Astrophysical solutions in Randall-Sundrum gravity

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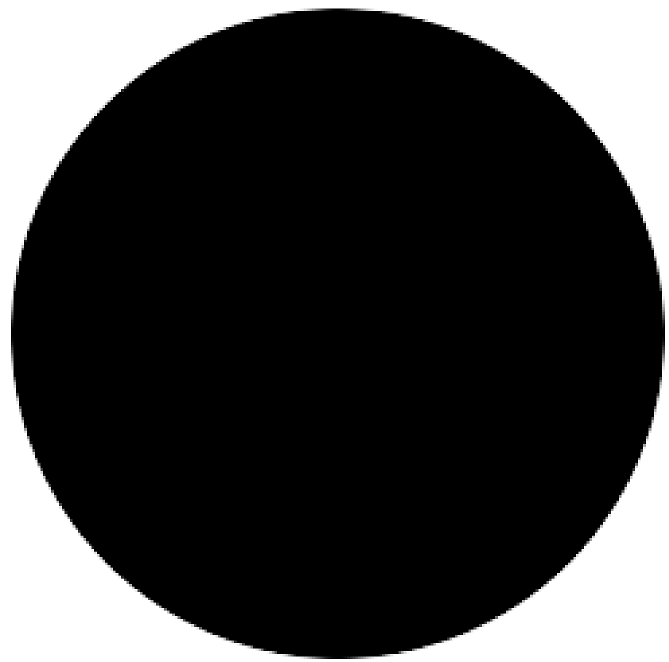
Introduccion

- Motivation
  - The standard model and the general relativity represents the two great theories in fundamental physics. The success of general relativity is beyond any doubt, however due to its inconsistency with quantum mechanics, it is not possible to ensure that this theory keeps its original structure at high energies.
  - One of the goals of the current study is to see what features of theories beyond Einstein could lead to an answer to any of the open problems in astrophysics (dark matter) or cosmology (dark energy)
  - In this talk: Astrophysics in the brane world
  - Some preliminary results about extra dimensional consequences on compact stellar structures.

Black holes, neutron stars, quark stars



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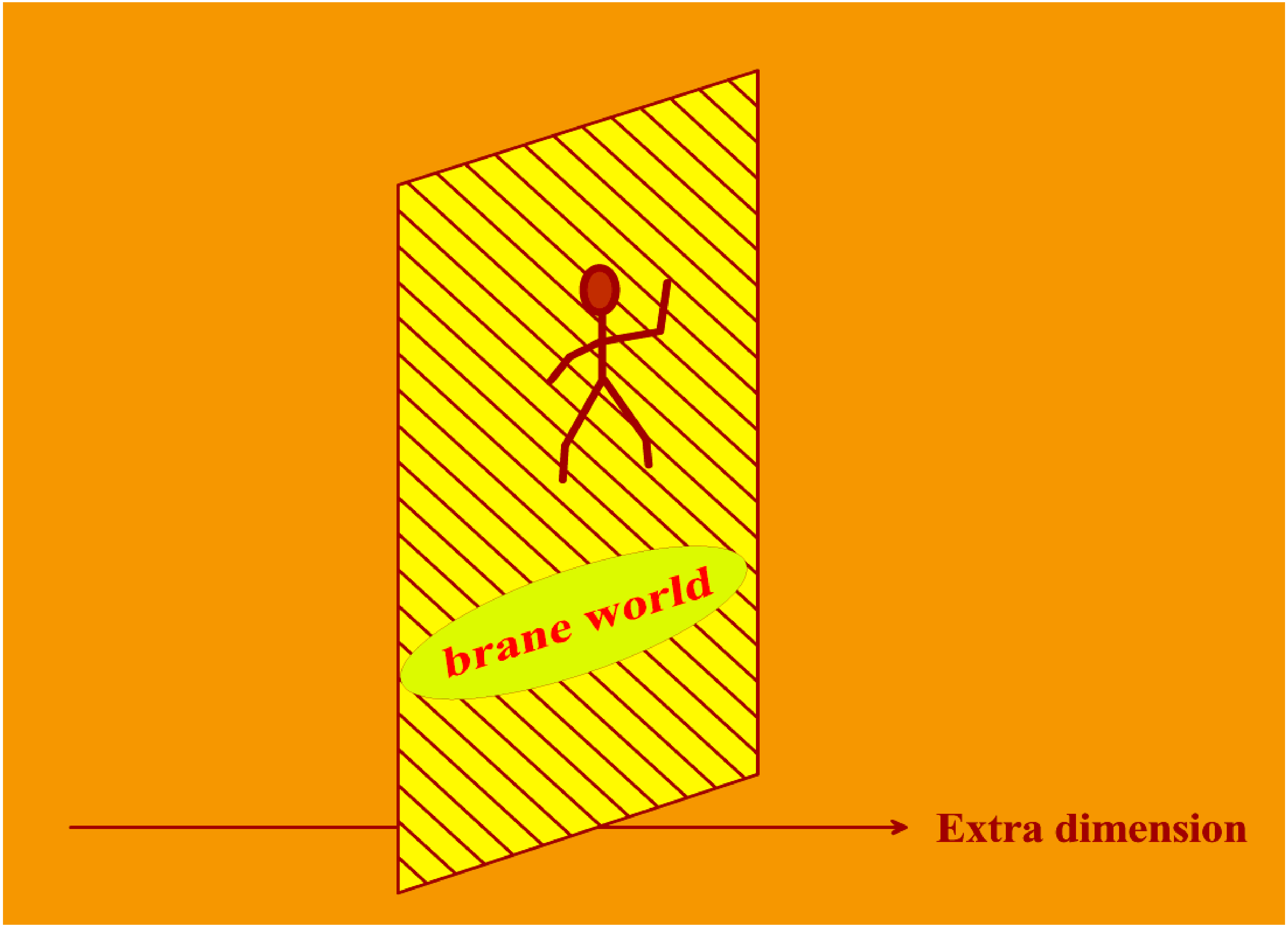
The Braneworld



