Reconstructing dark energy

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The accelerating Universe

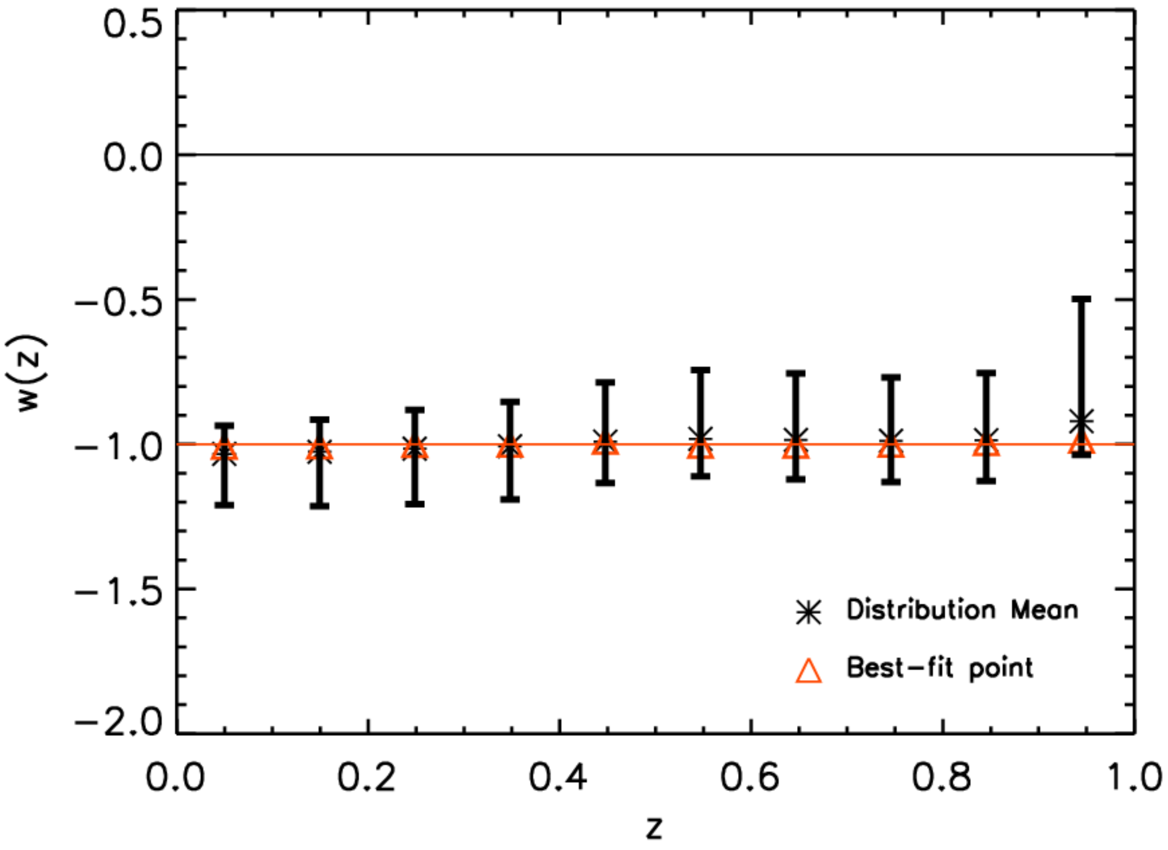
Dark energy: a component with (possibly time dependent) negative pressure

$$\frac{d}{dt}\left(\frac{\dot{a}}{a}\right) = -\frac{4\pi G}{3}\rho(1+3w)$$

acceleration
for $w(z) = p/\rho < -1/3$

So far, all data compatible with the hypothesis of a cosmological constant ($w = -1$)

Zunckel & Trotta, astro-ph/0702695



Observational techniques

Number of assumptions

- Weak gravitational lensing

  Challenging control of systematics
- Baryonic acoustic oscillations

  Less accurate, but systematics free
- Integrated Sachs-Wolfe effect

  Limited by cosmic variance
- SNe luminosity distance

  SNe variability, evolution
- Cluster abundance

  Do we understand clusters? Calibration

Desiderata for dark energy probes

Discriminative power

- *$w(z)$ vs Λ*
- *Dark energy vs gravity*
- *Growth of structures vs geometrical tests*

Reliability

- *Robust to systematics*
- *Based on well known physical observables*
- *Multiple, independent techniques*

Flexibility

- *Maximise discovery space*

Probes of DE based on standard candles & rulers

Luminosity distance (SNIa):

$$D_L(z) = \left(\frac{L}{4\pi\ell}\right)^2 = \frac{c}{100h}(1+z)\int_0^z \frac{1}{H(x)}dx$$

Angular diameter distance (CMB, BAO⊥):

$$D_A(z) = \frac{d}{\theta} = (1+z)^{-2} D_L(z)$$

Hubble function (BAOr):

$$\begin{aligned} H^2(z) &= (1-\Omega_m-\Omega_{DE})(1+z)^2 + \Omega_m(1+z)^3 \\ &+ \Omega_{DE}\exp\left[3\int_0^z \frac{1+w(z)}{1+z}dz\right] \end{aligned}$$

