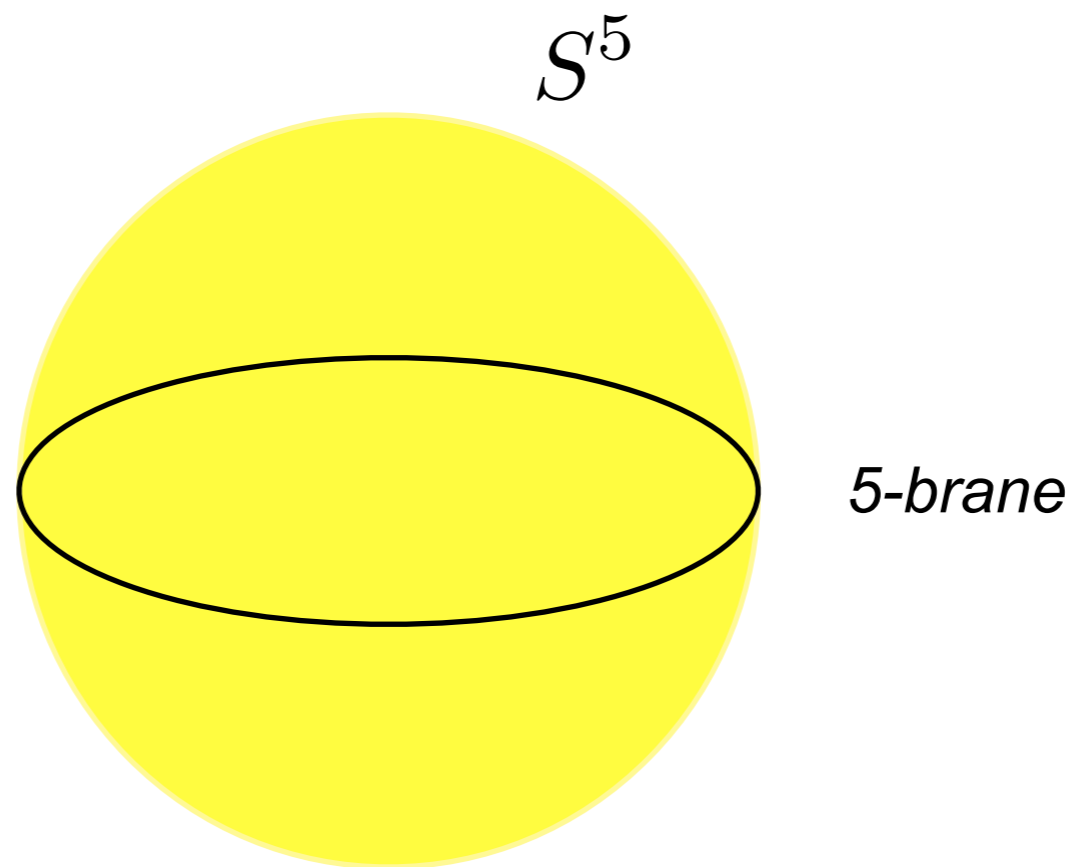
so the nearly-massless graviton has **non-universal couplings**  
to the other fields !

## The String-theory embedding



*Karch and Randall* proposed to embed their model in IIB string theory, by inserting 5-branes in the  $AdS_5 \times S^5$  geometry of D3-branes.

The geometry of a 5-brane in the probe limit is  $AdS_4 \times S^2$



The exact geometry of these configurations was discovered some years ago  
by *D'Hoker, Estes and Gutperle*

**Q:** Is the graviton in these geometries “localized” ?

**A:** **No**; but it does obtain an arbitrarily-small mass in  
a curved 10d spacetime.



The **EDG** solutions are  $AdS_4 \times S^2 \times S^2$  fibrations over a surface  $\Sigma$  .

They depend on **two harmonic functions**  $h_1, h_2$  subject to certain global consistency conditions.

metric :  $ds^2 = f_4^2 ds_{AdS_4}^2 + f_1^2 ds_{S_1^2}^2 + f_2^2 ds_{S_2^2}^2 + 4\rho^2 dzd\bar{z}$  ,

$$f_4^8 = 16 \frac{N_1 N_2}{W^2} , \quad f_1^8 = 16 h_1^8 \frac{N_2 W^2}{N_1^3} , \quad f_2^8 = 16 h_2^8 \frac{N_1 W^2}{N_2^3}$$

dilaton :  $e^{4\phi} = \frac{N_2}{N_1}$

where :  $W = \partial h_1 \bar{\partial} h_2 + \bar{\partial} h_1 \partial h_2 = \partial \bar{\partial} (h_1 h_2)$  ,

$$N_1 = 2h_1 h_2 |\partial h_1|^2 - h_1^2 W , \quad N_2 = 2h_1 h_2 |\partial h_2|^2 - h_2^2 W .$$

There are also 3-form and 5-form backgrounds, and 1/4 unbroken supersymmetry.

The solutions of interest have  $\Sigma =$  infinite strip  
 with  $h_1, h_2$  obeying N or D conditions, possibly  
 with isolated singularities on the boundary, e.g.