Einstein field equations on the brane

The Einstein field equations on the brane may be written as a modification of the standard field equations [Shiromizu et al 2002]

5D Einstein equations:

$$G_{ab} + \Lambda_5 g_{ab} = \kappa_5^2 T_{ab}; \quad \kappa_5 = 8\pi G_5 \quad a = 0, ...4 \quad (Bulk)$$

$$G_{\mu\nu} = -8\pi T_{\mu\nu}^{T} - \Lambda g_{\mu\nu}, \quad \mu = 0, ...3 \quad (Brane)$$

where the energy-momentum tensor has new terms carrying bulk effects onto the brane:

$$T_{\mu\nu} \rightarrow T_{\mu\nu}{}^{T} = T_{\mu\nu} + \frac{6}{\sigma} S_{\mu\nu} + \frac{1}{8\pi} \mathcal{E}_{\mu\nu}$$

Here σ is the brane tension

The new terms and are the high-energy corrections $S_{\mu\nu}$ and the projection of the bulk Weyl tensor on the brane $\mathcal{E}_{\mu\nu}$

$$S_{\mu\nu} = \frac{1}{12} T_\alpha^{\ \alpha} T_{\mu\nu} - \frac{1}{4} T_{\mu\alpha} T^\alpha_\nu + \frac{1}{24} g_{\mu\nu} \left[3 T_{\alpha\beta} T^{\alpha\beta} - (T_\alpha^{\ \alpha})^2 \right]$$

$$-8\pi \mathcal{E}_{\mu\nu} = -\frac{6}{\sigma} \left[\mathcal{U} (u_\mu u_\nu + \frac{1}{3} h_{\mu\nu}) + \mathcal{P}_{\mu\nu} + \mathcal{Q}_{(\mu} u_{\nu)} \right]$$

$$\begin{array}{rcl} \mathcal{U} & \rightarrow & Dark\ \ radiation \\ \mathcal{P}_{\mu\nu} & \rightarrow & Anisotropic\ \ stress \\ \mathcal{Q}_\mu & \rightarrow & Energy\ \ flux \end{array}$$

An open problem

The nonclosure of the braneworld equations represents an open problem in braneworld stars (a better understanding of the bulk geometry and proper boundary conditions is required). The source of this problem: the projection of the bulk Weyl tensor on the brane $\mathcal{E}_{\mu\nu}$

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$$\mathcal{E}_{\mu\nu} = 0 \qquad \mathit{INCORRECT!}$$

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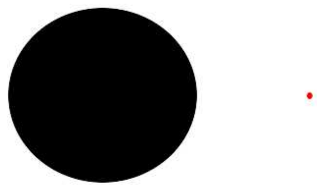
Many ways have been taken:

$$\mathcal{E}_{\mu\nu} = 0 \qquad INCORRECT!$$

$$\mathcal{P}_{\mu\nu} = 0 \qquad Too\ \ Strong!$$

Compact stars

The simplest solution: “vaccum”



$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

- Dadhich,Maartens,Papadopoulos and Rezania (DMPR Solution):

$$e^{\nu^+} = e^{-\lambda^+} = 1 - \frac{2\mathcal{M}}{r} + \frac{q}{r^2}, \quad \mathcal{U}^+ = -\frac{\mathcal{P}^+}{2} = \frac{4}{3}\pi q \sigma \frac{1}{r^4},$$

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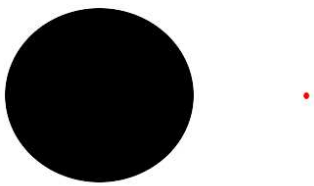

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- Casadio, Fabbri and Mazzacurati (CFM Solution)

$$e^{\nu^+} = \left[\frac{\eta + \sqrt{1 - \frac{2\mathcal{M}}{r}(1+\eta)}}{1+\eta}\right]^2, \quad e^{\lambda^+} = \left[1 - \frac{2\mathcal{M}}{r}(1+\eta)\right]^{-1},$$

$$\frac{16\pi\mathcal{P}^+}{k^4\sigma} = -\frac{\mathcal{M}(1+\eta)\eta}{\eta + \sqrt{1 - \frac{2\mathcal{M}}{r}(1+\eta)}} \frac{1}{r^3}, \quad \mathcal{U}^+ = 0,$$

The interior: the simplest distribution

$$ds^2 = e^{\nu}dt^2 - e^{\lambda}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Perfect fluid

The interior: the simplest distribution

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right)$$

Perfect fluid+high energy terms

Too complicated!

The interior: the simplest distribution

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Perfect fluid+high energy terms+dark radiation/pressure

Too complicated!
THERE IS NOT SOLUTION! (Indefinity system)

The interior: the simplest distribution

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Perfect fluid+high energy terms+dark radiation/pressure

Too complicated!
THERE IS NOT SOLUTION! (Indefinity system)
However....we found a general effective 4D solution!