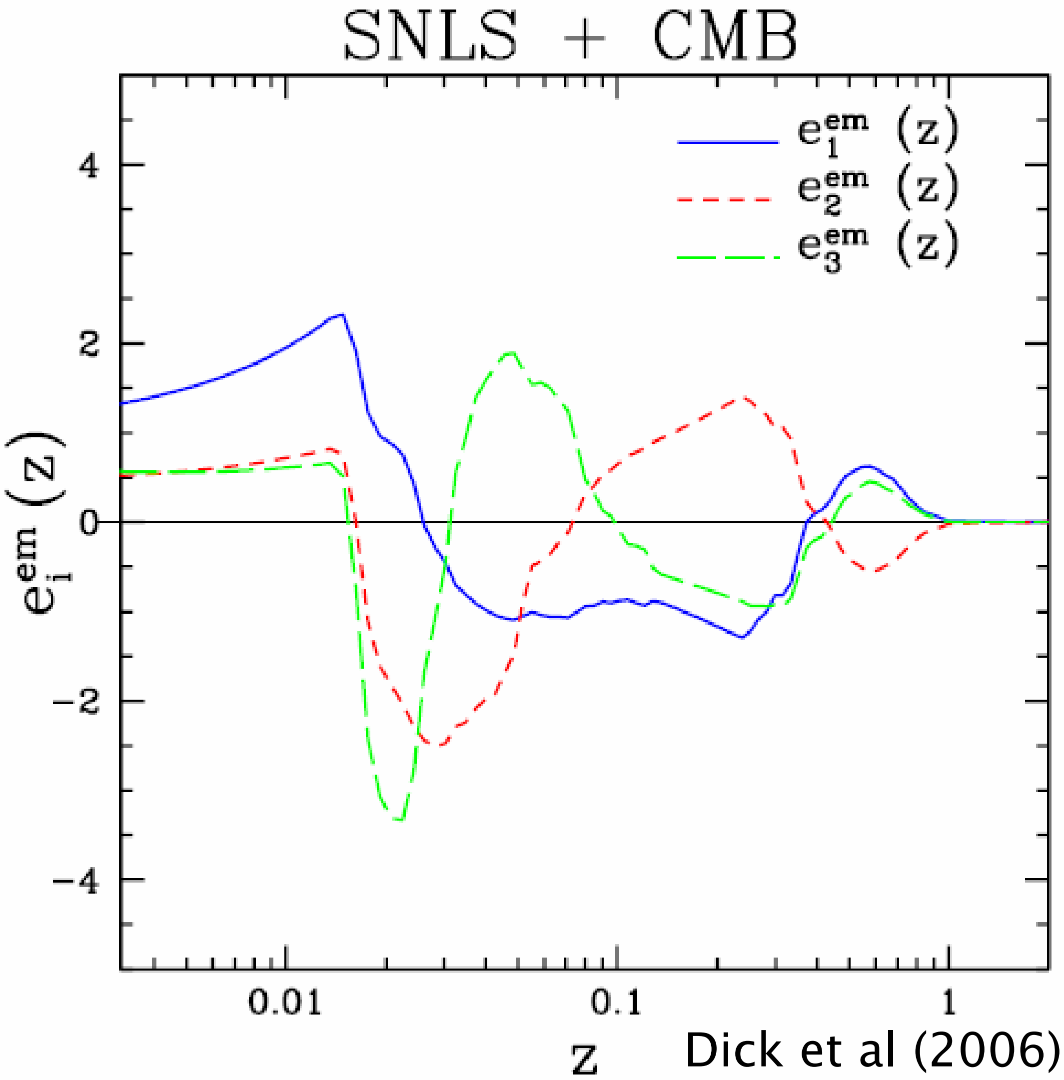
Reconstructing dark energy

- *Parametric methods might lead us astray, especially if they are purely phenomenological*
- *Popular methods:*

