Decoherence from Isocurvature Perturbations in Inflation

Gerasimos Rigopoulos (Helsinki)

Work in collaboration with Tomislav Prokopec (Utrecht): astro-ph/0612067

Also with Jurjen Koksma (Utrecht): in progress

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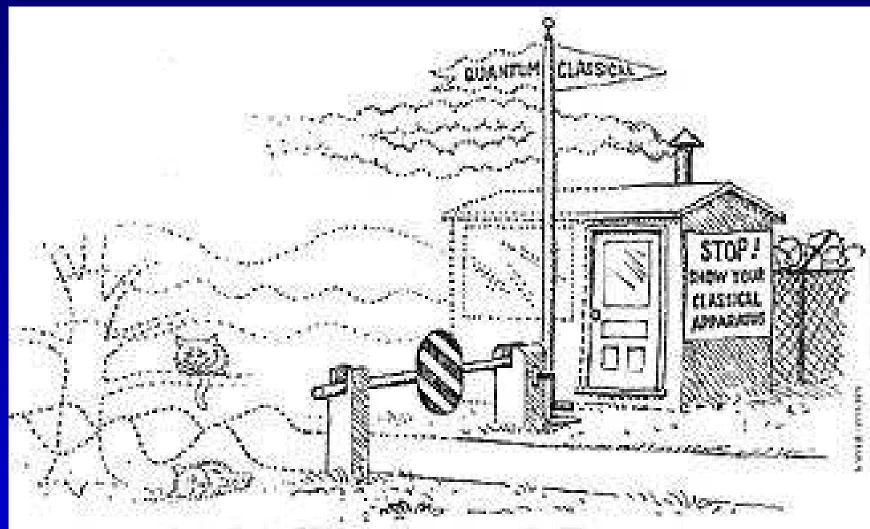

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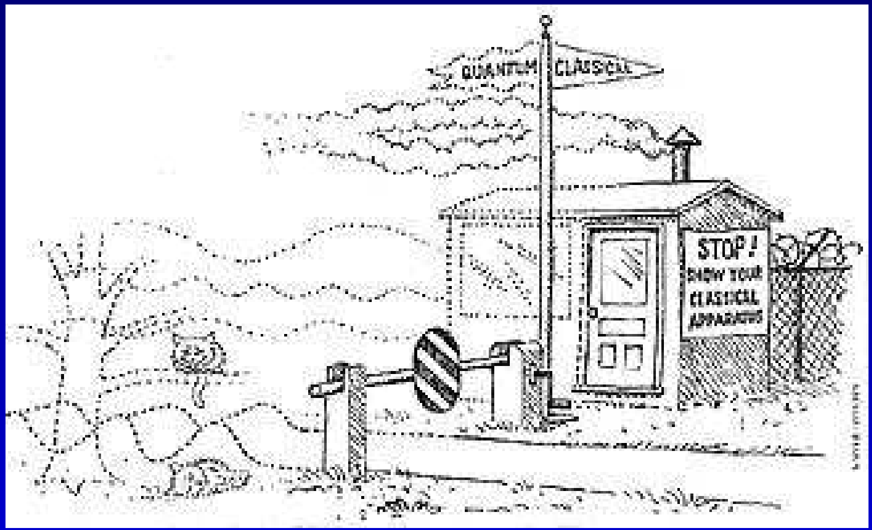

- *Collapse is an arbitrary deus ex machina outside of quantum mechanics*

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System - Measuring Apparatus - Environment

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However: Coupling to an *unobservable* (= trace out) environment can dynamically diagonalize ρ

$$\rho \longrightarrow \begin{pmatrix} |a|^2 & 0 \\ 0 & |b|^2 \end{pmatrix}$$

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Can decoherence considerations be applied to Inflationary Perturbations?

