Prescriptions and Superselection Sectors in Loop Quantum Cosmology

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Motivation

1) In LQC there exist several prescriptions to describe quantum mechanically a flat, homogeneous and isotropic spacetime.
2) All of them provide the same physical picture for states sharply peaked at classical trajectories.
3) However, may these different proposals result in an ambiguity whose effects could arise in other quantum regimes?
4) Could be their differences detectable?

Classical system & Quantum framework

Classical system

1) Flat open FRW model. Fiducial cell $\mathcal{V}$ of volume V_0.
2) Momentum constraint (homogeneity) and gauge constraint (diagonal gauge) vanish.
3) 1 geom. d.o.f.: $\{c, p\} = 8\pi\gamma G/3$ + 1 matter d.o.f.: $\{\phi, p_\phi\} = 1$.

The Hamiltonian constraint with flat slices is:

$$C = -6\gamma^{-2}c^2\sqrt{|p|} + 8\pi G p_\phi^2 V^{-1}, \quad V := |p|^{3/2}.$$

Quantum framework

1) Matter: standard representation $\mathcal{H}_{\rm matt}^{\rm kin} = L^2(\mathbb{R}, d\phi)$, $\hat{p}_\phi = -i\partial_\phi$.
2) Geometrical contribution: polymeric representation in the improved dynamics scheme

$$\mathcal{H}_{\rm grav}^{\rm kin} = \overline{\mathrm{span}\{|v\rangle, \, v \in \mathbb{R}\}}, \quad \langle v'|v\rangle = \delta_{v'v}, \quad \hat{\mathcal{N}}_{\bar{\mu}}|v\rangle = |v+1\rangle.$$

Quantum Constraint

1) The constraint can be regarded as: $8\pi G(-\hbar^2\hat{\Theta}\otimes\mathbb{I}+\mathbb{I}\otimes\hat{p}_\phi^2)$
2) The operator $\hat{\Theta}=-\hat{\mathcal{N}}_{2\bar{\mu}}f(\hat{v})\hat{\mathcal{N}}_{2\bar{\mu}}-\hat{\mathcal{N}}_{-2\bar{\mu}}f(\hat{v})\hat{\mathcal{N}}_{-2\bar{\mu}}+f_o(\hat{v})$, with
$$f(v)=\frac{3\pi G}{4}(v^2-2-\alpha)+O(v^{-2}),\quad f_o(v)=\frac{3\pi G}{2}(v^2-\alpha)+O(v^{-2}),$$
is a selfadjoint operator that only relates states with support in (semi)lattices $\mathcal{L}_\varepsilon$, with $\varepsilon\in(0,4]$.
3) The restriction of $\hat{\Theta}$ to each $\mathcal{L}_\varepsilon$ has continuous spectrum equal to $\mathbb{R}^+$. Its degeneracy is at most twofold (prescription dependent).
4) In a suitable representation ($\{v,b\}=4$)
$$\hat{\Theta}=-12\pi G\left[\frac{\alpha+1}{4\cosh^2(2x)}+\partial_x^2\right],\quad x:=\ln[\tan(b/4)]/2$$

Prescriptions

APS: Ashtekar, Pawłowski & Singh

Given $B(v) := (27/8)|v|\left||v+1|^{1/3} - |v-1|^{1/3}\right|^3$, and $\tilde{f}(v) = (3\pi G/8)|v|\left||v+1| - |v-1|\right|$:

$f(v) = [B(v+2)]^{-1/2}\tilde{f}(v)[B(v-2)]^{-1/2}$; $f_o(v) = B(v)^{-1}[\tilde{f}(v+2)+\tilde{f}(v-2)]$,

$\mathcal{L}_\varepsilon$ are lattices (*generic*) in general. For $\varepsilon = 2, 4$ they reduce to semilattices (*exceptional*).

Besides, a simple calculation yields $\alpha = 5/9$.

sLQC: Ashtekar, Corichi & Singh

$$f(v) = (3\pi G/4)\sqrt{|v+2|}|v|\sqrt{|v-2|}, \quad f_o(v) = (3\pi G/2)v^2,$$

This prescription shares the superselection sectors with APS. For this prescription $\alpha = 0$.

