Introduction to loop Quantum Cosmology

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A broad perspective on the challenges of Quantum Cosmology

Focus on the structure and successes of loop quantum Cosmology.

Organization:

1. Historical & Conceptual Setting
2. Structure of Loop Quantum Cosmology
3. Illustrative Applications: Inflation, QFT on Cosmological QSTs, ...

1. Historical & Conceptual Setting

- Deepest Feature of General Relativity: Gravity encoded in Geometry.

Space-time geometry became a physical and dynamical entity. Spectacular consequences: Cosmology, Black holes, Gravitational Waves. Impressive mathematical applications: Geometric Analysis.

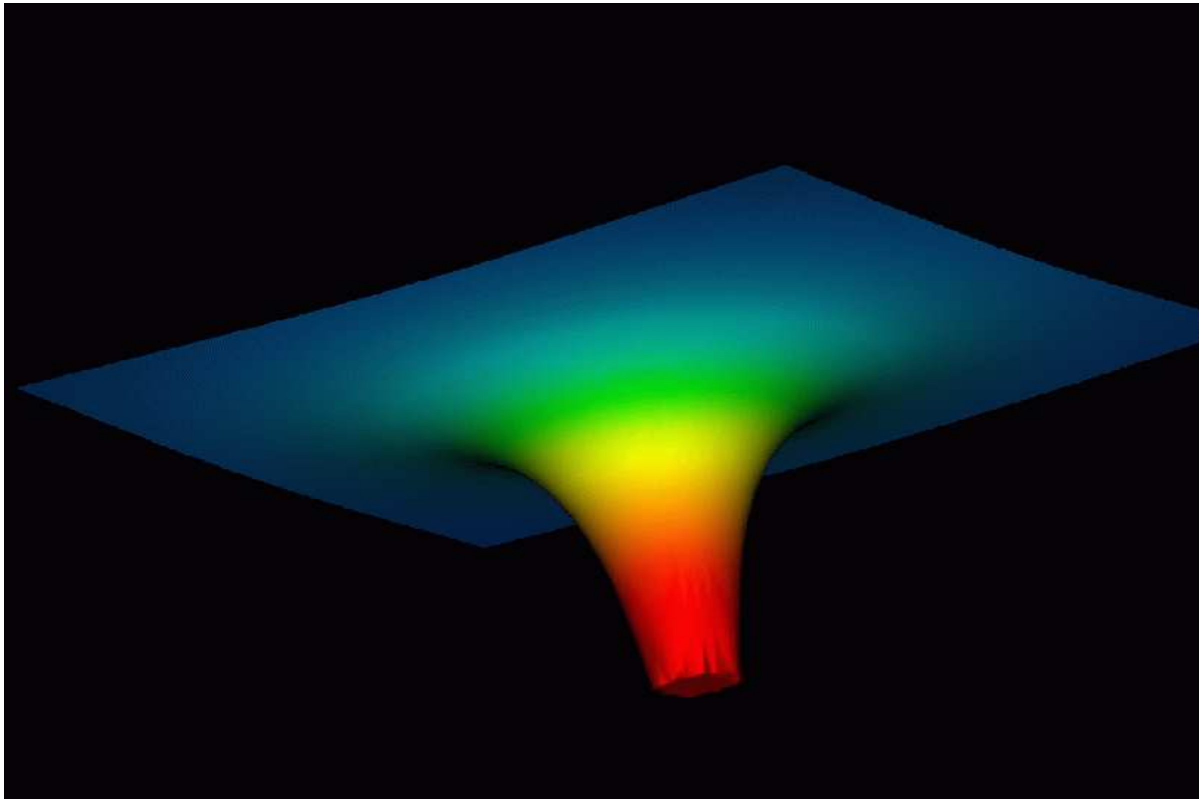
- But this fusion comes with a price: Now space-time itself ends at singularities (also in inflationary scenarios; (Borde, Guth & Vilenkin)). Big Bang thought of as the Beginning and black hole singularities as the End.

- In particular, the assumption of spatial homogeneity & isotropy implies that the metric has the FLRW form: $ds^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\vec{x}^2$

$a(t)$: Scale Factor; Volume $\sim [a(t)]^3$; Curvature $\sim [a(t)]^{-n}$

Einstein Equations $\Rightarrow$ volume $\rightarrow 0$ and Curvature $\rightarrow \infty$: BIG BANG!!
Classically: PHYSICS STOPS!!

The Big Bang in classical GR



Artist's conception of the Big-Bang. Credits: Pablo Laguna.

In classical general relativity the fabric of space-time is violently torn apart at the Big Bang singularity.

- Expectation: Just an indication that the theory is pushed beyond its domain of validity. Example: H-atom. Energy unbounded below in the classical theory; instability. Quantum theory: $E_o = -\frac{me^4}{2\hbar^2}$

- Is this the case? If so, what is the true physics near the Big Bang? Need a theory which can handle both strong gravity/curvature and quantum physics, i.e., Quantum Gravity.
Classical singularities are gates to Physics Beyond Einstein.

- Serious Challenge to LQG since the Gravity-Geometry duality lies at the heart of this approach. UV-IR Challenge: Do Quantum Geometry effects resolve the big bang singularity? If they are so strong as to overwhelm classical gravitational pull near the singularity, why aren't there observable deviations from GR today. The UV-IR tension!

- In cosmological models Quantum Physics does not stop at singularities. Quantum Riemannian geometry extends its life. Rather startling perspectives on the nature of space-time in LQG. Models simple but, in contrast to string theory, encompass physically most interesting singularities.

- Some Long-Standing Questions expected to be answered by Quantum Gravity from first principles:

⋆ How close to the big-bang does a smooth space-time of GR make sense? (Onset of inflation?)
⋆ Is the Big-Bang singularity naturally resolved by quantum gravity? Or, Is a new principle/ boundary condition at the Big Bang essential?
⋆ Is the quantum evolution across the ‘singularity’ deterministic?
(answer ‘No’ e.g. in the Pre-Big-Bang and Ekpyrotic scenarios)
⋆ What is on the other side? A quantum foam? Another large, classical universe? ...
(Fascinating history within classical GR: de Sitter, Tolman, Gamow, Dicke, Sakharov, Weinberg ...)

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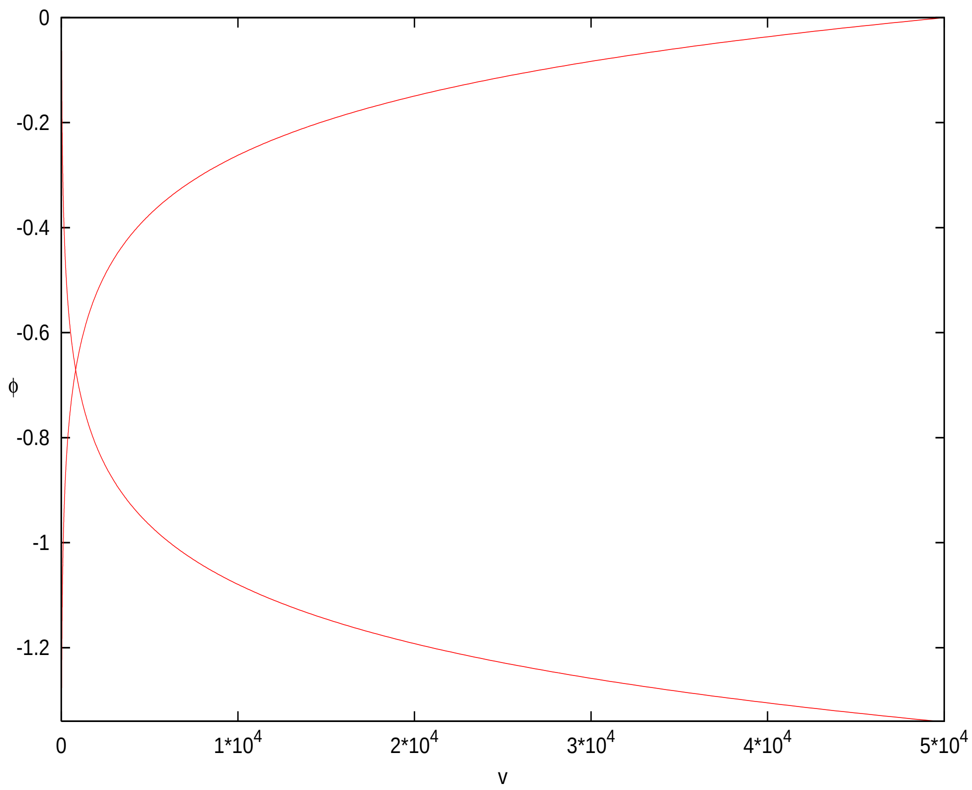
- Emerging Scenario: vast classical regions bridged deterministically by quantum geometry. No new principle needed. (AA, Bojowald, Chiou, Corichi, Pawlowski, Singh, Vandersloot, Wilson-Ewing,...)