The harmonic functions for this choice are:

$$h_1 = \left[ -i\alpha_1 \sinh(z - \beta_1) - \gamma_1 \ln \left( \tanh\left(\frac{i\pi}{4} - \frac{z - \delta_1}{2}\right) \right) \right] + \text{c.c.} ,$$

$$h_2 = \left[ \alpha_2 \cosh(z - \beta_2) - \gamma_2 \ln \left( \tanh\left(\frac{z - \delta_2}{2}\right) \right) \right] + \text{c.c.} .$$

Reduction of eigenmode equation:  $\psi(y^a) = Y_{l_1 m_1} Y_{l_2 m_2} \psi_{l_1 l_2}(z, \bar{z})$   
 leads to a **Laplace-Beltrami** spectral problem on  $\Sigma$  :

$$\frac{2h_1 h_2}{\partial \bar{\partial}(h_1 h_2)} \partial \bar{\partial} \tilde{\psi}_{00} = (m^2 + 2) \tilde{\psi}_{00} , \quad \text{where} \quad \tilde{\psi}_{00} \equiv h_1 h_2 \psi_{00} .$$

The norm is  $\|\psi\|^2 = \int_{\Sigma} d^2 z |W h_1 h_2| |\psi_{l_1 l_2}|^2 = \int_{\Sigma} d^2 z \left| \frac{W}{h_1 h_2} \right| |\tilde{\psi}_{l_1 l_2}|^2$

and the b.conditions for  $\psi_{00}$  are *Neumann*.

The simplest **Janus** solution  $[\gamma_i = 0]$  is a *dilaton domain wall*

$$L^4 = 16 |\alpha_1 \alpha_2| \cosh \Delta \phi$$

↑  
radius of asymptotic  $AdS_5 \times S^5$

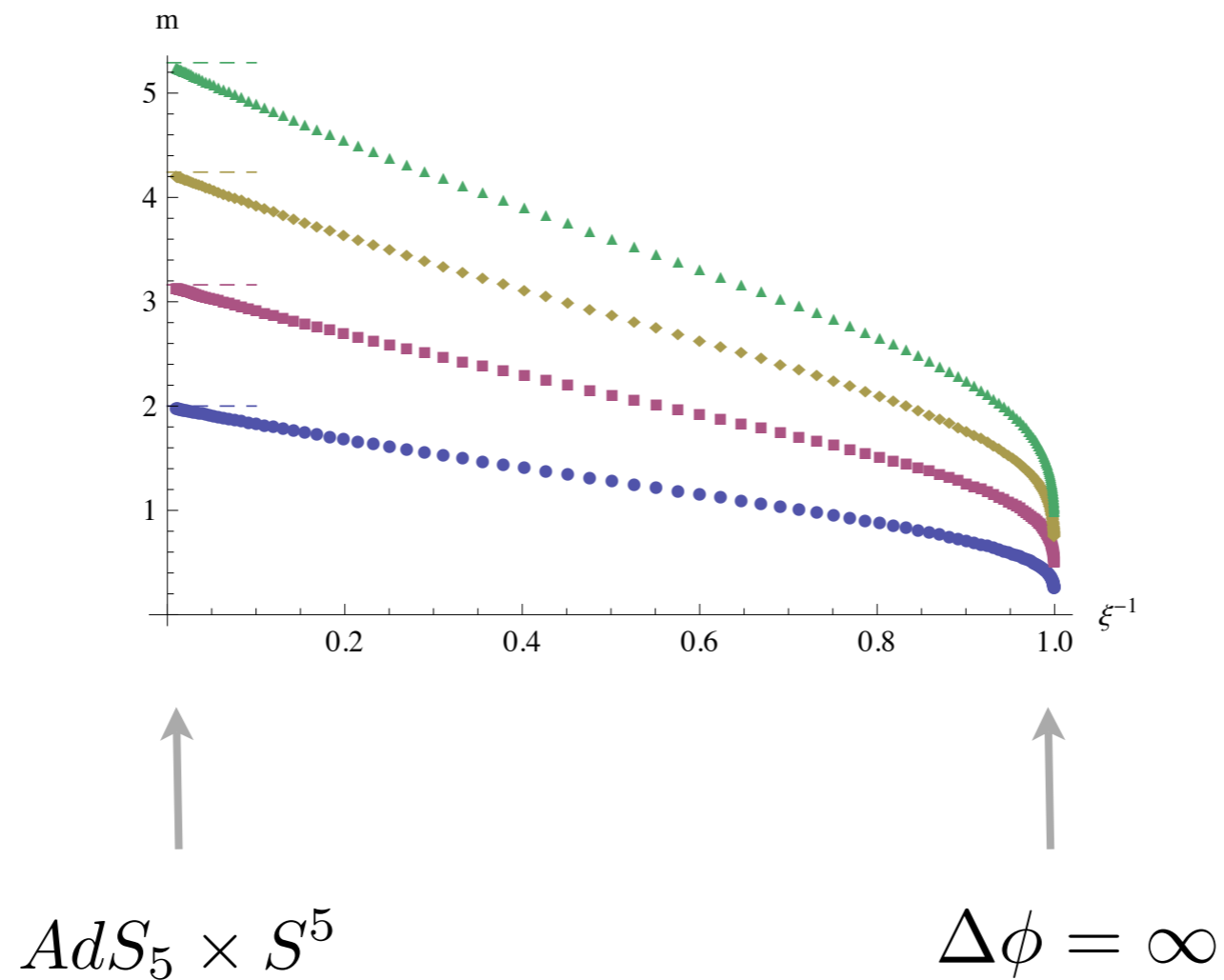
$$e^{2\phi_{\pm}} = \left| \frac{\alpha_2}{\alpha_1} \right| e^{\pm \Delta \phi}$$