Brandenberger, Laflamme & Mijic
Polarski, Starobinsky, Lesgourgues , Kiefer, Lohmar
Lombardo
Martineau
Burgess, Holman & Hoover

Isocurvature and Adiabatic perturbations (I)

Two fields with $V(\varphi, \chi)$: $\frac{d\phi}{dN} \simeq \frac{\partial_\varphi V}{3H^2}\,, \quad \frac{d\chi}{dN} \simeq \frac{\partial_\chi V}{3H^2}\,, \quad H^2 \simeq \frac{1}{3M_p^2} V$

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As usual: $\phi(\mathbf{x}, t) = \phi(t) + \delta\phi(\mathbf{x}, t)\,, \quad \chi(\mathbf{x}, t) = \chi(t) + \delta\chi(\mathbf{x}, t)\,,$
$ds^2 = -(1 + 2\Phi(\mathbf{x}, t))dt^2 + a^2(t)(1 - 2\Phi(\mathbf{x}, t))d\mathbf{x}^2$

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The Lagrangian of gauge invariant perturbations is:

$$\boxed{L = \int d^3x \ \tfrac{1}{2}\partial_\eta \mathbf{q} \partial_\eta \mathbf{q} - \tfrac{1}{2}\mathbf{q}\left(-\nabla^2 + (aH)^2 \mathbf{\Omega}\right)\mathbf{q}}$$

$$\mathbf{q} = a(\delta\varphi + \tfrac{\dot\varphi}{H}\Phi)\hat{\mathbf{e}}_\varphi + a(\delta\chi + \tfrac{\dot\chi}{H}\Phi)\hat{\mathbf{e}}_\chi \equiv q_\varphi \hat{\mathbf{e}}_\varphi + q_\chi \hat{\mathbf{e}}_\chi$$

$$\mathbf{\Omega} \simeq \frac{\partial_a \partial_b V}{H^2} \mathbf{e}_a \otimes \mathbf{e}_b - (2 - \epsilon)\mathbf{1} - 6\epsilon\, \mathbf{e}_1 \otimes \mathbf{e}_1 + \ldots\,,$$

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The inclusion of Φ produces a coupling between the two degrees of freedom even if there is no coupling in $V(\varphi,\chi)$

Isocurvature and Adiabatic perturbations (II) . . .

Define Adiabatic/Isocurvature directions:

$$\mathbf{e}_1 = \cos\theta\,\mathbf{e}_\varphi + \sin\theta\,\mathbf{e}_\chi \equiv \frac{\dot\varphi}{\sqrt{\dot\varphi^2+\dot\chi^2}}\,\mathbf{e}_\varphi + \frac{\dot\chi}{\sqrt{\dot\varphi^2+\dot\chi^2}}\,\mathbf{e}_\chi$$

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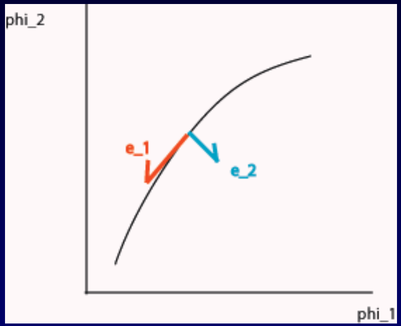

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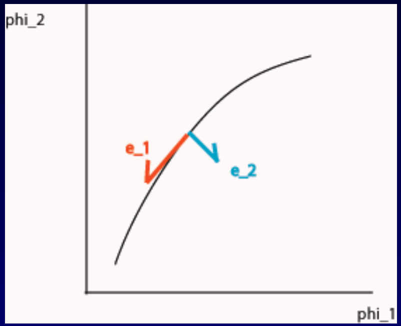

In this basis:

$$L = \int d^3x\ \frac{1}{2}\left(\partial_\eta \mathsf{q} + \mathsf{Z}\mathsf{q}\right)^{\mathsf{T}}\left(\partial_\eta \mathsf{q} + \mathsf{Z}\mathsf{q}\right) - \frac{1}{2}\mathsf{q}^{\mathsf{T}}\left(-\nabla^2 + (aH)^2\mathsf{\Omega}\right)\mathsf{q}$$

$$\mathsf{q} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}, \qquad \mathsf{q}_m = \mathbf{e}_m \cdot \mathbf{q}, \qquad \mathsf{\Omega}_{mn} = \mathbf{e}_m \mathbf{\Omega} \mathbf{e}_n \qquad (m, n = 1, 2)$$

$$\mathsf{Z} = \partial_\eta\theta \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

