an invitation to

SPINFOAM COSMOLOGY

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(LOOP) QUANTUM GRAVITY & COSMOLOGY

Ashtekar's talk / Rovelli's talk

canonical / covariant quantization

gravity → quantum gravity

symmetry reduction ↓

cosmology → quantum cosmology

- Wheeler, DeWitt, Misner (1967) / Hartle, Hawking (1983) & Vilenkin (1984)
- Bojowald (1999), Ashtekar et al. (2001) / a loopy path integral ?

Bojowald's talk   singularity resolution

FROM CANONICAL TO COVARIANT

- Idea: define a **path integral for general covariant system.** Ashtekar, Campiglia, Henderson '09
  The situation is different wrt the usual path integral, because there is not a preferred choice of time.

$$\langle\vec{\lambda}_F|\vec{\lambda}_0\rangle_{\rm phy} = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\alpha \langle\vec{\lambda}_N|e^{i\alpha\mathcal{C}_H}|\vec{\lambda}_0\rangle$$

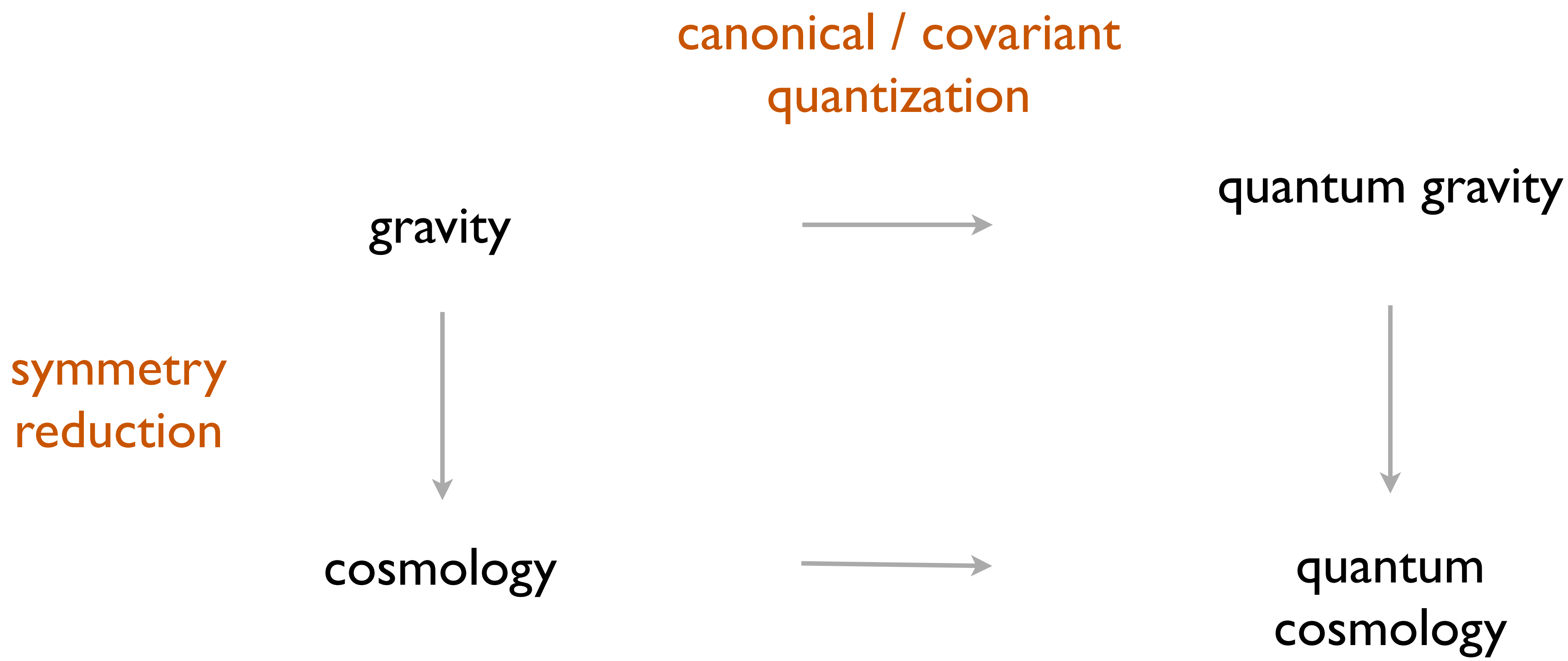
- Final goal: compute transition amplitude between two states of the geometry, and **connect with spinfoam** theory. Henderson, Rovelli, Vidotto, Wilson-Ewing '10

$$\langle\vec{\lambda}_F|\vec{\lambda}_0\rangle_{\pm\delta} = \frac{i}{2\pi}\sum_{M=0}^{\infty}\sum_{\vec{\lambda}_1\ldots\vec{\lambda}_{M-1}}\prod_f A_f(\vec{\lambda}_f)\prod_v A_v(\vec{\lambda}_f)$$