Spherically symmetric static distribution

Schwarzschild-like coordinates

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right)$$

A perfect fluid (General Relativity)+high energy corrections

$$-8\pi \left(\rho + \frac{1}{\sigma} \left(\frac{\rho^2}{2} \right) \right) = -\frac{1}{r^2} + e^{-\lambda} \left(\frac{1}{r^2} - \frac{\lambda'}{r} \right),$$

$$-8\pi \left(-p - \frac{1}{\sigma} \left(\frac{\rho^2}{2} + \rho p \right) \right) = -\frac{1}{r^2} + e^{-\lambda} \left(\frac{1}{r^2} + \frac{\nu'}{r} \right),$$

$$-8\pi \left(-p - \frac{1}{\sigma} \left(\frac{\rho^2}{2} + \rho p \right) \right) = \frac{1}{4} e^{-\lambda} \left[2\nu'' + \nu'^2 - \lambda'\nu' + 2\frac{(\nu' - \lambda')}{r} \right],$$

$$p' = -\frac{\nu'}{2}(\rho + p).$$

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$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right)$$

A perfect fluid (General Relativity)+high energy corrections+Weyl functions

$$-8\pi \left(\rho + \frac{1}{\sigma} \left(\frac{\rho^2}{2} + 6\mathcal{U} \right) \right) = -\frac{1}{r^2} + e^{-\lambda} \left(\frac{1}{r^2} - \frac{\lambda'}{r} \right),$$

$$-8\pi \left(-p - \frac{1}{\sigma} \left(\frac{\rho^2}{2} + \rho p + 2\mathcal{U} \right) + \frac{\mathcal{P}}{\sigma} \right) = -\frac{1}{r^2} + e^{-\lambda} \left(\frac{1}{r^2} + \frac{\nu'}{r} \right),$$

$$-8\pi \left(-p - \frac{1}{\sigma} \left(\frac{\rho^2}{2} + \rho p + 2\mathcal{U} \right) - \frac{\mathcal{P}}{2\sigma} \right) = \frac{1}{4} e^{-\lambda} \left[2\nu'' + \nu'^2 - \lambda'\nu' + 2\frac{(\nu' - \lambda')}{r} \right],$$

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Spherically symmetric static distribution

From the field equations we have

$$
\begin{aligned}
e^{-\lambda} &= 1 - \frac{8\pi}{r}\int_0^r r^2 \left[\rho + \frac{1}{\sigma}\left(\frac{\rho^2}{2} + \frac{6}{k^4}\mathcal{U}\right)\right] dr, \\
\frac{8\pi}{k^4}\frac{\mathcal{P}}{\sigma} &= \frac{1}{6}\left(G_1^1 - G_2^2\right), \\
\frac{6}{k^4}\frac{\mathcal{U}}{\sigma} &= -\frac{3}{\sigma}\left(\frac{\rho^2}{2} + \rho p\right) + \frac{1}{8\pi}\left(2G_2^2 + G_1^1\right) - 3p.
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With $G_1^1 = -\frac{1}{r^2} + e^{-\lambda}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right); \quad G_2^2 = \frac{1}{4}e^{-\lambda}\left[2\nu'' + \nu'^2 - \lambda'\nu' + 2\frac{(\nu' - \lambda')}{r}\right].$

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Hence we have three unknow functions $\{\nu(r),\, \rho(r),\, p(r)\}$ satisfying one equation:

$$p' = -\frac{\nu'}{2}(\rho + p)$$

Indefinity system.

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Indefinity system $\Rightarrow$ It is necessary to prescribe additional information.

Generating a constraint

Let us see the "solution" for the geometric function

$$e^{-\lambda} = 1 - \frac{8\pi}{r} \int_0^r r^2 \left[\rho + \frac{1}{\sigma} \left(\frac{\rho^2}{2} + \frac{6}{k^4} \mathcal{U} \right) \right] dr,$$

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It can be written as

$$e^{-\lambda} = \underbrace{1 - \frac{8\pi}{r}\int_0^r r^2 \rho dr}_{General\ Relativity}$$ **+ "DEFORMATIONS"**

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$$\underbrace{e^{-\lambda} = 1 - \frac{8\pi}{r}\int_0^r r^2\rho dr}_{General\ Relativity}$$ **+ "DEFORMATIONS"**

The deformation undergone by the geometric function λ produces anisotropic consequences, as can be seen through

$$\frac{8\pi}{k^4}\frac{\mathcal{P}}{\sigma} = \frac{1}{6}\left(G_1^1 - G_2^2\right),$$

Generating a constraint

The "DEFORMATIONS" **must vanish** when $\sigma^{-1} \to 0$ (Low energy limit). However it does not happen: **the low energy limit problem**.

$$\textbf{DEFORMATIONS} = e^{-I} \int_0^r \frac{e^I}{(\frac{\nu'}{2} + \frac{2}{r})} \left[H(p,\rho,\nu) + \frac{8\pi}{\sigma} \left(\rho^2 + 3\rho p\right) \right] dr,$$

with $$H(p,\rho,\nu) \equiv 8\pi 3p - \left[\mu'(\frac{\nu'}{2} + \frac{1}{r}) + \mu(\nu'' + \frac{\nu'^2}{2} + \frac{2\nu'}{r} + \frac{1}{r^2}) - \frac{1}{r^2} \right],$$

where $$I \equiv \int \frac{(\nu'' + \frac{\nu'^2}{2} + \frac{2\nu'}{r} + \frac{2}{r^2})}{(\frac{\nu'}{2} + \frac{2}{r})} dr, \qquad \mu \equiv 1 - \frac{8\pi}{r} \int_0^r r^2 \rho dr.$$

The function $H(p,\rho,\nu)$ measure the anisotropic consequence due to bulk effects on p, ρ, ν.

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To ensure General Relativity $$H(p,\rho,\nu) = 0$$

Minimal geometric deformation

When we impose $H(p,\rho,\nu)=0$, the geometric function ν does not produce any anisotropic consequence, hence the only source for anisotropy in the brane is the **minimal** deformation undergone by λ:

$$e^{-\lambda}=\underbrace{1-\frac{8\pi}{r}\int_0^r r^2\rho dr}_{General\ Relativity}\ \underbrace{+H-deformations}_{\frac{anisotropic\ consequence\ due}{to\ bulk\ effects\ on\ p,\ \rho\ and\ \nu}}\ \underbrace{+\frac{1}{\sigma}-deformations}_{High\ Energy\ Terms}$$

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$\Rightarrow$ **Anisotropy minimally projected onto the brane**

Minimal geometric deformation

- For non-uniform distributions ($\rho(r)$) the constraint $H(p, \rho, \nu) = 0$ is not enough and the system remains indefinite.