$w(z) = w_{eff}$
$w(z) = w_0 + (1\text{-}a)w_a$

eigenmodes or
PCA analysis

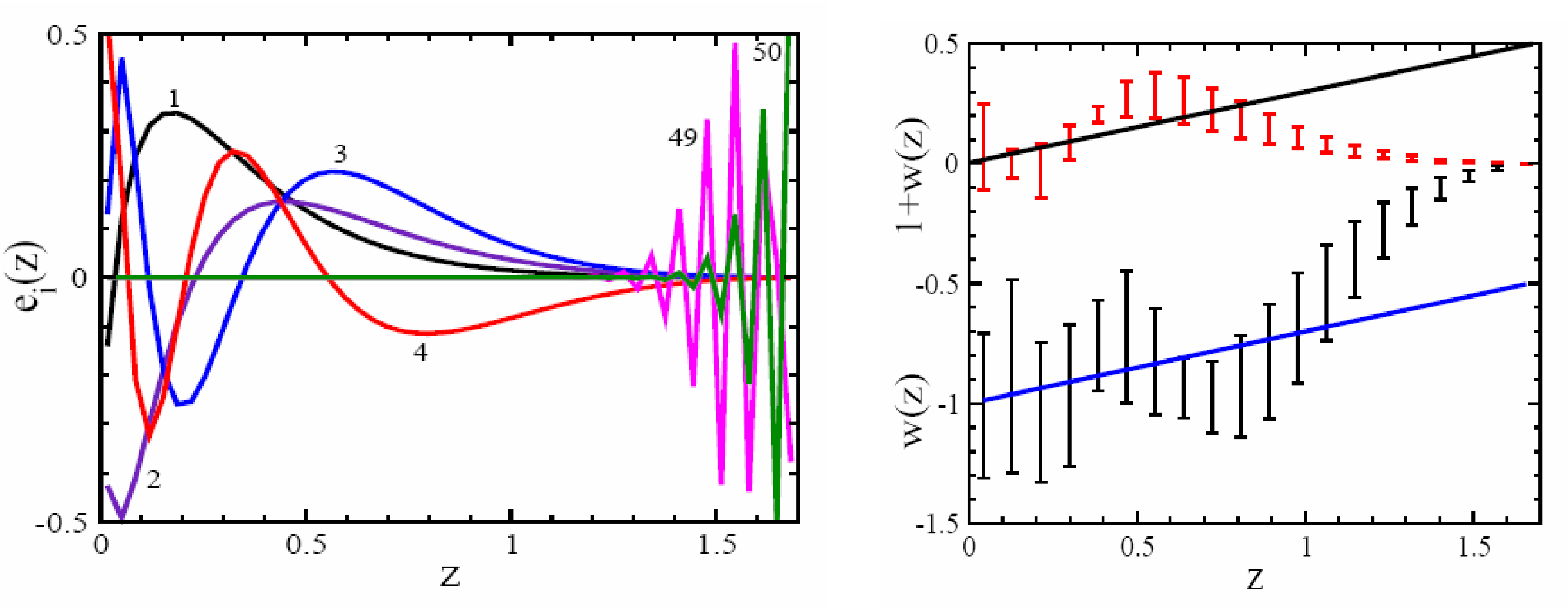
Huterer & Starkman, 2003
Simpson & Bridle, 2006
Dick et al., 2006



Dick et al (2006)

PCA reconstruction bias

- *Principal Component Analysis aims at reducing the noise of the reconstruction. The price you pay is bias*



Huterer & Starkmann (2003)

There is no inference without assumptions

State your assumptions!

Requirement for DE reconstruction

- *Maximum freedom in w(z)*
- *Unbiased reconstruction with suitably large errors*
- *Optimal efficiency in extracting info on w(z) from noisy data*
- *Robust wrt to noise, does not overfit*
- *Adjust itself to structure in the data*
- *Analogy: deconvolution in spectral analysis (due to the integral relating observables and w(z))*

A possible (Bayesian) solution:
Maximum Entropy Bayesian Reconstruction
Zunckel and Trotta (2007)
"Reconstructing the history of dark energy using maximum entropy", MNRAS, 380, 3, 865
astro-ph/0702695

A new reconstruction technique

- *Goal: maximum freedom in w(z) while avoiding overfitting*

Bayes' Theorem:

$$\overset{\text{Posterior}}{P(\mathbf{w}|\mathbf{D})} = \frac{\overset{\text{likelihood}}{P(\mathbf{D}|\mathbf{w})}\overset{\text{Prior}}{\pi(\mathbf{w})}}{P(\mathbf{D})}$$

- *The "maximum entropy" prior: a carefully chosen regularizing prior*

$$\pi\left(\mathbf{w}|\alpha,\mathbf{m}\right) \propto \exp\left(\alpha \underbrace{S(\mathbf{w},\mathbf{m})}_{\text{"Entropy" between w and a "default" model m}}\right)$$

Regularization parameter

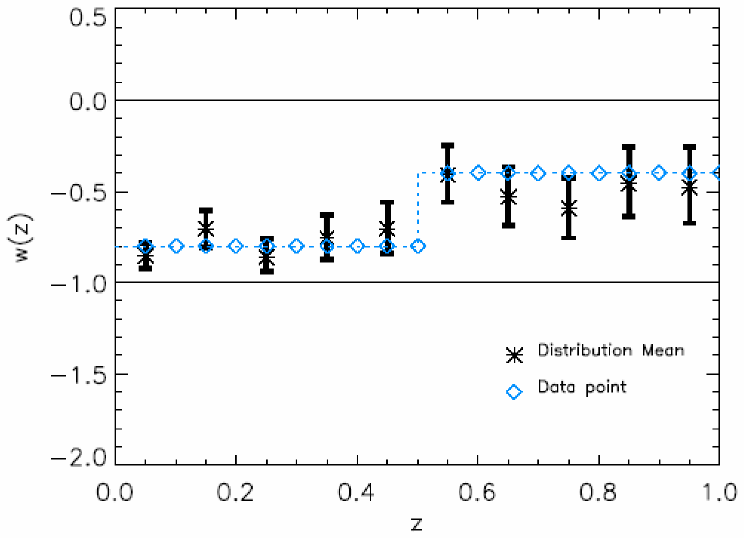
"Entropy" between w and a "default" model m