- sum over two-complexes ↔ sum over # of transitions
  sum over colorings ↔ sum over the values of the volume λ_i
  spinfoam vertices ↔ transitions
  spinfoam faces ↔ sequences of steps without transitions

To built a vertex we ask for
1. Locality
2. H should be a density
3. Lorentz invariance

(LOOP) QUANTUM GRAVITY & COSMOLOGY



- MOTIVATIONS
  - Which is the relationship between LQC and the full LQG?
  - Can we describe the full quantum geometry at the bounce?
  - Can we include "naturally" inhomogeneities ?

(LOOP) QUANTUM GRAVITY & COSMOLOGY

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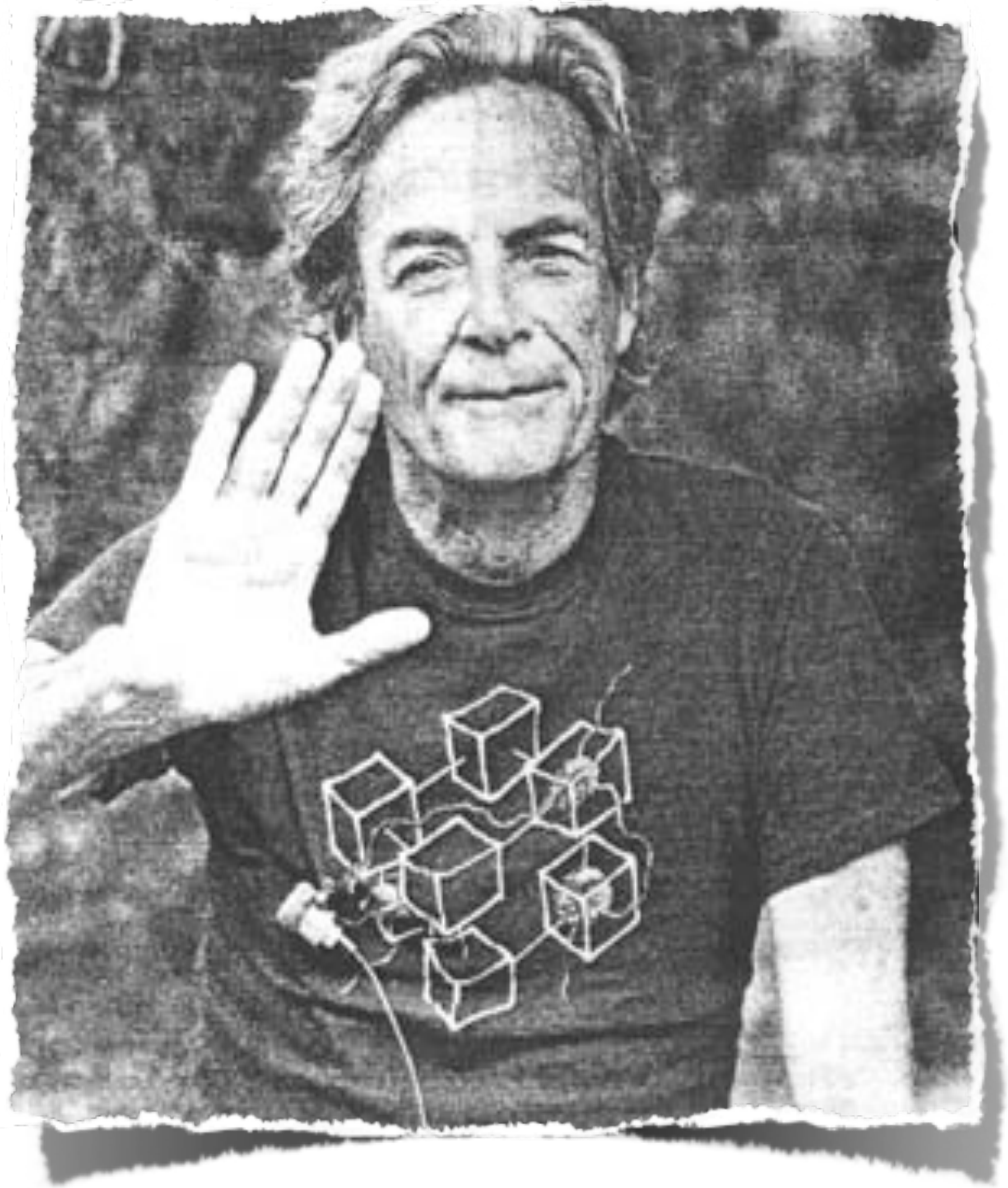
- How cosmology can be obtained from the full quantum gravity theory? Bianchi, Rovelli, FV '10
- RESULTS
  - There are approximations in the quantum theory that yield cosmology.
  - The theory recover general relativity in the semiclassical limit, also for non-trivial solutions (de Sitter).

