Codimension-2 brane cosmology

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Ch. Charmousis, A.P., arXiv:0804.2121[hep-th]
Ch. Charmousis, G. Kofinas, A.P., arXiv:0907.1640 [hep-th]

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Outline

1. Motivations
   - Why modify gravity at large scales?
   - Self-acceleration and self-tuning
2. Codimension-2 braneworlds
   - Codimension-2 distributions
   - Explicit vacua and their self-properties
   - Types of boundary conditions
3. Codimension-2 cosmology
   - FRW-like equations
   - Self-tuning scenario
   - Self-accelerating cosmology

Outline

Gravity at small and large scales

- GR in excellent agreement with precise gravity tests (Damour, PDG 2007)

  Equivalence principle: 10^{-13} accuracy
  Dynamics of gravitational field: 10^{-5} accuracy

- However, the large scale observations puzzling

  Composition of the Universe

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- The last component the most puzzling for both
  ★ high energy physics
  ★ cosmology

Cosmological constant problem(s)

- Field theoretic prediction far from observable value

$$\Lambda_{natural} \sim M_{Pl}^4 \qquad \ldots \qquad \Lambda_{obs} \sim 10^{-120} M_{Pl}^4$$

- No known symmetry which could enforce a vanishing vacuum energy and remain consistent with other observations

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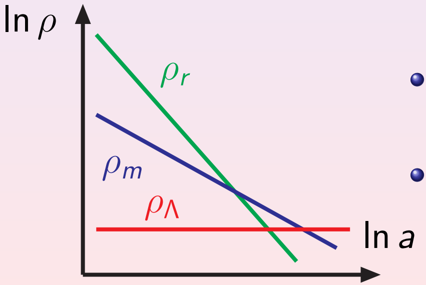

- Cosmological constant finally dominates
- Why are we so close to the point that ρ_m and ρ_Λ intersect?

$$\Omega_\Lambda \sim \Omega_{DM} \sim 10^{-1}\, \Omega_b \quad !!!$$

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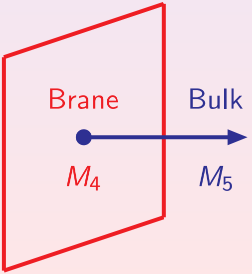

- Matter quantum loops induce an $M_4^2 \int \sqrt{g_4} R_4$ term on the brane

- There is a self-accelerating branch with (Deffayet, 2000)

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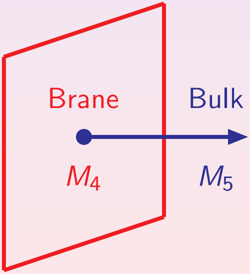

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(however, with a ghost ...)

Self-tuning

- Mechanism to make vacuum energy not gravitate
- In brane worlds: transfer curvature from the brane to bulk
- Self-tuning: Curvature of the brane insensitive to its vacuum energy with no brane-bulk fine tuning

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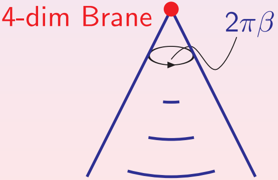
- Of special interest are the codimension-2 branes (Chen, Luty, Ponton, 2000)



- In 2D sources do not curve the space but only introduce a deficit/excess angle

$$\mathrm{Circ} \equiv 2\pi\beta = 2\pi - 4Gm$$

Outline

Are codimension-2 distributions well defined?

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- Sources should be regularised and the result will depend on the precise regularisation we pick
- Exception: pure vacuum energy branes

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- If we increase the differential complexity of the theory, the above can be avoided!
- Proposal: extension of bulk GR to Lovelock gravity (Bostock, Gregory, Navarro, Santiago, 2004)

Lovelock gravity

- Lovelock's theorem (1973): In $D = 4$ GR is the unique tensor theory which is
  1. Symmetric, $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$
  2. Divergence free, $\nabla^{\mu} G_{\mu\nu} = 0$
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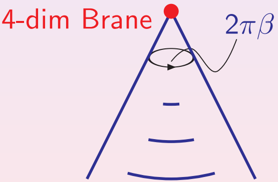
- In $D > 4$ the theory is extendable to $(\text{curvature})^2$ etc. additions. E.g. in $D = 6$ the unique theory is

$$\mathcal{S} = \frac{1}{16\pi G_6} \int d^6x \sqrt{-g} \left[R + \frac{\alpha}{6} (R^2 - 4R_{MN}^2 + R_{MNK\Lambda}^2) \right]$$

Gauss-Bonnet additional term

Brane gravity in a Gauss-Bonnet bulk

- Distributional description of general codim-2 sources possible!
- Brane junction conditions dictate an induced Einstein equation



$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

$$G_N = \frac{3G_6}{4\pi\alpha(1-\beta)}$$

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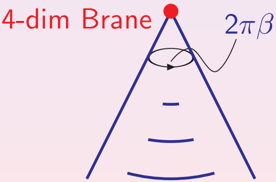

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu} + \frac{3}{2\alpha} g_{\mu\nu}$$

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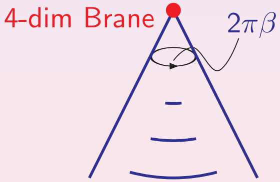

$$G_N = \frac{3G_6}{4\pi\alpha(1-\beta)}$$

$$W_{\mu\nu} = "K^2_{\mu\nu}" \text{ terms}$$

$K_{\mu\nu}$: brane extrinsic curvature

Maximally symmetric solutions

- There exist exact brane solutions for $K_{\mu\nu} = 0$:
  Double Wick rotated Gauss-Bonnet black holes

$$ds^2 = r^2\left(-dt^2 + e^{2Ht}d\vec{x}^2\right) + \frac{dr^2}{V(r)} + c^2 V(r) d\theta^2$$