↙  
dilaton jump

The spectral equation reduces to a ODE with 4 regular singular points (**Heun's equation**) which can be solved easily with fast numerics

The results are not particularly exciting for our purposes here:

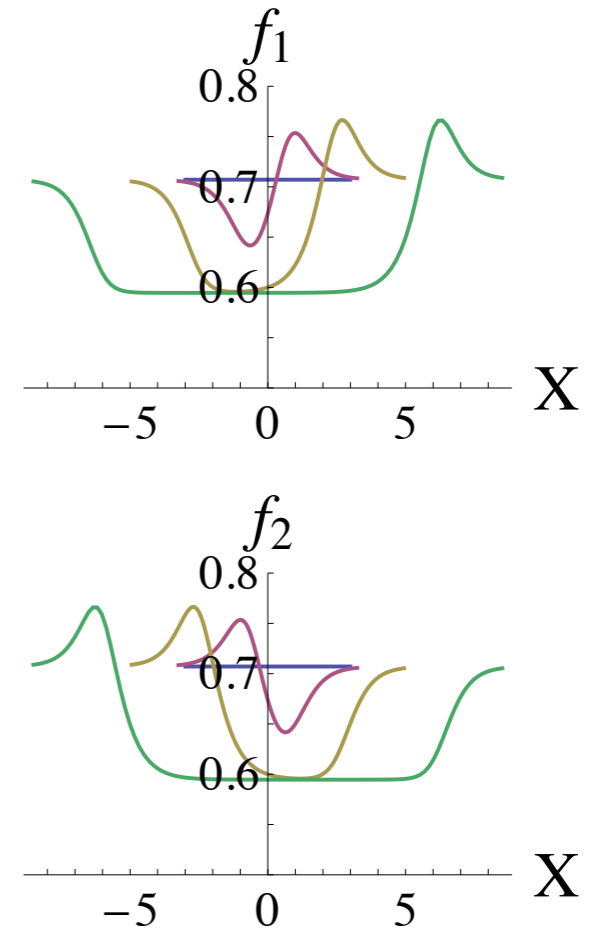
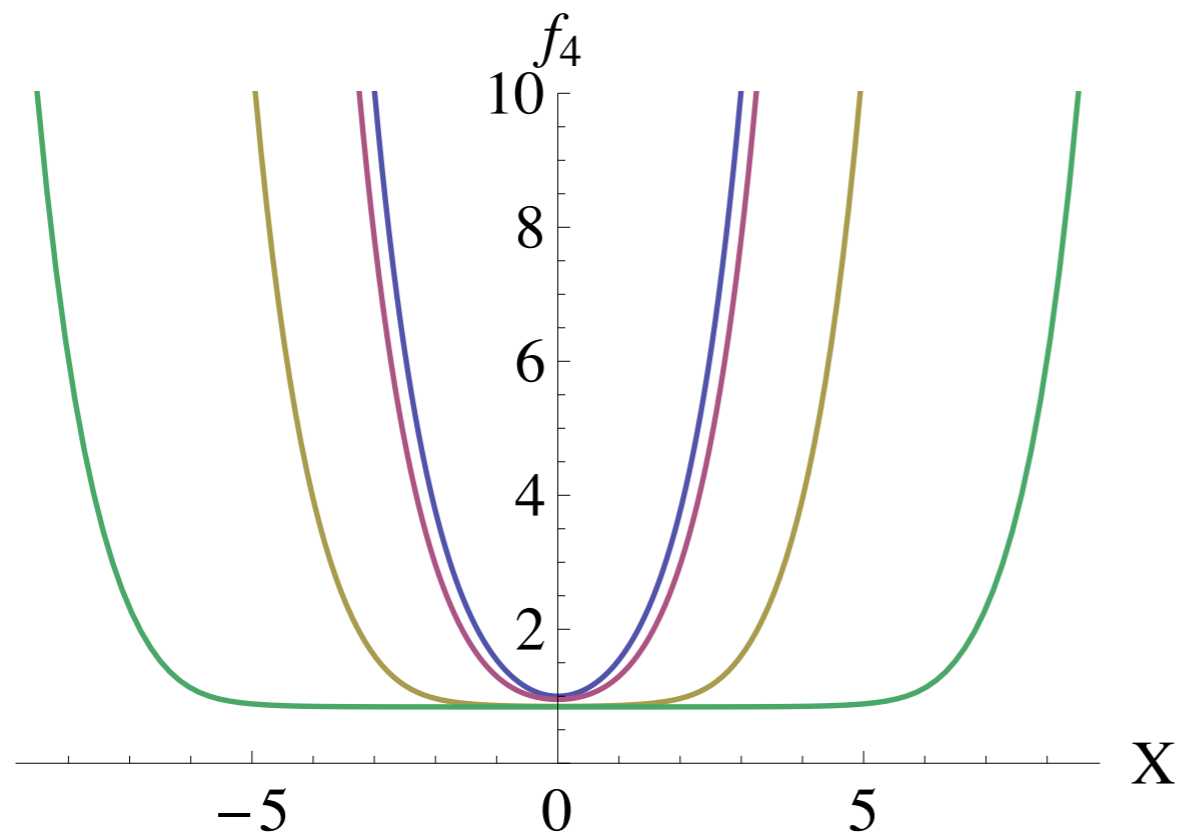
There is one free parameter,  $(\tanh \frac{\Delta\phi}{2})^2 \equiv \xi^{-1} \in [0, 1)$