Minimal geometric deformation

- For non-uniform distributions ($\rho(r)$) the constraint $H(p, \rho, \nu) = 0$ is not enough and the system remains indefinite.
- For uniform distributions ($\rho \neq \rho(r)$), the constraint $H(p, \rho, \nu) = 0$ solves the nonclosure problem in the brane.

The Schwarzschild's solution

Let us construct the braneworld version of Schwarzschild's solution, which is given by

$$e^{\nu} = B^2 \left(3\sqrt{1 - \frac{R^2}{C^2}} - \sqrt{1 - \frac{r^2}{C^2}} \right)^2, \quad e^{-\lambda_S} = 1 - \frac{r^2}{C^2},$$

$$\rho = \frac{3}{8\pi C^2}, \quad p(r) = \rho \left[\frac{\sqrt{1 - \frac{r^2}{C^2}} - \sqrt{1 - \frac{R^2}{C^2}}}{3\sqrt{1 - \frac{R^2}{C^2}} - \sqrt{1 - \frac{r^2}{C^2}}} \right],$$

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When $H(p, \rho, \nu) = 0$ is imposed

$$e^{-\lambda_S} \Rightarrow \; e^{-\lambda} = 1 - \frac{r^2}{C^2} + \frac{1}{\sigma} Deformations$$

The Schwarzschild's solution

From the field equations we have the Weyl functions

$$
\begin{aligned}
\mathcal{P}(r) \quad = \quad & \frac{1}{6\,r^2\,\left(C^2-r^2\right)\left(C^2-r^2-3\,C^2\alpha\right)\left[3\,r^2+C^2\left(6\alpha\,-2\right)\right]^2} \\
& \Big\{9\,r^2\Big(1-\frac{r^2}{C^2}\Big)^2\,\big[-4\,\left(-5+3\,\alpha\,\right)\,C^4 \\
& +C^2\,\left(\left(-25+21\,\alpha\,\right)\,r^2-18\,R^2\right)+6\,\left(r^4+3\,r^2\,R^2\right)\big] \\
& -g(r)\,\big[24\,\left(-7+9\,\alpha\,\right)\,C^8-2\,C^6\,\left(\left(-320+282\,\alpha\,\right)\,r^2+81\,\left(-1+\alpha\,\right)\,R^2\right) \\
& -6\,C^2\,r^4\,\left(5\,\left(-13+6\,\alpha\,\right)\,r^2+3\,\left(-41+9\,\alpha\,\right)\,R^2\right)+C^4\,r^2\,\big(7\,\left(-118+75\,\alpha\,\right)\,r^2 \\
& \;+9\,\left(-67+36\,\alpha\,\right)\,R^2\big)-9\,\left(4\,r^8+33\,r^6\,R^2\right)\big]\Big\}\,, \\
\mathcal{U}(r) \quad = \quad & \frac{1}{12\,C^4\,\left(C^2-r^2\right)\,\left[\left(-2+6\,\alpha\right)\,C^2+3\,r^2\right]^2\left(C^2-r^2-3C^2\alpha\right)} \\
& \Big\{9\,\left(C^2-r^2\right)\,\big[32\,\left(-1+3\,\alpha\right)\,C^6-3\,r^6+72\,r^4\,R^2 \\
& +C^2\,r^2\,\left(\left(-61+3\,\alpha\right)\,r^2+108\,\left(-1+\alpha\right)\,R^2\right) \\
& -12\,C^4\,\left(8\,\left(-1+\alpha\right)\,r^2+3\,\left(-1+3\,\alpha\right)\,R^2\right)\big] \\
& +2\,C^4\,\left(5\,C^2-3\,r^2\right)\,\big[4\,\left(-5+3\,\alpha\right)\,C^4 \\
& \;+C^2\,\left(\left(23-15\,\alpha\right)\,r^2+18\,R^2\right)-3\,\left(r^4+6\,r^2\,R^2\right)\big]\,g(r)\Big\}
\end{aligned}
$$

The Schwarzschild's solution

Using the Reissner-Nördstrom-like solution

$$e^{\nu^+} = e^{-\lambda^+} = 1 - \frac{2\mathcal{M}}{r} + \frac{q}{r^2},$$

$$\mathcal{U}^+ = -\frac{\mathcal{P}^+}{2} = \frac{4}{3}\pi q \sigma \frac{1}{r^4},$$

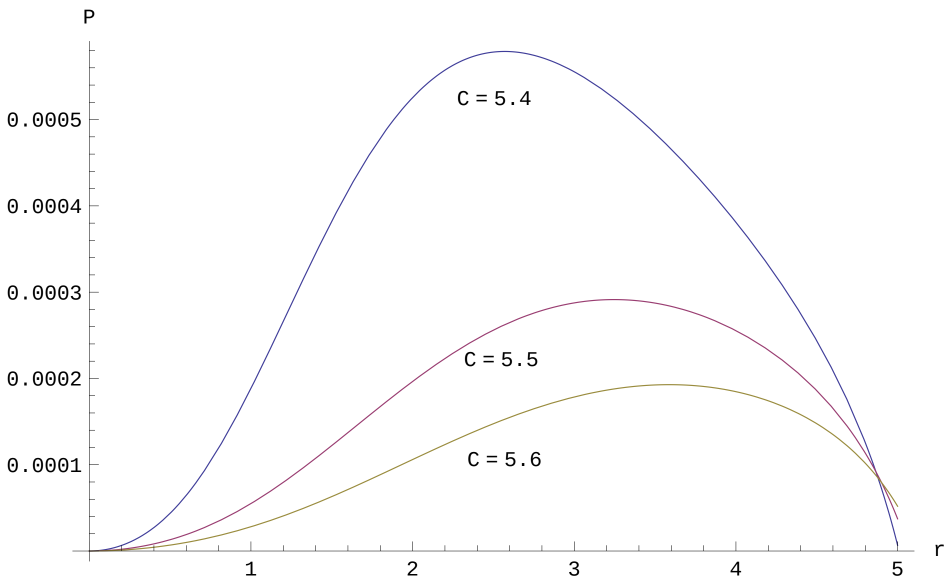
and considering the matching condition $[ds^2]_\Sigma = 0$ at the stellar surface Σ, we have