- In the classical theory, don’t need full Einstein equations in all their complexity. Almost all work in physical cosmology based on homogeneous isotropic models and perturbations thereon. At least in a first step, can use the same strategy in the quantum theory: mini and midi-superspaces.

The Simplest Model

The k=0, $\Lambda = 0$ FRW Model coupled to a massless scalar field ϕ. Instructive because every classical solution is singular. Provides a foundation for more complicated models.



Classical trajectories

Older Quantum Cosmology (DeWitt, Misner, Wheeler ... 70's)

- Since only finite number of DOF $a(t), \phi(t)$, field theoretical difficulties bypassed; analysis reduced to standard quantum mechanics.

- Quantum States: $\Psi(a, \phi)$; $\hat{a}\Psi(a, \phi) = a\Psi(a, \phi)$ etc.

Quantum evolution governed by the Wheeler-DeWitt differential equation

$$\ell_{\rm Pl}^4 \, \frac{\partial^2}{\partial a^2}(f(a)\Psi(a, \phi)) = \mathrm{const}\, G\; \hat{H}_\phi \, \Psi(a, \phi)$$

Without additional assumptions, e.g. matter violating energy conditions, singularity is not resolved. Precise Statement provided by the consistent histories approach (Craig & Singh).

General belief since the seventies: This is a real impasse because of the von-Neumman's uniqueness theorem (last lecture). How could LQC escape this conclusion?

2. Structure of LQC

- In WDW theory one did not have access to well-defined kinematics. In LQG we do. Furthermore, background independence (Diff invariance) selects the kinematical framework uniquely! (Recall the (Lewandowski, Okolow, Sahlmann, Thiemann; and Fleishchhack) Theorems from the last lecture.)

- Using the symmetry reduced version $\mathfrak{a}_{\rm red}$ of the holonomy-flux algebra $\mathfrak{a}$ and the 'same' positive linear functional as in full LQG but now on $\mathfrak{a}_{\rm red}$ led to the Kinematical framework for LQC (AA, Bojowald, Lewandowski). The analog of the continuity assumption of von Neumann fails in LQC $\Rightarrow$ von-Neumann's uniqueness result naturally bypassed. New Quantum Mechanics! ($\mathcal{H} \neq L^2(\mathbb{R})$ used in the Wheeler-DeWitt theory, but rather $\mathcal{H} = L^2(\bar{\mathbb{R}}_{\rm Bohr})$. The only common vector is the Zero element!)

- Recently, this representation was again shown to be uniquely selected by the residual diffeomorphism freedom in LQC. (AA, Henderson)

LQC Dynamics

• Because the Hilbert space $\mathcal{H} = L^2(\bar{\mathbb{R}}_{\rm Bohr})$ of LQC kinematics is so different from that of the Wheeler DeWitt theory, the Wheeler-DeWitt differential operator fails to be well defined on it. Have to construct the Hamiltonian constraint operator from scratch. Quite subtle. Final result: The WDW differential operator is replaced by a difference operator.
(AA, Bojowald, Lewandowski, Pawlowski, Singh)

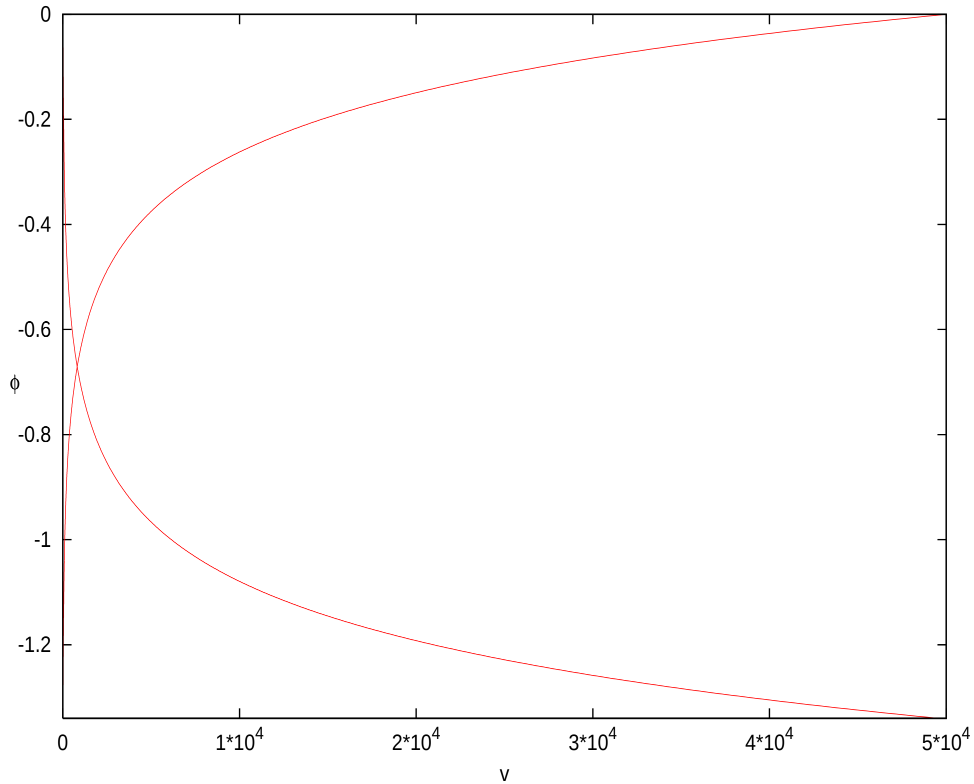
$$C^+(v)\,\Psi(v{+}4,\,\phi){+}C^o(v)\,\Psi(v,\phi){+}C^-(v)\Psi(v{-}4,\,\phi)\;=\;\gamma\ell_P^2\;\hat{H}_\phi\Psi(v,\phi)\quad(\star)$$

• The step size in $(\star)$ is governed by the area gap Δ of LQG. Reason: Recall that there is no local connection operator in LQG; only holonomy operators are well defined. Situation same in LQC. The curvature term in the hamiltonian constraint operator defined using holonomies around loops enclosing minimum area, Δ.

• Good agreement with the WDW equation at low curvatures but drastic departures in the Planck regime precisely because the WDW theory ignores quantum geometry (area gap Δ.)