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- "Potential" $V(r)$ depends on α, Λ_6 and μ ("black hole" mass)
- Brane junction condition

$$H^2 = \underbrace{-\frac{1}{2\alpha}}_{\text{Geometric contribution}} + \underbrace{\frac{8\pi G_N}{3} T}_{\text{Brane content}}$$

Self-accelerating and self-tuning examples

- Self-acceleration exists for solutions with $T = 0$, e.g.

$$\mu = 0 \quad , \quad \alpha < 0 \quad , \quad \Lambda_6 = \frac{5}{2|\alpha|} \ (\mathrm{Born-Infeld\ limit})$$

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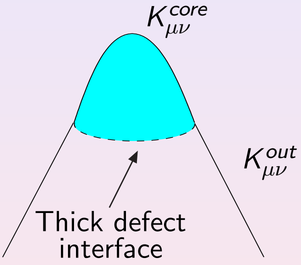
- Self-tuning exists for solutions with $H \approx 0$, when β self-tunes to cancel T, e.g.

$$\Lambda_6 = 0 \quad , \quad \alpha < 0 \quad , \quad \mu|\alpha|^{-3/2} \gg 1$$

- Then α is not constrained and G_N changes

Types of boundary conditions



- In general $K_{\mu\nu}^{core} \neq K_{\mu\nu}^{out}$
- Singularities structure is mixed codimension-2 *and* codimension-1 (Kanno, Soda, 2004)
- If ones want to focus to pure codimension-2, $K_{\mu\nu}$ should be continuous (topological boundary conditions of Charmousis, Zegers, 2005)
- $W_{\mu\nu}$ term not depending on β (different from Bostock et al)

Outline

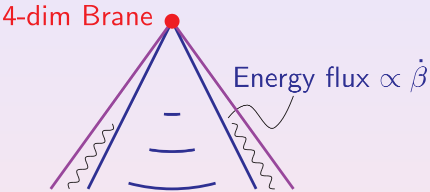
FRW-like equations on codimension-2 branes

- Brane Einstein equations + leading order of bulk equations on the brane: 3 independent equations

$$
\begin{aligned}
H^2 &= \frac{8\pi G_N}{3}\rho - \frac{\kappa}{a^2} - \frac{1}{2\alpha} + A^2 \\
\frac{\ddot{a}}{a} &= -\frac{4\pi G_N}{3}(\rho + 3P) - \frac{1}{2\alpha} + \frac{3P}{\rho}A^2 \\
\dot{\rho} &+ 3H(\rho + P) + \frac{\rho\,\dot{\beta}}{\beta(1-\beta)} = 0
\end{aligned}
$$

- New degrees of freedom $\beta(t)$ and $K_i^j = A(t)\,\delta_i^j$
- One free function - the system is not closed!

Self-tuning scenario



- Vacuum energy can be (in principle) relaxed when β changes
- Bulk boundary conditions necessary to determine $\beta(t)$
- $G_N \propto (1-\beta)^{-1}$ is time dependent

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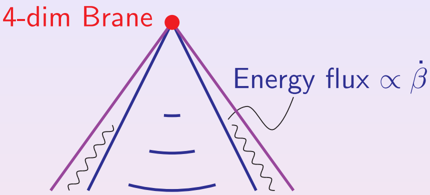

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- Bulk boundary conditions necessary to determine $\beta(t)$
- $G_N \propto (1-\beta)^{-1}$ is time dependent
- In the following $\beta =$constant
  [This is a necessity if one regularises the codim-2 brane by a ring codim-1 brane]

Self-accelerating cosmology

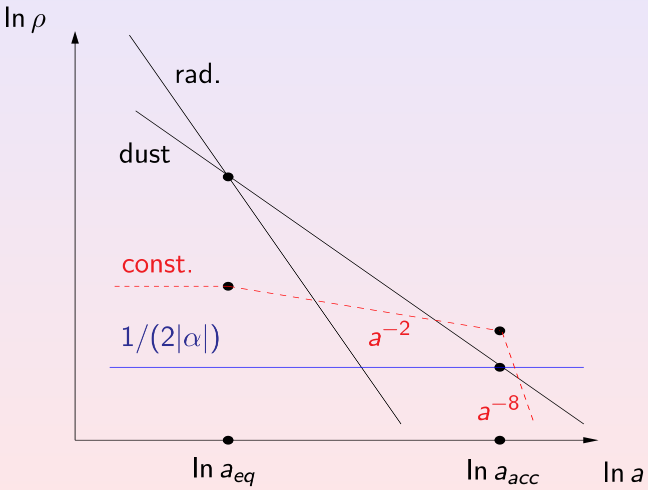
- For β =constant the system is closed
- Friedmann equation for tensionless branes with $P = w\ \rho$

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{\kappa}{a^2} - \frac{1}{2\alpha} + \underbrace{C^2\ \rho^{\frac{2(1-3w)}{3(1+w)}}}_{\propto\ a^{-2(1-3w)}}$$

- Late geometrical acceleration if $\alpha < 0$
- Correction term behaves like a mirage fluid

w	ρ	Correction
-1	const.	a^{-8}
0	a^{-3}	a^{-2}
$1/3$	a^{-4}	const.

A possible evolution



Conclusions

- Distributional description of codim-2 possible in 6D Lovelock gravity
- There exist self-accelerating and self-tuning solutions
- Friedmann equation receives corrections
  - geometrically induced vacuum energy
  - mirage fluid depending on w

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  - geometrically induced vacuum energy
  - mirage fluid depending on w
- Stability of these vacua should be studied (ghosts?)
- Challenge for phenomenology to constrain C^2
  $\Rightarrow$ not-trivial signature of codimension-2