$$4\,B^2\left(1 - \frac{R^2}{C^2}\right) = 1 - \frac{2\mathcal{M}}{R} + \frac{q}{R^2},$$

$$\frac{2\mathcal{M}}{R} = \frac{2M}{R} - \frac{1}{\sigma}\frac{9\,g(C)}{8\pi\,C^4} + \frac{q}{R^2},$$

$$\begin{aligned}\frac{q}{R^2} \quad &= \quad \frac{1}{\sigma}\,\frac{-3\,R^2\left(-1+64\,\pi^2\right)\left(8\,C^4-15\,C^2\,R^2+7\,R^4\right)}{1024\,C^4\,\pi^3\,(4\,C^4-7\,C^2\,R^2+3\,R^4)}\\ &+\frac{1}{\sigma}\,\frac{g(C)\ \left[-768\,C^4\pi^2+C^2\left(-5+896\pi^2\right)R^2+3\left(1-64\,\pi^2\right)R^4\right]}{1536\pi^3\,(4\,C^4-7\,C^2\,R^2+3\,R^4)}\end{aligned}$$

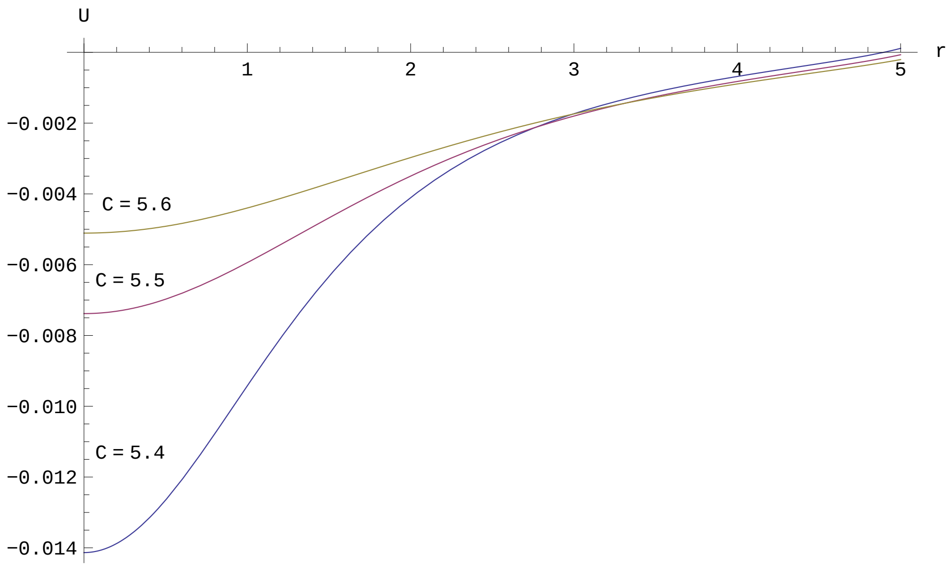
The Schwarzschild's solution



It can be seen that the anisotropic stress is proportional to the density.

–> the most compact distribution undergoes a higher anisotropic effect..

The Schwarzschild's solution



The Weyl function $\mathcal{U}$ is (almost) always negative.

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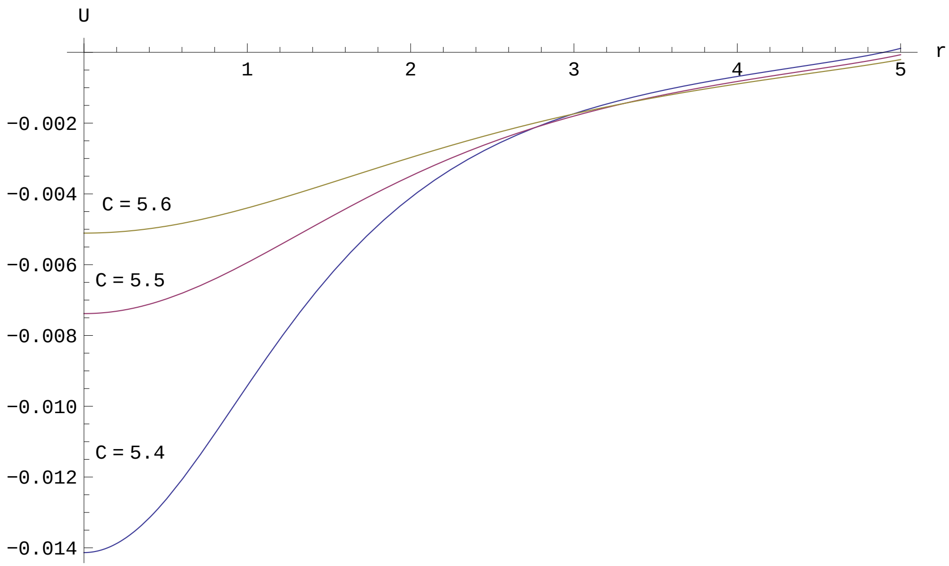

The Weyl function $\mathcal{U}$ is (almost) always negative.

$$\frac{6}{k^4}\frac{\mathcal{U}}{\sigma} = -\frac{3}{\sigma}\left(\frac{\rho^2}{2} + \rho p\right) + \frac{1}{8\pi}\left(2G^2_2 + G^1_1\right) - 3p.$$

Two sources for $\mathcal{U}$: high energy terms (always negative) and an anisotropic term.