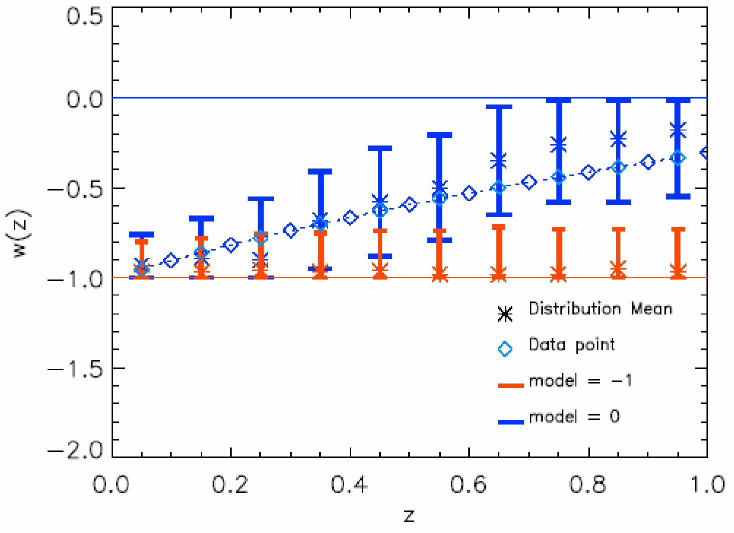
$$S(\mathbf{w},\mathbf{m}) = \sum_{j=1}^{N} w_j - m_j - w_j \log\left(\frac{w_j}{m_j}\right)$$

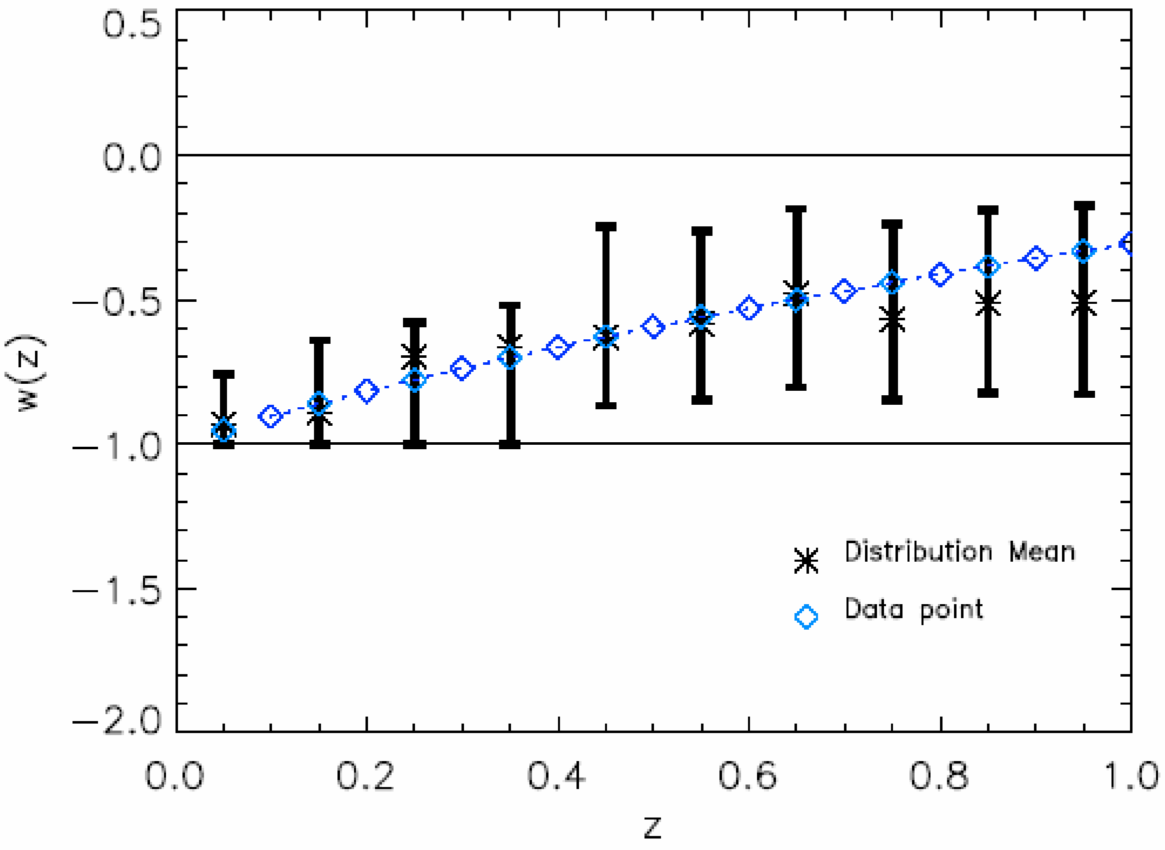
Skilling (1989)

