Determining the WIMP Mass from Direct Dark Matter Detection Data

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based on `arXiv:0707.0488 [astro-ph]`

Reconstructing the velocity distribution function of WIMPs
- Deriving $f_1(v)$ from the scattering spectrum
- Reconstructing $f_1(v)$ from experimental data

Determining the WIMP mass

Summary

Deriving $f_1(v)$ from the scattering spectrum

- Differential rate for elastic WIMP-nucleus scattering

$$\frac{dR}{dQ} = \mathcal{A} F^2(Q) \int_{v_{\rm min}}^{\infty} \left[\frac{f_1(v)}{v} \right] dv$$

Here

$$v_{\rm min} = \alpha \sqrt{Q}$$

is the minimal incoming velocity of incident WIMPs that can deposit the recoil energy Q in the detector.

$$\mathcal{A} \equiv \frac{\rho_0 \sigma_0}{2 m_\chi m_{\rm r}^2} \qquad \alpha \equiv \sqrt{\frac{m_{\rm N}}{2 m_{\rm r}^2}} \qquad m_{\rm r} = \frac{m_\chi m_{\rm N}}{m_\chi + m_{\rm N}}$$

ρ_0: WIMP density near the Earth
σ_0: total cross section ignoring the form factor suppression
$F(Q)$: elastic nuclear form factor

Deriving $f_1(v)$ from the scattering spectrum

- Normalized one-dimensional velocity distribution function

$$f_1(v) = \mathcal{N} \left\{ -2Q \cdot \frac{d}{dQ} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] \right\}_{Q=v^2/\alpha^2}$$

$$\mathcal{N} = \frac{2}{\alpha} \left\{ \int_0^\infty \frac{1}{\sqrt{Q}} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] dQ \right\}^{-1}$$

- Moments of the velocity distribution function

$$\langle v^n \rangle = \mathcal{N}(Q_{\rm thre}) \left(\frac{\alpha^{n+1}}{2} \right) \left[\frac{2\, Q_{\rm thre}^{(n+1)/2}}{F^2(Q_{\rm thre})} \left(\frac{dR}{dQ} \right)_{Q=Q_{\rm thre}} + (n+1) I_n(Q_{\rm thre}) \right]$$

$$\mathcal{N}(Q_{\rm thre}) = \frac{2}{\alpha} \left[\frac{2\, Q_{\rm thre}^{1/2}}{F^2(Q_{\rm thre})} \left(\frac{dR}{dQ} \right)_{Q=Q_{\rm thre}} + I_0(Q_{\rm thre}) \right]^{-1}$$

$$I_n(Q_{\rm thre}) = \int_{Q_{\rm thre}}^\infty Q^{(n-1)/2} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] dQ$$

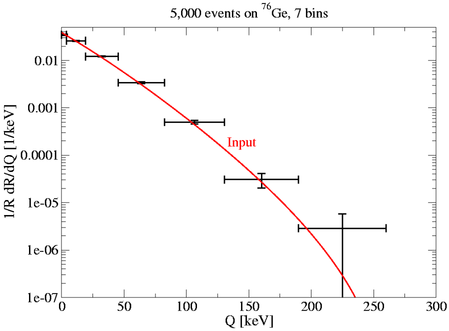
[M. Drees and C. L. Shan, JCAP 0706, 011]

Reconstructing $f_1(v)$ from experimental data

- Experimental data

$$Q_n - \frac{b_n}{2} \le Q_{n,i} \le Q_n + \frac{b_n}{2} \qquad i = 1,\ 2,\ \cdots,\ N_n,\ n = 1,\ 2,\ \cdots,\ B$$

- Theoretically predicted scattering spectrum



Reconstructing $f_1(v)$ from experimental data

- Ansatz: in the nth Q-bin

$$\left(\frac{dR}{dQ}\right)_n \equiv \left(\frac{dR}{dQ}\right)_{Q \simeq Q_n} = \tilde{r}_n e^{k_n (Q - Q_n)} \equiv r_n e^{k_n (Q - Q_{s,n})}$$

$$\tilde{r}_n \equiv \left(\frac{dR}{dQ}\right)_{Q = Q_n} \qquad r_n \equiv \frac{N_n}{b_n}$$