Prescriptions

MMO: Mena-Marugán, Martín-Benito & Olmedo

In this case, if $g(v) := \sqrt{3/2}|v|^{1/3}B(v)^{-1/6}$, and $s_\pm(v) := \mathrm{sgn}(v \pm 2) + \mathrm{sgn}(v)$, then:

$$
\begin{aligned}
f(v) &= (\pi G/12)g(v+2)g(v-2)g^2(v)s_+(v)s_-(v), \\
f_o(v) &= (\pi G/12)g^2(v)\{[g(v+2)s_+(v)]^2 + [g(v-2)s_-(v)]^2\},
\end{aligned}
$$

$\mathcal{L}_\varepsilon$ are always semilattices. Besides, $\alpha = 5/3$.

sMMO

$$
\begin{aligned}
f(v) &= (3\pi G/16)\sqrt{|v+2|}|v|\sqrt{|v-2|}s_+(v)s_-(v), \\
f_o(v) &= (3\pi G/16)|v|\left[|v+2|s_+^2(v) + |v-2|s_-^2(v)\right].
\end{aligned}
$$

This prescription shares the superselection sectors with MMO. In addition, $\alpha = 0$.

Generalized eigenfunctions $(e_k^\varepsilon|\hat{\Theta} = (e_k^\varepsilon|12\pi G k^2$

Semilattices: $\mathcal{L}_\varepsilon^\pm = \{\pm(\varepsilon + 4n), n \in \mathbb{N}\}$

Eigenfunctions $e_k^\varepsilon(v)$ determined by 1 piece of data $e_k^\varepsilon(\varepsilon) \in \mathbb{R}^+$. Asymptotic standing wave $\lim_{v\to\infty} e_k^\varepsilon(v) = 2[\underline{e}_k(v)e^{i\phi_k^\varepsilon} + \underline{e}_{-k}(v)e^{-i\phi_k^\varepsilon}]$.

Lattices: $\mathcal{L}_\varepsilon = \{\varepsilon + 4n, n \in \mathbb{Z}\}$

Half of the basis determined by 2 pieces of data (suitable selected). The remaining half is determined by orthogonal completion. The construction involves: i) specification of the asymptotic data of 2 complex, auxiliary eigenfunctions; ii) complex rotation, addition and normalization; iii) extension to the symmetric domain $\mathcal{L}_\varepsilon \cup \mathcal{L}_{4-\varepsilon}$.

Efficiency

In generic cases, we finally extract the data of 4 real standing waves (single exceptional situations), and we evaluate them in certain domain twice larger with respect to the except. cases (higher error). The numerical cost is then **8x** higher than in the exceptional cases.

Physical states & numerics

Physical states

We have considered a 2-parameter $\{k_o, \sigma\}$ family of logarithmic normal distributions (with k_o and σ determined by $\langle \hat{p}_\phi \rangle$ and $\langle \Delta \hat{p}_\phi \rangle$).
The analyzed sector: $\langle \Delta \hat{p}_\phi \rangle / \langle \hat{p}_\phi \rangle \in [0.05, 0.25]$ and $\langle \hat{p}_\phi \rangle \in [30\hbar, 500\hbar]$.

Numerics

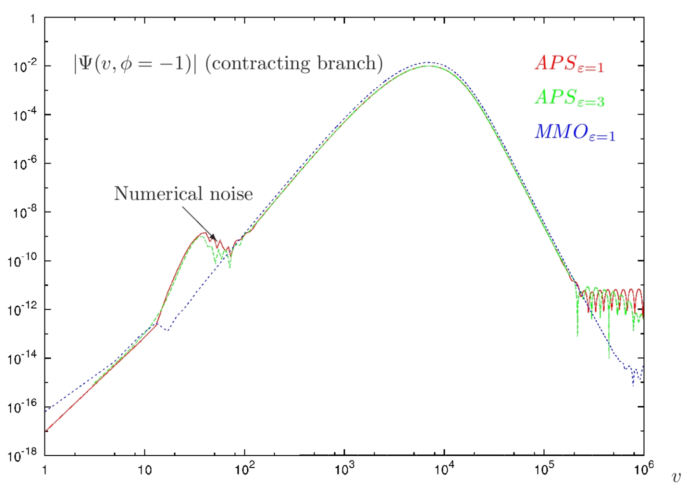
The evaluation of

$$\Psi(v, \phi) = \int_{k \in \mathbf{D}} dk \tilde{\Psi}(k) e_k^\varepsilon(v) e^{i\omega(k)\phi}, \quad \omega(k) = \sqrt{12\pi G}|k|,$$

consists in a discretization of the integral and a reduction of its domain to a finite interval (the main error source still ruled by $(e_k^\varepsilon|$).
In degenerate situations, the integration is approx. **3x** more expensive than in nondegenerate cases: $(e_k^\varepsilon|$ is twice larger and is complex.