The only interesting limit is one in which a **linear-dilaton dimension decompactifies**, and the geometry becomes

$$AdS_4 \times \mathbb{R}_\phi \times_w \tilde{S}^5$$

$$\Delta\phi = 0, 1, 4, 10$$

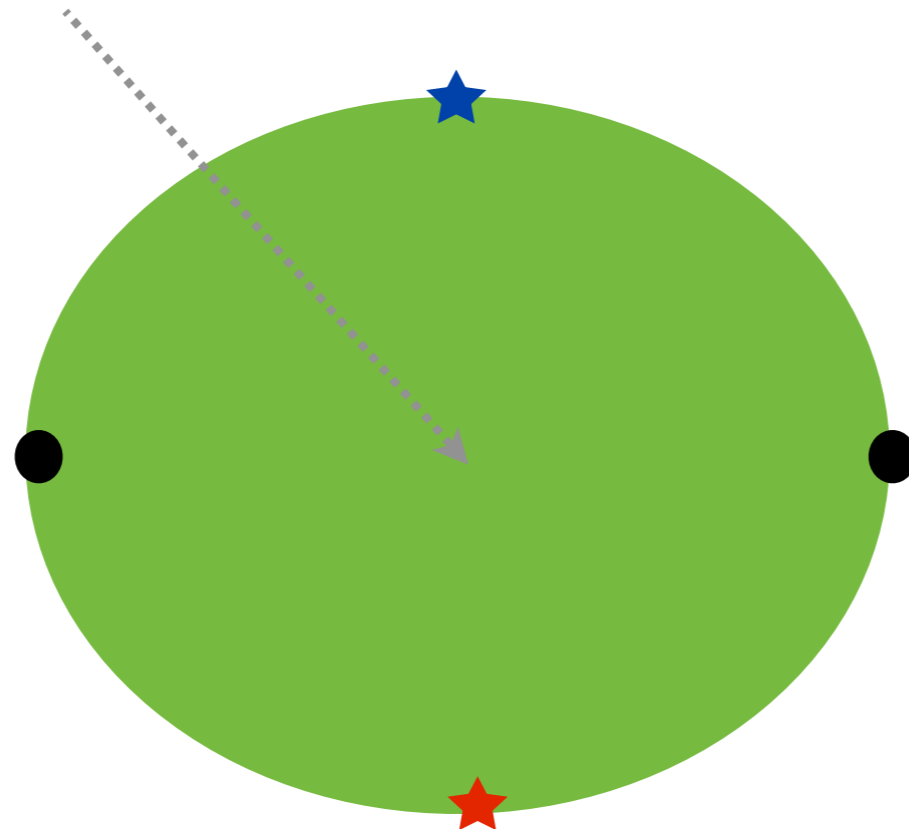


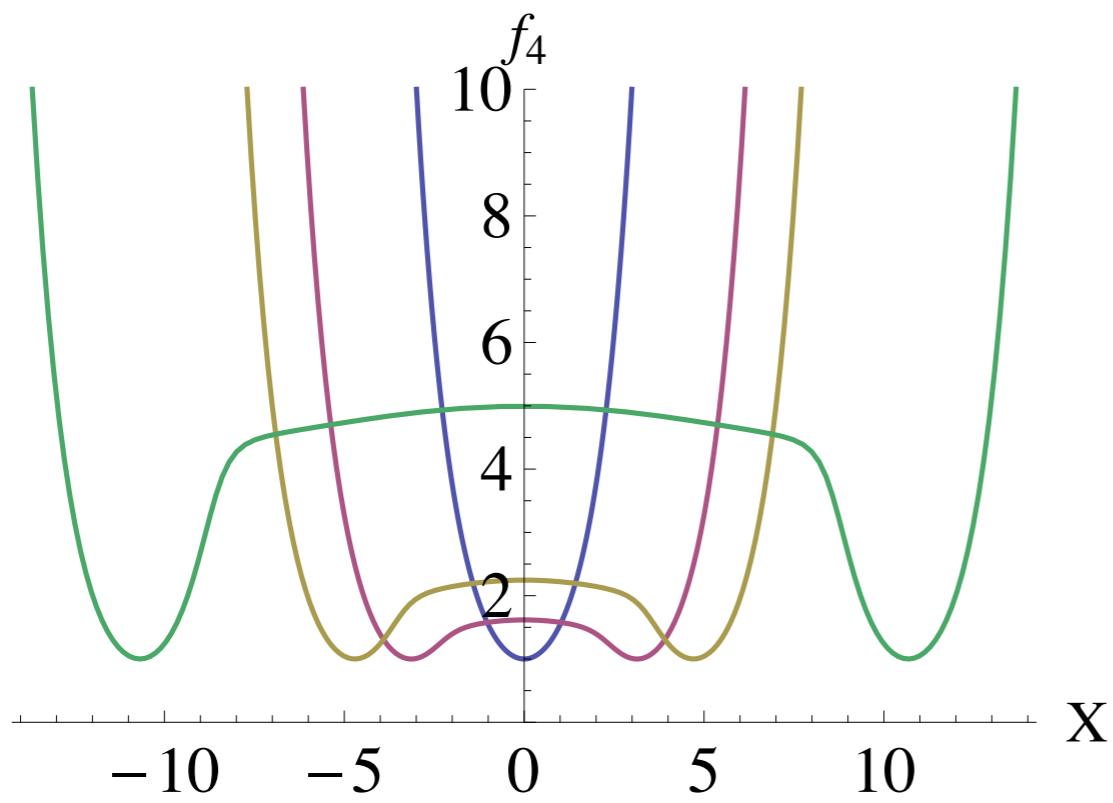
The “problem” is that the dilaton has **no (super)potential**, so its domain wall spreads to infinite thickness.