APPROXIMATIONS

everything we know is only some kind of approximation

- GRAPH TRUNCATION
- BOUNDARY STATES
- VERTEX EXPANSION
- SEMICLASSICALITY

⇒



APPROXIMATIONS

- GRAPH TRUNCATION ↔ number of d.o.f we want to describe
  - *example:* a direct graph corresponding to a triangulated **3-sphere**
- BOUNDARY STATES ↔ coherent states peaked on a given geometry
  - we choose an **homogeneous and isotropic** geometry
- VERTEX EXPANSION ↔ the dynamics is coded by interaction vertices
  - we consider the 1st order ↔ single vertex
- SEMICLASSICALITY ↔ coherent states + large distance
  - large distance $\Rightarrow$ large spin j (a rough graph truncation is well defined)

Graph Expansion ⟷ Mode Expansion

- **Restrict the states to a fixed graph with a finite number N of nodes.**
  This defines an approximated kinematics of the universe,
  inhomogeneous but truncated at a finite number of cells.

Rovelli, Vidotto '08
Rovelli, Battisti, Marcianò '09

- **The graph captures the large scale d.o.f. obtained averaging the metric** over the faces of a cellular decomposition formed by N cells.

- **The full theory can be regarded as an expansion for growing N.**
  The full theory may be recovered by adding degrees of freedom
  one by one, starting from the cosmological ones.

- **Every regular graph leads to the same result in the semiclassical regime!** Vidotto '11

COHERENT STATES

- **Spinnetwork states** $|\Gamma, j_\ell, v_n\rangle \in \tilde{\mathcal{H}} = \bigoplus_\Gamma \mathcal{H}_\Gamma$ $\mathcal{H}_\Gamma = L_2[SU(2)^L/SU(2)^N]$
- **Coherent states** $|\Gamma, \eta_\ell, \xi_\ell, \vec{n}_\ell, \vec{n}'_\ell\rangle$

Livine, Speziale '07
Bianchi, Magliaro, Perini '10

- **Geometrical interpretation** for the $(\vec{n}_\ell, \vec{n}'_\ell, \eta_\ell, \xi_\ell)$ labels:

$\vec{n}_\ell, \vec{n}'_\ell$ are the 3d normals to the faces of the cellular decomposition;
ξ_ℓ ↔ extrinsic curvature at the faces and η_ℓ↔ area of the face $z_\ell = \xi_\ell + i\eta_\ell$

- **Homogeneous coherent states** $|\Gamma, z\rangle$

Marcianò, Magliaro, Perini '11

$\vec{n}_\ell,\ \vec{n}'_\ell$ fixed by requiring a regular cellular decomposition

$$\xi_\ell = \xi \qquad \eta_\ell = \eta \qquad z_\ell = z \qquad z = \xi + i\eta$$

- in terms of the scale factor $Re(z) \sim \dot{a}$ $\sqrt{Im(z)} \sim a$

TRANSITION AMPLITUDE

- CLASSICAL DYNAMICS $S_H = const \int dt\,\left(a\dot{a}^2 + \frac{\Lambda}{3}a^3\right)\Big|_{\dot{a}=\pm\sqrt{\frac{\Lambda}{3}}a} = const\,\frac{2}{3}\sqrt{\frac{\Lambda}{3}}\,(a_f^3 - a_i^3)$
- QUANTUM DYNAMICS $W(a_f, a_i) = e^{\frac{i}{\hbar}S_H(a_f,a_i)} = W(a_f)\overline{W(a_i)}$
- LOOP DYNAMICS $\langle W|\psi_{H_{(z,z')}}\rangle = W(z,z') = W(z)\,\overline{W(z')}$

Bianchi, Rovelli, FV '10

Bianchi, Krajeski, Rovelli, FV '11

term that yields the cosmological constant ↓

Transition amplitude with cosmological constant :

$$Z_{\mathcal{C}} = \sum_{j_f, \mathrm{v}_e} \prod_f (2j+1) \prod_e e^{i\lambda\,\mathrm{v}_e} \prod_v A_v(j_f, \mathrm{v}_e)$$

$$A_v(j_f, \mathrm{v}_e) \longrightarrow W_v(H_\ell) = \langle A|\psi_{H_\ell}\rangle$$

boundary state $\psi \in \mathcal{H}$

Vertex amplitude: $W_v(H_\ell) = \int_{SL(2,\mathbb{C})^N} dG_n \prod_\ell P_t(H_\ell\,,\, G_{s(\ell)}G^{-1}_{t(\ell)})$

where $P_t(H,G) = \sum_j (2j+1)e^{-2t\hbar j(j+1)}\,\mathrm{Tr}\left[D^{(j)}(H)Y^\dagger D^{(j^+,j^-)}(G)Y\right]$