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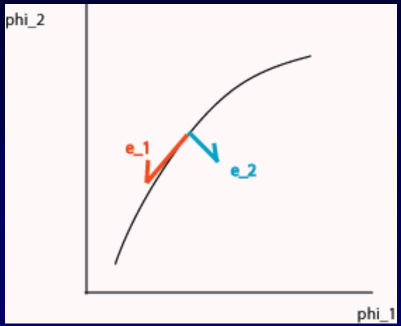

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Then, using the conjugate momentum $\pi(\mathbf{x}) = \frac{\partial\mathcal{L}}{\partial(\partial_\eta q(\mathbf{x}))}$ the Hamiltonian can be written as:

$$H = \int d^3x\ \tfrac{1}{2}\pi^{\mathsf{T}}\pi - \tfrac{1}{2}\pi^{\mathsf{T}}\mathsf{Z}\mathsf{q} - \tfrac{1}{2}\mathsf{q}^{\mathsf{T}}\mathsf{Z}^{\mathsf{T}}\pi + \tfrac{1}{2}\mathsf{q}^{\mathsf{T}}\left(-\nabla^2 + (aH)^2\Omega\right)\mathsf{q}$$

…and their Schrödinger equation

The quantum Hamiltonian $\hat{H}$ is obtained by:

$$\pi(\mathbf{x}) \to -i\hbar\frac{\delta}{\delta q(\mathbf{x})} \quad \Leftrightarrow \quad [\mathsf{q}(\mathbf{x}), \pi(\mathbf{y})] = i\hbar\delta(\mathbf{x}-\mathbf{y})$$

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With the vacuum ansatz. . . :

$$\Psi = N \exp\left(-\frac{1}{2} \int d^3x d^3y \; \mathsf{q}^{\mathsf{T}}(\mathbf{x})\mathsf{B}(\mathbf{x} - \mathbf{y})\mathsf{q}(\mathbf{y})\right)$$

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$$\boxed{i\hbar\ \partial_\eta\mathsf{B}(\mathbf{x}-\mathbf{y}) \ = \ \hbar^2\left(\int d^3z\,\mathsf{B}(\mathbf{x}-\mathbf{z})\mathsf{B}(\mathbf{z}-\mathbf{y})\right) + i\hbar\,[\mathsf{B}(\mathbf{x}-\mathbf{y}),\,\mathsf{Z}] + \left(\nabla^2_{\mathbf{x}} - (aH)^2\Omega\right)\delta^{(3)}(\mathbf{x}-\mathbf{y})}$$

$$i\hbar\partial_\eta \ln N = \frac{\hbar}{2}\int d^3x\,\mathsf{B}(0)$$

System - Apparatus - Environment

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A gravitational "Apparatus" - Inflationary perturbations are detectable through their gravitational effect:

$$\hat{\mathcal{R}} = \frac{H}{\sqrt{\dot{\varphi}^2+\dot{\chi}^2}} \frac{\hat{\mathsf{q}}_1}{a} \,, \quad \frac{d}{dN}\hat{\mathcal{R}} = 2\frac{\nabla V \cdot \mathbf{e}_2}{\dot{\varphi}^2+\dot{\chi}^2} \frac{\hat{\mathsf{q}}_2}{a}$$

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System - apparatus - Environment = q_1 - $\mathcal{R}$ - q_2

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The full density matrix is: $\rho(\mathsf{q}_1, \mathsf{q}_2, \bar{\mathsf{q}}_1, \bar{\mathsf{q}}_2) = \Psi(\mathsf{q}_1 \mathsf{q}_2)\Psi^\star(\bar{\mathsf{q}}_1, \bar{\mathsf{q}}_2)$

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The reduced density matrix is obtained by tracing out q_2:

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For the vacuum state we find:

$$\tilde{\rho}(u, \Delta) = \exp\left[-\frac{1}{2}(u, \Delta)\mathsf{C}\begin{pmatrix} u^\star \\ \Delta^\star \end{pmatrix}\right]$$

$$\mathsf{C} \equiv \begin{pmatrix} 2\left(\Re[\mathsf{B}_{11}] - \frac{\Re[\mathsf{B}_{12}]^2}{\Re[\mathsf{B}_{22}]}\right) & i\left(\Im[\mathsf{B}_{11}] - \frac{\Re[\mathsf{B}_{12}]\Im[\mathsf{B}_{12}]}{2\Re[\mathsf{B}_{22}]}\right) \\ i\left(\Im[\mathsf{B}_{11}] - \frac{\Re[\mathsf{B}_{12}]\Im[\mathsf{B}_{12}]}{2\Re[\mathsf{B}_{22}]}\right) & \frac{1}{2}\left(\Re[\mathsf{B}_{11}] + \frac{\Im[\mathsf{B}_{12}]^2}{\Re[\mathsf{B}_{22}]}\right) \end{pmatrix}$$

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If $\mathsf{C}_{11}/\mathsf{C}_{22} \to 0$, the *off-diagonal* Δ terms become unimportant

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$$\boxed{\mathsf{B}_{11} \simeq \frac{aH}{\hbar}\left(\frac{2\pi}{\Gamma(\nu_1)^2}\left(\frac{k}{2aH}\right)^{2\nu_1} + i\,\left(\tfrac{3}{2} - \nu_1\right)\right)}$$

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where

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After a few efolds: $\Im[\mathsf{B}_{12}] \gg 2\sqrt{\Re[\mathsf{B}_{11}]\Re[\mathsf{B}_{22}]}$

Entanglement Entropy

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The Wigner Function: $W(q,\pi) = \int d\pi d\pi^\star\, \rho\, e^{-i\Delta\pi^\star - i\Delta^\star\pi} \Rightarrow$

$$W(q,\pi) \propto \exp\Big(- \mathsf{q}\frac{1}{\Delta_{\mathsf{q}}^2}\mathsf{q}^\star - (\pi - \pi_{cl})\,\frac{1}{\Delta_\pi^2}\,(\pi - \pi_{cl})^\star \Big)$$

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Entropy:

$$S = \mathrm{tr} \ln \left[2\Delta_{\mathsf{q}} \Delta_\pi / \hbar \right] = V \int \frac{d^3k}{(2\pi)^3} \, s_{\mathbf{k}} = V \int \frac{d^3k}{(2\pi)^3} \, \frac{1}{2} \ln \left[\frac{4\mathsf{C}_{22}}{\mathsf{C}_{11}} \right]$$

$$\boxed{s_{\mathbf{k}} = (3+\alpha) \ln \left(2 \frac{aH}{k} \right) + \ln \left[\frac{\mu^2 - m^2}{6H^2} \sin 2\theta_0 \, (3-\beta) \, \frac{\Gamma(\nu_1)\Gamma(\nu_2)}{2\pi} \right]}$$

$\alpha = \frac{4}{3}\epsilon - \frac{\mu^2 + m^2}{3H^2}$

Entanglement Entropy

The Wigner Function: $W(q,\pi) = \int d\pi d\pi^\star \, \rho \, e^{-i\Delta\pi^\star - i\Delta^\star\pi} \Rightarrow$

$$\boxed{W(q,\pi) \propto \exp\Big(-\, \mathsf{q} \frac{1}{\Delta_{\mathsf{q}}^2} \mathsf{q}^\star - (\pi - \pi_{cl}) \frac{1}{\Delta_\pi^2} (\pi - \pi_{cl})^\star \Big)}$$