Non-uniform distributions

Using the conservation equation

$$\nabla^{\mu} T_{\mu\nu}^{\;\;T} = 0$$

we obtain

$$\mathcal{U}' + \frac{\nu'}{2} 4\mathcal{U} - 2\mathcal{P}' - \frac{\nu'}{2} 2\mathcal{P} - 6\frac{\mathcal{P}}{r} = -(\rho + p)\rho' \frac{k^4}{2}$$

This equation is a lineal combination of the field equations, so there is not new information from it. However we can learn that the density gradients are a source for Weyl stresses in the interior!

$$\rho = \rho(r) \rightarrow \mathcal{U} \quad and/or \quad \mathcal{P} \neq 0$$

Non-uniform distributions

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Non-uniform distributions

- In the case of non-uniform distributions, the constraint $H(p, \rho, \nu) = 0$ is not enough to close the system of equations. However...
- Any general relativistic solution satisfy the condition $H(p, \rho, \nu) = 0$.
- This constraint represents a "natural" way to obtain the braneworld version of any general relativistic solution.

Non-uniform distributions

Let us pick a general relativistic solution:

$$\rho(r) = \frac{C\ \left(9 + 2\,C\,r^2 + C^2\,r^4\right)}{7\,\pi\,(1 + C\,r^2)^3}; \quad p(r) = \frac{2C(2 - 7Cr^2 - C^2r^4)}{7\pi(1 + Cr^2)^3}; \quad e^{\nu} = A(1 + Cr^2)^4$$

The braneworld solution is found through

$$e^{-\lambda(r)} = 1 - \frac{2\tilde{m}(r)}{r}$$

where the interior mass function is given by

$$\begin{aligned} \tilde{m}(r) \quad &= \quad m(r) - \frac{1}{\sigma}\left(\frac{2}{7}\right)^2 \frac{Cr}{2\pi}\left[\frac{240 + 589Cr^2 - 25C^2r^4 - 41C^3r^6 - 3C^4r^8}{3(1 + Cr^2)^4(1 + 3Cr^2)}\right. \\ &\left. - \frac{80}{(1 + Cr^2)^2}\frac{arctg(\sqrt{C}r)}{(1 + 3Cr^2)\sqrt{C}r}\right], \end{aligned}$$

$$m(r) = \int_0^r 4\pi r^2 \rho dr = \frac{4}{7}Cr^3\frac{(3 + Cr^2)}{(1 + Cr^2)^2}, \quad GR\ \ mass\ \ \ function.\ \ Durgapal{-}Fuloria\ (1983).$$

the interior Weyl functions are

$$
\begin{aligned}
\mathcal{P}(r) \quad = \quad & \frac{32}{441 r^3 (1+Cr^2)^6 (1+3Cr^2)^2} \left[Cr \left(180 + 2040 Cr^2 + 8696 C^2 r^4 \right. \right. \\
& \left. + 16533 C^3 r^6 + 12660 C^4 r^8 + 146 C^5 r^{10} - 120 C^6 r^{12} + 9 C^7 r^{14} \right) \\
& \left. - 60 \sqrt{C} (1+Cr^2)^3 (3 + 26 Cr^2 + 63 C^2 r^4) arctg(\sqrt{C} r) \right] ,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{U}(r) \quad = \quad & \frac{32}{441 r (1+Cr^2)^6 (1+3Cr^2)^2} \left[C^2 r \left(795 + 4865 Cr^2 + 10044 C^2 r^4 \right. \right. \\
& \left. + 6186 C^3 r^6 - 373 C^4 r^8 - 219 C^5 r^{10} - 18 C^6 r^{12} \right) \\
& \left. - 240 C^{3/2} (1+Cr^2)^3 (5 + 9 Cr^2) arctg(\sqrt{C} r) \right] .
\end{aligned}
$$

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$$\left.+16533C^3r^6+12660C^4r^8+146C^5r^{10}-120C^6r^{12}+9C^7r^{14}\right)$$
$$-60\sqrt{C}(1+Cr^2)^3(3+26Cr^2+63C^2r^4)arctg(\sqrt{C}r)\Big],$$

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.

EXACT SOLUTION!!!

Bulk Effects ?

The general relativistic solution for p and ρ is given by

$$\rho(r) = \frac{C\ \left(9 + 2\,C\,r^2 + C^2\,r^4\right)}{7\,\pi\,(1 + C\,r^2)^3}; \qquad p(r) = \frac{2C(2 - 7Cr^2 - C^2r^4)}{7\pi(1 + Cr^2)^3};$$

Where is the bulk effect on $p(r)$ and $\rho(r)$?

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Where is the bulk effect on $p(r)$ and $\rho(r)$?

Using the matching conditions we will have $C \longrightarrow C(\sigma)$

Bulk Effects $C \to C(\sigma)$

Matching our solution with the Reissner-Nördstrom–like solution

$$e^{\nu^+} = e^{-\lambda^+} = 1 - \frac{2\mathcal{M}}{r} + \frac{q}{r^2}; \quad \mathcal{U}^+ = -\frac{\mathcal{P}^+}{2} = \frac{4}{3}\pi q \sigma \frac{1}{r^4} \quad \frac{Dadhich,\ Maartens,}{Papadopoulos,\ Rezania\ (2000).}$$