Adding one type of 5-branes does not help: the dilaton adjusts to ( $\infty$  ly) small or large value, so as to minimize 5-brane tension.

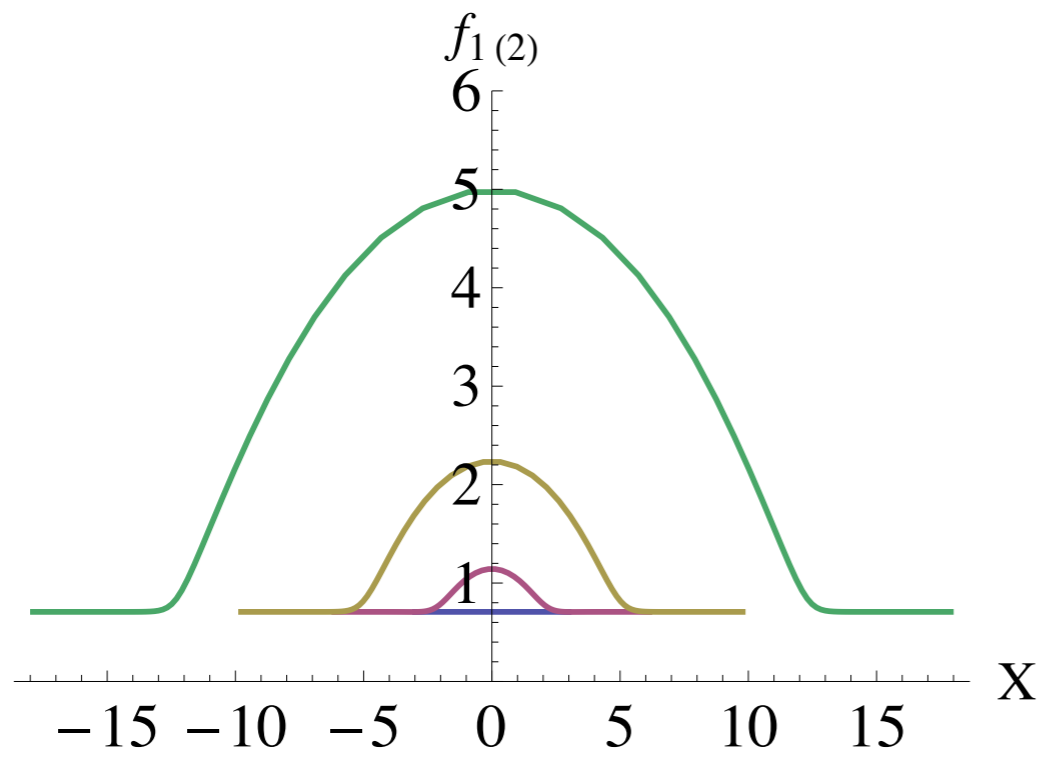
The only interesting limit is one with both NS5 and D5 charges, and with  $\frac{Q_5}{Q_3} \gg 1$ .

Inspection of the geometry shows that this leads to a bubble of (almost flat)  $\mathbb{R}^{1,9}$  in the central region.