$(\pi_{1cl} = -\mathsf{q}_1 \Im(\mathsf{B}_{11})\hbar)$

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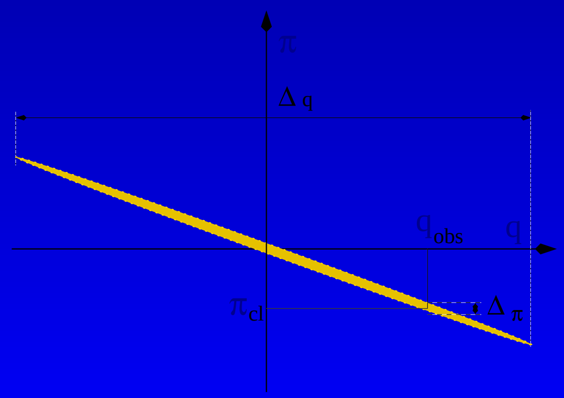
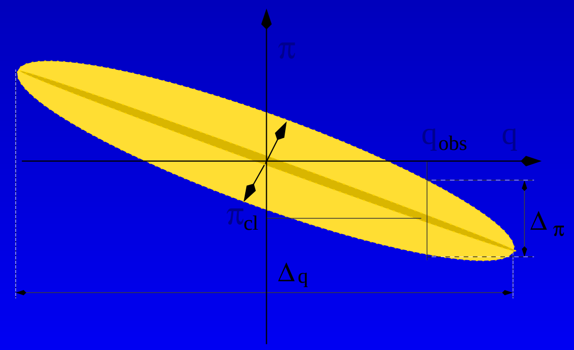
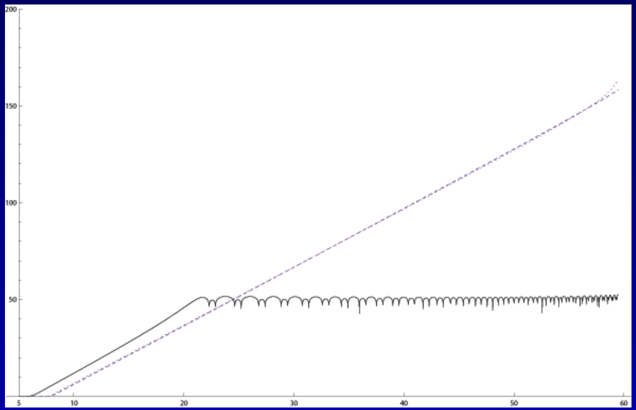


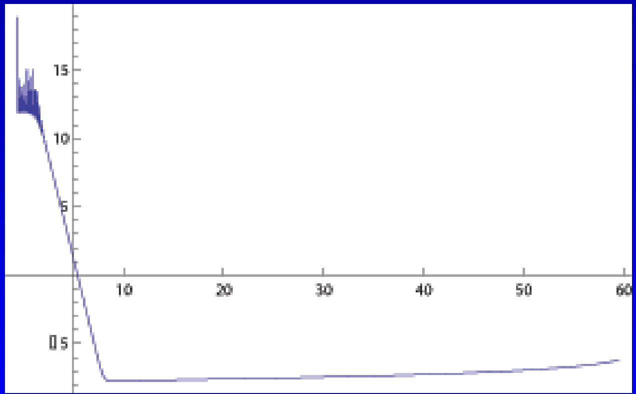


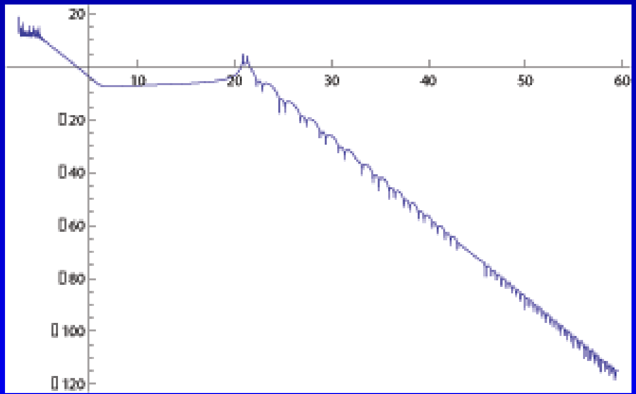
Numerics

Decoherence persists for a wide range of mass ratios

- $\mu = 1.05m$
- $\mu = 10m$







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- Decoherence can be quantified by the entropy of adiabatic perturbations