Performance on synthetic data

- *The parameter α is adjusted through the data themselves. The model can also be included in the parameter space*





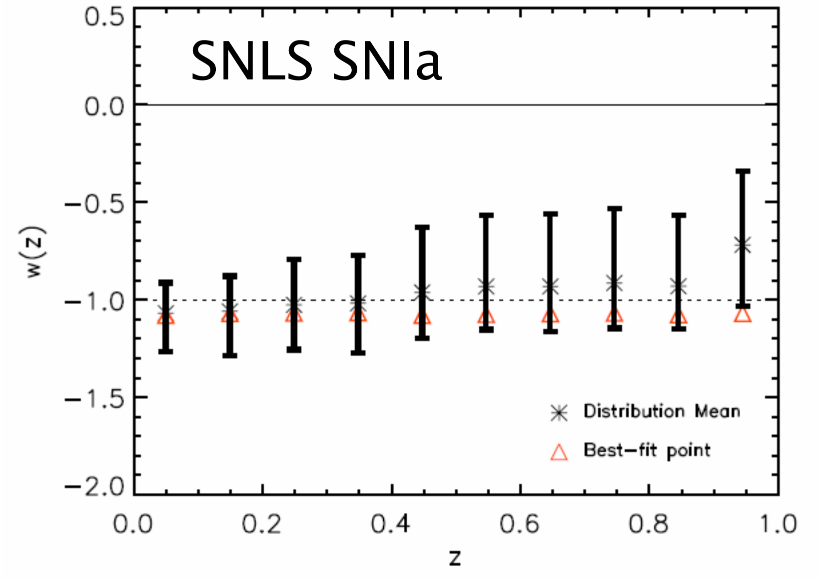


After marginalization on prior model

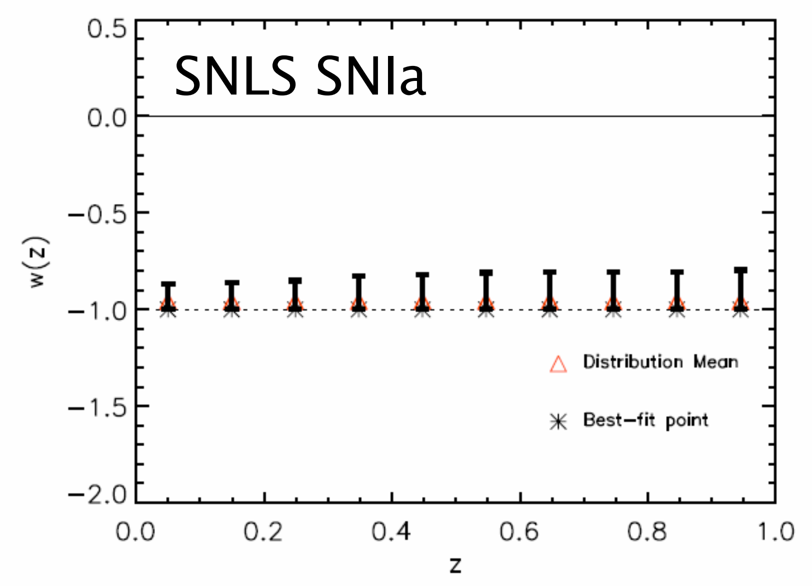
Results from CMB+SN+SDSS(LRG) (April 2007)

Data used: supernovae type Ia, acoustic oscillations and CMB

CONSERVATIVE
(model marginalized, –2 < w < 0)



AGGRESSIVE
(model = –1, –1 < w < 0)



Conclusion:
the cosmological constant, w = – 1, remains the best description

Zunckel & Trotta (2007)