- Recoil spectrum at $Q = Q_n$

$$\tilde{r}_n = \frac{N_n}{b_n}\left(\frac{\kappa_n}{\sinh \kappa_n}\right) \qquad \kappa_n \equiv \left(\frac{b_n}{2}\right) k_n$$

- Logarithmic slope and shifted point in the nth Q-bin

$$\overline{Q}_n - Q_n = \frac{b_n}{2}\left(\coth \kappa_n - \frac{1}{\kappa_n}\right) \qquad \overline{Q}_n = \frac{1}{N_n}\sum_{i=1}^{N_n} Q_{n,i}$$

$$Q_{s,n} = Q_n + \frac{1}{k_n} \ln\left(\frac{\sinh \kappa_n}{\kappa_n}\right)$$

Reconstructing $f_1(v)$ from experimental data

- Reconstructing the one-dimensional velocity distribution

$$f_{1,r}(v_{s,\mu}) = \mathcal{N}\left[\frac{2Q_{s,\mu}r_\mu}{F^2(Q_{s,\mu})}\right]\left[\frac{d}{dQ}\ln F^2(Q)\bigg|_{Q=Q_{s,\mu}} - k_\mu\right]$$

$$v_{s,\mu} = \alpha\sqrt{Q_{s,\mu}}$$

$$\mathcal{N} = \frac{2}{\alpha}\left[\sum_a \frac{1}{\sqrt{Q_a}\,F^2(Q_a)}\right]^{-1}$$

- Determining the moments of the velocity distribution

$$\langle v^n\rangle = \alpha^n\left[\frac{2Q_{\rm thre}^{1/2}r_{\rm thre}}{F^2(Q_{\rm thre})} + I_0\right]^{-1}\left[\frac{2Q_{\rm thre}^{(n+1)/2}r_{\rm thre}}{F^2(Q_{\rm thre})} + (n+1)I_n\right]$$

$$I_n = \sum_a \frac{Q_a^{(n-1)/2}}{F^2(Q_a)} \qquad\qquad r_{\rm thre} = \left(\frac{dR}{dQ}\right)_{Q=Q_{\rm thre}}$$

[M. Drees and C. L. Shan, JCAP 0706, 011]

Determining the WIMP mass

- Using two different target nuclei

$$\langle v^n \rangle = \alpha_X^n \left[\frac{(n+1) I_{n,X}}{I_{0,X}} \right] = \alpha_Y^n \left[\frac{(n+1) I_{n,Y}}{I_{0,Y}} \right]$$

Determining the WIMP mass

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- WIMP mass

$$m_\chi = \frac{\sqrt{m_X m_Y} - m_X \mathcal{R}_n}{\mathcal{R}_n - \sqrt{m_X / m_Y}} \qquad \mathcal{R}_n \equiv \frac{\alpha_Y}{\alpha_X} = \left(\frac{I_{n,X}}{I_{0,X}} \cdot \frac{I_{0,Y}}{I_{n,Y}} \right)^{1/n} \quad (n \neq 0,\ -1)$$

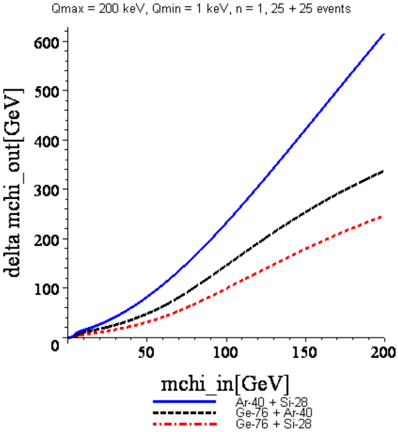
- 1-σ statistical error

$$\sigma(m_\chi) = \frac{\mathcal{R}_n \sqrt{m_X / m_Y}\, |m_X - m_Y|}{\left(\mathcal{R}_n - \sqrt{m_X / m_Y} \right)^2} \times \frac{1}{|n|} \left[\frac{\sigma^2 \left(I_{n,X} \right)}{I_{n,X}^2} + \frac{\sigma^2 \left(I_{0,X} \right)}{I_{0,X}^2} - \frac{2 \mathrm{cov} \left(I_{0,X}, I_{n,X} \right)}{I_{0,X} I_{n,X}} + (X \longrightarrow Y) \right]^{1/2}$$