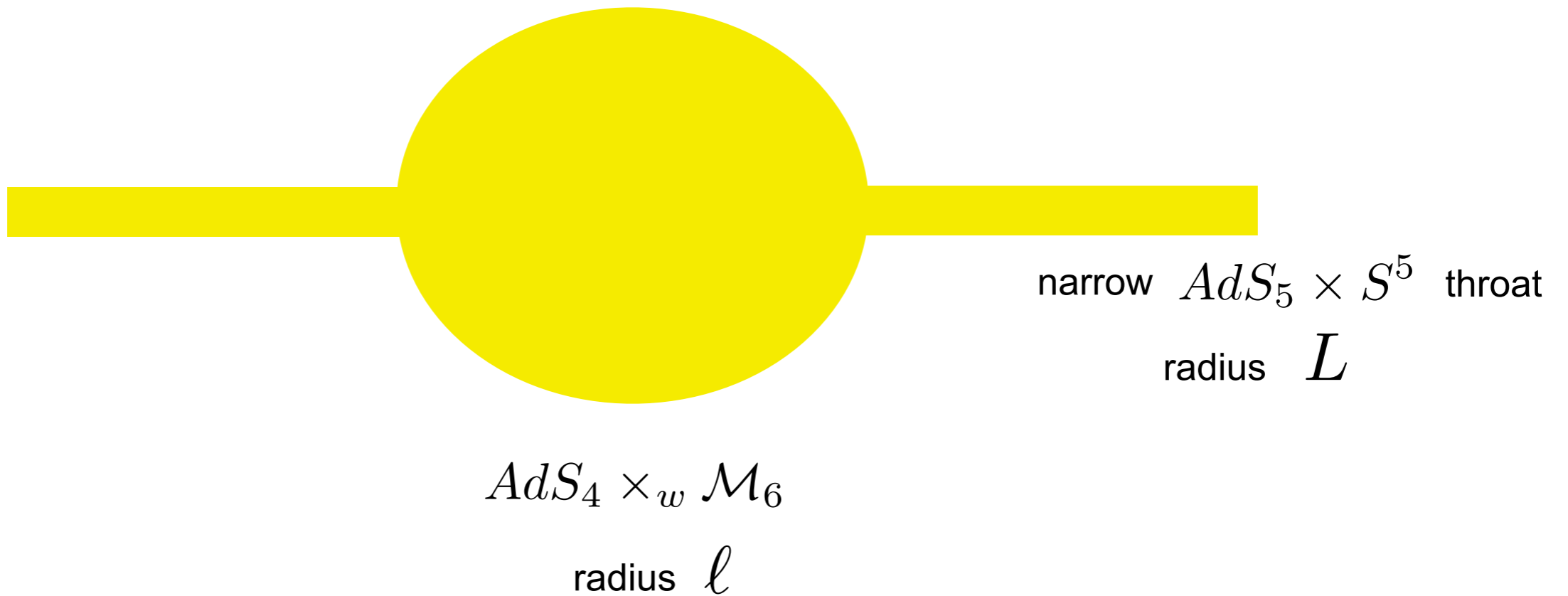
warp factor



sphere radii



The 10d geometry looks like this:



graviton mass  $\sim \frac{L}{\ell} \ll 1$

This is as expected by KR, except that the backreaction decompactifies the 10d spacetime

The dual holographic interpretation is that of a domain wall in N=4 super Yang-Mills.

The domain wall degrees of freedom are much more numerous than those of the bulk. The 3d energy-momentum tensor is thus almost (but not exactly) conserved.

Actually the limit  $Q_3 \rightarrow 0$  is smooth: **transverse space compactifies**,  
the asymptotic regions  $AdS_5 \times S^5$  go over to smooth  $AdS_4 \times \mathcal{D}_6$  caps

These  $AdS_4 \times_w \mathcal{M}_6$  solutions must be **gravity duals to**  
**3-dimensional (super)conformal field theories**

Which ones ?

By studying the flat-space configurations, *Gaiotto and Witten* have proposed the existence of a class of interacting SCFTs in three dimensions that they called

$$T_{\rho}^{\hat{\rho}}(SU(N))$$

They are in 1-to-1 correspondence with solutions of Nahm's equations:

$$\frac{dX^a}{dt} = i\epsilon_{abc}[X^b, X^c]$$

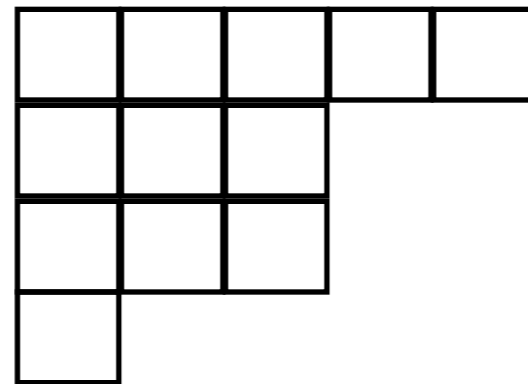
on the interval, with boundary conditions that are simple poles,

$$X^a \sim \frac{J^a}{t} \longleftarrow \begin{array}{l} \text{N-dimensional generators} \\ \text{of } SU(2) \end{array}$$

