# *Coupled Quintessence with Exponential Potentials*

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# OUTLINE

- Introduction
- Scalar-tensor Coupling
- Phenomenological Model
- Our Model: Curvaton-like Model
- Conclusions

## Introduction

- A light scalar field called Quintessence has been proposed to explain cosmic acceleration.
- The models with uncoupled quintessence cannot explain acceleration since:

$$w_\phi \longrightarrow w$$

- We study some models of interacting dark matter/dark energy, with an interaction term $Q$
  - Scalar-tensor coupling $Q = C\rho_c\dot{\phi}$
  - Phenomenological Model $Q = \alpha H\rho_c$
  - Curvaton-like Model $Q = \Gamma\rho_c$

## Introduction

- Mathematical Background
  - The evolution equations are:

$$
\begin{aligned}
H^2 &= \frac{\kappa}{3}(\rho_c + \rho_\gamma + \rho_\phi) \\
\dot{H} &= -\frac{\kappa}{2}\left[\rho_c + \frac{4}{3}\rho_\gamma + \dot{\phi}^2\right] \\
\dot{\rho}_c + 3H\rho_c &= -Q \\
\dot{\rho}_\phi + 3H(1 + w_\phi)\rho_\phi &= Q
\end{aligned}
$$

## Introduction

- The Klein-Gordon equation is:

$$\dot{\phi}(\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi}) = Q$$

- The potential is:

$$V = V_0 \exp[-\kappa\lambda\phi]$$

# Scalar-tensor Coupling, $Q = C\rho_c\dot{\phi}$

- MODEL: Quintessence + matter

Defining:

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \quad y = \frac{\kappa\sqrt{V}}{\sqrt{3}H}$$

The equations become:

$$\begin{aligned} x' &= -3x + \lambda\sqrt{\frac{3}{2}}y^2 + \frac{3}{2}x(1 + x^2 - y^2) + \beta(1 - x^2 - y^2) \\ y' &= -\lambda\sqrt{\frac{3}{2}}xy + \frac{3}{2}y(1 + x^2 - y^2) \end{aligned}$$

with $\beta = \sqrt{\frac{3}{2}}\frac{C}{\kappa}$

## Scalar-tensor Coupling, $Q = C\rho_c\dot{\phi}$

We found two late-time attractor admitting acceleration:

| | $w_\phi$ | Acceleration |
|---|---|---|
| 1 | $\frac{\lambda^2}{3} - 1$ | Yes for $\lambda^2 < 2$ |
| 2 | $\frac{12\beta(\lambda\sqrt{6}-2\beta)}{108-12\beta(\lambda\sqrt{6}-2\beta)}$ | Yes for $\beta < 0$ |

Table 1. Attractors for the Scalar-tensor model

# Phenomenological Model, $Q = \alpha H \rho_c$

- MODEL: Quintessence + matter

The dynamical system is:

$$
\begin{aligned}
x' &= -3x + \lambda\sqrt{\frac{3}{2}}y^2 + \frac{3}{2}x(1+x^2-y^2) \\
&+ \frac{\alpha}{2x}(1-x^2-y^2) \\
y' &= -\lambda\sqrt{\frac{3}{2}}xy + \frac{3}{2}y(1+x^2-y^2)
\end{aligned}
$$

## Phenomenological Model, $Q = \alpha H \rho_c$

We found two late time attractors:

| | $w_\phi$ | Acceleration |
|---|---|---|
| 1 | $\frac{\lambda^2}{3} - 1$ | Yes for $\lambda^2 < 2$ |
| 2 | $\frac{\alpha\lambda^2}{(3+\alpha)^2-\alpha\lambda^2}$ | Yes for $\alpha < 0$ |

Table 2. Attractors for the phenomenological model.

## Curvaton-like Model, $Q = \Gamma\rho_c$

- IDEA: CDM decays into Dark Energy (Curvaton decays into radiation, astro-ph/0211602).
- The constant decay rate of dark matter into dark energy is $\Gamma > 0$.
- The potential is given by $V = V_0 \exp(-\lambda\kappa\phi)$.
- The equations are:

$$\dot{\rho}_c + 3H\rho_c = -\Gamma\rho_c$$
$$\dot{\rho}_\phi + 3H(1 + w_\phi)\rho_\phi = \Gamma\rho_c$$

## Curvaton-like Model, $Q = \Gamma\rho_c$

- The phase space is now 3-dimensional, we need a new variable:

$$z = \frac{H_0}{H + H_0} \text{ and } \gamma = \frac{\Gamma}{\mathrm{H}_0}$$

So, the system is:

$$\begin{aligned} x' &= -3x + \lambda\sqrt{\frac{3}{2}}y^2 + \frac{3}{2}x(1 + x^2 - y^2) \\ &- \frac{\gamma(1 - x^2 - y^2)}{2x}\frac{z}{z-1} \\ y' &= -\lambda\sqrt{\frac{3}{2}}xy + \frac{3}{2}y(1 + x^2 - y^2) \\ z' &= z(1-z)\left[\frac{3}{2}(1 + x^2 - y^2)\right] \end{aligned}$$

