# Critical Gravity in D-dimensions

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# Outline

1. Motivation
2. Einstein Gravity: Action, spectrum ...
3. Critical theory in D=3: Chiral gravity
4. Quadratic Curvature Critical Gravities

# Based On

This talk is based on:

- S. Deser, H. Liu, H. Lu, C. N. Pope, T. C. S. and B. Tekin, *"Critical Points of D-Dimensional Extended Gravities,"* Phys. Rev. D **83**, 061502 (2011) [arXiv:1101.4009 [hep-th]].
- I. Gullu, M. Gurses, T. C. S. and B. Tekin, *"AdS waves as exact solutions to quadratic gravity,"* Phys. Rev. D **83**, 084015 (2011) [arXiv:1102.1921 [hep-th]].

# Motivation

- **Problem:** Einstein's gravity

$$I = \frac{1}{\kappa}\int d^4x\sqrt{-g}\,R,$$

  is not renormalizable ('t Hooft and Veltman, 1974; Deser and van Nieuwenhuizen, 1974). ($\kappa \equiv 16\pi G$)
- **Effective field theory perspective:** High energies $\Rightarrow$ Higher curvature terms; i.e. $\alpha R^2$, $\beta R^{\mu\nu}R_{\mu\nu}$, $\gamma R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$, ... .
- **Result:** In addition to determining the high energy behavior, the theory gains additional features such as particle spectrum change.
- **Question:** Are there any special points in the extended parameter space having nontrivial change in the features of the theory like particle spectrum, energy of the excitations, energy of the black holes.
- This "critical" points may be interesting in quantizing gravity.

- Einstein's gravity

$$I = \frac{1}{\kappa}\int d^4x\sqrt{-g}\,R \qquad \underset{\delta}{\Rightarrow} \qquad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0,$$

  which describes evolution of $g_{\mu\nu}$.

- One can consider this theory as the fluctuations of massless spin-2 field $h_{\mu\nu}$ around the background $\bar{g}_{\mu\nu}$ solving $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0$:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}.$$

- Noninteracting theory can be found by looking at the linearized order in $h_{\mu\nu}$: In flat background, $h_{\mu\nu}$ satisfies

$$\bar{\Box}h_{\mu\nu} = 0,$$

  in the gauge $\frac{\partial}{\partial x^\mu}h^\mu{}_\nu = \frac{1}{2}\frac{\partial}{\partial x^\nu}h$. ($\bar{\Box} \equiv \frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x^\mu}$)

# Critical theories in $D = 3$ - I

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- Adding cosmological constant: Theory has black hole solution (Banados, Teitelboim and Zanelli, 1992).

$$I = \frac{1}{\kappa} \int d^3x \sqrt{-g} \left( R + \frac{2}{\ell^2} \right).$$

- Adding higher derivative terms: Theory has unitary massive helicity-2 excitations with mass $m^2 = \frac{\mu^2}{\kappa^2}$ (Deser, Jackiw and Templeton, 1982).

$$I = \int d^3x \left[ -\sqrt{-g} \frac{1}{\kappa} R + \frac{1}{2\mu} \varepsilon^{\mu\nu\rho} \left( \Gamma^{\alpha}_{\mu\beta} \partial_\nu \Gamma^{\beta}_{\rho\alpha} + \frac{2}{3} \Gamma^{\alpha}_{\mu\gamma} \Gamma^{\gamma}_{\nu\beta} \Gamma^{\beta}_{\rho\alpha} \right) \right].$$

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- Adding cosmological constant + higher derivative terms: The mass of the excitation modified as $m^2_{AdS} = \left(\frac{\mu^2}{\kappa^2} - \frac{1}{\ell^2}\right)$.

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- For generic values of $\ell$ and $\mu$, the unitarity of the degree of freedom and the positivity of the energy of the black hole is in conflict.

- BUT, if the parameters of the theory tuned as $\mu\ell = \kappa$; massive mode becomes massless, on-shell energy of this mode becomes zero $\Rightarrow$ degree of freedom becomes pure gauge $\Rightarrow$ conflict in the unitarity of the excitations and the positivity of the black hole is removed (Li, Song and Strominger, 2008).

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# Quadratic curvature gravity in $D = 4$

- In $D = 4$, quadratic curvature modification of Einstein's gravity:

$$I = \int d^4x \sqrt{-g} \left[ \frac{1}{\kappa}(R - 2\Lambda_0) + \alpha R^2 + \beta R^{\mu\nu} R_{\mu\nu} \right].$$

- Flat background analysis; i.e. $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$: Renormalizable, but not unitary (Stelle, 1977).
- Spectrum of the theory: massless spin-2, massive helicity-0 and massive helicity-2 excitations.
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# Critical gravity in $D = 4$

- Lu and Pope (2011) apply the ideas in three dimensions to four dimensions:

$$I = \int d^4x \sqrt{-g} \left[ \frac{1}{\kappa} \left( R + \frac{1}{2\kappa\alpha} \right) + \alpha R^2 - 3\alpha R^{\mu\nu} R_{\mu\nu} \right].$$

- The critical theory involves only massless spin-2 excitation with zero on-shell energy.
- The energy of the Schwarzschild-de Sitter black hole is zero.