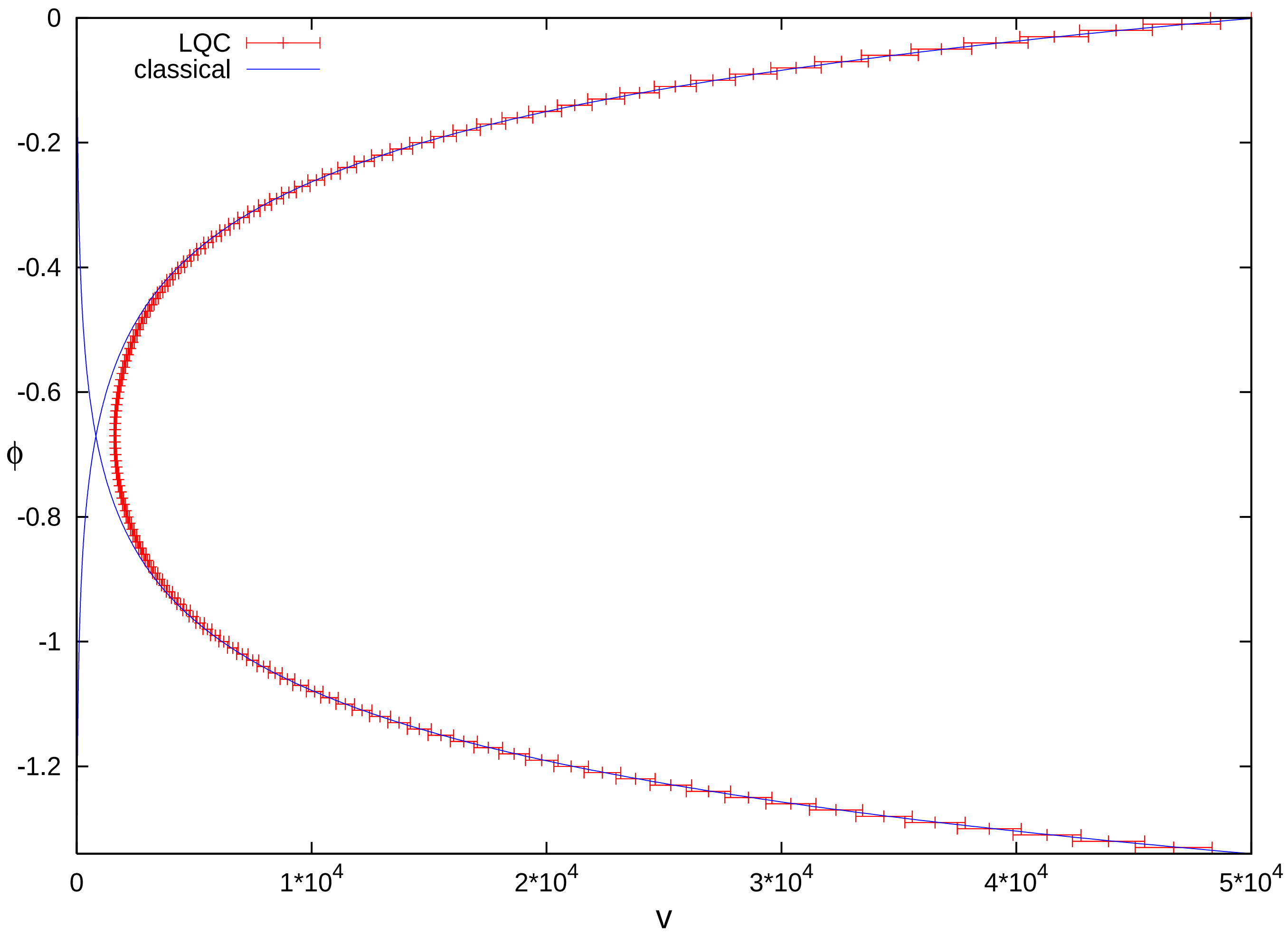
LQC: k=0 FLRW Model

FLRW, k=0 Model coupled to a massless scalar field ϕ. Instructive because every classical solution is singular. Provides a foundation for more complicated models.



Classical Solutions

k=0 LQC

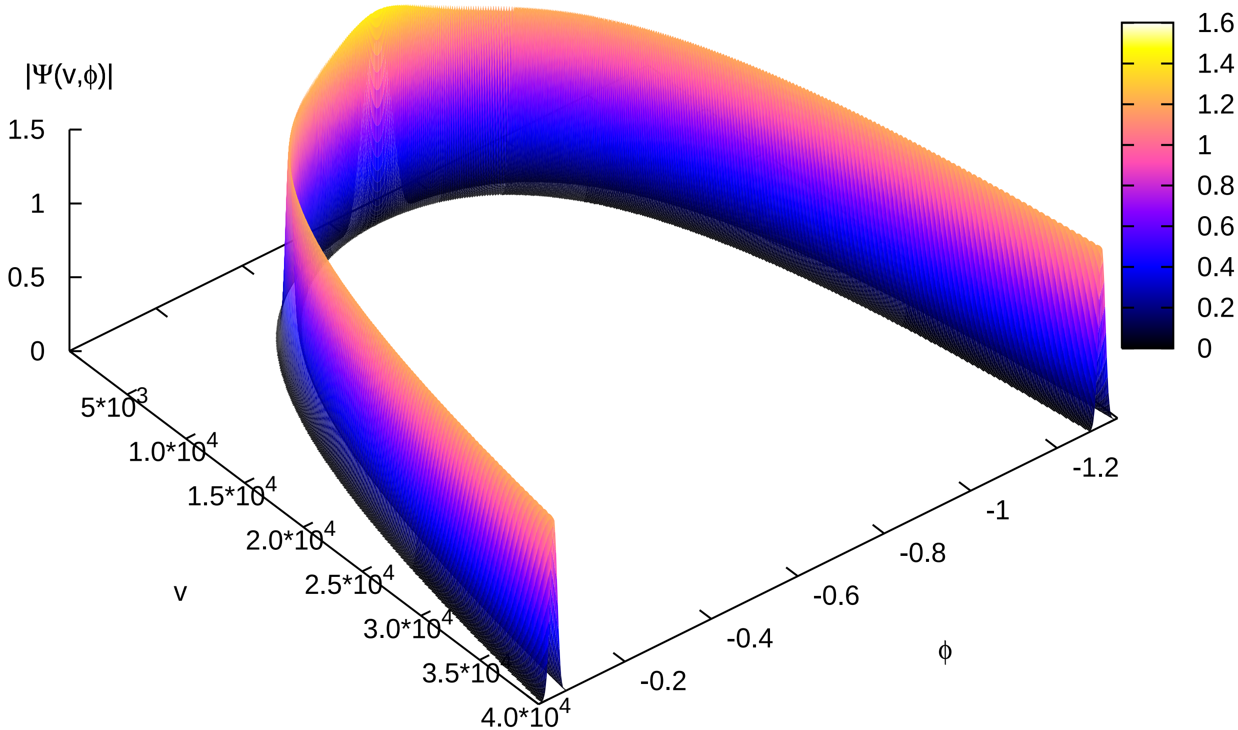


(AA, Pawlowski, Singh)

Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories.

Gamow's favorite paradigm realized.

k=0 LQC



Absolute value of the physical state $\Psi(v, \phi)$

(AA, Pawlowski, Singh)

k=0 Results

Assume that the quantum state is semi-classical at a late time and evolve backwards and forward. Then: (AA, Pawlowski, Singh)

• The state remains semi-classical till *very* early and *very* late times, i.e., till $R \approx 1/lp^2$ or $\rho \approx 0.01\rho_{\rm Pl}$. $\Rightarrow$ We know 'from first principles' that space-time can be taken to be classical during the inflationary era (since $\rho \sim 10^{-12}\rho_{\rm Pl}$ at the onset of inflation).

• In the deep Planck regime, semi-classicality fails. But quantum evolution is well-defined through the Planck regime, and remains deterministic unlike in other approaches. No new principle needed.

• No unphysical matter. All energy conditions satisfied. But the left side of Einstein's equations modified because of quantum geometry effects (discreteness of eigenvalues of geometric operators.): Main difference from WDW theory.

k=0 Results

- To compare with the standard Friedmann equation, convenient to do an algebraic manipulation and move the quantum geometry effect to the right side. Then:

$$(\dot{a}/a)^2 = (8\pi G\rho/3)[1 - \rho/\rho_{\rm max}] \quad \text{where } \rho_{\rm max} \sim 0.41\rho_{\rm Pl}.$$

Big Bang replaced by a quantum bounce.