This problem has been solved by *Kronheimer and Nakajima*

One can associate a partition of  $N$  with each choice of the  $J^a$

e.g.  $\rho: 12 = 5 + 3 + 3 + 1$



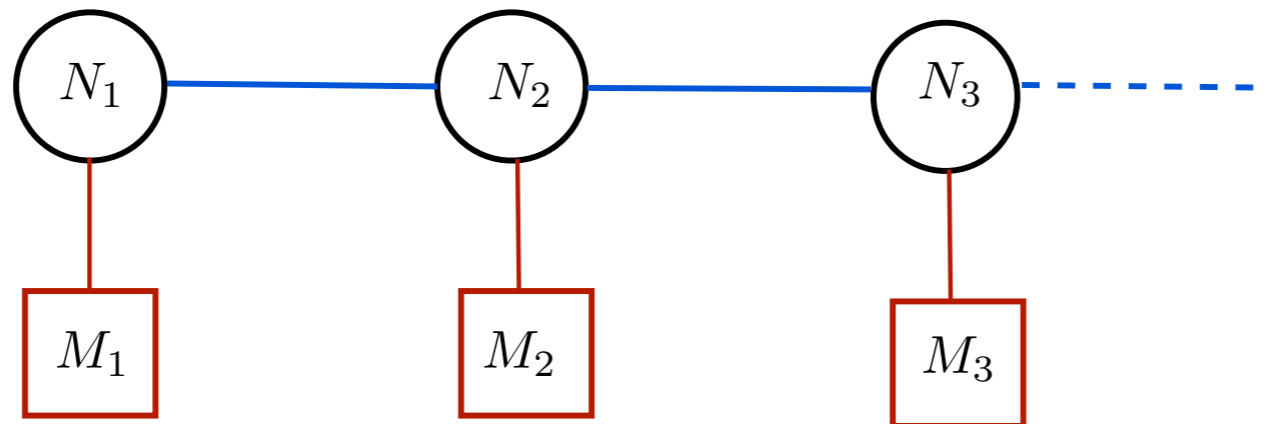
$K$  &  $N$  have shown that solutions exist iff  $\rho^T > \hat{\rho}$

where these are the two partitions at the interval ends.

(CB, J.Hoppe, B. Pioline '00)

By computing these partitions from the 5-brane charges, could show that supergravity knows about this constraint !

The underlying gauge theories are described by *linear quivers*

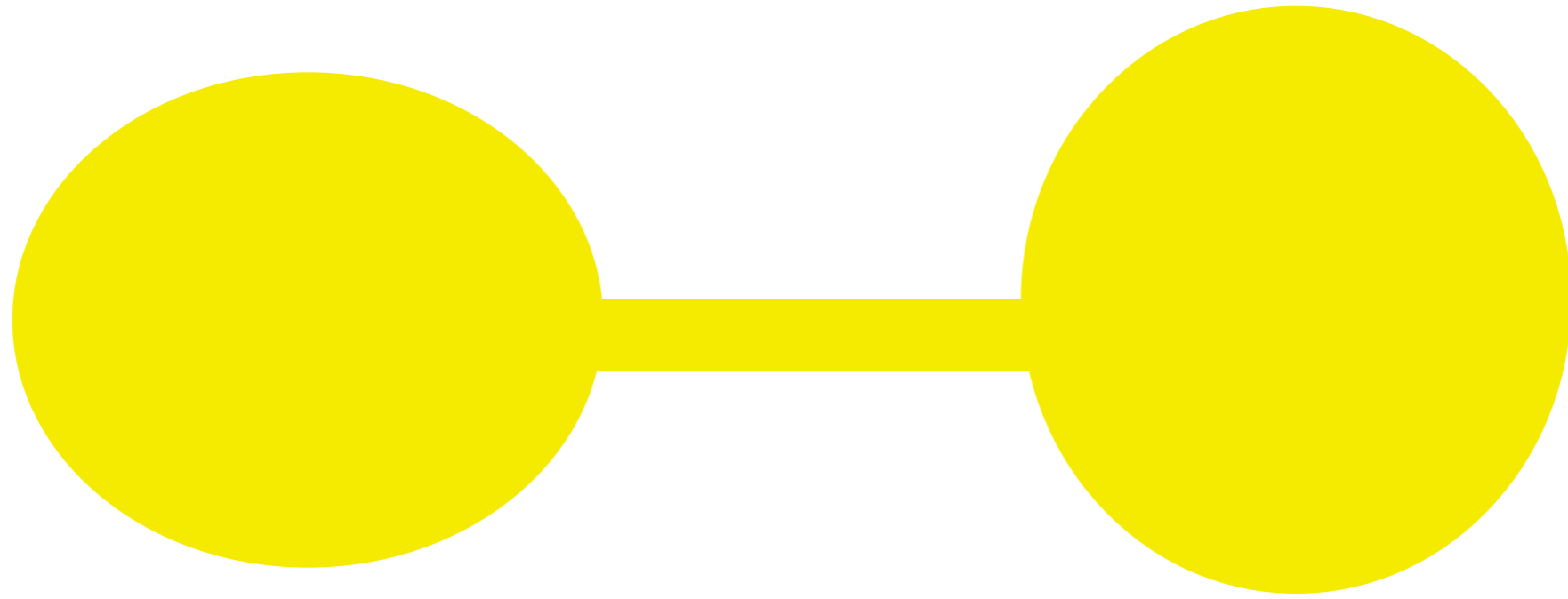


$$U(N_1) \times U(N_2) \times U(N_3) \times \dots$$

Another interesting limit  $\hat{\rho} \simeq \rho^T$  correspond to **severing**

**one** (or more) **link**, by taking  $N_i \rightarrow 0$

This corresponds to factorizing the 5-brane singularities on the strip.