OUT OF THE PLANCK REGIME

$$W_v(z) = \sum_j \prod_{\ell=1}^{L} (2j_\ell + 1)\, \mathrm{H}\;\; \exp[-2t\hbar j_\ell(j_\ell+1) - i\tilde{z}j_\ell - i\lambda \mathrm{v}_o j^{\frac{3}{2}}]$$

$$j \sim j_o + \delta j$$

- max(real part of the exponent) gives where the gaussian is peaked;

$$j_o = \frac{Im(z)}{4t\hbar}$$

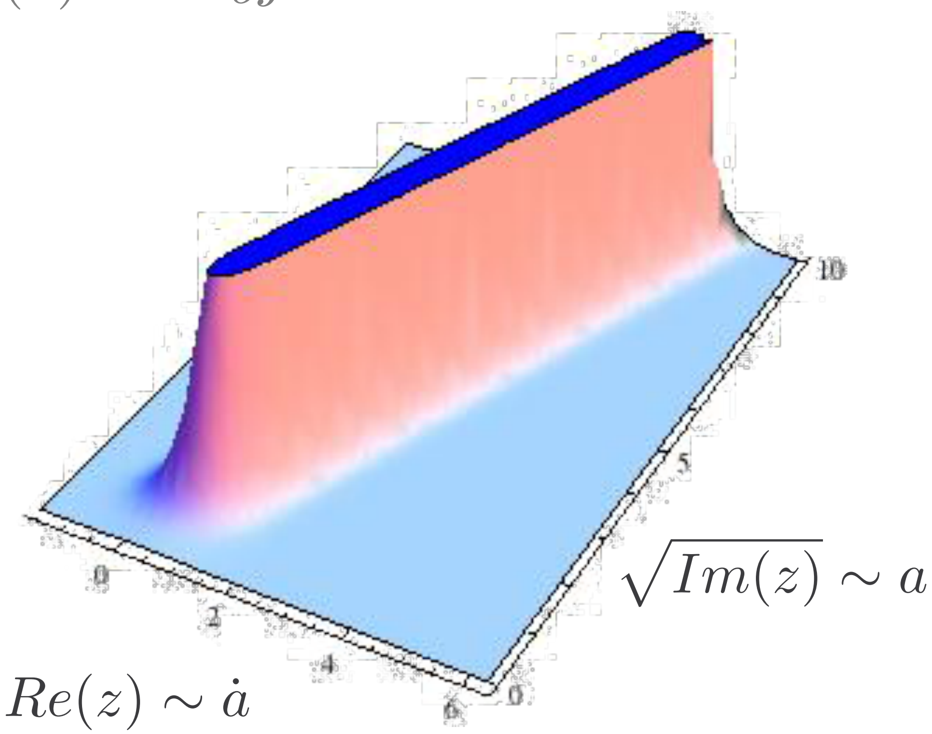
- imaginary part of the exponent =2kπ gives where the gaussian is not suppressed.

$$Re(z) + \lambda \mathrm{v}_o j^{\frac{1}{2}} = 0.$$

- $Re(z) \sim \dot{a}$ and $\sqrt{Im(z)} \sim a$

- $$\frac{Re(z)^2}{Im(z)} = \frac{\lambda^2 \mathrm{v}_o^2}{4t\hbar} \longrightarrow \left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3}$$

with $\Lambda = const\, \lambda^2 G^2 \hbar^2$



SUMMARY

- **The approximations in the quantum theory can yield cosmology.** Rovelli, Vidotto
- Coherent states for cosmology. Marcianò, Magliaro, Perini, Rovelli, Vidotto; Borja, Diaz-Polo, Garay, Livine — Martin-Benito's talk
- There is a simple way to add the cosmological constant. Bianchi, Krajeski, Rovelli, Vidotto
- **The theory recovers general relativity in the semiclassical limit,** also for non-trivial solutions such as de Sitter space. Bianchi, Rovelli, Vidotto
- These results are independent from the (regular) graph used. Vidotto
- Connecting canonical and covariant in loop cosmology. Ashtekar, Campiglia, Calcagni, Gielen, Henderson, Nelson, Oriti, Rovelli, Vidotto, Wilson-Ewing