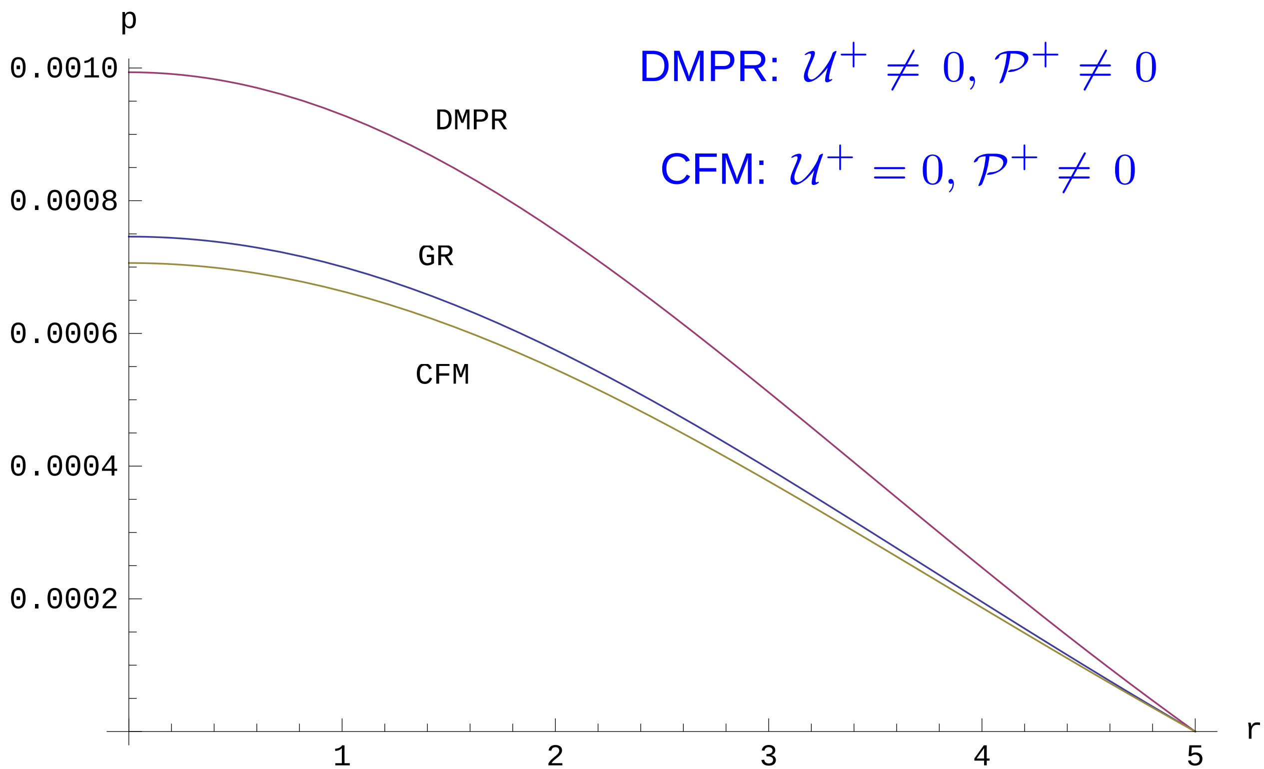
$$A(1 + CR^2)^4 = 1 - \frac{2\mathcal{M}}{R} + \frac{q}{R^2},$$

$$\begin{aligned}
\frac{2\mathcal{M}}{R} &= \frac{2M}{R} - \frac{1}{\sigma}\left(\frac{2}{7}\right)^2 \frac{C}{\pi}\left[\frac{240 + 589CR^2 - 25C^2R^4 - 41C^3R^6 - 3C^4R^8}{3(1+CR^2)^4(1+3CR^2)}\right. \\
&\left. - \frac{80}{(1+CR^2)^2}\frac{arctg(\sqrt{C}R)}{(1+3CR^2)\sqrt{C}R}\right] + \frac{q}{R^2}, \\
q &= \frac{-4R}{147(1+CR^2)^5(1+3CR^2)}\left[CR\left((-2 + CR^2 + 22C^2R^4 + 3C^3R^6)\right.\right. \\
& 84\pi R^2(1+CR^2)^2 + \frac{1}{\sigma}(-240 - 2749CR^2 - 5276C^2R^4 + 266C^3R^6 \\
& \left.\left. +372C^4R^8 + 27C^5R^{10})\right) + \frac{1}{\sigma}240\sqrt{C}(1+CR^2)^2(1+9CR^2)arctg(\sqrt{C}R)\right].
\end{aligned}$$