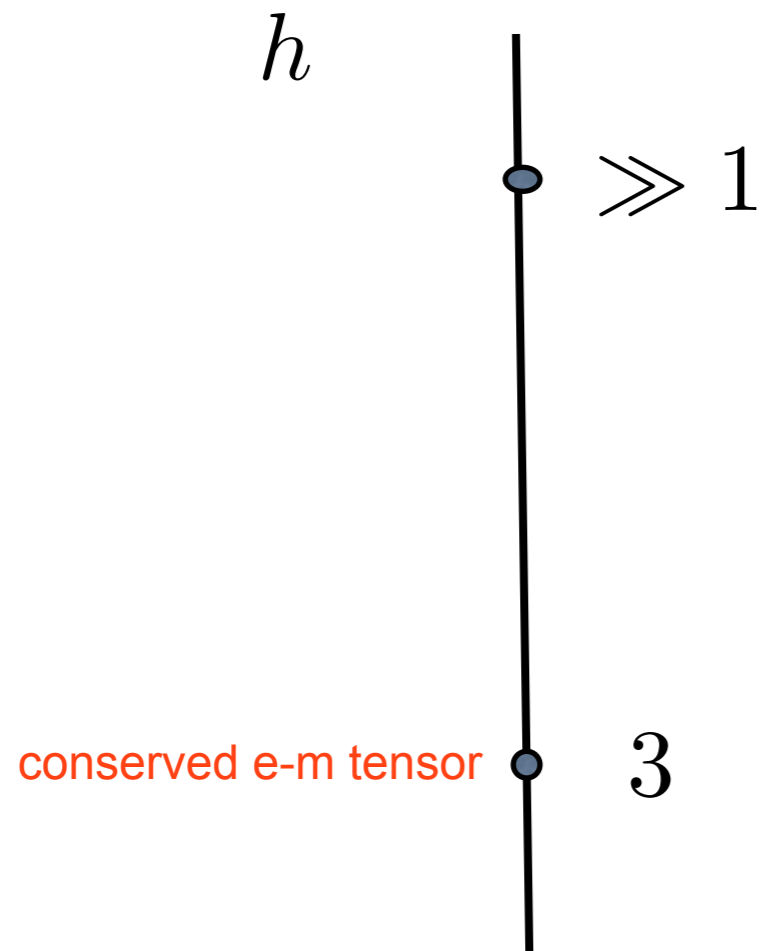


this is a string-theory wormhole

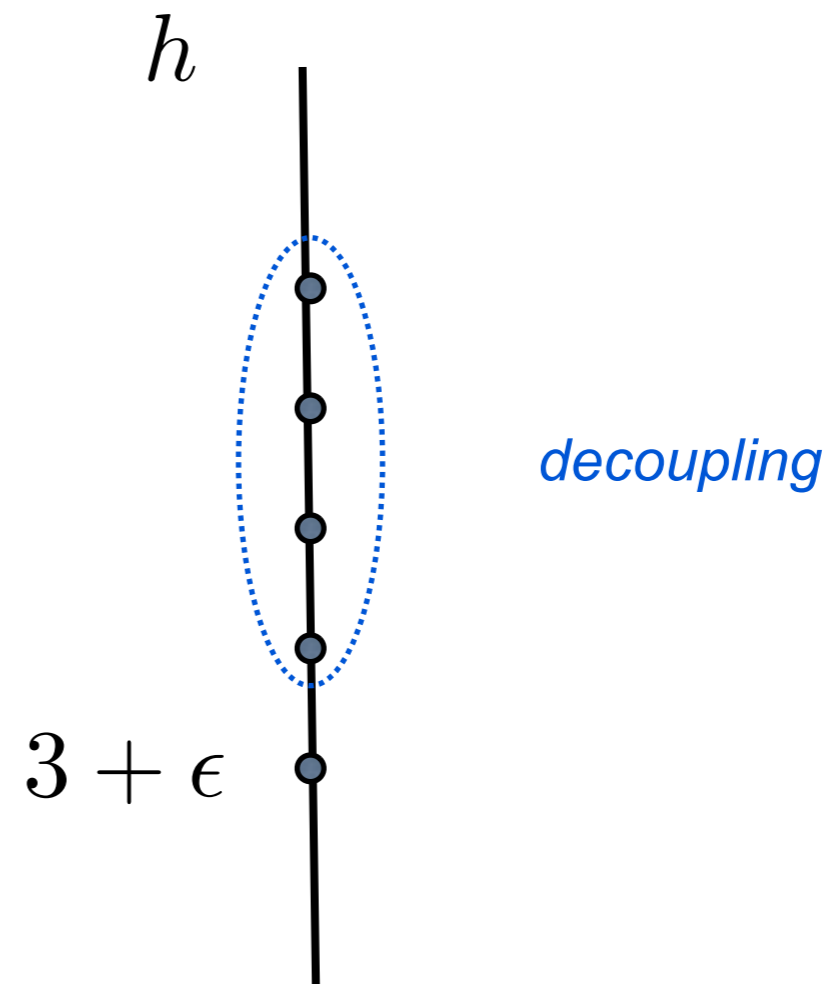


# Holographic comment

on massive AdS gravity theories:



CFT spectrum



defect CFT spectrum

*Thank you*