## Curvaton-like Model, $Q = \Gamma\rho_c$

| | $x$ | $y$ | $z$ | Stability | Acceleration |
|---|---|---|---|---|---|
| A | 1 | 0 | 0 | Saddle $\lambda > \sqrt{6}$<br>Unstable $\lambda < \sqrt{6}$ | No |
| B | -1 | 0 | 0 | Unstable $\lambda > -\sqrt{6}$<br>Saddle $\lambda < -\sqrt{6}$ | No |
| C | $\lambda/\sqrt{6}$ | $[1-\lambda^2/6]^{1/2}$ | 0 | Saddle $\lambda^2 < 6$ | Yes |
| D | $\sqrt{6}/2\lambda$ | $\sqrt{6}/2\lambda$ | 0 | Saddle $\lambda^2 > 3$ | No |
| E | 1 | 0 | 1 | Saddle $\sqrt{6} < \lambda < 3$<br>Stable $\lambda > 3$, $\gamma > 0$ | No |
| F | -1 | 0 | 1 | Saddle | No |
| G | $\lambda/\sqrt{6}$ | $[1-\lambda^2/6]^{1/2}$ | 1 | Stable $\lambda^2 < 6$, $\gamma > 0$ | Yes for $\lambda^2 < 2$ |

Table 3. Critical points for the Curvaton-like model

# Curvaton-like Model, $Q = \Gamma \rho_c$

- Late-time Attractor

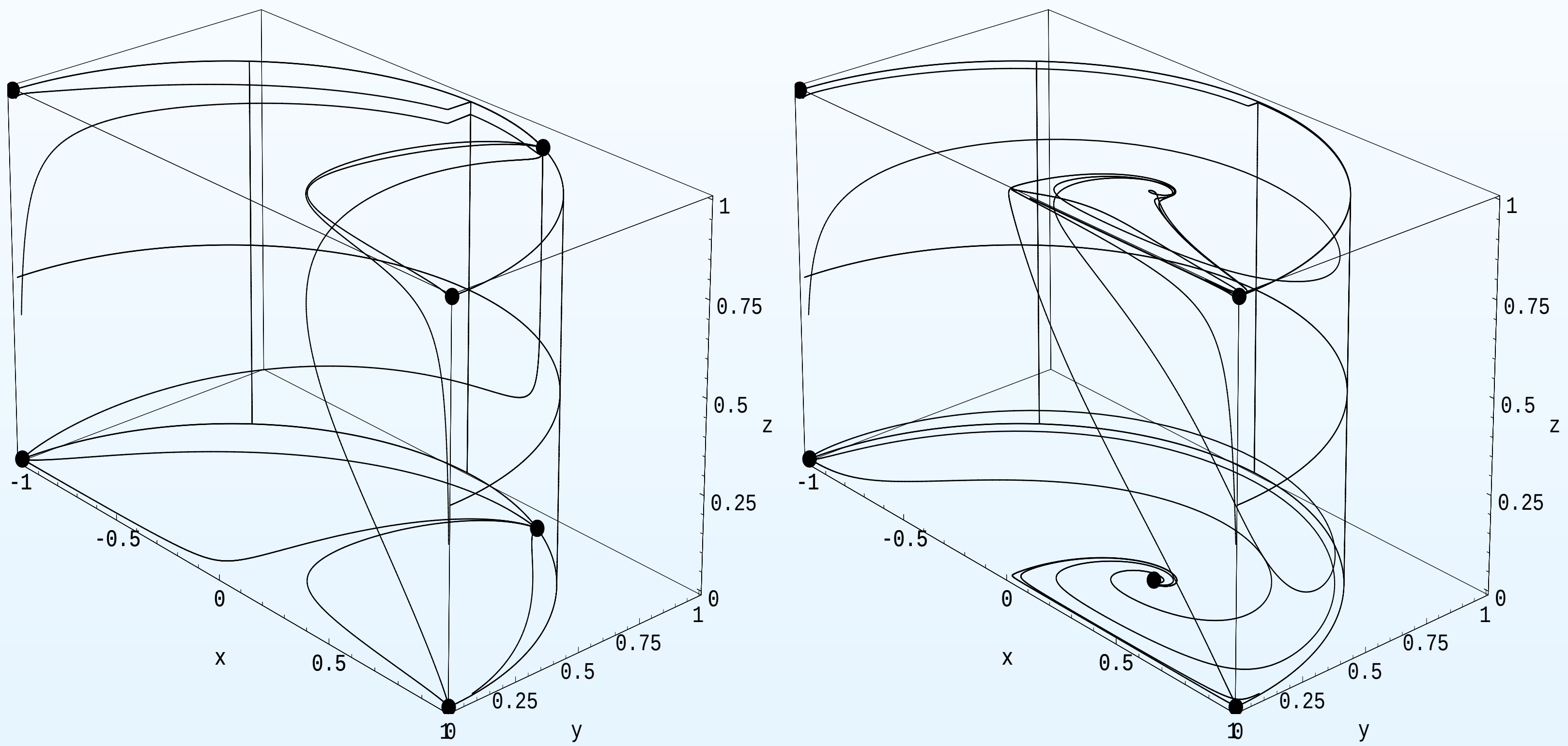


Fig. 1 Phase-space, the left hand plot is for $\lambda = 1$ and $\gamma = 10^{-6}$.
The right hand plot is for $\lambda = 4$

# Conclusions

- We found two late-time attractors.
- Just one admit accelerating solution.

Future work

- Perturbations need to be investigated for the model with late-time acceleration.
- Observations need to be used to constrain the parameters in the model.