On the Space of Generalized Fluxes for Loop Quantum Gravity

Carlos Guedes

Albert Einstein Institute, Potsdam, Germany

with B. Dittrich and D. Oriti
[to appear soon]

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Motivation

Loop Quantum Gravity
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Loop Quantum Gravity

- (Smeared) variables: holonomies $h_e[A] \in SU(2)$, fluxes $E(S, f) \in \mathfrak{su}^*(2)$
- Configuration space (projective limit): $\mathcal{A} = \varprojlim A_\gamma$
- Hilbert space (inductive limit):
  $$\mathcal{H}_0 = L^2(\mathcal{A}, d\mu_0) = \varprojlim \mathcal{H}_\gamma = (\bigcup_\gamma L^2(A_\gamma, d\mu_H))/\sim$$
Loop Quantum Gravity

- (Smeared) variables: holonomies $h_e[A] \in SU(2)$, fluxes $E(S, f) \in su^*(2)$
- Configuration space (projective limit): $\overline{A} = \lim_{\gamma} A_\gamma$
- Hilbert space (inductive limit):
  $$\mathcal{H}_0 = L^2(\overline{A}, d\mu_0) = \lim_{\leftarrow} \mathcal{H}_\gamma = (\bigcup_{\gamma} L^2(\overline{A}_\gamma, d\mu_H)) / \sim$$
- Group Fourier transform $\mathcal{F}_{\gamma}$ [hep-th/1004.3450]

$$\mathcal{F}_{\gamma} : \bigcup_{\gamma} \mathcal{H}_\gamma \xrightarrow{\mathcal{F}_{\gamma}} \bigcup_{\gamma} \mathcal{H}_{*\gamma}$$

$$\pi : \bigcup_{\gamma} \mathcal{H}_\gamma / \sim \xrightarrow{\pi} (\bigcup_{\gamma} \mathcal{H}_{*\gamma}) / \sim$$

$$\tilde{\mathcal{F}} : (\bigcup_{\gamma} \mathcal{H}_{*\gamma}) / \sim \xrightarrow{\tilde{\mathcal{F}}} \bigcup_{\gamma} \mathcal{H}_{*\gamma} / \sim$$

$$\bigcup_{\gamma} \mathcal{H}_{*\gamma} / \sim \simeq L^2(\overline{E}, d\mu_{*0}) ? \quad \overline{E} : \text{space of generalized fluxes}$$
Cyl is well-defined: norm and (pointwise)-product cylindrical consistent

$\Delta(\text{Cyl}) = \overline{A}$
Group Fourier transform (one edge):
\[ F_e : C(G) \rightarrow C_*(g^*) \]
\[ f(g) \mapsto \hat{f}(x) := F_e(f)(x) = \int_G dg \, f(g) \, e_g(x) \]

\[ \ast\text{-product: } e_{g_1} \ast e_{g_2} = e_{g_1 g_2} \]

\[ \mathcal{F}(f_1) \ast \mathcal{F}(f_2) = \mathcal{F}(f_1 \ast f_2) \rightarrow \text{dual to convolution} \]
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Convolution product \textit{not} cylindrically consistent unless \( G \) \textit{abelian}!
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\[ \text{SU}(2) \rightarrow \text{U}(1)^3 \rightarrow \text{U}(1) \text{ (quantization of linearized gravity)} \]
The space of generalized connections
Group Fourier transform
The space of generalized fluxes
Open Issues and Outlook

The space of generalized fluxes

Suppose $A \gamma$ are abelian groups, and let $A$ be the projective limit with projections $p_\gamma: A \to A \gamma$. Then the dual group $\hat{A}$ equals the inductive limit of the dual groups $\hat{A}_\gamma$. Consistency conditions: $5/6$
The space of generalized fluxes

- Push-forward through the Fourier transform: fail!
The space of generalized fluxes

- Push-forward through the Fourier transform: fail!
- Duality

**Theorem**

Suppose $\mathcal{A}_\gamma$ are abelian groups, and let $\overline{\mathcal{A}}$ be the projective limit with projections $p_\gamma: \overline{\mathcal{A}} \to \mathcal{A}_\gamma$. Then the dual group $\hat{\mathcal{A}}$ equals the inductive limit of the dual groups $\hat{\mathcal{A}}_\gamma$. 
Push-forward through the Fourier transform: fail!

Duality

**Theorem**

Suppose $A_\gamma$ are abelian groups, and let $\overline{A}$ be the projective limit with projections $p_\gamma : \overline{A} \rightarrow A_\gamma$. Then the dual group $\hat{A}$ equals the inductive limit of the dual groups $\hat{A}_\gamma$.

Consistency conditions:
\[ \hat{A} = \text{Hom}(\text{Hom}(\mathcal{P}, U(1))) \] (inductive limit): better characterization?

LQG kinematics treats \( A \) and \( E \) very asymmetrically!

Start from scratch encoding the conditions tailored to the fluxes: possible?

Loop Quantum Cosmology
  - Configuration space: \( \overline{\mathbb{R}}_{\text{Bohr}} \) (projective limit)
  - Flux interpretation for LQC?
  - Embed LQC into \( U(1) \)-LQG?
Open Issues and Outlook

- \( \widehat{A} = \text{Hom}(\text{Hom}(\mathcal{P}, U(1))) \) \text{(inductive limit)}: better characterization?
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Thank you for your attention!