Physical states

$|\Psi(k)|^2 = \frac{1}{\sqrt{2\pi}\sigma k} e^{-(\log k/k_0)^2/2\sigma^2}$, $\langle \hat{p}_\phi \rangle = 100\hbar$, $\langle \Delta\hat{p}_\phi \rangle / \langle \hat{p}_\phi \rangle = 0.1$ and $\varepsilon = 1$.



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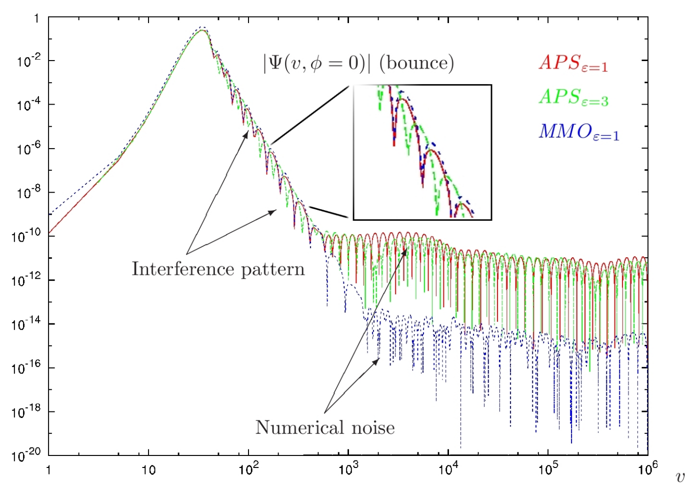


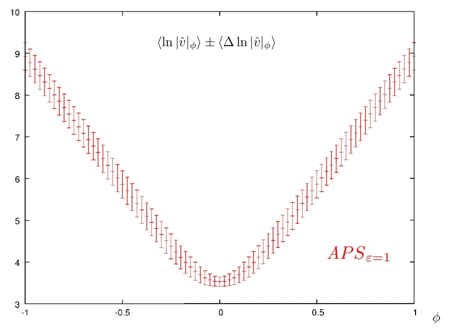
Natural observables:

The observables are:

a) A natural function of the volume $\log|\hat{v}|$.

b) The energy density of the universe $\hat{\rho} := \widehat{\frac{p_\phi^2}{2V^2}} = \frac{\hbar^2}{2}\left[\widehat{\frac{1}{V}}\right]\hat{\Theta}\left[\widehat{\frac{1}{V}}\right]$.

c) The Hubble parameter $\hat{H} := \frac{1}{4i\gamma\sqrt{\Delta}}(\hat{\mathcal{N}}_{4\bar{\mu}} - \hat{\mathcal{N}}_{-4\bar{\mu}})$.



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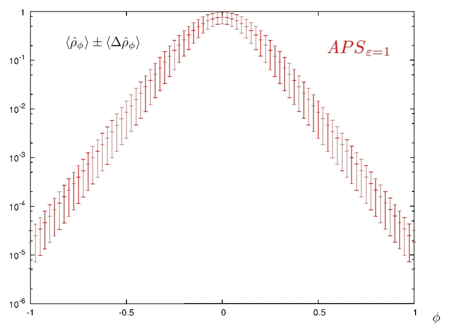

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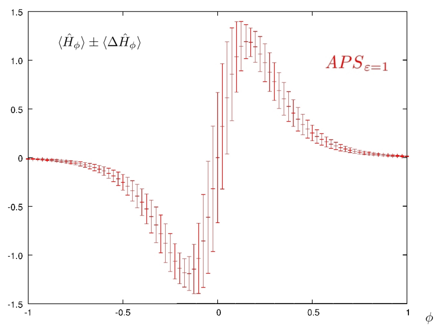
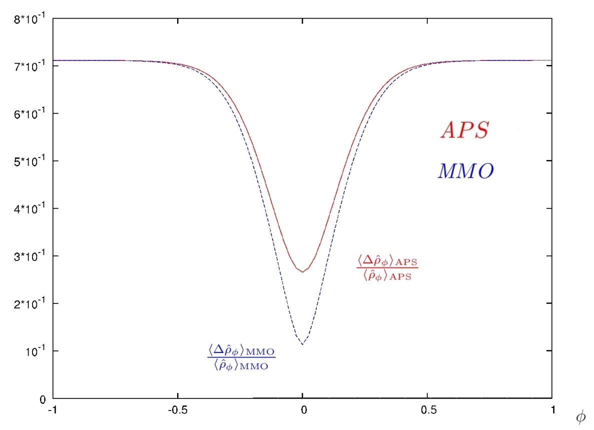