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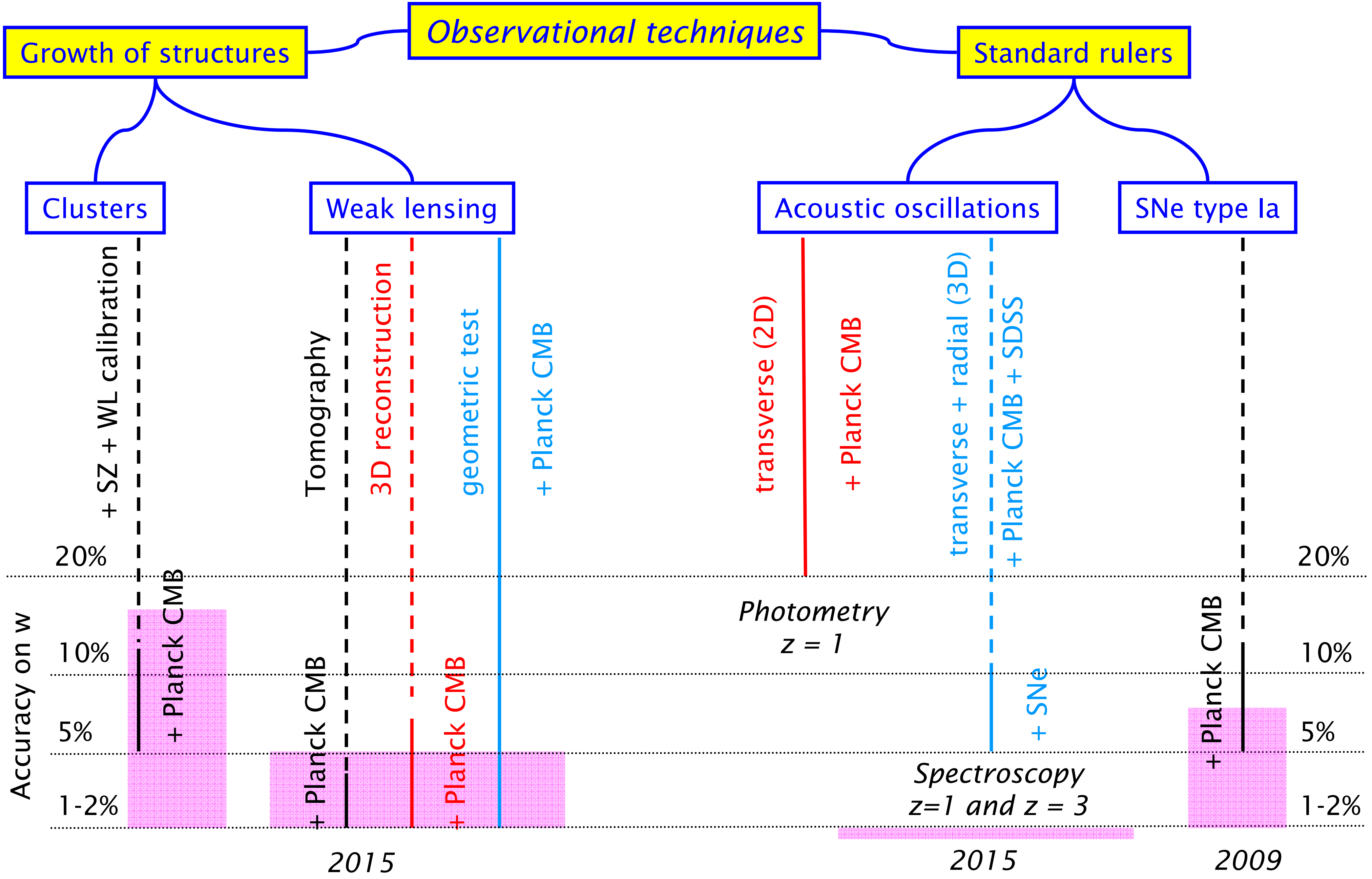
Dark energy discovery space

▨ systematics impact



Bayesian model comparison

Goal: to compare the "performance" of two models against the data. Do we need w(z)?

the model likelihood
("evidence")

$$\mathcal{P}(\mathrm{d}|\mathcal{M}) = \int_{\Omega} \mathrm{L}(\mathrm{d}|\boldsymbol{\theta}, \mathcal{M})\pi(\boldsymbol{\theta}, \mathcal{M})\mathrm{d}\boldsymbol{\theta}$$

the posterior prob'ty
of the model given the data

$$\mathcal{P}(\mathcal{M}|\mathrm{d}) \propto \mathcal{P}(\mathrm{d}|\mathcal{M})\pi(\mathcal{M})$$

The Bayes factor
(model comparison)

$$B_{01} = \frac{\mathcal{P}(\mathcal{M}_0|\mathrm{d})}{\mathcal{P}(\mathcal{M}_1|\mathrm{d})}$$