Role of dark radiation and dark pressure

DMPR: $\mathcal{U}^+ \neq 0$, $\mathcal{P}^+ \neq 0$

CFM: $\mathcal{U}^+ = 0$, $\mathcal{P}^+ \neq 0$



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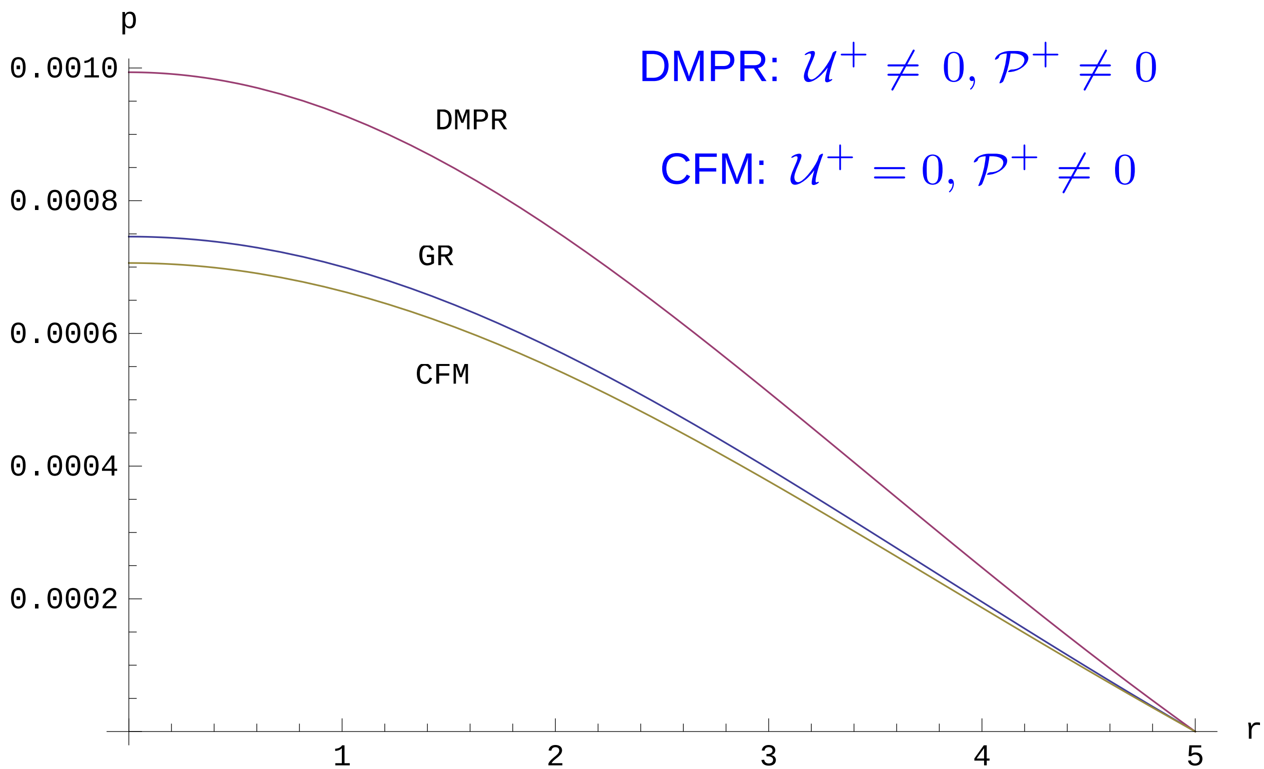
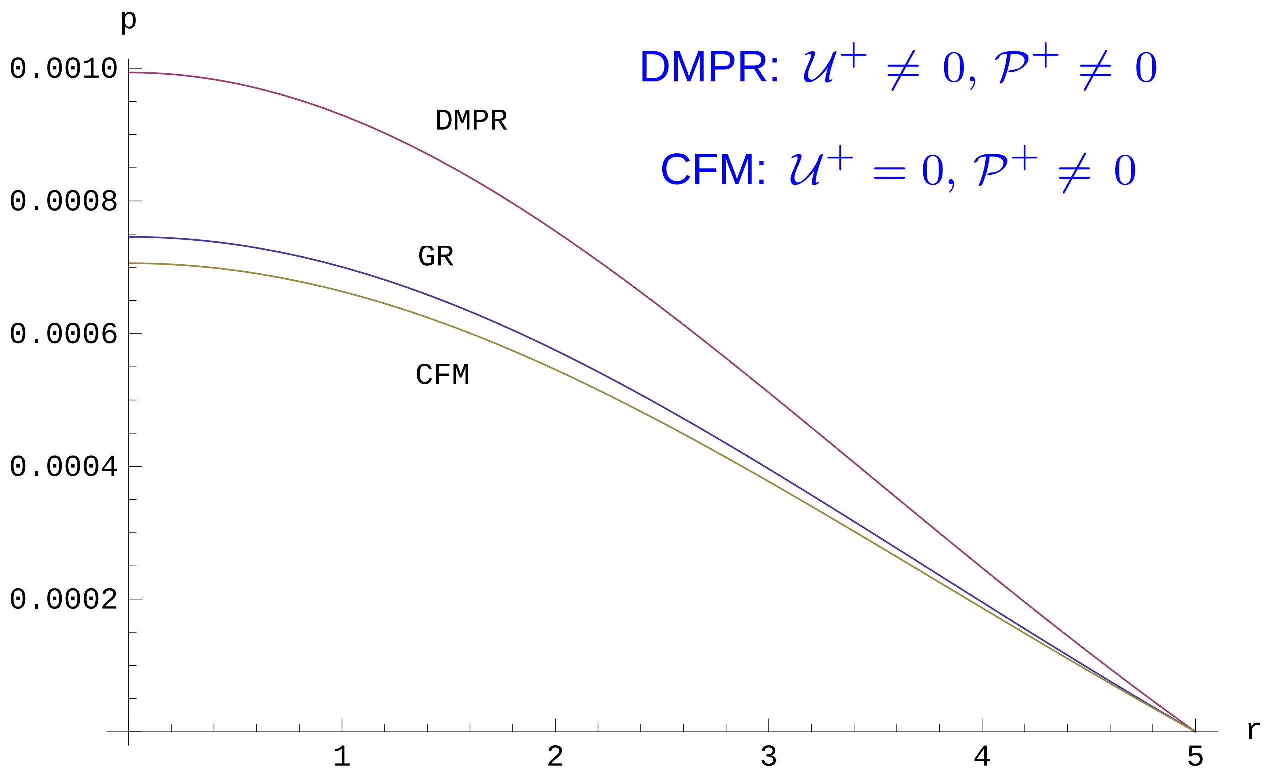


- The exterior dark radiation $\mathcal{U}^+$ always increases both the pressure and the compactness of the stellar structures.

Role of dark radiation and dark pressure



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- The exterior dark radiation $\mathcal{U}^+$ always increases both the pressure and the compactness of the stellar structures.
- The exterior dark pressure $\mathcal{P}^+$ always reduces them.

Conclusions

- The minimal geometrical deformation approach represents a natural way to see the five dimensional consequences on compact distributions.
- Both exterior Weyl functions $\mathcal{U}^+$ and $\mathcal{P}^+$ have well defined consequences on stellar structure.
- The exterior dark radiation $\mathcal{U}^+$ always increases both the pressure and the compactness of the stellar structures, and that the exterior dark pressure $\mathcal{P}^+$ always reduces them.
- An exterior solution with $\mathcal{U}^+ = 0$ and $\mathcal{P}^+ \neq 0$ surrounding a stellar distribution might be seen as an environment whose physical effects on the stellar structure are such that it can be considered as a region with *negative* effective pressure.