# Singular topologies in Group Field Theory (GFT)

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## Introduction: some general ideas about GFT

A Group Field Theory (GFT) is:

- a quantum field theory on a group manifold.
- a generalization of matrix models.
- dual to simplicial quantum gravity models and spinfoam models.

$\Rightarrow$ allows to implement a notion of sum over topologies, and a sum over geometries for each topology.

One of the main challenges in LQG and Spinfoam approaches to quantum gravity: continuum limit.

**Hope:** available QFT tools will help us understand the discrete to continuum transition.

# Introduction

In GFT, the topology of spacetime is dynamical.

The sum over topologies includes:

- all topological manifolds (of a given dimension $d$)
- but also degenerate configurations i.e. with topological singularities : points whose neighborhoods are not homeomorphic to $\mathbb{R}^d$.

$\Rightarrow$ 1st structure to be recovered on the way to the continuum limit: topological manifold.

In a sense, GFT provides us with a mechanism for the emergence of the topological structure of spacetime.

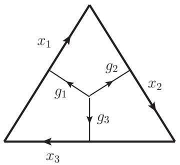
# Outline

# Boulatov model: kinematics

A model for Euclidean 3d quantum gravity: $\mathrm{SU}(2)$ or $\mathrm{SO}(3)$. [Boulatov '92]

- Scalar field on $\mathrm{SU}(2)^3$ ($L^2$ with respect to the Haar measure), or $\mathfrak{su}(2)^3$, mapped to one another by a group Fourier transform: [Baratin, Oriti '10]

$$\varphi(g_1, g_2, g_3) \longleftrightarrow \widehat{\varphi}(x_1, x_2, x_3) \tag{1}$$




- Closure constraint:

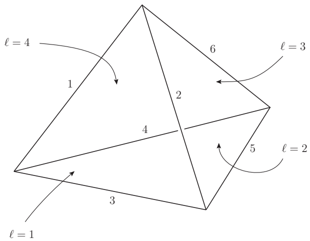
$$\begin{aligned} \text{"}\; x_1 + x_2 + x_3 &= 0 \;\text{"} \\ \varphi(hg_1, hg_2, hg_3) &= \varphi(g_1, g_2, g_3) \end{aligned}$$

## Boulatov model: dynamics

- Action:

$$S_{kin}[\varphi] = \int [\mathrm{d}g_i]^3 \, \varphi(g_1, g_2, g_3)\varphi(g_1, g_2.g_3),$$

$$S_{int}[\varphi] = \lambda \int [\mathrm{d}g_i]^6 \, \varphi(g_1, g_2, g_3)\varphi(g_3, g_5, g_4)\varphi(g_5, g_2, g_6)\varphi(g_4, g_6, g_1)$$




- Partition function: $\mathcal{Z} \equiv \int \mathrm{d}\mu_{inv}(\varphi, \overline{\varphi}) \, \mathrm{e}^{-S[\varphi]} = \sum_{\mathcal{G}} \frac{\lambda^{\mathcal{N}_{\mathcal{G}}}}{\mathrm{sym}(\mathcal{G})} \mathcal{A}_{\mathrm{PR}}(\mathcal{G}) \Rightarrow$ Sum over discrete quantum spacetimes (simplicial complexes), with Ponzano-Regge weights.

## Singularities

Q: Which discrete (topological) structures are we summing over?

- Topological manifolds (spacetime interpretation)
- Structures with topological singularities (no spacetime interpretation):
    - Point sigularities
    - Extended singularities

$\Rightarrow$ Two strategies:

- Find a physical interpretation for the singular topologies
- Find a way to get rid of these pathological structures
    - Impose additionnal combinatorial conditions, so that (some of) these structures are not generated.
    - Show that their amplitudes are highly suppressed in some regime.

# 1st step: Colored GFT

- From one to four fields, with color labels $\ell$:

$$\phi_\ell \,, \qquad \ell \in \{1, \cdots, 4\} \tag{2}$$

- Restrict the interaction to fields with 4 different colors:

$$S[\phi] = \sum_\ell \int |\phi_\ell|^2 + \lambda \int \phi_1\phi_2\phi_3\phi_4 + c.c \tag{3}$$

$\Rightarrow$ amplitudes unchanged, but restricted class of simplicial complexes summed over:

**Theorem** [Ferri, Cagliardi '80s; Gurau '10; Caravelli '10]

Simplicial complexes generated by the colored model are triangulations of pseudo-manifolds (i.e. with at most pointlike singularities), and the singularities are located on the vertices of the triangulation.

## 2nd step: bounding the remaining singular topologies

Principal tool: $1/N$ expansion [Gurau '10 '11; Gurau, Rivasseau '11]

- Analog of the size N of a matrix $\rightarrow$ ultra spin cut-off $\Lambda$ on $\delta$-functions:

$$\delta(g) = \sum_{j \in \mathbb{N}/2} (2j+1)\chi_j(g) \rightarrow \delta^{\Lambda}(g) = \sum_{j \leq \Lambda} (2j+1)\chi_j(g) \tag{4}$$

- Appropriate scaling of $\lambda$ such that:

$$\mathcal{Z} = [\delta^{\Lambda}(\mathbf{1})]^2 \mathcal{Z}_0(\lambda\bar{\lambda}) + O([\delta^{\Lambda}(\mathbf{1})]^1) \tag{5}$$

[Gurau '11]

$\mathcal{Z}_0$ contains only triangulations of the sphere, hence manifolds

# Optimal bounds

- Genus $g$ of a closed surface: number of "holes". ex: $g_{sphere} = 0$ ; $g_{donut} = 1$; etc.
- Bubble $\equiv$ boundary of a small neighborhood around a vertex $v$ of the triangulation.
  $v$ is regular $\Leftrightarrow$ the bubble around $v$ is a sphere $\Leftrightarrow$ $g = 0$
- A measure of the *manifoldness* of the triangulation: sum over the genera of the bubbles.

## Amplitude $\mathcal{A}$ of a triangulation [Oriti, SC '11]

$$\mathcal{A} = O([\delta^{\Lambda}(\mathbf{1})]^{2-2\sum_{b\in\mathcal{B}_\ell} g_b}) . \quad (6)$$

N.B. This result is optimal.

## Corollary

Dominating graphs are manifolds (i.e. $g_b = 0\,,\ \forall b \in \mathcal{B}_\ell$), and moreover amplitudes of pseudo-manifolds are at most in $O(1)$.

# Conclusion and perspectives

- Topological singularities in the Boulatov model tackled in two steps:
  - Combinatorial restricions $\Rightarrow$ only point singularities.
  - Bounds on bubbles $\Rightarrow$ amplitudes of pseudo-manifolds suppressed.
- w.i.p and future work:
  - Detail study of the divergent amplitudes: leading order, and first order corrections.
  - Generalization to 4D BF and gravity models.

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Thank you for your attention