Comparison between expectation values

$|\Psi(k)|^2 = \frac{1}{\sqrt{2\pi}\sigma k} e^{-(\log k/k_0)^2/2\sigma^2}$, $\langle \hat{p}_\phi \rangle = 100\hbar$, $\langle \Delta\hat{p}_\phi \rangle / \langle \hat{p}_\phi \rangle = 0.1$ and $\varepsilon = 1$.

Relative dispersions (for $\hat{\rho}_\phi$)



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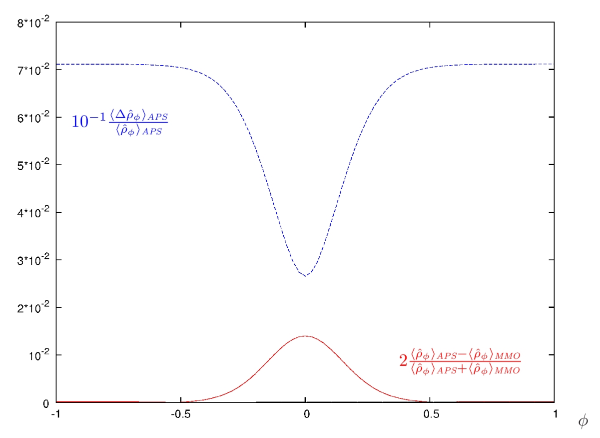
Dispersions vs. differences (for $\hat{\rho}_\phi$)



The observable: $(\widehat{\Delta\Theta}_{AB})^2|_\phi := (\hat{\Theta}_A - \hat{\Theta}_B)^2|_\phi$

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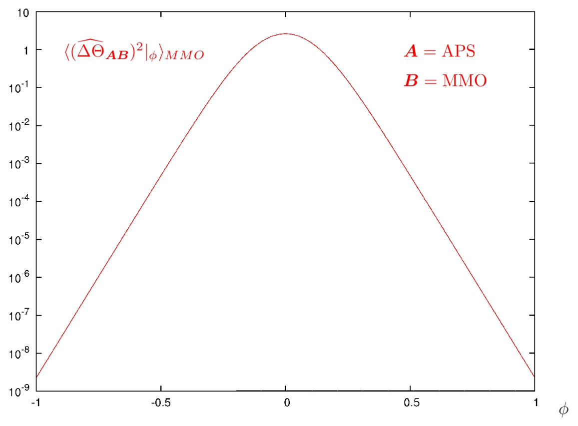

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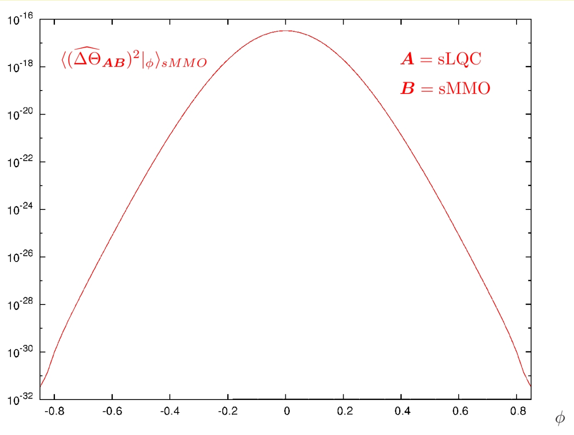

Conclusions

1) The calculation of the basis of generalized eigenfunction is (numerically) more precise and efficient in semilattices than in lattices.

2) The operator $(\widehat{\Delta\Theta_{AB}})^2|_\phi$ measures quantitative differences between prescriptions.

3) Some natural observables are insensitive to these differences (dispersions $\gg$ difference between expectation values).

4) The difference between prescriptions is essentially the absolute value of the wave function (interference pattern at the bounce).