[C. L. Shan, arXiv:0707.0488]

Determining the WIMP mass

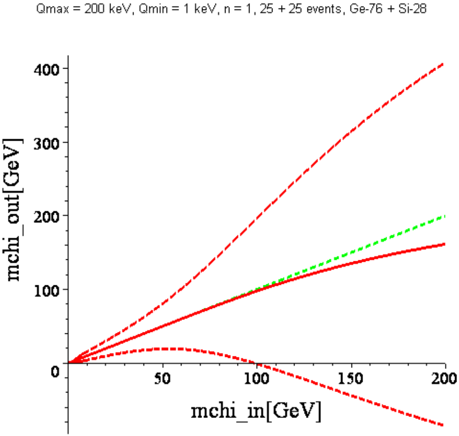
- 1-σ statistical error for different combinations
  ($1 - 200$ keV, $n = 1$, 25 + 25 events)



[C. L. Shan, arXiv:0707.0488]

Determining the WIMP mass

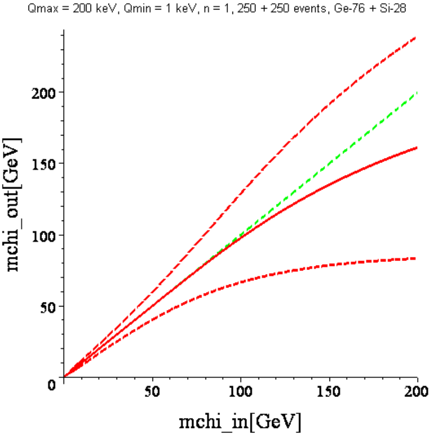
- Reproduced WIMP mass
  (1 – 200 keV, $n = 1$, ^{76}Ge + ^{28}Si, 25 + 25 events)



[C. L. Shan, arXiv:0707.0488]

Determining the WIMP mass

- Reproduced WIMP mass
  ($1 - 200$ keV, $n = 1$, $^{76}Ge + ^{28}Si$, $250 + 250$ events)



[C. L. Shan, arXiv:0707.0488]

Determining the WIMP mass

- With $Q_{\rm thre} > 0$

$$\mathcal{R}_n(Q_{\rm thre}) = \left[\frac{2Q_{{\rm thre},X}^{(n+1)/2} r_{{\rm thre},X} + (n+1) I_{n,X} F_X^2(Q_{{\rm thre},X})}{2Q_{{\rm thre},X}^{1/2} r_{{\rm thre},X} + I_{0,X} F_X^2(Q_{{\rm thre},X})}\right]^{1/n} \Big(X \longrightarrow Y\Big)^{-1}$$

- Choosing $n = -1$

$$\mathcal{R}_{-1}(Q_{\rm thre}) = \frac{r_{{\rm thre},Y}}{r_{{\rm thre},X}} \left[\frac{2Q_{{\rm thre},X}^{1/2} r_{{\rm thre},X} + I_{0,X} F_X^2(Q_{{\rm thre},X})}{2Q_{{\rm thre},Y}^{1/2} r_{{\rm thre},Y} + I_{0,Y} F_Y^2(Q_{{\rm thre},Y})}\right]$$

$$\sigma(\mathcal{R}_{-1}) = \mathcal{R}_{-1}\left\{\left[\frac{I_{0,X} F_X^2(Q_{{\rm thre},X})}{2Q_{{\rm thre},X}^{1/2} r_{{\rm thre},X} + I_{0,X} F_X^2(Q_{{\rm thre},X})}\right]^2 \times \left[\frac{\sigma^2(r_{{\rm thre},X})}{r_{{\rm thre},X}^2} + \frac{\sigma^2(I_{0,X})}{I_{0,X}^2} - \frac{2{\rm cov}(r_{{\rm thre},X}, I_{0,X})}{r_{{\rm thre},X} I_{0,X}}\right] + (X \longrightarrow Y)\right\}^{1/2}$$

[C. L. Shan, arXiv:0707.0488]

Summary

- By using experimental data with different detector materials we can determine the WIMP mass.

- The larger the mass difference between two target nuclei, the smaller the statistical error will be.

- Our method is model-independent and needs only measured recoil energies.

- With 200 keV maximal measuring energy and 25 events from each experiment, we can already extract meaningful information about the WIMP mass.