# Critical gravity in $D$ dimensions - I

- The most general quadratic curvature action in $D$ dimensions is

$$I = \int d^D x \sqrt{-g} \Big[ \frac{1}{\kappa}(R - 2\Lambda_0) + \alpha R^2 + \beta R^{\mu\nu} R_{\mu\nu} + \gamma \left( R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4 R^{\mu\nu} R_{\mu\nu} + R^2 \right) \Big].$$

- Gauge choice $\bar{\nabla}^\mu h_{\mu\nu} = \bar{\nabla}_\nu h$. With this gauge choice, field equation for the massive helicity-0 mode

$$\left[ (4\alpha(D-1) + D\beta)\bar{\Box} - (D-2)\left( \frac{1}{\kappa} + 4f\Lambda \right) \right] h = 0.$$

- Set $4\alpha(D-1) + D\beta = 0$ and take $\frac{1}{\kappa} + 4f\Lambda \neq 0$, no helicity-0 mode.

- The remaining field equations are

$$-\frac{\beta}{2}\left( \bar{\Box} - \frac{4\Lambda}{(D-1)(D-2)} - M^2 \right)\left( \bar{\Box} - \frac{4\Lambda}{(D-1)(D-2)} \right) h_{\mu\nu} = 0,$$

where $M^2 \equiv -\frac{1}{\beta}\left( c + \frac{4\Lambda\beta}{(D-1)(D-2)} \right)$.

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- The energy of the black holes and the on-shell energy of the excitations are proportional to $M^2$.
- The "critical point" is $M^2=0$, and the action takes the form

$$I=\int d^Dx\sqrt{-g}\left[(R-2\Lambda_0)+\gamma C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma}\right], \qquad \Lambda_0=\frac{(D-1)(D-2)}{8\gamma(D-3)}.$$

Here, $C_{\mu\nu\rho\sigma}$ is the Weyl tensor, for which

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# Log mode of critical gravity

- Double pole structure yields logarithmic modes which have a different asymptotic behavior than Einstein modes at spacelike infinity in radial coordinate (Alishahiha *et al*, 2011; Gullu *et al*, 2011; Bergshoeff *et al*, 2011).
- Take the $D$-dimensional AdS-wave metric ansatz in the Kerr-Schild form

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + 2V\lambda_\mu\lambda_\nu,$$

where $\lambda_\mu\lambda^\mu = 0, \quad \lambda^\mu\nabla_\mu\lambda^\nu = 0$, and $\bar{g}_{\mu\nu}$ is the metric of the AdS space in the coordinates

$$d\bar{s}^2 = \frac{\ell^2}{z^2}\left(dz^2 + 2dudv + \sum_{n=1}^{D-3}(dx^n)^2\right).$$

- Constrain coordinate dependence of $V$ by requiring $\nabla_\mu\left(V\lambda_\mu\lambda_\nu\right) = 0 \quad \Rightarrow \quad \partial_v V = 0 \quad \Rightarrow$ curvature tensors become linear in $V \quad \Rightarrow$ Field equations are linear in $V \quad \Rightarrow$ solving field equations

$$V\left(u,z\right) = d_1\left(u\right)z^{D-3} + \frac{d_2\left(u\right)}{z^2} + \frac{1}{D-1}\left(c_1\left(u\right)z^{D-3} - \frac{c_2\left(u\right)}{z^2}\right)\ln\left(\frac{z}{\ell}\right).$$

- The theory is NOT *unitary* in the presence of the Log modes. Removing Log modes by boundary conditions yields a *trivial* theory with just vacuum state (Porrati and Roberts, 2011).

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# Critical gravity and conformal gravity

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- Lu, Pang and Pope observed that $\Lambda_0 = \Lambda_0(\gamma)$ is the same as in critical gravity and the boundary conditions are the ones removing Log modes (arXiv:1106.4657 [hep-th]).
- Thus, trivial nature of the critical gravity is basically a reflection of the equivalance between two parts of the critical gravity action:

$$I = \int d^4x \sqrt{-g}\left[(R-2\Lambda_0) + \gamma C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma}\right], \qquad \Lambda_0 = \frac{3}{4\gamma}.$$

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# Conclusions

- We extend $D = 4$ critical gravity of Lu and Pope to $D$ dimensions which reduces to New Massive Gravity at the Proca limit for $D = 3$ (Deser *et al*, 2011) $\Rightarrow$ these theories involve massless spin-2 excitation just like Einstein's gravity.
- We find Log modes of the critical gravity (Alishahiha *et al*, 2011; Gullu *et al*, 2011; Bergshoeff *et al*, 2011) $\Rightarrow$ the theory is *not* unitary due to appearance of Log modes.
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# Conclusions

- We extend $D = 4$ critical gravity of Lu and Pope to $D$ dimensions which reduces to New Massive Gravity at the Proca limit for $D = 3$ (Deser *et al*, 2011) $\Rightarrow$ these theories involve massless spin-2 excitation just like Einstein's gravity.
- We find Log modes of the critical gravity (Alishahiha *et al*, 2011; Gullu *et al*, 2011; Bergshoeff *et al*, 2011) $\Rightarrow$ the theory is *not* unitary due to appearance of Log modes.
- Critical gravity may be interesting in AdS/LCFT context.
- Thank you for your attention!