- The matter density operator $\hat{\rho} = \frac{1}{2}\,(\hat{V}_\phi)^{-1}\,\hat{p}_{(\phi)}^2\,(\hat{V}_\phi)^{-1}$ has an absolute upper bound on the physical Hilbert space (AA, Corichi, Singh):

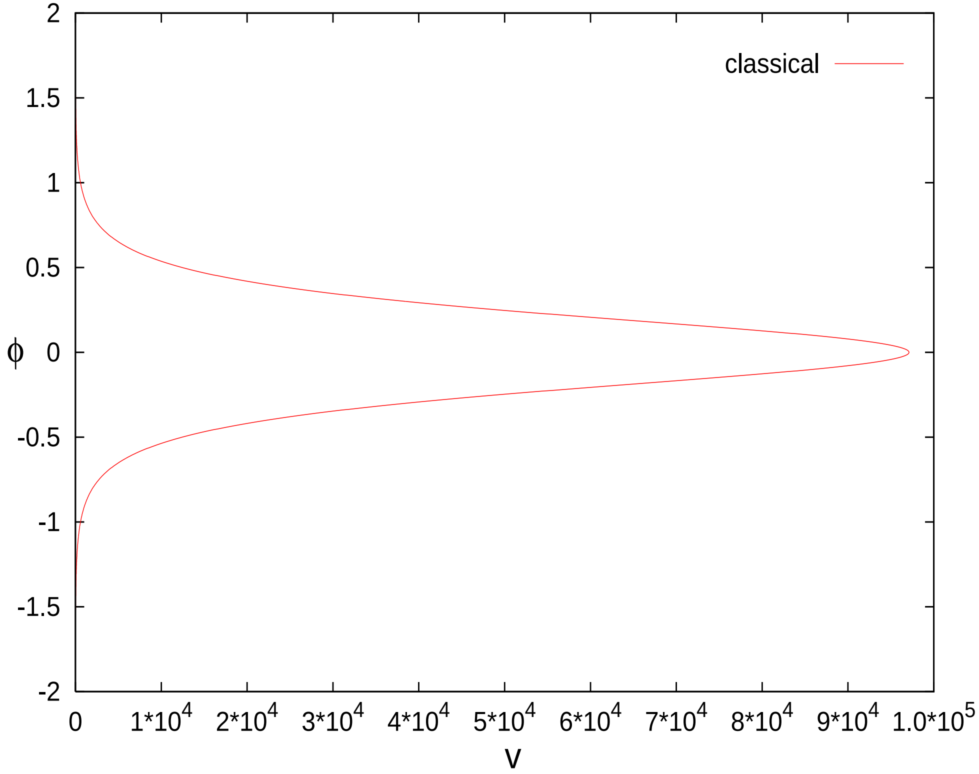
$$\rho_{\rm sup} = \sqrt{3}/16\pi^2\gamma^3 G^2\hbar \approx 0.41\rho_{\rm Pl}!$$

Provides a precise sense in which the singularity is resolved.
(Brunnemann & Thiemann)

- Quantum geometry creates a brand new repulsive force in the Planck regime, replacing the big-bang by a quantum bounce. Physics does not end at singularities. A robust super-inflation phase immediately after the bounce.

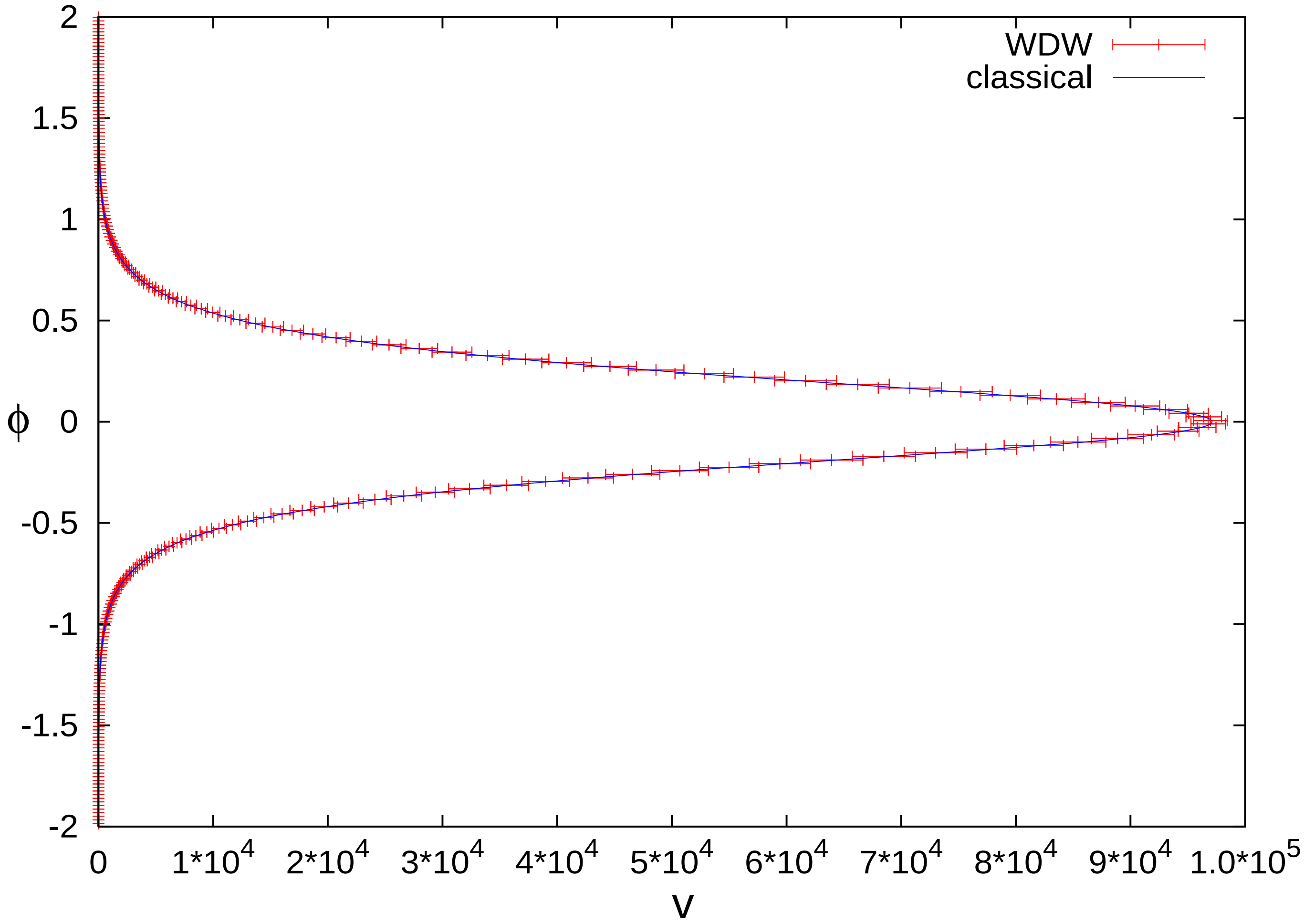
The k=1 Closed Model: Bouncing/Phoenix Universes.

Another Example: $k = 1$ FLRW model with a massless scalar field ϕ. Instructive because again every classical solution is singular; scale factor not a good global clock; More stringent tests because of the classical re-collapse. (Le Maître, Tolman, Sakharov, Dicke,...)