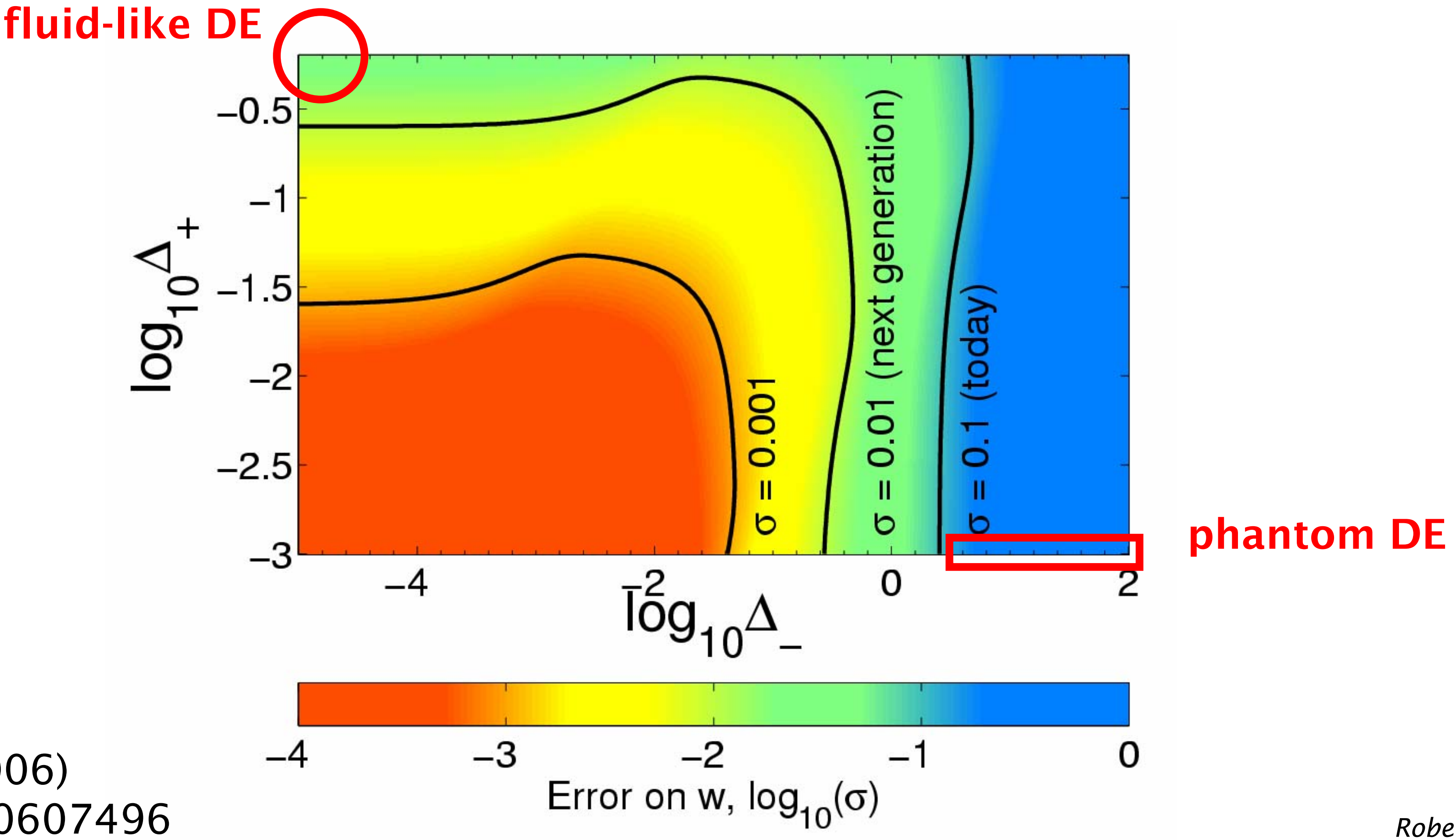
Interpretation: Jeffreys' scale for the strength of evidence

$\lvert \ln B_{01} \rvert$	Odds	Probability	Equiv σ	(My) Interpretation
< 1.0	*< 3:1*	*< 0.750*	*1.15*	*not worth the mention*
< 2.5	*< 12:1*	*0.923*	*1.77*	*positive*
< 5.0	*< 150:1*	*0.993*	*2.70*	*moderate*
>5.0	*> 150:1*	*> 0.993*	*> 2.70*	*strong*

Ruling in Λ: A Bayesian perspective

- *Which dark energy models can be ruled out with moderate evidence (lnB>3) compared to Λ for a given accuracy σ?*