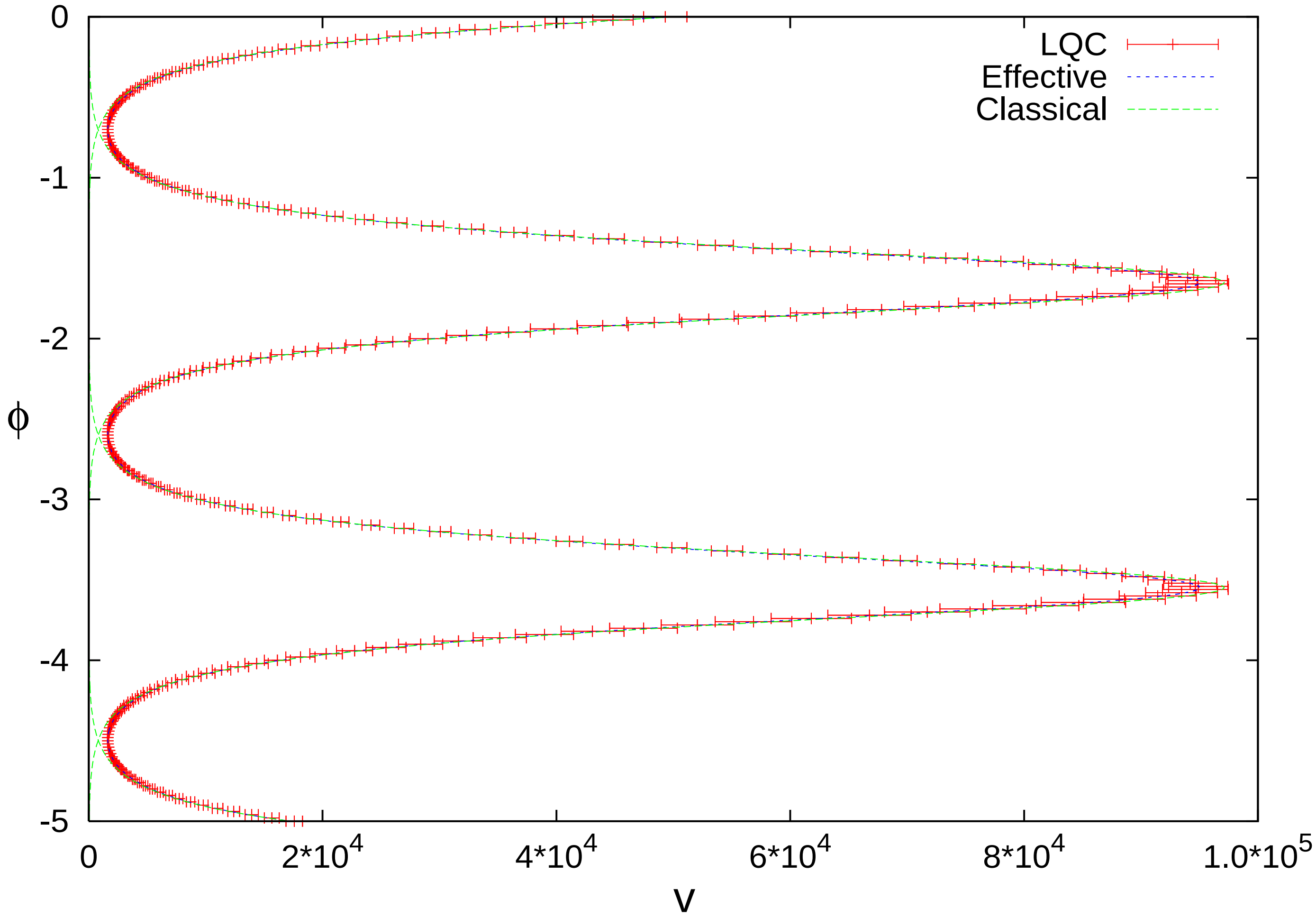
Classical Solutions

k=1 Model: WDW Theory



Expectations values and dispersions of $\hat{V}|_{\phi}$.

k=1 Model: LQC



Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories.

(AA, Pawlowski, Singh, Vandersloot)

k=1: Domain of validity of classical GR

(AA, Pawlowski, Singh, Vandersloot)

- Classical Re-collapse: The infrared issue.

$$\rho_{\rm min} = (3/8\pi G a_{\rm max}^2)\left(1 + O({\ell_{\rm pl}}^4/a_{\rm max}^4)\right)$$

So, even for a very small universe, $a_{\rm max} \approx 23\ell_{\rm pl}$, agreement with the classical Friedmann formula to one part in 10^5. Classical GR an excellent approximation for $a > 10\ell_{\rm pl}$. For macroscopic universes, LQC prediction on recollapse indistinguishable from the classical Friedmann formula.

- Quantum Bounces: The ultra-violet issue

For a universe which attains $v_{\rm max} \approx 1\,{\rm Gpc}^3$,

$$v_{\rm min} \approx 6 \times 10^{18}{\rm cm}^3 \approx 10^{117}{\ell_{\rm pl}}^3\text{:}\;\; 6km \times 18km \times 54km \text{ Mountain!}$$

What matters is curvature, which enters Planck regime at this volume.

Generalizations

- Inclusion of Λ (A B P): $\surd$ (Infrared limit trickier)
  Inclusion of a $m^2\phi^2$ inflationary potential (A P S): $\surd$

- More general singularities: In presence of 'exotic' matter, new types of singularities can arise. Ex: At finite proper time, scale factor may blow up, along with similar behavior of density or pressure (Big rip).Quantum geometry resolves all strong singularities in homogeneous isotropic models with $p = p(\rho)$ matter (Singh).

- Beyond Isotropy and Homogeneity:
  Bianchi Models (A W-E): $\surd$ (Anisotropies & Grav Waves)
  The Gowdy model (G M-B M W-E): $\surd$ (Inhom and Grav Waves.)

These results by AA, Bentevigna, Garay, Martin-Benito, Mena, Pawlowski, Singh, Vandersloot, Wilson-Ewing, ... show that the singularity resolution is quite robust. Anytime a physical observable reaches the Planck regime, the repulsive effect from quantum geometry effect becomes dominant and dilutes it. A general 'singularity resolution theorem'?

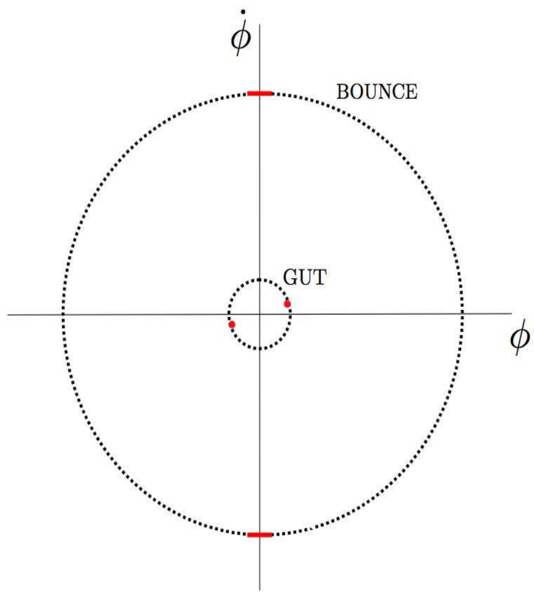
3. Illustrative Applications: I

• Inflationary scenarios (k=0, FLRW with a scalar field) have had tremendous success with the 7year WMAP data & structure formation. Natural question: How generic is the necessary slow roll inflationary phase?

• Start with generic data at the bounce. Evolve. Will it enter slow roll at the $\sim$ GUT energy scale determined by the 7 year WMAP data ($\rho \approx 7.32 \times 10^{-12} {m_{\rm Pl}}^4$) ? Note: 11 orders of magnitude from the bounce to the onset of the desired slow roll!

• Answer: Yes in LQC.

For $m^2\phi^2$ potential, the relative induced Liouville volume of the initial data at the bounce that, upon LQC evolution does not achieve a slow roll compatible with 7 year WMAP data is $< 3 \times 10^{-6}$.



Illustrative Applications: II

- QFT in Cosmological Quantum Space-times (AA, Kaminski, Lewandowski).

Apparent tension because underlying structures are so different: The issue of time (proper/conformal versus relational); background space-time (classical FLRW versus a probability distribution given by the quantum state); . . .

Yet, through systematic approximations, one arrives at the QFT in CST as practised by cosmologists starting from QFT on QST. Several conceptual and a few technical issues had to be resolved. Step by step procedure $\Rightarrow$ in and near the Planck era, we can drop the unreliable approximation, work at the 'higher level' of QFT in QST and analyze the evolution in the relational time ϕ of the full quantum state, e.g., $|v, \phi, \varphi_i\rangle$ given by the (appropriately truncated) Hamiltonian constraint.

- Cosmological Perturbation Theory (Agullo, AA, Nelson, ...)

QFT in QST well suited for studying cosmological perturbations from the bounce to the onset of inflation (if there is an inflaton and a suitable potential); or those generated in the contracting pre-bounce phase (e.g., as suggested by Brandenberger in his "Matter Bounce Scenario). Removes the criticism that one applies QFT in classical space-times in domains where quantum gravity effects should be important (trans-Planckian problems). Phenomenological ramifications are being studied. Ex: New avenues to Non-Gaussianity.

Illustrative Applications: III

- Bousso's Covariant Entropy Bound

Conjecture (Simplest Version): The matter entropy flux across $\mathcal{L}(\mathcal{B})$ is bounded by

$$S := \int_{\mathcal{L}(\mathcal{B})} S^a dA_a \leq \ (A_{\mathcal{B}}/4{\ell_{\rm pl}}^2).$$

Violated in the radiation filled FLRW space-time but in the Planck regime.

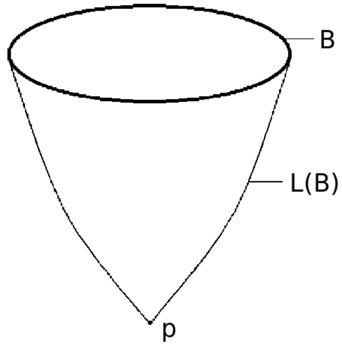
- Curious features: i) Requires a notion of entropy current; ii) Refers to quantum gravity; iii) Requires a classical geometry. Consequently, quite difficult to test in practice!

- LQC provides a near ideal arena.

Answer: $S < 0.976\ (A_{\mathcal{B}}/4{\ell_{\rm pl}}^2)$ (AA, Wilson-Ewing)

The bound is satisfied in LQC!

- Illustrates that the entropy bound need not be a fundamental ingredient in the construction of the theory. It can simply arise in suitable regimes because of other fundamental considerations such as quantum geometry.



Illustrative Applications: IV

- Cosmological Spin Foams (AA, Campiglia, Henderson, Nelson, Rovelli, Vidotto, Wilson-Ewing) Very significant recent advances in Spin Foam Models and Group Field Theory in full LQG **(Rovelli's lectures)**. But several important issues remain.

- In cosmological models, these issues have been addressed rigorously by recasting the well-defined Hamiltonian theory as a sum over quantum histories. Answers provide clear support for the spin-foam paradigm and provides concrete hints for further work.

Application of Loop Quantum Gravity to Cosmological Settings has provided fresh insights into many long standing conceptual questions of QG and Cosmology. In addition, the field has begun to provide phenomenological results for confronting quantum gravity with observations.

4. Discussion: Merits and Limitations of QC

One's first reaction: Symmetry reduction gives only toy models! Full theory much richer and much more complicated. But examples can be powerful.

- Full QED versus Dirac's hydrogen atom.
- Singularity theorems versus first discoveries in simple models.
- BKL behavior: homogeneous Bianchi models.

Do *not* imply that behavior found in examples is necessarily generic. Rather, they can reveal important aspects of the full theory and should not be dismissed a priori.

One can work one's way up by considering more and more complicated cases. (e.g. the Gowdy models have infinite degrees of freedom). At each step, models provide important physical checks well beyond formal mathematics. Can have strong lessons for the full theory.

Summary

• Quantum geometry creates a brand new repulsive force in the Planck regime, replacing the big-bang by a quantum bounce. Repulsive force rises and dies *very* quickly but makes dramatic changes to classical dynamics. (‘Origin’: quantum corrections to Einstein’s equations due to area gap.) Physics does not end at singularities.

• A large number of cosmological models have been analyzed; all strong curvature singularities are removed in LQC. Emerging scenario: Anytime a curvature scalar threatens to diverge, quantum gravity repulsion kicks in and cures the UV problem of GR. Yet agreement with GR in the IR regime. General “Singularity Resolution” theorems?

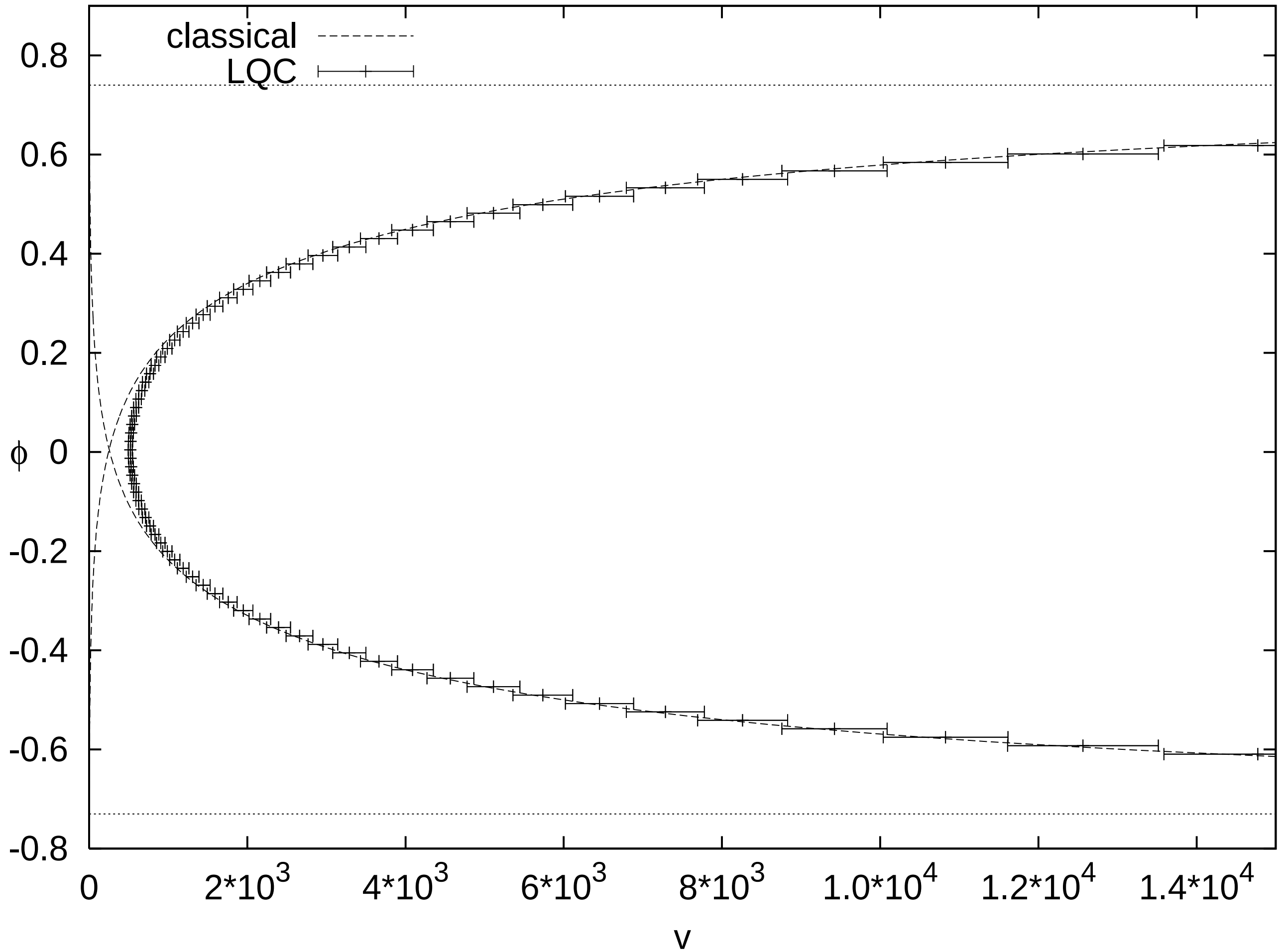
• LQC mature enough for applications: path integrals and spin foams; QFT on cosmological quantum space-times; Probability of inflation; restoration of Bousso’s entropy bound in the radiation-filled FLRW model AA & Wilson-Ewing; Probability of inflation; Cosmological perturbations (Agullo, AA, Barrau, Bojowald, Calcagni, Mielczarek, Tsujikawa, . . .).

• Recent, detailed Review on LQC: AA & Singh, arXiv 1108.089 (CQG at Press).

APPENDIX

This is supplementary material that complements and completes what I discussed in my third talk at the 6th Agean Summer School.