$$-1 - \Delta_- \leq w_{\rm eff} \leq -1 + \Delta_+$$



Trotta (2006)
astro-ph/0607496

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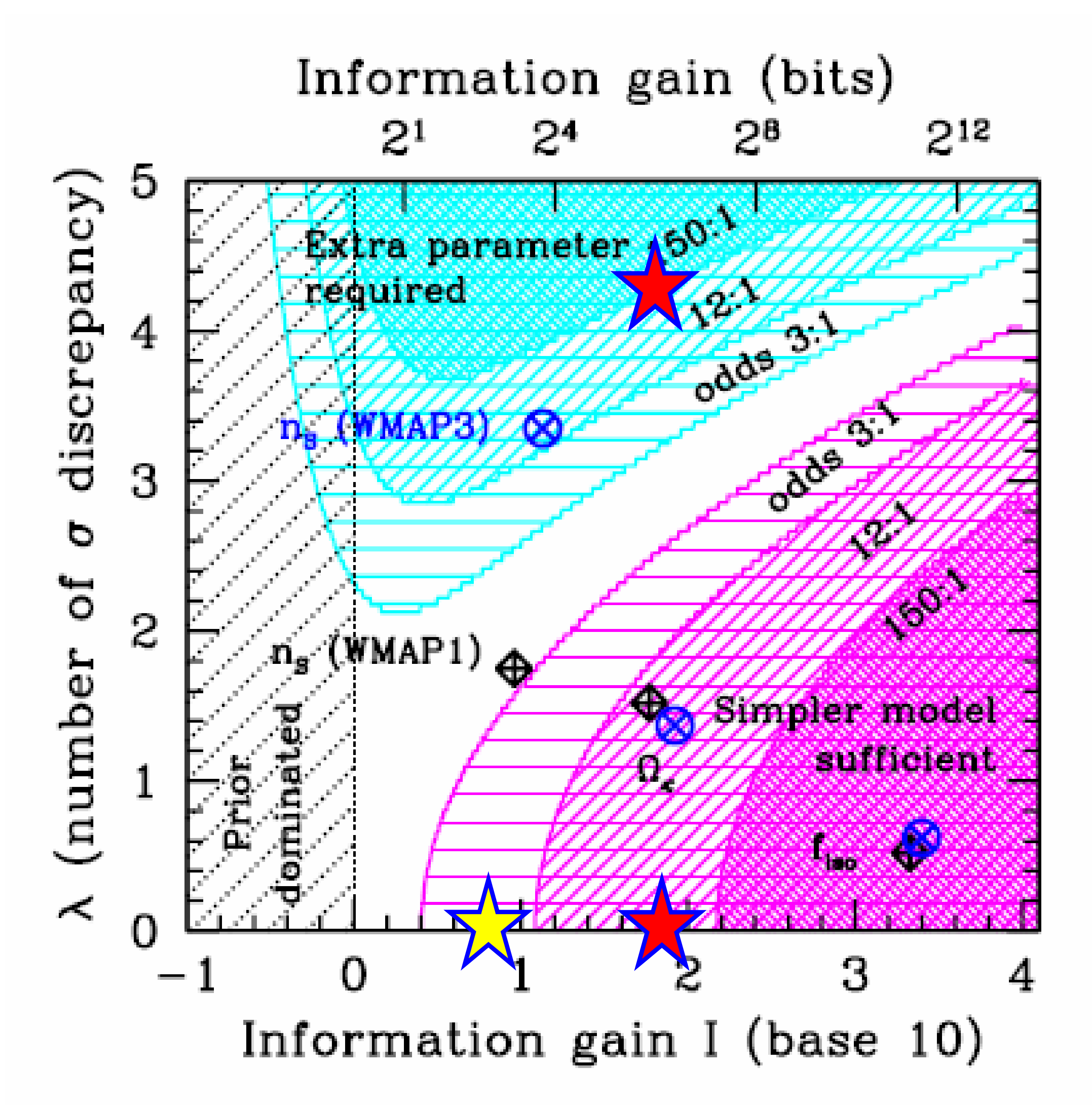
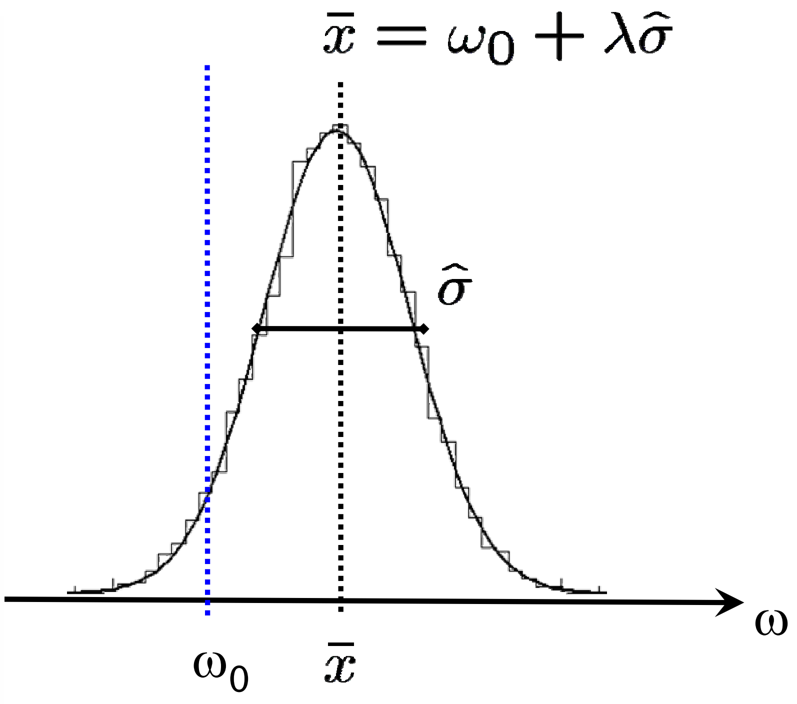
Bayesian model selection

Trotta 2005, astro-ph/0504022
Trotta 2007, astro-ph/0703063





Strength of evidence (Bayes factor) in favour of w=-1 compared to -1/3 < w < -1

Closing remarks

- *Preparing for the unexpected*
  - What will be the most interesting questions in 2010?
  - Dark energy could surprise us again: maximise the discovery potential
- *Developping know-how*
  - Indispensable tools on the road to even larger surveys
- *Making the most of the data*
  - Statistical tools for optimal parameter inference: eg, MaxEnt method
  - Model selection approach, surveys optimization
- *Plenty of other science!*
  - Next generation of surveys will provide extremely high quality data for numerous astronomical and astrophysical studies