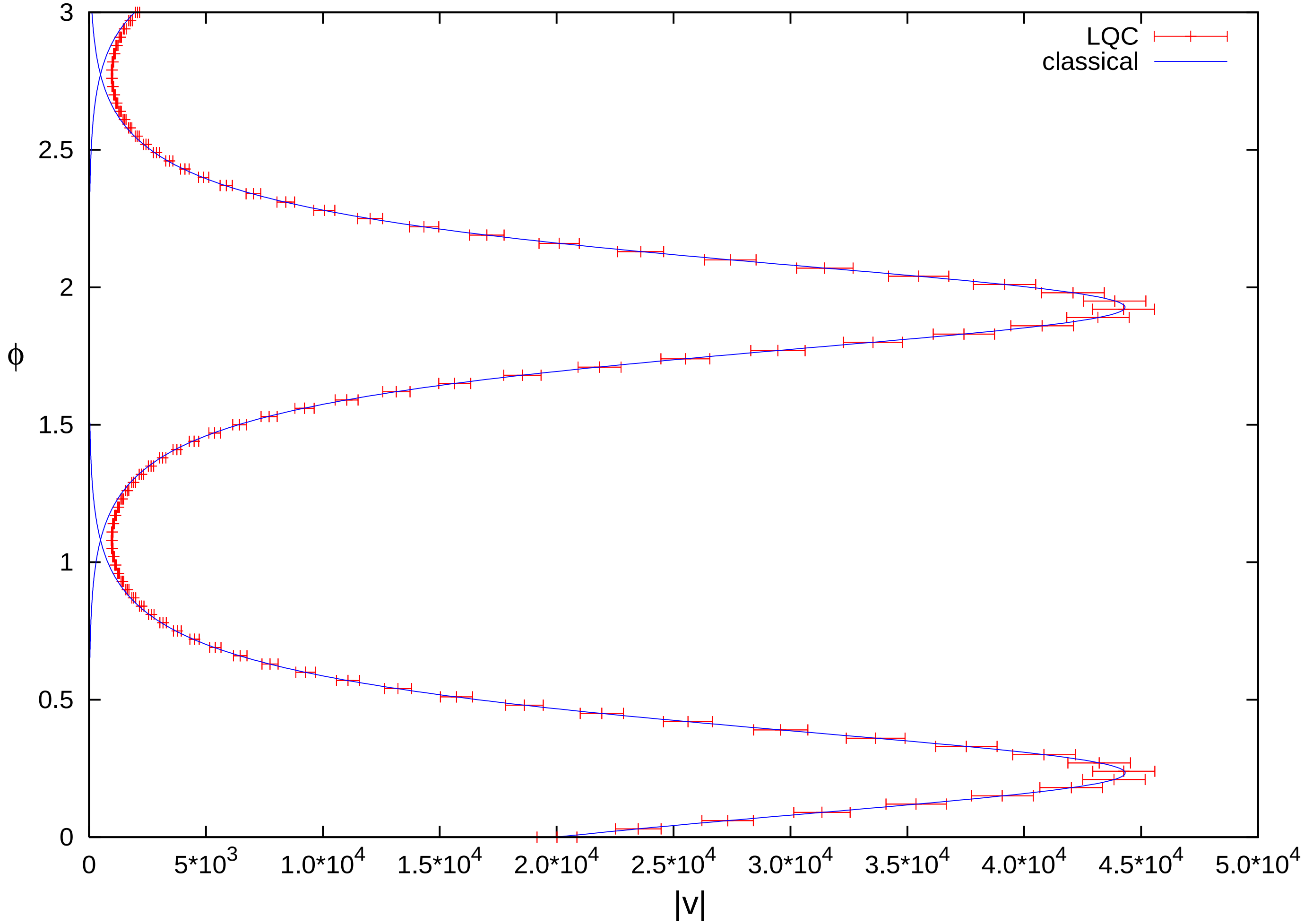
k=0 Model with Positive Λ



Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories.

(AA, Pawlowski)

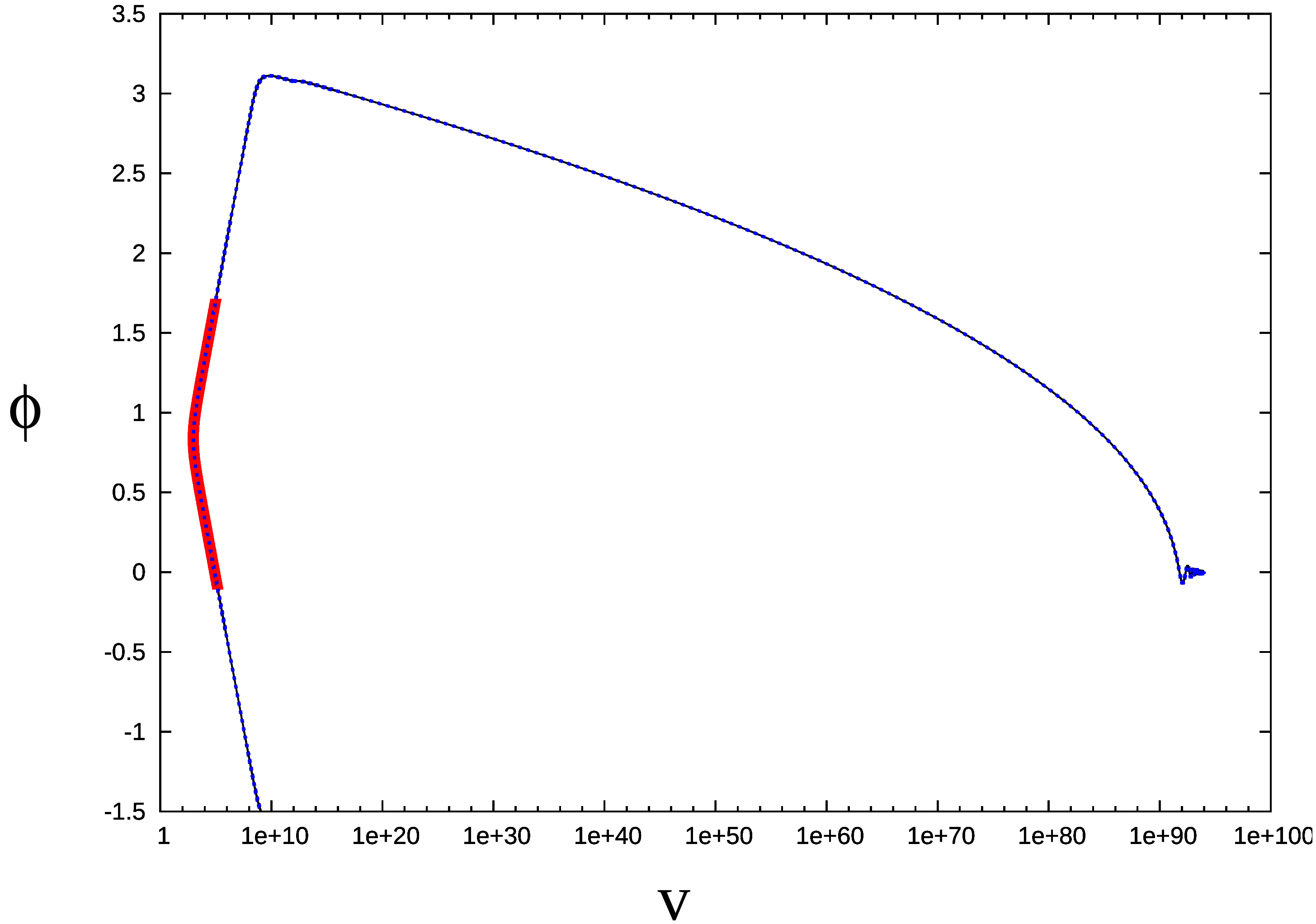
k=0 Model with Negative Λ



Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories.

(Bentevigna, Pawlowski)

Inflation



Expectations values and dispersions of $\hat{V}|_\phi$ for a massive inflaton ϕ with phenomenologically preferred parameters (AA, Pawlowski, Singh).

Path Integrals: 3 slides

• Apparent Tension: Major departures from Einstein's theory near the big bang seem surprising at first in the path integral approach where quantum corrections normally become significant only when the action is comparable to $\hbar$!

• But if one starts from the Hamiltonian theory,
the path integral measure not always dictated by $e^{iS_{\rm Cl}}$
Ex: Free non-relativistic particle moving on a Riemannian manifold (DeWitt).

With $H = -(\hbar^2/2m) g^{ab} D_a D_b$, the Feynman procedure leads to

$$\langle q_f, t_f | q_i, t_i \rangle = \int D[q(t)]\; e^{iS} \quad \text{where} \quad S[q(t)] = (1/2) \int {\rm d}t\, m\, g_{ab}\, \dot q^a\, \dot q^b + \hbar^2 (R/6m)$$

• In GR: Additional complication. No external time!
Result: transition amplitudes replaced by Extraction amplitudes that determine the dynamical content of the theory: In the FLRW models with a scalar field:

$$\mathcal{E}(v_f, \phi_f;\, v_i, \phi_i) = \int {\rm d}\alpha \langle {\rm v_f}, \phi_{\rm f} |\, {\rm e}^{{\rm i}\alpha \hat{\rm C}} |\, {\rm v_i}, \phi_{\rm i} \rangle$$

so that

$$\Psi_{\rm phys}(v, \phi) = \sum_{v'} \int {\rm d}\phi\, \mathcal{E}({\rm v}, \phi; {\rm v}', \phi')\, \Psi_{\rm kin}({\rm v}, \phi'), \text{ and}$$

$$(\Phi_{\rm phys},\, \Psi_{\rm phys}) := \sum_{v,\, v'} \int {\rm d}\phi\, {\rm d}\phi'\, \bar\Phi_{\rm kin}({\rm v}, \phi)\, \mathcal{E}({\rm v}, \phi; {\rm v}', \phi') \Psi_{\rm kin}({\rm v}', \phi')$$

From the Hamiltonian Theory to Path Integrals

- Start with: $\mathcal{E}(v_f, \phi_f;\, v_i, \phi_i) = \int \mathrm{d}\alpha \langle \mathrm{v_f}, \phi_\mathrm{f}\, | \mathrm{e}^{\mathrm{i}\alpha\hat{\mathrm{C}}} |\, \mathrm{v_i}, \phi_\mathrm{i} \rangle$ Treat $\alpha\hat{C}$ as a fictitious Hamiltonian and the mathematical 'evolution' it generates for $\Delta t = 1$. Then follow Feynman to write $e^{i\alpha\hat{C}} = [e^{i\epsilon\alpha\hat{C}}]^N$ with $\epsilon = 1/N$; insert a complete basis between each exponential to rewrite $\mathcal{E}$ as a sum over quantum paths in the phase space:

$$\mathcal{E}(v_f, \phi_f; v_i, \phi_i) = \int \mathrm{d}\alpha \ \int [\mathcal{D}\mathrm{v_q}(\tau)]\ [\mathcal{D}\mathrm{b_q}(\tau)]\ [\mathcal{D}\mathrm{p}(\tau)]\ [\mathcal{D}\phi(\tau)]\ \ \mathrm{e}^{\frac{\mathrm{i}}{\hbar}\bar{\mathrm{S}}}$$

- Quantum paths $\Rightarrow$ Sum involved paths only with $v \in 4n\ell_o$ and $b \in (0, \pi/\ell_o)$, where $\ell_o^2 =$ Area gap. None of these paths passes through the classical singularity ($b = \infty$)! $\Rightarrow$ Singularity Resolution.

- Can address the tension more directly by using a trick from the path integral framework of a particle on a circle. Can simply rewrite the path integral as an integral over all phase space paths. Then,

$$\mathcal{E}(v_f, \phi_f; v_i, \phi_i) = \int \mathrm{d}\alpha \ \int [\mathcal{D}\mathrm{v}(\tau)]\ [\mathcal{D}\mathrm{b}(\tau)]\ [\mathcal{D}\mathrm{p}(\tau)]\, [\mathcal{D}\phi(\tau)]\ \ \mathrm{e}^{\frac{\mathrm{i}}{\hbar}\mathrm{S}},$$

where

$$S = \int_0^1 \mathrm{d}\tau \left(\mathrm{p}\dot{\phi} - \tfrac{1}{2}\mathrm{b}\dot{\mathrm{v}} - \alpha\left(\mathrm{p}^2 - 3\pi \mathrm{G}\mathrm{v}^2 \frac{\sin^2 \ell_\mathrm{o}\mathrm{b}}{\ell_\mathrm{o}^2}\right)\right) \ \neq \mathrm{S_{EH}}.$$

- Now all paths are allowed but weighted by a quantum corrected action. Captures quantum geometry effects, as it must.

Steepest Descent and WKB

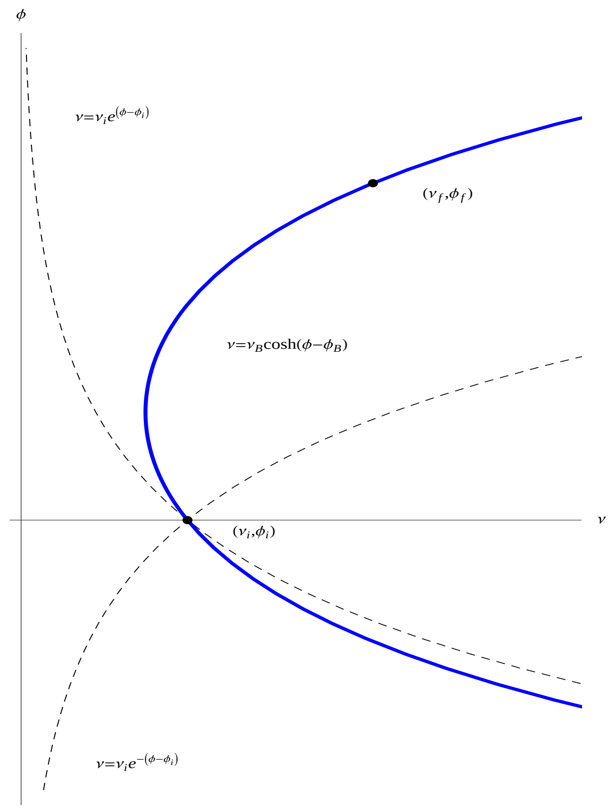
- Subtlety in using the WKB approximation: Now the action has a $\hbar$-dependent term because of ℓ_o. So, the standard (Rovelli, §3.2, §5.2), $\hbar$-expansion acquires subtleties.

$$\hbar \to 0, \gamma \to \infty \text{ such that } \ell_o \sim \sqrt{\gamma^3 \hbar G} \text{ is kept fixed.}$$

Then we obtain a well-defined WKB expansion.

- Non-trivial check: The leading order WKB term yields an excellent approximation to the (numerically computed) exact result away from the ‘classically’ forbidden region.

- Summary: there is no tension between the path integral and Hamiltonian frameworks. LQC Perspective: Incorrect to start with the Einstein-Hilbert action on classical geometries. Rather, to correctly handle uv issues, have to keep track of quantum geometries. Then the weight associated with the classically singular paths is negligible; bouncing solutions of effective LQC equations dominate.



Precise relation between LQC and the WDW Theory

Question analyzed in detail for the k=0 model. (Corichi, Singh, AA). Expect the answer to be the same for others.

Start with the 'same physical state at time $\phi = \phi_o$' and evolve using LQC or WDW theory. Then:

- Certain predictions of LQC approach those of the WDW theory as the area gap λ goes to zero:
  Given a semi-infinite 'time' interval $\Delta\phi$ and $\epsilon > 0$, there exists a $\delta > 0$ such that $\forall\lambda < \delta$, 'physical predictions of the two theories are within ϵ of each other.'

- However, approximation is *not* uniform. The WDW theory is *not* the limit of sLQC:
  Given $N > 0$ however large, there exists a ϕ such that
  $\langle\hat{V}_\phi\rangle_{\rm sLQC} - \langle\hat{V}_\phi\rangle_{\rm WDW} > N$.
  LQC is *fundamentally* discrete.